July 1992 No. 347

THE BARTEN-GORMAN MODEL IN THE AIDS FRAMEWORK

By

M. Luisa Ferreira and C. Federico Perali */
University of Wisconsin-Madison

Abstract
The Barten-Gorman model has been difficult to estimate due to the problems in identifying all demographic parameters and the highly nonlinear nature of the specification. Restrictions on the demographic parameters that allow the identification of the model and make it estimable are derived. An empirical application shows that the Barten-Gorman model behaves well and preferred to the nested Translating and Scaling specifications.

Key Words: Barten-Gorman Model, Identification, Concavity and Demographics.

---

*/ Both authors are graduate students, Department of Agricultural Economics, University of Wisconsin-Madison. The authors would like to thank, without implicating, Rueben C. Buse, and Thomas L. Cox for their comments.

1. Introduction

Incorporating demographic or other factors into a demand system is gaining renewed interest due to the results derived by Lewbel (1985). Using a somewhat cryptic notation, Lewbel shows how to modify complete demand systems with the introduction of demographic characteristics that interact with prices and income in an almost unlimited variety of functional specifications. Such modifications include the popular Barten (1964) scaling and translating specifications, first introduced by Pollak and Wales (1981), as special cases. Lewbel describes the properties that the modifying functions must maintain to ensure integrability of the modified demand system. This guarantees that the original preference structure is preserved and that the modified model is still theoretically plausible. These conditions are necessary to compute the true equivalence scales and to derive correct measures of welfare.

This study concentrates on the Barten-Gorman modifying specification which combines both translating and scaling. Deaton and Muellbauer (1986) shows that this model is metrically superior to competing specifications for generating equivalence scales. Similar to the Slutsky decomposition of income and substitution effects, the Barten-Gorman specification translates the budget line through the fixed cost element (translating) and rotates the budget constraint by modifying the effective prices with the related substitution effects via demographic characteristics (scaling).

The Barten construction is motivated by Gorman's (1976) observation that "when you have a wife and a baby, a penny bun costs threepence." However, some characteristics may affect consumption only through income effects, or may show both a translating and a scaling effect. Only a model that nests both could resolve this empirical question.

In the past, the Barten-Gorman model was not very popular because its estimation is highly nonlinear and the model was thought to have serious identification problems. Moreover, Lewbel states that there is no guaranteee that all parameters can be identified, just as there is no guarantee that all the parameters in any demand system can be identified (1985:5). Deaton and

Muellbauer assert that in practical applications it will always be extremely difficult to estimate the parameters of the Gorman-Barten model (1986:740). Chung (1987) circumvents the problem by arbitrarily introducing into the translating function demographic variables different from the ones incorporated into the scaling function. Davidson and McKinnon (1981) identify the artificial parameters used to compound alternative specification hypotheses into a nesting model by estimating the artificial parameters conditional on the knowledge of the estimated parameters of the single alternatives (Atkinson 1970).

In the present study we specify the Barten-Gorman model in a theoretically plausible way. We use the property of homogeneity of degree one in p of the cost function to derive the identifying restrictions that make the model estimable. We complete the description of the Barten-Gorman model by providing the expression to analyze the concavity properties. We also derive the expressions for the elasticities.

The paper first lays out the notation and definitions. The third section specifies the model. The fourth section demonstrates how all the demographic parameters can be identified. The next section derives the conditions to test concavity in the Barten-Gorman model. The sixth section applies the model to the estimation of the demand for food at home and food away from home in the United States during the period 1953-1988. The conclusions complete the discussion.

2. Notation and Definitions

Set the basic notation according to the following definitions:

$h = (h_1,..,h_H) \in \mathbb{R}_+^H$ ($\mathbb{R}$ the real numbers, $\mathbb{R}_+$ the non negative reals) is the index vector for demographic profiles at year h=1,..,H: h=0 designates the reference profile, h=1 designates the profile chosen for comparison;

p_j = the price of commodity i=1,..,n assumed constant across profiles;

$p = (p_1,..,p_n) \in \mathbb{R}_+^n$;

q_i^h = the quantity of the **i**th commodity consumed by the **h**th demographic profile;

$q \quad = (q_1,..,q_n) \in \Re_+^n$;

w_i^h = the budget share of the **i**th commodity by the **h**th profile;

$y^h \quad$ = the total expenditures of the **h**th demographic profile (income in short);

$d_r \quad$ = the **r**th demographic characteristic;

$d \quad = (d_1,..,d_R) \in \Re_+^R$;

δ_{ir} = the scaling demographic parameter for the **i**th commodity and the **r**th characteristic;

$\delta_i = (\delta_1,..,\delta_R) \in \Re^R$ the R vector of scaling parameters for the **i**th commodity;

$m_i(d;\delta_i): \Re_+^R \rightarrow \Re$ = the scaling function specific to the **i**th commodity;

τ_{ir} = the translating demographic parameter for the **i**th commodity and the **r**th characteristic;

$\tau_i = (\tau_1,..,\tau_R) \in \Re^R$ the R vector of translating parameters for the **i**th commodity;

$t_i(d;\tau_i): \Re^R \rightarrow \Re$ = the translating function specific to the **i**th commodity;

$C(u,p)$ = the minimum cost of attaining utility level u at prices p. By definition, $y^h = C(u,p)$. This cost function is assumed to be twice continuously differentiable and theoretically plausible.

$V(y,p)$ = the indirect utility function at income y and prices p.

$\Phi(u)$ = the level of utility of the reference demographic profile.

3. Specification of the Barten-Gorman Model

Following Lewbel (1985), consider the relation:

$$y = C(u,p,d) = f\{C^*[u,h(p,d)], z(p,d), d\} \tag{1}$$

where $C^*(u,p^*)$ is a well-behaved expenditure function, $y^* = C^*[u,h(p,d)] = C^*(u,p^*)$ is the minimum expenditure necessary to attain utility level u at some scaled prices $p_i^* = h_i(p,d)$ and translated prices $p_i^T = z_i(p,d)$ for some vector valued functions h and z.

Using the facts $y=C=f$ and $y^*=C^*$, the Barten-Gorman specification[1] is obtained from equation (1) using the following $f(y,p,d)$ modifying function:

$$y = C(u,p,d) = f(y^*, z(p,d), d) = y^* P^T \quad with \quad P^T = \prod_i \left(z_i(p_i,d)\right)^{t_i(d)} .$$

This expression corresponds to the Barten (1964) specification with the addition of fixed overheads P^T for "necessary" or "subsistence" quantities (Gorman 1976). Several different demographic specifications can be derived by making explicit assumptions about the functions $h(p,d)$ and $z(p,d)$. The specifications are:

(a) $h(p,d)=z(p,d)=p$ → budget share Translating

(b) $h(p,d)=z(p,d)=pm=p^*$ → budget share Reverse Gorman

(c) $h(p,d)=p^*$ and $z(p,d)=1$ → budget share Scaling

(d) $h(p,d)=p^*$ and $z(p,d)=p$ → budget share Gorman.

These definitions comply with Pollak and Wales (1981) terminology. For empirical convenience[2] the translating demographic functions $t_i(d)$ are specified as $t_i(d)=\Sigma_r \tau_{ir} \ln(d_r)$ and the scaling demographic functions $m_i(d)$ as $m_i(d)=\prod_r d_{ir}^{\delta ir}$, for $r=1,..,n$.

Assume quasi-homothetic preferences as described by the demographically modified Gorman Polar cost function:

$$C(u,p,d) = \left(A(p,d)\ (\phi(u))^{B(p,d)}\right) P^T . \tag{2}$$

The linear in logarithm analog is:

---

1 The Barten-Gorman demographic specification that we refer to in this paper is the Reverse Gorman modifying structure.

2 It should be emphasized that the choice of the functional form of the demographic functions is not restricted to any particular form partly because only the relative magnitudes of the estimated demographic functions have a meaningful interpretation. The researcher, however, can specify a more complex form such as a translog if interested in modelling economies of scale.

$$\ln C(u,p,d) = (\ln A(p,d) + B(p,d) \ln \phi(u)) + \ln P^T$$

where:

$$\ln A(p,d) = \alpha_0 + \sum_i \alpha_i \ln p_i^* + .5 \sum_i \sum_j \gamma_{ij}^* \ln p_i^* \ln p_j^*$$

$$B(p,d) = \beta_0 \prod_{i=1}^{n} (p_i^*)^{\beta_i} .$$

The corresponding Barten-Gorman AIDS indirect utility function is given by:

$$\ln V = \frac{\ln y^* - (\alpha_0 + \sum_i \alpha_i \ln p_i^* + .5 \sum_i \sum_j \gamma_{ij} \ln p_i^* \ln p_j^*)}{\beta_0 \prod_i (p_i^*)^{\beta_i}} \qquad \textbf{(3)}$$

where $\gamma_{ij}=\gamma_{ij}^*+\gamma_{ji}^*$, $V=\phi(u)$ and $\ln y^*=\ln y - \Sigma_i t_i(d) \ln p^*$ from equation (2). This extension permits distinguishing between the intercept shifting function and the translating function. Moreover, it allows deriving profile specific income elasticities. Roy's identity yields the Barten-Gorman AIDS budget shares:[3]

$$w_i = \alpha_i + t_i(d) + \sum_j \gamma_{ij} \ln p_j^* + \beta_i \ln \left(\frac{y^*}{A'(p,d)} \right) . \qquad \textbf{(4)}$$

Lewbel shows that a theoretically plausible specification of a modified Marshallian share demand system can be obtained from equation (1) by applying the following transformation (1985, Theorem 4):
where $\Sigma_i t_i(d)=0$ due to the homogeneity restrictions.

It is important to note that the system represented in equation (4) is not a unique specification of the Barten-Gorman. Many other specifications can be obtained by applying Lewbel's technique. Consider, for example, the

3 The term $\ln A'(p,d)$ is the same as $\ln A(p,d)$ with γ_{ij} in place of γ_{ij}^*. Henceforth, to simplify notation $\ln A(p,d)$ will be used in lieu of $\ln A'(p,d)$.

$$w_i = \frac{\partial f(y^*,p,d)}{\partial y^*} \frac{y^*}{y} \sum_j^n \frac{\partial h_j(p,d)}{\partial p_i} \frac{p_i}{p_j^*} w_i^*(y^*,p^*) + \frac{\partial f(y^*,p,d)}{\partial p_i} \frac{p_i}{y}$$

$$= (1 - \sum_i t_i(d))\ w_i^*(y^*,p^*) + t_i(d) = w_i^* + t_i(d)$$

following exponential specification of the h(p,d) function $h_i(p,d) = \exp(p_i m_i(d)) = \exp(p_i^*)$. The derived Barten-Gorman shares are:

$$w_i = \alpha_i + \frac{p_i^*\ t_i(d)}{y^*} + \sum_j \gamma_{ij} \ln p_j^* + \beta_i \ln\left(\frac{y^*}{A(p,d)}\right). \tag{5}$$

This specification is interesting because the translation term looks much like the committed quantity term of the linear expenditure system. However, the overhead is not fixed. The supernumerary quantities increase as the ratio p^*/y^* also increases. Hence the degree to which a good is perceived as a necessity is subjective and varies from individual to individual (Lewbel 1985).

The Barten-Gorman model in equation (4) nests the following demographic specifications:

(a) Scaling

$$w_i = \alpha_i + \sum_j \gamma_{ij} \ln p_j^* + \beta_i \ln\left(\frac{y^*}{A(p,d)}\right) \tag{6}$$

where $h(p,d)=p^*=pm$, and $\ln y^*=\ln y$;

(b) Translating

$$w_i = \alpha_i + t_i(d) + \sum_j \gamma_{ij} \ln p_j + \beta_i \ln\left(\frac{y^*}{A(p)}\right) \tag{7}$$

where $h(p,d)=p$, and $\ln y^*=\ln y-\ln P^T$.

Integrability requires that these specifications be estimated including y^* and without linearization of the deflating index A(.). This guarantees recovering exactly the underlying modified cost and indirect utility function which can then be used to derive equivalence scales and to make welfare comparisons that are fully cardinal.

4. Identification

From the previous section, recall the Barten-Gorman specification of $f(y^*,p^*,d)$ where $h(p,d)=z(p,d)=pm=p^*$:

$$C(u,p,d) = f(y^*,p^*,d) = y^* \left[\prod_i h_i \, (p,\ d)^{t_i(d)} \right] = y^* \, p^T \tag{8}$$

that nests both the scaling and translating specifications. A straightforward application of Lewbel's Theorem 1 (1985) leads to the following proposition.

Proposition 1.

Let $C^*(u,p^*)$ be a cost function homogeneous of degree one in prices. Then $C(u,p,d)$ will maintain the same property if and only if (iff):

$$\sum_{j=1}^{n} \frac{\partial h_i(p,d)}{\partial p_j} \frac{p_j}{p_i^*} = \frac{\partial h_i(p,d)}{\partial p_i} \frac{p_i}{p_i^*} = 1 \ , \quad since \ \frac{\partial h_i(p,d)}{\partial p_j} = 0 \quad \forall \, j \neq i \ ,$$

$$1 - \sum_{j=1}^{n} \frac{\partial f(y^*,p,d)}{\partial p_j} \frac{p_j}{f(y^*,p,d)} = 1 - \sum_{i=1}^{n} t_i(d) =$$

$$= \frac{\partial f(y^*,p,d)}{\partial y^*} \frac{y^*}{y} = 1, \quad iff \ \sum_{i=1}^{n} t_i(d) = 0 \ . \|$$

Using Euler's law, the legitimate cost function $C(u,p,d)$ can be rewritten as:

$$C(u,p,d) = \sum_i \frac{\partial C}{\partial p_i} p_i = \frac{\partial f}{\partial C^*} \sum_i \frac{\partial C^*}{\partial p_i^*} \frac{\partial p_i^*}{\partial p_i} p_i + \sum_i \frac{\partial f}{\partial p_i} p_i \ . \tag{9}$$

Given the Barten-Gorman specification of C(u,p,d) in equation (8), the Euler relation in (9) becomes:

$$f(y^*,p,d) = \sum_i \frac{\partial C}{\partial p_i} p_i = P^T \sum_i \frac{\partial C^*}{\partial p_i^*} p_i^* + C \sum_i t_i(d) \ , \tag{10}$$

where:

$$\frac{\partial f}{\partial y^*} = P^T; \quad \frac{\partial p_i^*}{\partial p_i} = m_i(d); \quad \sum_i \frac{\partial f}{\partial p_i} p_i = C \sum_i t_i(d) \ .$$

Equation (10) gives the fundamental economic relationship that is used to derive the econometric restrictions at the share demand level. The homogeneity, or invariance, property implicit in the budget constraint modified by the Barten-Gorman specification is a sufficient condition to identify all demographic parameters. It makes the derived share demand models econometrically tractable.

By applying Shephard's lemma to (10) and dividing both sides by $C(u,p,d)=C^*(u,p^*)P^T$, we obtain:

$$\sum_i w_i = 1 \ .$$

Hence, homogeneity of degree one in prices of the cost function or of degree zero in prices and income of the share demands require:

$$\sum_i t_i(d_j) = 0.$$

In the Barten-Gorman framework the number of "free" demographic parameters doubles from r(n-1), as in scaling or translating, to 2xr(n-1). The procedure of specifying the same demographic functions for scaling and translating gives rise to problems of perfect correlation between the translating and scaling set of parameters. Moreover, this procedure would imply the behavioral assumption that the translating effect is the same as the scaling effect. Thus, this approach would result in a loss of economic information.

Instead, the hypothesis that the translating demographic effects differ statistically from the scaling effects is interesting and can be tested by using the nested procedure suggested by Pollak and Wales (1981). In line with previous work (Atkinson 1970; Davidson and McKinnon 1981; and Pesaran and Deaton 1981), Pollak and Wales (1981) propose the following parsimonius solution to estimate the compound Barten-Gorman model:

$$\ln m_i = v_i \ln m_i^*$$

$$t_i = (1-v_i)\ln m_i^* \quad \textit{for } i=1,..,n\ ; \ \ \ln m_i^* = \sum_r \delta_{ir} \ln d_r\ .$$

Since the m_i and t_i functions include the same parameters and have the same functional form, the approach generates only r(n-1) "free" demographic slopes and n artificial parameters v_i. An additional benefit of this structure is that it nests the translating and scaling hypothesis as special cases. If $v_i=1$, the model collapses to scaling; if $v_i=0$, it collapses to translating. These restrictions are amenable to standard null hypothesis tests against the more general compound specification.

To evaluate the implications of (10) on this specification, it is insightful to reconsider the share equations of the Barten-Gorman AIDS model:

$$w_i = \alpha_i + (1-v_i)\ln m_i^* + \sum_j \gamma_{ij}\ (\ln p_j + v_j \ln m_j^*) + \beta_i \ln\left(\frac{y^*}{A(p,d)}\right). \tag{11}$$

where:

$$\ln y^* = \ln C(u,p,d) - \sum_i \Big[(1-v_j) \ln m_j^* (\ln p_j + v_j \ln m_j^*) t_j \Big] =$$

$$= \ln C(u,p,d) - \sum_i \Big[\ln p_j \ln m_j^* + v_j (\ln m_j^*)^2 - v_j \ln p_j \ln m_j^* - v_j^2 (\ln m_j^*)^2 \Big].$$

Let us examine the necessary conditions for an AIDS Barten-Gorman system to generate a cost function homogeneous of degree 1 in prices. As seen before, this is obtained by ensuring that the shares add to one. In order to recognize the restrictions on the v_i's and τ_{jr}'s parameters when adding up is applied to equation (11), it is convenient to isolate three elements GT(.), GS(.) and GCI(.) as follows:

$$GT(d;v,\boldsymbol{\delta}) = \sum_i \Big[(1-v_i) \ln m_i^* \Big] = 0 \ ,$$

$$GS(d;\boldsymbol{\gamma},v,\boldsymbol{\delta}) = \sum_i \Big[\sum_j \boldsymbol{\gamma}_{ij} \ v_j \ \ln m_j^* \Big] = 0 \ ,$$

$$GCI(d,p;\boldsymbol{\gamma},v,\boldsymbol{\delta}) = \sum_i \Big[\boldsymbol{\beta}_i \ \ln y^* \Big] = \ln y^* \sum_i \boldsymbol{\beta}_i = 0.$$

Each of these terms is examined sequentially.

(1) The GT(.) term

$$\sum_i ((1-v_i) \ln m_i^* = \sum_i \ln m_i^* - \sum_i v_i \ \ln m_i^* = 0,$$

hence, homogeneity of C(u,p,d) in p and adding up imply:

$$\sum_i \ln m_i^* = \sum_i \sum_r \boldsymbol{\delta}_{ir} \ln d_r = 0 \quad \rightarrow \quad \textit{for each } r, \ \ln d_r \sum_i \boldsymbol{\delta}_{ir} = 0, \textit{ and}$$

$$\sum_i v_i \ln m_i^* = \sum_i v_i \Big(\sum_r \boldsymbol{\delta}_{ir} \ln d_r \Big) = 0, \quad \rightarrow \quad \textit{for each } r, \ \ln d_r \sum_i v_i \ \boldsymbol{\delta}_{ir} = 0,$$

which implies:

$$\sum_i v_i\, \delta_{ir} = 0, \quad \textit{and}$$

$$v_n = \frac{-\left(\sum_{i=1}^{n-1} v_i\, \delta_{ir}\right)}{\delta_{nr}}\,. \tag{12}$$

(2) The GS(.) term

$$\sum_i \left[\sum_j \gamma_{ij}\; v_i \sum_r \delta_{ir} \ln d_r\right] = 0, \textit{ hence, for each } r\textit{:}$$

$$\ln d_r \sum_i \gamma_{ij} \sum_i v_j\, \delta_{jr} = 0, \quad \rightarrow \quad \sum_i v_j\, \delta_{jr} = x$$

where x is a finite constant. Note that the restriction on the function GT$(d;v,\delta)$ requires x be equal to 0. Hence, as before:

$$v_n = \frac{-\left(\sum_{i=1}^{n-1} v_i\, \delta_{ir}\right)}{\delta_{nr}}\,. \tag{13}$$

(3) The GCI(.) term

It can be easily shown that the function GCI$(d,p;\gamma,v,\delta)$ does not generate further restrictions.

In matrix notation, the identifying restrictions of the Barten-Gorman model can be written as $\Delta\iota=0$, and $\Delta T=0$, where Δ is a rxn matrix of demographic parameters with n being the number of equations, ι is a nx1 vector of ones, T is a n column vector of v_i parameters and 0 is a rx1 vector of zeros.

It is important to note that, given this set of restrictions, the artificial parameters v_i of the Barten-Gorman model are overidentified. To clarify, note that the condition $\Delta\iota=0$ is derived from the homogeneity condition of degree one in prices of the cost function. This implies that Δ is of rank n-1. Hence, the parameter v_n can take infinitely many solutions and there is not a unique way to reconcile the values of v_n. Remarkably, it is neither necessary nor interesting to recover the value of v_n uniquely from the product $v_n\boldsymbol{\delta}_{nr}$. Due to the homogeneity of degree zero in prices of the demand system only n-1 of the artificial parameters, v_n, have to be uniquely identified to fully compound the translating and scaling effects in the Barten-Gorman framework.

The existence of at least one solution is ensured by the fact that the rank of $\boldsymbol{\Delta}$ is $\leq r$. Observe that the system would be otherwise consistent if all the v_i are equal. This option is not satisfactory since v would act as a normalization that could take any value. Finally, note that neither equation (12) nor equation (13) imply linear or nonlinear restrictions on the parameters v_i's *per se* when the v_i's are equal.

Equations (12) and (13) are the restrictions needed to identify and estimate the Barten-Gorman AIDS model. They are normalizations derived from the homogeneity requirement of the cost function. Therefore, these restrictions do not alter the economic content of the model.

5. Concavity

Following Lewbel's proof of Theorem 3, let $C^*(u,h(p,d))=C^*(u,p^*)=y^*$ be a modified cost function and let $C(u,p,d)=f[C^*(u,h(p,d)),p^*,d)]$. For all arbitrary n vectors v, C(u,p,d) is concave in p if and only if [4]

$$\mathbf{v}' \frac{\partial^2 C(u,p,d)}{\partial p \partial p'} \mathbf{v} \leq 0,$$

where:

[4] As in Lewbel (1985), all gradient vectors, Jacobian and Hessian are represented as derivatives or by vectors.

$$v' \frac{\partial^2 C(u,p,d)}{\partial p \partial p'} v = \sum_{i=1}^{n} \left\{ \left(\frac{\partial C^*}{\partial p_i^*}\right)^2 \left[\frac{\partial^2 f(y^*,p,d)}{\partial y^{*2}} \left(\left(\frac{\partial h_i(p,d)}{\partial p}\right)' v\right)^2\right] + \right.$$

$$\left(\frac{\partial C^*}{\partial p_i^*}\right) \left[\frac{\partial f(y^*,p,d)}{\partial y^*} \left(v' \frac{\partial^2 h_i(p,d)}{\partial p\, \partial p'} v\right)^2\right] +$$

$$\left. \frac{1}{n} \left[v' \frac{\partial^2 f(y^*,p,d)}{\partial p\, \partial p'} v\right] + \left[\frac{\partial f}{\partial y^*} v' \frac{\partial h}{\partial p} \frac{\partial^2 C^*}{\partial p^*\, \partial p^{*\prime}} \frac{\partial h}{\partial p'} v\right]\right\} =$$

$$= X^2 B_1 + X B_2 + B_3 + B_4 \quad for\ X = \frac{\partial C^*}{\partial p_i}.$$

Assume that $f[C^*(u,h(p,d)),p^*,d]$ describe Barten-Gorman preferences as:

$$f(y^*,p^*,d) = y^* \left[\prod_{i=1}^{n} (p_i\, m_i)^{t_i}\right].$$

Note that $B_1=0$ since

$$\frac{\partial^2 f}{\partial y^{*2}} = 0,$$

and $B_2=0$ because $\Sigma_i\, t_i=1$. In fact,

$$\sum_{i=1}^{n} \left(\frac{\partial^2 h_i(p,d)}{\partial p \partial p'}\right) = \sum_{i=1}^{n} \frac{t_i\, P^T}{p_i^2} (1-t_i).$$

Thus, $C(u,p,d)$ is concave in p if $B_3 \le 0$ and C^* in B_4 is also concave given that y^* is only differentiable once and $\partial h/\partial p_i > 0$ because p lies in the positive orthant and m>0 is an exponential function.[5] Thus, in a Barten-Gorman context a sufficient test for concavity reduces to testing the concavity in p of both f and C^*.

---

5 Observe that for more general forms of $f(y^*,p^*,d)$ where, for example, y^* is twice differentiable due to the introduction of a function of demographics that also shifts the parameters, then the conditions for $C(u,p,d)$ to be concave in p are more restrictive (See Lewbel Theorem 3).

Assume again a Gorman Polar form for the cost function whose elements are specified by the AIDS model. Straightforward differentiation shows that

$$\frac{\partial \ln f(y^*,p,d)}{\partial \ln p_i}$$

gives the Barten-Gorman specification of the AIDS model presented in equation (11) and that

$$\frac{\partial \ln f^2(y^*,p,d)}{\partial \ln p_i \, \partial \ln p_j^{\prime}} = s_{ij}$$

gives the compensated component s_{ij} of the Slutsky equation.

As suggested by Deaton and Muellbauer (1980), it is possible to obtain a simpler algebraic expression to test for concavity by adopting the following transformation:

$$k_{ij} = s_{ij} \frac{p_i p_j}{y} = \frac{\partial w_i}{\partial \ln p_j} + w_i w_j - \Lambda_{ij} \, w_i \,, \quad \forall_{ij}$$

and,

$$\epsilon_{ij} = \frac{k_{ij}}{w_i} = \frac{1}{w_i}\left(\gamma_{ij} + \beta_i \beta_j \ln\left(\frac{y^*}{A(p,d)}\right) + w_i w_j - \Lambda_{ij} \, w_i\right)$$

where ϵ_{ij} is the compensated price elasticity and $\Lambda_{ij}=1$ for i=j and 0 otherwise is the Kronecker operator. The concavity of C^* implies the concavity of f since f is continuous, monotonic and separable in C^* and P^T as shown above. This completes the proof.

6. Application to Food at Home and Food Away from Home in the USA During the Period 1954-1990

6.1 Estimation

The application is carried out estimating a complete demand system over the period 1953-1988 whose separable components are food-at-home, food-away-from-home, and non-food. In recent years the empirical examination of the food-at-home/food-away-from-home issue has received increasing attention. From the point of view of welfare measurement, the decomposition between food and non-food is interesting because it is ethically in line with the Engel way of associating utilities with well-being.

In the data set, personal consumption expenditure represents income. Expenditure information was obtained from the National Income and Product Accounts of the United States as published by the United States Department of Commerce. Price indices were derived from the annual city averages of consumer price indices from the regular urban National Statistical Accounts with base years 1983-84. The demographic variables included in the model are the percentage of the U.S. population falling in the 0-15 age (D1) category and the percentage of U.S. population enrolled in schools in each year (D2). Demographic information was drawn from Current Population Reports of the U.S. Bureau of the Census. Descriptive statistics of the data used in the analysis are presented in the Data Appendix.

The stochastic Barten-Gorman model is given by:

$$E(w_i) = \alpha_i + t_i(d) + \sum_j \gamma_{ij} \ln p_j^* + \beta_i \ln\left(\frac{y^*}{A(p,d)}\right). \tag{14}$$

We assume that the errors across equations (ϵ_i) are normally distributed, with a constant covariance matrix Ω. They are uncorrelated over time, but correlated in each period:

$$E(\epsilon_{ir}\epsilon_{js}) = \begin{cases} \sigma_{ij} & for\ r = s \\ 0 & for\ r \neq s \end{cases}$$

Moreover, all variables affecting demand are assumed as exogenous.

The system of equations (14) formed by food-at-home (FH), food-away-from-home (FAH) and non-food (NF) was estimated jointly using maximum likelihood (ML) estimation. Because the adding up restrictions were imposed to identify the parameters in the model, $\iota' w_i=1$ and $\iota'\epsilon_i=0$, the covariance matrix is singular and the system is estimated by invariantly dropping the non-food equation.

Following Atkinson (1970), Pollak and Wales (1981), and Davidson and McKinnon (1981), the Barten Gorman model can be compounded in a parsimonious fashion that allows testing the specification of both the nested and non-nested models. This is accomodated by defining the demographic functions as follows:

$$t_i(d) = (1 - v_i) \sum_i \delta_{ir} \ln d_r = \sum_i \tau_{ir} \ln d_r \quad for \quad \tau_{ir} = (1 - v_i)\, \delta_{ir}\,, \ and$$

$$m_i(d) = v_i \sum_i \delta_{ir} \ln d_r$$

for some constant v_i.

The models were estimated with the maintained hypothesis of homogeneity and symmetry. Adding up was explicitly imposed since the model is non-linear. As shown in section 3, the artificial parameters v_i of the Barten-Gorman model are overidentified, but all demographic parameters can be uniquely identified. The derivation of the income, price and demographic elasticities is presented in Appendix A.

6.2 Results

Three demographic specifications were estimated. The parameter estimates for the Translating (T), Scaling (S), and Barten-Gorman (BG) specification can be found in Appendix B. To test for the econometric superiority of one demographic specification over the other we rely upon the Likelihood Ratio. The statistical difference of the v terms from either 1 or 0 is non-informative since the v's are not exactly identified.

The likelihood ratio tests of translating against Barten-Gorman, and Barten-Scaling against Barten-Gorman, are shown in Table 1. The values of the test fail to reject the null hypothesis that scaling and translating are as good as the Barten-Gorman. Nevertheless, according to the likelihood dominance criterion introduced by Pollak and Wales (1991), the Barten-Gorman model is econometrically superior to both. The values of the likelihood functions are presented in Appendix B, Table B.1.

Table 1. Likelihood Ratio Test for demographic specification

Demographic Specification		$LR=2(L^*-L)$	$X^2(.01;d.f.)$	$X^2(.05;d.f)$
BG vs BS	d.f. = 2	1.12	9.21	5.99
BG vs T	d.f. = 2	1.23	9.21	5.99

Note: L^* is the unrestricted log-likelihood value. $X^2(s;d.f.)$ where s=significance level and d.f.=number of restrictions.

If the model is to be used for estimating equivalence scales and money metrics for utility, Blackorby and Donaldson (1988) and Lewbel (1989) point out that the money metric representations should be concave at all price levels. This requires that the Slutsky matrix must be negative definite at all prices. Concavity ensures that social judgements do not contradict distributional judgments derived from a social welfare function which is quasi-concave when each of its arguments is concave.

We test for "single-peaked" preferences, by computing the eigenvalues of the Slutsky matrix incorporating demographic factors.[6] In all the three demographic specifications the test for the violation of the second order conditions was performed at all data points and at the data means following the procedure explained in section 5. No violations in sign were encountered. All eigenvalues were negative.

The elasticities for the Scaling, Translating and Barten-Gorman models are presented in Table 2. Comparing the results provides some insights into whether or not the estimated elasticities are sensitive to the demographic specification.

6 Note that non-positive compensated elasticities is a necessary (minimal) but not sufficient condition for negative semi-definiteness of the Slutsky matrix.

Table 2. Elasticities Estimates for the Translating, Scaling and Barten-Gorman Models
Almost Ideal Model

	Food at Home (fh)			Food \ Home (fah)			Others (oth)		
	T	S	BG	T	S	BG	T	S	BG
pfh	-.556 (.095)	-.558 (.095)	-.569 (.106)	-.035 (.197)	-.032 (.199)	-.019 (.196)	.116 (.021)	.116 (.021)	.119 (.022)
pfah	-.012 (.068)	-.011 (.069)	-.007 (.067)	-.088 (.450)	-.091 (.456)	-.113 (.009)	.009 (.019)	.009 (.019)	.009 (.018)
poth	.568 (.101)	.569 (.103)	.575 (.108)	.123 (.273)	.123 (.277)	.132 (.258)	-.125 (.014)	-.125 (.014)	-.128 (.015)
x	.381 (.078)	.380 (.078)	.396 (.083)	.864 (.092)	.867 (.094)	.879 (.101)	1.14 (.012)	1.14 (.012)	1.133 (.013)
D1	-.330 (.126)	-.332 (.125)	-.378 (.226)	.102 (.222)	.106 (.019)	.105 (.041)	.061 (.020)	.060 (.230)	.07 (.218)
D2	.364 (.123)	.371 (.226)	.333 (.227)	-.451 (.226)	-.453 (.129)	-.464 (.308)	-.043 (.019)	-.044 (.019)	-.036 (.059)

Note: Asymptotic standard errors are in parentheses. The price elasticities are compensated.

The results indicate that the statistical and economic differences between the estimated elasticities across demographic specifications are not significant. The estimates are consistent with the theory and conform with estimates from other time-series studies. Though the elasticity estimates do not vary statistically and economically across demographic specifications, it is worth noting that the estimated elasticities are similar for the Translating and Scaling but differ somewhat from the Barten-Gorman results. This is an indication that the Barten-Gorman model significantly captures the economic information conveyed by either Translating or Scaling.

7. Conclusions

This study provides a theoretical consistent specifcation of the Barten-Gorman model, illustrates how it can be identified and estimates it in the AIDS framework. We use the homogeneity property of the modified Gorman cost function to derive conditions that allow identifying all demographic parameters. We also show how to test for concavity in the Barten-Gorman context.

With the present data set the Likelihood ratio test did not indicate the Barten-Gorman model as statistically superior. Neverthless, the Barten-Gorman specification can still be deemed as more interesting. In fact, the v's artefact permits singling out the price and income component of the demographic effect and distinguishing whether a variable affects consumption through an income or a price effect. This knowledge is usually not available *a priori*. On this ground, the Barten-Gorman model is preferred to a less theoretically rich specification such as Translating and Scaling.

Appendix A. Derivation of the Income, Price and Demographic Elasticities for the Barten-Gorman model

Recall from equation (11) the estimated version of the Barten-Gorman model:

$$w_i = \boldsymbol{\alpha}_i + t_i + \sum_j \boldsymbol{\gamma}_{ij} \ln p_j^* + \boldsymbol{\beta}_i \ln\left(\frac{y^*}{A(p,d)}\right) =$$

$$= \boldsymbol{\alpha}_i + (1-v_i)\ln m_i + \sum_j \boldsymbol{\gamma}_{ij}(\ln p_j + \ln m_i) + \boldsymbol{\beta}_i\Big((\ln y - \sum_i (1-v_i)\ln m_i(\ln p_i + \ln m_i)) - \ln A(p,d)\Big) \qquad \textbf{(A.1)}$$

where:

$$\ln A(p^*) = \ln A(p,d) = \boldsymbol{\alpha}_0 + \sum_i \boldsymbol{\alpha}_i \ln p_i^* + .5 \sum_i \sum_j \boldsymbol{\gamma}_{ij}^* \ln p_i^* \ln p_j^* \quad \textit{and,}$$

for i=1,..,n, n being the number of equations and r=1,..,R, R being the number

$$\ln m_i(d) = \sum_r \boldsymbol{\delta}_{ir} \ln d_r$$

of demographic variables.

Let:
A = nx1 vector of $\boldsymbol{\alpha}$ parameters,
$\boldsymbol{\Gamma}$ = nxn matrix of $\boldsymbol{\gamma}$ parameters,
$\boldsymbol{\Delta}$ = nxR matrix of $\boldsymbol{\delta}$ demographic parametrs,
B = nx1 vector of $\boldsymbol{\beta}$ parameters,
$\boldsymbol{\Lambda}$ = nxR matrix of $\boldsymbol{\delta}$v parameters,[7]
P = nx1 vector of the means of the natural log of the price variables,
D = Rx1 vector of the means of the natural log of the demographic variables,
Y = 1x1 scalar denoting the mean of the natural log of income, and
W = nx1 vector of shares,
Q = nx1 vector of the natural log of quantities,
$\boldsymbol{\iota}$ = nx1 vector of 1.

Define:
M = $\boldsymbol{\Delta}$*D = nx1 vector valued demographic function,
S = $\boldsymbol{\Lambda}$*D = nx1 vector valued scaling component of M,
T = M-S = nx1 vector valued translating component of M, and
P^*= P+S = nx1 vector valued function of demographically modified prices.

8 Given the overidentification of the Barten-Gorman model, the elements of the row of the $\boldsymbol{\Lambda}$ matrix corresponding to the omitted equation are not uniquely separable into its $\boldsymbol{\delta}$ and v components.

In matrix notation, equation (A.1) can be rewritten as:

$$W = \mathrm{A} + (M-S) + (\Gamma * P^*) + \mathrm{B}\left(\left(Y - \iota'(P^* .* (M-S)\right) - \left(\mathrm{A}'P^* + .5 * P^{*\prime}\Gamma P^*\right)\right).$$

where .* is the element-wise multiplication operator.

Using the definition of a share as $w_i = p_i q_i / y$, specify the following relationship in matrix notation:

$$Q = \ln W + Y - P .$$

Then, use the rule of vector differentiation to derive expenditure, price and demographic elasticity.

Expenditure Elasticity η

$$\eta = \frac{\partial Q}{\partial Y} = \iota + \frac{\partial \ln W}{\partial Y} = \iota + \frac{\partial \ln W}{\partial W}\frac{\partial W}{\partial Y} = \iota + \frac{\mathrm{B}}{W} .$$

Uncompensated Price Elasticity ϵ_u

$$\epsilon_u = \frac{\partial Q}{\partial P} = \frac{\partial \ln W}{\partial W}\frac{\partial W}{\partial P} = \frac{1}{W}\left[\Gamma - \mathrm{B}\left(\mathrm{A} + (M-S) + (\Gamma * P^*)\right)'\right] - \mathrm{I}$$

where I is an nxn identity matrix.

Compensated Price Elasticity ϵ

$$\epsilon = \epsilon_u + \eta\, W'$$

using the Slutsky relationship.

Demographic Elasticity

$$\xi = \frac{\partial Q}{\partial D} = \frac{\partial \ln W}{\partial W}\frac{\partial W}{\partial D} =$$

$$= \frac{1}{W} .* \left((\Delta - \Lambda) + (\Gamma * \Lambda) - \mathrm{B} .* \left((\Lambda' * (\mathrm{A} * \iota'))\right)' + \left((\Delta - \Lambda)' * (P^* * \iota')\right)\right)' .$$

Appendix B.

Table B.1. Values of the Likelihood Functions

Demographic Specification	
Barten-Gorman (BG)	356.83
Barten Scaling (BS)	356.27
Translating (T)	356.21

Table B.2. Parameter Estimates

param	T	S	BG
α_1	0.4482 (0.0384)	0.4456 (0.0373)	0.3883 (0.0695)
α_2	0.01645 (0.0212)	0.01816 (0.0213)	0.02271 (0.0242)
γ_{11}	-0.01280 (0.0119)	-0.01291 (0.0118)	-0.01146 (0.0121)
γ_{12}	-0.09936 (0.0124)	-0.09921 (0.0125)	-0.09737 (0.0135)
γ_{22}	0.05143 (0.0140)	0.05170 (0.0136)	0.04603 (0.0139)
β_1	0.04717 (0.0251)	0.04733 (0.0248)	0.04607 (0.0235)
β_2	-0.00739 (0.0052)	-0.00756 (0.0051)	-0.00673 (0.0056)
δ_{11}	-1.605 (1.764)	-0.05244 (0.0198)	-0.9252 (11.57)
δ_{12}	1.422 (1.497)	0.05772 (0.0195)	-0.7986 (3.653)
δ_{21}	0.5119 (0.6813)	0.005695 (0.0123)	3.085 (3.698)
δ_{22}	-0.7067 (0.5806	-0.02506 (0.0125)	-0.9219 (2.721)
v_1			1.042 (0.0911)
v_2			1.076 (0.0765)

Note: Standard deviations are in parentheses

Appendix C. Summary Statistics - Years 1953 to 1988

Variable	Unit	Minimum	Maximum	Mean	Std Dev
total PC Expenditure	$bil	232.6	3235.1	1070.61	904.2203
share(food at home)	%	0.1151	0.2056	0.1604	0.0264
share(food away from home)	%	0.0529	0.0606	0.0554	0.0019
share(non food)	%	0.7347	0.8302	0.7843	0.0274
p(food at home)	$	29.5	116.6	50.2916	1.6450
p(food away from home)	$	21.5	121.8	44.6150	1.8094
p(non food)	$	23.3238	124.688	45.1786	1.7558
population 0-15 of age	%	0.2285	0.3306	0.2827	1.1421
pop enrolled in school	%	0.2047	0.2943	0.2584	1.0905

REFERENCES

Atkinson, A. 1970. "A Method for Discriminating between Models." *Journal of the Royal Statistical Society*. 32:323-53.

Barten, A.P. 1964. "Family Composition, Prices and Expenditure Patterns." In *Econometric Analysis for National Economic Planning: 16th Symposium of the Colston Society*, ed. by P. Hart, G. Mills, and J.K. Whitaker. London: Butterworth.

Blackorby, C. and D. Donaldson. 1988. "Money Metric Utility: A Harmless Normalization?" *Journal of Economic Theory*. Vol.46:120-129.

Chung, C. 1987. "Modelling Demand Systems with Demographic Effects." McKethan-Matherly Discussion Paper MM25, University of Florida, Gainesville.

Davidson, R. and J. MacKinnon. 1981. "Several Tests for Model Specification in the Presence of Alternative Hypothesis." *Econometrica*. Vol.49:781-794.

Deaton, A. and J. Muellbauer. 1986. "On Measuring Child Costs: With Application to Poor Countries". *Journal of Political Economy*. Vol.94:720-744.

Deaton, A. and Muellbauer, J. 1980. "An Almost Ideal Demand System", *American Economic Review*, 70:312-326.

Gorman, W. 1976. "Tricks with Utility Functions." In *Essays in Economic Analysis:Proceedings of the 1975 AUTE Conference, Sheffeld,* ed. by M.J.Artis and A.R.Nobay. Cambridge: Cambridge University Press.

Lewbel, A. 1985. "A Unified Approach to Incorporating Demographic or other Effects into Demand Systems." *Review of Economic Studies*. Vol.52:1-18.

________. 1989. "Household Equivalence Scales and Welfare Comparisons." *Journal of Public Economics*. Vol 39:377-391.

Pesaran, M. and A. Deaton. "Testing Non-Nested Nonlinear Regression Models." *Econometrica. Vol 46*:677-694

Pollak, R. and T. Wales. 1981. "Demographic Variables in Demand Analysis." *Econometrica*. 49(6):1533-1559.

________. 1991. "The Likelihood Dominance Criterion. A New Approach to Model Selection." *Journal of Econometrics*. 47:227-242.