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THE STONE'S INDEX APPROXIMATION AND THE DEMOGRAPHICALLY AUGMENTED ALMOST IDEAL DEMAND SYSTEM. HOW CORRECT IS IT?

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Abstract: One of the advantages of the AIDS model is that it is simple to estimate when in the linear approximate form. To avoid non-linear estimation it h as been common practice to approximate the price Index by the Stone's index. This note shows that there are several unrecognized difficulties in estimating the AIDS linearized form. The selection of the Stone's index approximation is incorrect in the framework of the demographically augmented AI demand system.

Key Words: Stone's Index, Integrability, Demographically Augmented Demand Systems

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The Stone's Index Approximation and the Demographically Augmented Almost Ideal Demand System. How Correct is It?

1. Introduction

The Almost Ideal Demand System (AIDS) model was first introduced and estimated by Deaton and Muellbauer (1980). One of the advantages of the AIDS model is that it is simple to estimate when in the linear approximate form (LA/AIDS). To avoid nonlinear estimation it has been common practice to follow their methodology in approximating the price index by Stone's Index. Deaton and Muellbauer (p. 317) argue that Stone's index constitutes an "excellent approximation", although they noted that "it must be emphasized that [this] exists only as an approximation and will be accurate in specific circumstances, albeit widely occurring ones in time series estimation." However, there are several unrecognized difficulties with this common procedure for estimating AIDS in its linearized form.

Cross-section data, with detailed demographic information raises the question of the validity of using Stone's approximation in demographically augmented demand systems. The AIDS model has been used extensively in recent years (Blanciforti and Green 1983; Chalfant 1987; Green and Alston 1990, and many others). All these studies use Stone's Index to avoid non-linear estimation. Without further justification it is common to read: "A linearized index [such as Stone's Index using scaled prices] is assumed to provide an acceptable approximation to ln P." (Gould *et al.* 1991). Chalfant (1987:234) says: "Given the high correlation to be expected between price indexes, the selection [of the Stone's price approximation] is likely to be unimportant. At the same time, it does not seem to be a poor approximation to treat (y/P) as exogenous." This note will show otherwise.

2. The Stone's Index Approximation Revisited

Consider the AIDS model with no demographic variables. The derived budget shares are given by:

$$w_i(p,y) = \alpha_i + \sum_j \gamma_{ij} \ln p_j + \beta_i(\ln y - \ln P)$$

where $\ln P = \sum_i \alpha_i \ln p_i + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j$, w_i is the budget share for the ith commodity, with price p_i, and y is the income, or total expenditure. The Stone's index is defined in its logarithmic form as $\ln P^S = \sum_i w_i \ln p_i$. Assume that prices are highly collinear, such that P is exactly proportional to P^S. Then one can write $P = \xi P^S$. If ln P is substituted by $\ln P^S$ [1] then:

$$w_i(p,y) = (\alpha_i - \beta_i \ln \xi) + \sum_j \gamma_{ij} \ln p_j + \beta_i(\ln y - \ln P^S),$$

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[1] Referred to in the literature as the LA/AIDS.

and the parameters of the "true" AIDS model can be recovered from the estimates of the LA/AIDS model, since the LA/AIDS can be expressed in terms of the AIDS model. However, there are some other problems with the LA/AIDS estimation. The shares (the dependent variable in the system) are part of the definition of the Stone's index, calling for the resolution of a simultaneity problem. In time-series data simultaneity is commonly avoided by the use of lagged shares, and, in cross section data by using the difference between the share of household h and the mean of shares for all households. Browning and Meghir (1991) is an exception. They explicitly consider this simultaneity problem, and estimate the demand system iteratively. They conclude that "using the Stone price index approximation with no iteration is not acceptable; this is in contrast to the experience on aggregate time series data." (p. 935) Thus, an LA/AIDS model may be satisfactory in time series data where demographics is ignored. In cross-sectional data where the richness of demographic information is exploited, Stone's approximation may lead to serious difficulties that have not been clearly recognized in the literature.

3. Demographically Augmented AIDS

In the case of demographically augmented demand systems the problem is even more acute. The problem with Stone's index approximation is that there is no known relationship between the parameters of the LA/AIDS model and the AIDS model. Furthermore, the theoretical properties of the LA/AIDS model are not known (Green and Alston 1991). When using some popular demographically augmented demand systems it can be shown that the AIDS parameters cannot be identified from the parameters of the LA/AIDS even in the case of $P = \xi P^S$. This is the case with the popular Barten (1964) and Barten-Gorman (Pollak and Wales 1981) specifications. Lemma 1 illustrates this assertion for the case of Barten scaled shares.

Lemma 1

In the AIDS model with Barten scaling when prices are highly collinear such that $P = \xi P^S$, approximating $\ln P^*$ by $\ln P^S$ does not allow recovering the parameters of the AIDS model.

Proof:

Let the Barten scaled AIDS shares be given by:

$$w_i(p,y,d) = \alpha_i + \ln t_i + \sum_j \gamma_{ij} \ln p_j^* + \beta_i(\ln y - \ln P^*))$$

where $\ln P^* = \sum_i \alpha_i \ln p_i^* + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln p_i^* \ln p_j^*$, $p_i^* = p_i m_i$, $m_i = m_i(d)$ is the scaling function, and d is a vector of demographic variables.

Rewrite $\mathrm{Ln}\, P^*$ as $\sum_i \alpha_i \ln p_i + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j + \sum_i \alpha_i \ln m_i + \sum_i \sum_j \gamma_{ij} \ln m_i \ln p_j + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln m_i \ln m_j$, which is equivalent to $\ln P^* = \ln P + \sum_i \sum_j \gamma_{ij} \ln m_i \ln p_j + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln m_i \ln m_j$. If $\ln P^*$ is approximated by $\ln P^S$, the term $\sum_i \sum_j \gamma_{ij} \ln m_i \ln p_j + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln m_i \ln m_j$

is neglected. In the Barten-scaled AIDS model, even in the extreme case that $P = \xi P^S$, there is no consistent way to correctly recover the parameters of the "true" AIDS model. In this caseprice/expenditure elasticity computation could be highly misleading. When the objective is welfare comparisons, the situation is even more serious because one cannot recover the correct cost function. This is too high a price to pay for reducing <u>but not eliminating</u> non-linearities in the system.

Another approximation suggested in the literature is a Stones's index specific to each household, i.e., $\sum w_i \ln(p_i m_i)$. In this case there is also no known relationship between the parameters of the AIDS system and the LA/AIDS system. Furthermore, the model remains nonlinear.

When the AIDS system is translated, Stone's Index yields a correct approximation to ln P, provided that the demographically deflated income is not approximated by the actual income. This result is shown in lemma 2.

Lemma 2

In presence of translated shares, if $P = \xi P^S$, then the parameters of the AIDS model can be recovered from the LA/AIDS model.

Proof:

The AIDS translated shares can be written as:

$$w_i(.) = \alpha_i + \ln t_i + \sum_j \gamma_{ij} \ln p_j + \beta_i(\ln y - \sum_i \ln t_i \ln p_i - \ln P)$$

where t_i are the translating functions. In this case the LA/AIDS model is exact and given by:

$$w_i(.) = (\alpha_i - \xi\, \beta_i) + \ln t_i + \sum_j \gamma_{ij} \ln p_j + \beta_i(\ln y/\ln P^S - \sum_i \ln t_i \ln p_i).$$

The approximation is correct because the ln P term does not depend on the demographic variables. It is worth noting that in this case the system remains nonlinear (in the variables), but the nonlinearities are substantially smaller than in the original budget share equation. However, if one substitutes for both the ln P term and the term $\sum \ln t_i \ln p_i$, then there is no way to recover the cost function, and the LA/AIDS does not correctly approximate the AIDS system.[2] Furthermore, the term $\sum \ln t_i \ln p_i$ is essential to ensure integrability of the demand system (Ferreira and Perali 1992:5, proposition 1). In this case the system is linear. However, even if the estimated demand parameters of the LA/AIDS are a good approximation to the AIDS parameters, one cannot recover the cost function, and thus consistent welfare estimations are impossible.

This result can be extended to all possible demographic specifications of the AIDS system.

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[2] This was the approach followed in Heien and Wessells (1991).

Theorem

When the general price index term P in the AIDS is a function of demographic variables, an approximation of this function with P^S will always lead to a model which is not integrable. Moreover, there is no known relationship between the parameters of the AIDS and the LA/AIDS model.

Proof:

The proof is trivial. By definition $\ln P^* = f(p,d)$. If this function cannot be separated in p and d then we are approximating a function of p and d by a function of only p. Assume now that we can write $\ln P^*$ as $f_1(p) + f_2(p,d)$ (as discussed on lemma 1). Note that the remainder term is a function of the demographic variables (and might also be a function of the prices, as in the Barten Scaling case). When $\ln P^*$ is approximated by $\ln P^S$, this remainder term is ignored. Even if ξ is known there is no way to identify the relationship between the AIDS and the LA/AIDS parameters.

4. Conclusions

One of the advantages of the AIDS model is that it is simple to estimate when in its linear approximate form. To avoid non-linear estimation it has been common practice to approximate the price Index by Stone's index. However, there are several unrecognized difficulties in estimating the AIDS linearized form in demographically augmented systems. First, Stone's Index approximation to ln P is a good approximation only if the prices are highly collinear. Second, Stone's Index is a weighted sum of prices, the weights being the shares (the left hand-side variables). This is likely to cause inconsistency of the parameter estimates, unless the whole system is estimated simultaneously. Last, but not least, when the objective is welfare comparisons, integrability of the demand system must be satisfied. As this note shows, in the framework of the demographically augmented AI demand system, the "true AIDS" model cannot be recovered from the parameter estimates of the "Linear Approximated AIDS" model. Welfare comparisons, as well as demand estimates, may produce theoretically undesirable outcomes.

REFERENCES

Barten, A. (1964) "Family Composition, Prices and Expenditure Patterns", in P.E. Hart, G. Mills and J.K. Whittaker (eds), *Econometric Analysis for National Economic Planning*, Colston Research Society, Butterworth, London.

Blanciforti, L. and R. Green (1983) "An Almost Ideal Demand System Incorporating Habits: An Analysis of Expenditures on Food and Aggregate Commodity Groups", *The Review of Economics and Statistics*, 69:511-515.

Browning, M. and C. Meghir (1991) "The Effects of Male and Female Labor Supply on Commodity Demands", *Econometrica*, 59:925-952.

Chalfant, J. (1987) "A Globally Flexible, Almost Ideal Demand System", *Journal of Business and Economic Statistics*, 5:233-242.

Ferreira, M. and C. Perali (1992) "Tricks with the Demographically Modified Rotterdam and AIDS Models: Implications for Welfare Measurement", University of Wisconsin-Madison, Department of Agricultural Economics, Staff Paper 348.

Gould, B., T. Cox, and F. Perali (1991) "Demand for Fats and Oils", *American Journal of Agricultural Economics*, 73:212-221.

Green, R. and J. Alston (1990) "Elasticities in AIDS Models", *American Journal of Agricultural Economics*, 72:442-445.

Heien, D. and C. Wessells (1990)",Demand Systems Estimation with Microdata: A Censored Regression Approach", *Journal of Business and Economic Statistics*, 8:365-371.

Pollak, R. and T. Wales (1981) "Demographic Variables in Demand Analysis", *Econometrica*, 49:1533-1551.