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TRICKS WITH THE DEMOGRAPHICALLY MODIFIED ROTTERDAM AND AIDS MODELS: IMPLICATIONS FOR WELFARE MEASUREMENT

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Abstract
The use of *ad hoc* models in demand analysis with demographic factors can lead to erroneous specifications. We show that modified *ad hoc* models are often incorrect and term them "pseudo models." Modified demand systems should be derived using a theoretically plausible specification ensuring integrability and the possibility of deriving "exact" welfare measures, true cost of living indices and equivalence scales.

Key words: demographic translating and scaling, Rotterdam, Aids, welfare

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1. Introduction

Many demand studies (Pollak and Wales (1981), Rossi (1988), Gould *et al.* (1991)) have shown that extraneous factors such as demographic or quality characteristics significantly affect commodity demands. The theoretical foundations for introducing extraneous factors into a demand system, while preserving the original preference structure and the integrability of demand, lies on the seminal work conducted by Pollak and Wales (1981), and Lewbel (1985). Rigorous applications of the technique and the power of its generality has not been fully explored.

One of the most frequent transgressions occurs when *ad hoc* demand systems are used as the demand kernel. The modified version of *ad hoc* systems is not, in general, theoretically plausible because the form of the underlying utility function is not known. Furthermore, these models "allow for virtually any set of interactions between demographic and price effects but do not have any general applicability being specific to the given starting model." (Lewbel, 1985). For these reasons, the choice of *ad hoc* models as demand kernels can lead to erroneous specifications of the modified demand system. We illustrate this using the Rotterdam model, one of the most widely used *ad hoc* specifications, as an example.

Often, presumed true modified models are in effect "pseudo models". We use this term for those models that we deem to be correct when, indeed, they are not. Our main objective, hence, is to make the applied researcher aware of the fact that pseudo models, implicitly, bear specific behavioral assumptions, that may lead to incorrect measures of welfare, and, moreover, to misleading interpretations.

As Deaton and Muellbauer (1980) and Mountain (1988) pointed out, a first differenced AIDS model has the same right-hand side as the Rotterdam model. This fact, however, does not imply that the Rotterdam model possesses the integrability property (Mountain, 1988). In general, this is not considered a major shortcoming. Integrability becomes a requirement when demographic factors are incorporated since this leads directly into issues of welfare measurement and aggregation. In order to satisfy integrability, the specification of the demand system must be theoretically consistent. This can be achieved exclusively by adopting demand models derived from known preferences.

Ad hoc models can be interpreted as reduced form specifications in the sense that no exact *a priori* information available from the theory is used in the specification process. Prior information from theory is available both when estimating a system of demand equations and when exogenous factors are introduced in the demand functions by means of affine transformations such as translating, scaling or Gorman. Demographic transformations impose a precise structure. If it is known *a priori*, then, it is possible to specify a reduced form that is unique and/or does not lead to overidentification.

The paper develops as follows. In the next section the classical approach is used to specify a modified Rotterdam demand system. The third section compares and contrasts Lewbel's technique to modify the utility derived Almost Ideal Demand System (AIDS). Section 4 discusses the consequences and differences for welfare measurement. The concluding remarks contrast the competing approaches and provide some useful guidelines for the applied researcher.

2. The modified Rotterdam model: pseudo-translating and pseudo-scaling

We first derive the absolute price version of the Rotterdam model. Consider the following demand specification, in implicit form:

$$q = q\ (p, y, d) \tag{1}$$

where q is a vector of n quantities, p is a vector of n prices, y is income and d is a vector of r demographic characteristics, τ_{ir} is the translating demographic parameter for the ith commodity and the rth characteristic and $t_i(d;\tau_i):\Re_+^R \rightarrow \Re$ is the translating function specific to the ith commodity. For simplicity, and without loss of generality, we linearly specify $t_i(d)$ as $t_i(d)=\Sigma_r \tau_{ir} \ln d_r$.

In line with the classical approach of the Rotterdam School, we proceed without assuming a particular utility function. Consider a double-logarithmic form for equation (1) as, for example, in Theil (1980) and Goldberger (1989):

$$\ln q_i = \sum_r \tau_{ir} \ln d_r + \sum_{j=1}^{N} \epsilon_{ij}^{u} \ln p_j + \eta_i \ln y \tag{2}$$

here τ_{ir} is the demographic elasticity of the r^{th} extraneous factor with respect to the i^{th} good. Totally differentiate equation (2) and insert the Slutsky decomposition in elasticity form, $\epsilon_{ij}{}^{u}=\epsilon_{ij}{}^{c}-\eta_i w_j$, where $\epsilon_{ij}{}^{c}$ is the compensated cross price elasticity and w_j is the budget share of the j^{th} good, to yield:

$$dln\ q_i = \sum_i \tau_{ir} dln\ d_r + \sum_{j=1}^{N} \epsilon_{ij}{}^{C}\ dln\ p_j + \eta_i (dln\ y - \sum_{j=1}^{N} w_j\ dln\ p_j) \tag{3}$$

In order to impose the symmetry condition more easily, multiply equation (3) by w_i as:

$$w_i dln\ q_i = \sum_i w_i\ \tau_{ir} dln\ d_r + \sum_{j=1}^{N} w_i \epsilon_{ij}{}^{C}\ dln\ p_j + w_i \eta_i\ (dln\ y - \sum_{j=1}^{N} w_j\ dln\ p_j)\ . \tag{4}$$

The globally flexible Rotterdam model (Barnett, 1979) is obtained by approximating the log differential with the corresponding discrete change, replacing w_i with a Divisia approximation $w_i{}^{*}$ and by grouping the coefficients as:

$$w_i^{*} \Delta \ln q_i = \sum_i \delta_{ir} \Delta \ln d_r + \sum_j \gamma_{ij}\ \Delta \ln p_j + \beta_i\ (\Delta \ln y - \sum_j w_j^{*}\ \Delta \ln p_j) \tag{5}$$

where, $\beta_i = w_i^{*}\eta_i$, $\gamma_{ij} = w_i^{*}\epsilon_{ij}{}^{C}$ *and* $\delta_{ir} = w_i^{*}\tau_{ir}$.

This is what we term the pseudo-translating specification of the Rotterdam model. It is a pseudo model because the translating function translates only the intercept but does not correct income for necessary fixed costs (Pollak and Wales, 1981).

Assume to introduce the demographic modification by imposing structure on the demographic elasticities. Following Pollak and Wales' (1981) specification of the translated quantities, and considering a linear specification for the translating function -- $t_i(d)=\Sigma_r \lambda_{ir} d_r$ -- we obtain the following specification for the demographic elasticity:

$$\epsilon^{ir} = \frac{\lambda_{ir} d_r}{q_i} + \eta_i\, y\, (-\sum_i p_i\, \lambda_{ir}\, d_r)$$

Two points are worth mentioning. First, if we introduce the above elasticity formula into (5), then it is not possible to derive translating of the quantities. Second, as it will be clearer later, translating of the quantities and translating of the shares correspond to distinct transformations of the cost function. One transformation is in q space, while the other is in ln(q) space. In the case of the Rotterdam model, we do not know the underlying cost function. Thus, it is not possible to know how to demographically translate in the space consistent with the cost function underlying the Rotterdam model.

Let C(u,p,d) be the cost function for a household having utility level u, facing prices p, and with demographic characteristics d, and V(y,p,d) the indirect utility function for the same household having income y. Assume that both the modified cost function and indirect utility function are twice continuosly differentiable. Define an additively translated cost function as follows:

$$C(u,p,d) = C(u,p) + \sum_i p_i\, t_i(d)\ .$$

Proposition 1. An *ad hoc* demand system is correctly **translated** if the income term is also translated.

Proof. Invert C(u,p,d) to derive V(y,p,d) and apply Roy's identity to obtain the marshallian demand:

$$q_i(y,p,d) = q_i^*(y - \sum_i p_i\, t_i(d), p) + t_i(d) \ . \tag{6}$$

Note that the translated cost function is **uniquely** recovered up to a constant from the demand function. ‖

Equation (6) reproduces Pollak and Wales (1981) specification of a translated system. However, they did not stress the importance of a deflated income term to ensure integrability. The integrability of a demand system requires that the term $y^*=y-\Sigma_i p_i t_i(d)$ is included in the estimated demand equation. This condition is necessary to ensure integrability but is not sufficient since the empirical system may not be well-behaved. Note also that the content of the proposition applies to non *ad hoc* models as well. This proof condenses in a simple fashion some of Lewbel's results (1985) presented in his Theorem (4), Theorem (8) and Theorem (10).

Notice further that the result in proposition 1 is not expressed in terms of the logarithm of quantity in analogy with the Rotterdam or other double-log specifications. There exist no cost functions, transformed via an additive or multiplicative translating function, that support a demand function specified directly in the logarithms. This assertion bears implications that will be examined in the next section.

Let us now analyze how to incorporate Barten scales (Barten, 1964) in the Rotterdam model. Define δ_{ir} as the scaling demographic parameter for the ith commodity and the rth characteristic and $m_i(d;\delta_i):\Re_+^R \rightarrow \Re$ as the scaling function specific to the ith commodity. Consider the following specification of the $m_i(d)$ function analog to the translating specification:

$$m_i(d) = \prod_r d_r^{\delta_{ir}} \rightarrow \ln m_i(d) = \sum_r \delta_{ir} \ln(d_r) \tag{7}$$

Define the augmented demand equations as in Pollak and Wales:

$$q_i(p,y,d) = q_i^*(p^*,y)\ m_i(d) \tag{8}$$

where $p_i = p_i^*/m_i(d)$. Take logarithm of the scaled demand analogously to what has been done to derive equation (2) and obtain:

$$\ln q_i(p,y,d) = \ln m_i(d) + \ln q_i^*(p^*,y)\ . \tag{9}$$

Substitute the scaling function and insert the Slutsky relationship in elasticity form to obtain:

$$\ln q_i = \sum_r \boldsymbol{\delta}_{ir} \ln d_r + \sum_j (\epsilon_{ij}{}^c - \boldsymbol{\eta}_i w_j)(\ln p_j + \sum_r \boldsymbol{\delta}_{jr} \ln d_r) + \boldsymbol{\eta}_i \ln y\ . \tag{10}$$

Equation (10) is not estimable as it is. In the case of translated quantities, the demand system incorporates linearly the demographic function for the ith commodity in the intercept term. In addition, the income term is deflated non-linearly, both in the parameters and the variables, by a function of the demographic functions and the j prices for all the n goods. For scaled quantities, each price is scaled by the corresponding demographic function. In neither case the same demographic function appears twice in the same equation. In equation (10), the translating and scaling demographic function can be linearly combined and give rise to perfect collinearity. This is an irrefutable sign of misspecification.

Expression (10) is an hybrid of scaling and translating. A scaling specification should not incorporate any translating function. In this sense, equation (10) describes what we term a pseudo-scaling system. Similarly, expression (10) can also be termed as a pseudo-Gorman specification in the sense that it incorporates both scaling and translating but does not translates the income term. Not secondarily, the demand structure obtained in equation (10) implicitly imposes the unjustified restriction of $\boldsymbol{\delta}_{ir}=\boldsymbol{\delta}_{jr}$, which makes the translating function indistinguishable from the scaling function.

The translating and scaling demographic functions are distinct mathematical objects with a specific behavioral content.

To better understand the nature of the problem it is useful to derive expression (10) following an alternative approach. Construct the demographic elasticity of the i^{th} good with respect to the r^{th} factor $\epsilon_{ir}=(\partial \ln q_i/\partial \ln d_r)$ by totally differentiating the expression (9) with respect to $\ln d_r$:

$$\epsilon_{ir} = \delta_{ir} + \sum_j \epsilon^c{}_{ij}\delta_{jr} - \eta_i \sum_j w_j \delta_{jr} \tag{11}$$

Rewrite equation (11) in uncompensated terms as:

$$\epsilon_{ir} = \delta_{ir} + \sum_j \epsilon^u_{ij}\delta_{jr} \tag{12}$$

and in matrix notation $E_{ir}= \Omega_{ir}+ \Sigma_j\Gamma_{ij}\Omega_{jr}$. This result is the same as the one derived by Barten (1964) and Pollak and Wales (1981). This expression says that a change in "extraneous factors" manifests itself both **directly** via the term Ω_{ir} and **indirectly** through the interactions of the changes in the Ω_{jr} with the changes in prices p_j, namely Γ_{ij}, which are the two components of the normalized prices p^*. Note that the right-hand-side is not estimable because $\Omega_{jr}=\Omega_{ir}$ and E_{ir} is a linear relationship of Ω.

Insert equation (12) into equation (3) and derive:

$$dln\ q_i = \sum_r [\delta_{ir} + \sum_j \epsilon^c{}_{ij}\delta_{jr} - \eta_i \sum_j w_j\delta_{jr}]\ dln\ d_r + \sum_j \epsilon_{ij}{}^c\ dln\ p_j$$
$$+ \eta_i\ (dln\ y - \sum_j w_j\ dln\ p_j)$$

After some manipulations, rewrite the above equation as:

$$w_i^* \Delta \ln q_i = \sum_r w_i^* \delta_{ir} \Delta \ln d_r + \sum_j w_i^* (\epsilon_{ij}{}^c - \eta_i w_j^*) \ (\Delta \ln p_j + \sum_r \delta_{jr} \Delta \ln d_r)$$

$$+ w_i{}^* \eta_i \ \Delta \ln y$$

which corresponds to the Rotterdam extension of equation (10). It is easily recognizable that the first demographic term is the direct effect and the second interaction term is the indirect effect.

The two approaches have one characteristic in common. Both the derivation of equation (10) and the computation of the demographic elasticity as in equation (12) imply taking the logs of the scaled demand system as described in equation (8). This is the critical factor for a correct modification of Marshallian demand systems.

Proposition 2. A plausible scaled *ad hoc* demand system can be derived from equation (8) if equation (8) is rewritten as:

$$q_i^*(p^*, y) = \frac{q_i(p, y, d)}{m_i(d)} .$$

Proof. Assume a smooth cost function such that:

$$C(u,p,d) = C^*(u,p^*) . \tag{13}$$

Apply Shepard's lemma with respect to p_i:

$$\frac{\partial C}{\partial p_i} = \frac{\partial C^*}{\partial p_i^*} \frac{\partial p_i^*}{\partial p_i} = q_i^* \, m_i . \tag{14}$$

Take the log of expression (14) as $\ln q_i = \ln q_i{}^* + \ln m_i$. Redefine $\ln q_i$ as h_i and refer to proposition 1. It follows that integration does not uniquely recover up to a constant the original cost structure in equation (13) unless equation (14) is rewritten in q^* space as:

$$\frac{\partial c}{\partial p_i}\frac{1}{m_i} = \frac{\partial c^*}{\partial p_i^*}\frac{\partial p_i^*}{\partial p_i}\frac{1}{m_i} = q_i^* \ . \ \|$$

Note, however, that the system as in (14) is not directly estimable because q_i^* is not observable. Equation (8), hence, should be estimated without using a logarithmic transformation to linearize the system.

Further, express equation (8) in shares and note that in this form the logarithmic transformation is harmless. Two reasons justify this assertion: (a) Shepard's lemma applied to the logarithm of the cost function gives directly the shares, and (b) the share can be written as $w_i = q_i^* p_i^*/y$ by definition.

In the next section we show that misspecification errors arise when the underlying structure of preferences is not known. We use Lewbel's modifying technique to show how to introduce extraneous factors without altering the original preference structure. The example that we develop relates to the specification of the Barten-Gorman model from AIDS-Gorman preferences. In general, this procedure allows to derive several variations of modified demand systems which are integrable, theoretically plausible and appropriate for welfare measurement.

3. Derivation of some plausibly modified AIDS models

For the sake of contrast, this section develops Lewbel's technique to incorporate demographic or other extraneous effects into a demand system which is derived from a known cost function such as the AIDS. Following Lewbel (1985), consider the relation:

$$y = C(u,p,d) = f\{C^*[u,h(p,d),p^*,d]\} = f(y^*,p,d) \tag{15}$$

where $C^*(u,p^*)$ is a well-behaved expenditure function, $y^* = C^*[u,h(p,d)] = C^*(u,p^*)$ is the minimum expenditure necessary to attain utility level u at corrected prices $p_i^* = h_i(p,d)$, d is a rx1 vector of demographic characteristics, and i=1,..,n is a commodity index. The Reverse Gorman specification (Pollak and Wales) adds fixed costs for "necessary" or "subsistence" quantities $t_i(a)$ at corrected prices to the expenditure function. Using the facts $y=C(.)=f(.)$ and $y^*=C^*(.)$, the Reverse Gorman specification is obtained from equation (15) using the following f(.) and h(.) linear in p^* modifying functions:

$$h_i(p,d) = p_i m_i(d) \tag{16}$$

$$C(u,p,d) = f(y^*,p,d) = y^* P^T = y^* \left[\prod_i (h_i(p_i,d))^{t_i(d)} \right], \tag{17}$$

Note that the assumption of a linear h(.) function implies Leontief household technologies.

Recall that the translating demographic function $t_i(d)$ was specified as $t_i(d)=\Sigma_r \tau_{ir} \ln(d_r)$ and that the scaling demographic function $m_i(d)$ as $m_i(d)=\prod_r d_{ir}^{\delta_{ir}}$, for r=1,..,n. Note that these functional specifications were chosen because of their empirical convenience.[1]

Assume quasi-homothetic preferences as described by the demographically modified Gorman Polar cost function:

$$C(u,p,d) = \left(A(p,d)\ (\phi(u))^{B(p,d)}\right) P^T$$

and the linear in logarithm analog:

$$\ln C(u,p,d) = (\ln A(p,d) + B(p,d) \ln \phi(u)) + \ln P^T$$

1 It should be emphasized that the choice of the functional form of the demographic functions is not restricted to any particular form partly because only the relative magnitudes of the estimated demographic functions have a meaningful interpretation. The researcher, however, can specify a more complex form such as a translog if interested in modelling economies of scale.

where:

$$\ln A(p,d) = \ln A(p^*) = \alpha_0 + \sum_i \alpha_i \ln p_i^* + .5 \sum_i \sum_j \gamma_{ij}^* \ln p_i^* \ln p_j^*$$

$$B(p,d) = \beta_0 \prod_{i=1}^{n} (p_i^*)^{\beta_i}$$

The corresponding demographically modified AIDS indirect utility function is given by:

$$\ln V = \frac{\ln y^* - (\alpha_0 + \sum_i \alpha_i \ln p_i^* + .5 \sum_i \sum_j \gamma_{ij} \ln p_i^* \ln p_i^*)}{\beta_0 \prod_i (p_i^*)^{\beta_i}}$$

where $\gamma_{ij}=\gamma_{ij}^*+\gamma_{ji}^*$ and $V=\phi(u)$ and $\ln y^*$ is $\ln y^*=\ln y - \Sigma_i t_i(d) \ln p^*$ from equation (17). Roy's identity gives the Reverse-Gorman AIDS budget shares:[2]

$$w_i = \alpha_i + t_i(d) + \sum_j \gamma_{ij} \ln p_j^* + \beta_i \ln \left(\frac{y^*}{A'(p,d)} \right). \tag{18}$$

Lewbel's theorem 4 shows that a theoretically plausible specification of a modified Marshallian share demand system can be directly obtained from equation (18) by applying the following transformation:

$$\begin{aligned} w_i &= \frac{\partial f(y^*,p,d)}{\partial y^*} \frac{y^*}{y} \sum_j^n \frac{\partial h_j(p,d)}{\partial p_i} \frac{p_i}{p_j^*} w_i^*(y^*,p^*) + \frac{\partial f(y^*,p,d)}{\partial p_i} \frac{p_i}{y} \\ &= (1 - \sum_i t_i(d))\, w_i^*(y^*,p^*) + t_i(d) = w_i^* + t_i(d) , \end{aligned} \tag{19}$$

where $\Sigma_i t_i(d)=0$ due to the adding up restrictions.

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2 The term $\ln A'(p^*)$ is as $\ln A(p^*)$ with γ_{ij} in place of γ_{ij}^*.

It is important to note that the system represented in equation (19) is not a unique specification of the Reverse Gorman. Many other specifications can be obtained applying consistently Lewbel's technique. Consider, for example, the following exponential specification of the h(p,d) function in (16):

$$h_i(p,d) = \exp\ (p_i m_i(d)) = \exp\ (p_i^*)\ . \tag{20}$$

The derived Reverse Gorman shares are:

$$w_i = \alpha_i + \frac{p_i^*\ t_i(d)}{y^*} + \sum_j \gamma_{ij} \ln p_j^* + \beta_i \ln\left(\frac{y^*}{A(p,d)}\right). \tag{21}$$

This specification[3] is interesting because the translation term looks much like the committed quantity term of the linear expenditure system. However, the overhead is not fixed. The supernumerary quantities increase as the ratio p^*/y^* also increases. Hence the degree to which a good is perceived as a necessity is subjective and varies from individual to individual (Lewbel, 1985).

As they stand, systems (18) and (21) have a Reverse Gorman representation (Pollak and Wales, 1981) whose parameters are all identifiable (Perali, 1992). Both the modified Marshallian share demand systems proposed are integrable. This property assures the possibility to recover exactly the underlying modified cost and indirect utility function which can then be used to derive Hicksian welfare measures, cost of living indices and equivalence scales.

3 Note that when t_i=0 the model degenerates to scaling, while when m_j=0 the model collapses to translating.

4. Implications for welfare measurement

It is well known that *ad hoc* models are not suitable for welfare analysis. The original structure of preference is not known. Hence, Hicksian welfare measures cannot be exactly derived. Furthermore, equivalence scales cannot be computed, thus precluding the possibility of making welfare comparisons (Jorgenson (1990), Lewbel (1989)) or of analyzing the distribution of incomes "deflated" by the equivalence scales.

Note that a consistently modified AIDS demand system is integrable in the sense that it is possible to recover the original AIDS-Gorman preferences exactly. This does not imply that a "pseudo model" is not integrable. It could well be that the regularity conditions are empirically met. In this case, a utility function underlying the pseudo demand system does exist (Epstein, 1982). However, it is still not possible to recover the original preference structure. On the other hand, models derived from an assumed structure of preferences such as the AIDS or the Translog demand systems, are exempted from the above criticism. A bold application of Lewbel's technique, however, may conceal some pitfalls that could be crucial for exact welfare measurement.

Moreover, note that misspecification of the demographically modified *ad hoc* models could imply errors of measurement of unpredictable size. Such errors may significantly alter any economic, policy or welfare analysis that is based upon such models.

The modification of the h(.) function as shown in equation (16) is an example of the plasticity of Lewbel's modifying technique. Some new specifications may have more desirable properties or embed more interesting economic behavior than others. However, a sufficient condition for new demand systems to be suitable for welfare analysis is that they are derived using the modifying technique which guarantees integrability and theoretical consistency of the modified cost function.

Consider the following additively modified Reverse Gorman cost function similar to the one shown in Proposition 1:

$$C(u,p,d) = C^*(u,p) + \sum_i p_i\, t_i =$$

$$= \exp\left(\alpha_0 + \sum_i \alpha_i \ln p_i + .5 \sum_i \sum_j \gamma^*_{ij} \ln p_i \ln p_j + \beta_0 \prod_i p_i^{\beta_i} \ln u\right) \qquad (22)$$

$$+ \sum_i p_i\, t_i.$$

The additive form of the cost function does not allow to compute equivalence scales that are "true" in the sense that they are independent of the base level of income or utility which is referred to as the IB property (Lewbel, 1991). In fact, the equivalence scale, ES, that compares household 1 with a reference household h,

$$ES(u,p,d) = \frac{(C^*(u,p^*) + \sum_i p_i \ln t_i^1)}{(C^*(u,p^*) + \sum_i p_i \ln t_i^0)} \qquad (23)$$

does not possess the IB property because it is not independent of u. This is the content of Lewbel's (1991) Lemma 5. Note also that preferences described as in equation (22) are not invariant to monotonic transformations. As a consequence, specifications of Marshallian quantities must be derived first and then transformed into shares.

The result shown in equation (23) corresponds to an additively transformed cost function. Interestingly, a translating specification derived from a multiplicatively modified cost function does possess IB scales. This can be generalized to other models that nest translating as a special case.

Proposition 3. Assume a cost function that is multiplicatively translated. Then any model that nests translating as a degenerate case possesses the IB property.

Proof. Consider the following multiplicative cost function that incorporates both translating and scaling a la Reverse-Gorman used also in equation (16) and (17):

$$C^*(u,p^*) = C(u,p,d) \left[\prod_i (p_i \; m_i^*)^{(t_i^*)} \right]^{-1},$$

where,

$$m_i^*(d) = v_i \; m_i(d) \quad \text{and} \quad t_i^*(d) = (1 - v_i) \; m_i(d)$$

for some constants v_i.[4] Next, make the function C(u,p,d) independent of d by constructing a matrix valued function G(u,p) and aggregate all demographic terms along with the related interactions with prices into a vector valued function $m_0(p,d;v)$ as follows:

$$C(u,p,d) = G(u,p) \; m_0(p,m_i(d);v)$$

Then, true equivalence scales (TES) can be derived as:

$$TES(p,d) = \frac{(G(u,p) \; m_0^1(p,m_i(d^1);v)}{(G(u,p) \; m_0^0(p,m_i(d^0);v)} = \frac{m_0^1(p,m_i(d^1);v)}{m_0^0(p,m_i(d^0);v)} \; .$$

which is IB. ‖

Proposition 3 points out that not all classes of cost function generate equivalence scales that possess the IB property. This condition is necessary, though not sufficient, to identify the scales from demand data alone and to generate scales that are unique and "true" in a cost of living sense. Moreover, only IB scales represent a partial but theoretically admissible means to make welfare comparisons (Lewbel, 1989; 1991).

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4 Note that v_i=0 implies that the model degenerates to translating, and v_i=1 implies that the model collapses to scaling.

5. Conclusions

The main objective of this study is to show that modified demand systems should be estimated using a theoretically plausible specification derived from a known indirect utility or cost function, when the main objective is to perform welfare comparisons. Theoretical plausibility is obtained by applying Lewbel's technique (1985). By so proceeding, the integrability of the derived modified system is guaranteed and "exact" measures of welfare can be derived.

The analysis demonstrated that this objective is not achievable using the Rotterdam model or any other *ad hoc* demand model. From any Rotterdam style model, it is impossible to derive a utility level on which to base welfare measurement without strong assumptions. System of *ad hoc* demands do locally integrate to a cost function. However, this might not be very interesting and/or easy to use in applied welfare analysis. When the structure of the demographic modifications is not plausibly taken into account, then it is not possible to recover the transformed cost function uniquely.

Furthermore, the pseudo versions of *ad hoc* models are misspecified. Pseudo translating does not incorporate a translated income term. Pseudo scaling is, in effect, a Reverse-Gorman specification where the translating and scaling functions are incorrectly restricted to be the same. The specification problem arises because the demand models are not derived from an assumed preference structure that allows a consistent derivation of demand functions in either quantity or share space.

These observations add to some other theoretical advantages of AIDS over Rotterdam, or, more in general, of cost function derived demand models over *ad hoc* demand models. As pointed out by Deaton and Muellbauer (1980): "AIDS is of comparable generality to the Rotterdam, but has considerable advantages over it. AIDS satisfies the axioms of choice exactly; it aggregates perfectly over consumers without invoking parallel linear Engel curves; it has a functional form which is consistent with known household-budget data...".

On the other hand, any demand system derived from a known structure of preferences and consistently estimated is germane to welfare analysis. The present study showed that affine transformations of the cost function such as the class of multiplicative cost function modifications generate scales that possess the IB property. Non-affine transformations, on the contrary, (a) do not possess true scales if the demand system is estimated in its pseudo version, or (b) do not generate scales that possess the IB property.

This paper also shows how heterogenous preferences can be modelled using many different, theoretically consistent, formulations. They embed distinct behavioral assumptions whose welfare implications are not yet well understood.

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