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MODELLING RISK RESPONSE IN THE MARKETING
CHANNEL FOR BEEF: A MULTIVARIATE
GENERALIZED ARCH-M APPROACH

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Modelling Risk Response in the Marketing Channel for Beef:
A Multivariate Generalized ARCH-M Approach

I. Introduction

Following the pioneering work of Behrman and Just, numerous studies have attempted to quantify the role of risk in agricultural supply equations (Antonovitz and Green; Chavas and Holt). More recently, the effect of risk on marketing margins for agricultural products has been examined. Using a variant of Sandmo's model of the firm under output price uncertainty, Brorsen et al. show that marketing channel intermediaries also may be impacted by output (demand) price risk. Brorsen et al.'s model has been extended by Schroeter and Azzam to allow for possible noncompetitive behavior on the part of marketing firms.

Although previous studies have illustrated the potential for output price risk to influence margin behavior, more work is required. First, prior studies have not recognized that, in an expected utility framework, expectations must be taken with respect to both the mean and variance of output price. Accordingly, the appropriate dependent variable in risk-responsive margin equations is the expected price spread, as opposed to the observed price spread as used previously.^{1/} There also is a need to refine the procedures used to infer risk response in margin equations. Brorsen et al. and Brorsen, Chavas, and Grant used fixed-weight moving average methods to estimate risk effects. Although used extensively in applied work, simple extrapolative techniques often provide inaccurate results (Pagan and Ullah). Alternatively, Schroeter and Azzam used an ARCH (Autoregressive Conditional Heteroskedasticity) model. The ARCH approach is a clear improvement over ad hoc extrapolative procedures; Schroeter and Azzam, however, did not estimate the ARCH process simultaneously with their model's structural equations, the result being that the process generating price variability is not endogenous in their model.

The primary objective of this paper is to determine the role of risk in the marketing channel for beef. Like Schroeter and Azzam, we use a Generalized ARCH (GARCH) process to estimate risk response in a farm-retail margin equation. We go beyond their approach, however, in that the structural model's conditional covariance matrix is time varying. That is, our model treats risk as endogenous because the multivariate GARCH process used to infer risk response is estimated simultaneously with the structural equations. This study also parallels Brorsen, Chavas, and Grant in that, in addition to estimating retail demand and farm-retail margin equations, beef production (i.e., primary supply) also is endogenized. In as much as short-run beef supply also responds to (farm) price risk (Antonovitz and Green), we are able to assess market equilibrium risk impacts in the beef marketing channel.

This paper also addresses squarely the issue that, under risk aversion and output price uncertainty, the appropriate dependent variable in the margin equation is the expected price spread. Specifically, ex ante expectations of the mean and variance of retail price are obtained by using a rational-expectations setup (Diebold and Pauly). Because of the associated nonlinear cross-equation restrictions, the resulting rational-expectations model is a type of multivariate GARCH-in-Mean (GARCH-M) model.^{2/} Engle, Lilien, and Robins, Diebold and Pauly, and others have found strong GARCH-M effects in high-frequency, univariate, structural and non-structural models; this study, however, reports the first attempt to apply the GARCH-M approach to a formal multivariate structural system.

The focus on the beef marketing channel also is of interest. Over the past twenty years the beefpacking industry has undergone considerable change. Importantly, beefpacking plants increasingly combine slaughter and processing operations, the outcome being that over 90% of all beef is now sold in boxed

(i.e., fabricated) form (Johnson et al.). This result has implications for beef pricing because, unlike for carcass beef, comparatively little use is made of contracting or formula-based pricing in the boxed beef market (Hayenga and Schrader; Ward).^{3/} Moreover, there are no futures markets for boxed beef which could allow marketing intermediaries to spread price risks. These reasons, along with the fact that beefpacking is a high sales volume, low-margin industry, combine to suggest that the beef marketing channel may be quite sensitive to meat price variability (Ward, p. 170).

II. Conceptual Framework

The conceptual model developed here has its roots in Sandmo's model of the firm under output price uncertainty and is similar to the one presented in Brorsen et al. Following Gardner, Heien, Wohlgenant, and others, we assume market intermediaries operate effectively in a competitive environment.^{4/} The present model differs from Brorsen et al.'s, however, in that packer-processors are assumed to form rational-expectations of output price and price risk.

We assume m firms purchase a raw farm input, x , and transform it into a food product, q .^{5/} Other inputs, z , also are used in the production of q . Moreover, q is produced by using fixed proportions of x , the raw farm input; but other inputs, z , can be used in variable proportions. Under these assumptions, each firm's technology can be represented by a Leontief-type production function

$$(1) \quad q = \min[x/k, g(z)],$$

where k is the positive constant of fixed proportion. Letting w denote price of the farm product, and \underline{r} a price vector for inputs z , the cost function associated

with (1) is

$$(2) \quad C(w, \underline{r}, q) = \min_{x, z} [wx + \underline{r}'z \mid \text{s.t. (1)}] = wkq + \underline{r}'\underline{z}^*(\underline{r}, q),$$

where $x^*(w, q) = kq$ and $\underline{z}^*(\underline{r}, q)$ are cost-minimizing input demands for, respectively, farm and non-farm inputs. The standard properties of C include linear homogeneity, increasing and concave in (w, \underline{r}) , and increasing and convex in q . Firm profit is then given by

$$\pi = (p - kw)q - \tilde{C}(r, q)$$

where $\tilde{C}(r, q) = \underline{r}'\underline{z}^*(\underline{r}, q)$ denotes the cost function for non-farm inputs.

In addition, firms face a random (inverse) demand schedule

$$(3) \quad p = \bar{p}(Q, \underline{s}) + \gamma\tilde{\epsilon}_1,$$

where $Q = mq$ is industry output; \underline{s} is a vector of exogenous demand shifters; and $\tilde{\epsilon}_1$ is a random variable, where $E(\tilde{\epsilon}_1) = 0$ and $E(\tilde{\epsilon}_1^2) = 1$. Expected market price is then given by $E(p) = \bar{p}(Q, \underline{s})$ and retail price variance by $\text{var}(p) = \gamma^2$.

Under risk aversion, each firm's goal is to maximize expected utility of profit. Each packer-processor's objective is then characterized by

$$(4) \quad \max_q Eu[(p - kw)q - \tilde{C}(r, q)],$$

where $u(\pi)$ is a von Neumann-Morgenstern utility function with $du/d\pi > 0$ and $d^2u/d\pi^2 < 0$ under risk aversion. Expectations are taken with respect to the

random variable, retail price. The first-order condition associated with (4) is

$$(5) \quad E[u' \{ (p - kw) - \bar{c}(r, q) \}] = \bar{p}(Q, \underline{s}) - kw - \bar{c}(r, q) + \text{cov}(u', p)/Eu' = 0,$$

where $\bar{c}(r, q) = \partial \bar{C}(r, q)/\partial q$ and $\text{cov}(u', p) = \rho(\gamma^2 \cdot E[u' - Eu']^2)^{1/2}$ is the covariance between marginal utility and expected price, ρ being the correlation between u' and p . Equation (5) can be solved to obtain the firm's supply function. Alternatively, this firm-level supply equation can be inverted to obtain an expression for the expected farm-retail margin (Brorsen et al.)

$$(6) \quad \bar{p}(Q, \underline{s}) - kw = \bar{c}(r, q) + \delta^* \gamma,$$

where $\delta^* = -(Eu')^{-1} \rho(E[u' - Eu']^2)^{1/2}$. Because under risk aversion output price and marginal utility of profit are negatively correlated (Baron), δ^* will be positive for risk averse firms. Brorsen et al. show that under decreasing absolute risk aversion (DARA), a marginal increase in output price risk will increase the expected marketing margin. Also, because of the fixed factor of proportionality, kw is farm input price expressed in units equivalent to \bar{p} .

Assuming the industry behaves like a representative firm, the aggregate expected margin equation (e.g., inverse aggregate packer-processor supply) is

$$(7) \quad \bar{M} = \bar{p}(Q, \underline{s}) - kw = \Pi(\underline{r}, Q) + \delta_1 \gamma + \epsilon_2,$$

where \bar{M} denotes expected margin and a stochastic term, ϵ_2 , has been added as a prelude to econometric specification.^{6/} Brorsen et al. show that under DARA, $\partial \bar{M}/\partial Q > 0$ and $\partial \bar{M}/\partial r_j \geq 0$ (≤ 0) as $\partial Q/\partial r_j \geq 0$ (≤ 0).

III. Empirical Issues

The conceptual framework provides a basis for specifying margin equations with risk terms. Before proceeding, however, several issues regarding retail price expectations and time-varying risk measures must be addressed.

To begin, equation (7) shows that it is the difference between expected output price and farm price (i.e., the expected margin) that serves as the dependent variable in a margin equation with retail price uncertainty. Brorsen et al., Brorsen, Chavas, and Grant, and Schroeter and Azzam used the observed margin as the dependent variable. Although this substitution seems innocuous, the econometric implications are not insignificant. If the observed margin is used in place of the expected margin in (7), the margin equation's error process coincides with that of the (inverse) demand function. In other words, $\epsilon_{2t} = \gamma \tilde{\epsilon}_{1t}$, where a t subscript has been added to denote time^{2/8/}. Alternatively, ϵ_{2t} could be a separate error term (i.e., ϵ_{2t} is the result of errors in approximation, optimization, etc.), but appropriate estimates can only be obtained by using the joint error term $\lambda_t = \epsilon_{2t} + \gamma \tilde{\epsilon}_{1t}$ when actual output price is used in (7). Either way, the margin equation's error process depends on the demand equation's error process when observed output price is used in lieu of expected output price, an issue not explored in previous research.

Conversely, a method could be identified to determine directly ex ante expectations of output price and, consequently, expected margins. Such a method would preferably be consistent with the retail demand specification. In other words, output price expectations could be determined according to the Rational Expectations Hypothesis (REH). The REH has appeal because if output price is the primary "unknown" and is the underlying source of uncertainty for packer-processors, then it is logical to estimate price expectations in a manner

consistent with the specification of primary demand.

The REH can also be used to model ex ante expectations of price variance (Aradhyula and Holt; Antonovitz and Green). Of course this requires that the model's forecast error variances be time varying. In recent years, ARCH and GARCH models have been used to estimate time-varying conditional variances in single- (e.g., Engle; Bollerslev, 1986) and multi-equation (e.g., Bollerslev Engle, and Wooldridge; Bollerslev, 1990; Baillie and Myers) setups. GARCH (ARCH) models have appeal because they provide a time-series rationale for time-varying conditional variances. The GARCH (ARCH) approach to modelling second moments also is consistent with the REH because the information set, including lagged realizations and innovations of endogenous variables, coincides with that commonly used to form expectations of the mean (Diebold and Pauly).

Defining $\epsilon_{1t} = \gamma \tilde{\epsilon}_t$, a GARCH(p,q) process consistent with equation (3) is

$$(8) \quad \text{Var}(\epsilon_{1t} | \psi_{t-1}) = h_{1t} = \omega_0 + \sum_{j=1}^q \alpha_j \epsilon_{1t-j}^2 + \sum_{j=1}^p \beta_j h_{1t-j},$$

where $\omega_0 > 0$ and $\alpha_j > 0$, $\beta_j > 0$ for all j ; and ψ_{t-1} is the information set generated by all available information through time $t-1$. If $\beta_j = 0$ for all j , then (8) reduces to an ARCH(q) process. The square root of one-step-ahead predictions from (8) replace γ in (7) when estimating risk response in margin equations. The resulting model is a multivariate GARCH-M (ARCH-M) model because, under the REH, not only are demand and margin equation parameters shared, but parameters of the demand equation's GARCH (ARCH) process also are shared. Thus, the structure generating price risk is endogenous in a GARCH-M model.

IV. Econometric Methodology

The preceding discussion suggests that risk effects in the beef marketing channel can be modeled by using a GARCH-M framework. This section outlines estimation procedures used when the model's conditional covariance matrix is time varying. Recent advances in multivariate GARCH (ARCH) modelling include: Bollerslev, Engle, and Wooldridge's diagonal vech model; Baillie and Myers' positive definite model; and Bollerslev's (1990) constant conditional correlations model. We focus here on Bollerslev's (1990) approach.^{9/}

Bollerslev's (1990) setup holds conditional correlations among endogenous variables constant, but allows time-varying conditional covariances. Define \underline{y}_t as an $(N \times 1)$ vector of endogenous variables, $\underline{\epsilon}_t$ as a corresponding $(N \times 1)$ vector of innovations, and h_{ijt} as the ij 'th element of H_t . The conditional covariance between the i 'th and j 'th elements of $\underline{\epsilon}_t$, ϵ_{it} and ϵ_{jt} , is

$$(9) \quad h_{ijt} = \rho_{ij} (h_{iit} h_{jjt})^{1/2}, \quad i = 1, \dots, N, \quad j = i+1, \dots, N,$$

where $\rho_{ij} = \text{Corr}(\epsilon_{it}, \epsilon_{jt} | \psi_{t-1})$, the conditional correlation coefficient; $\rho_{ij} \in [-1, 1]$ for all i and j ; and $\rho_{ii} = 1$ for all i . The diagonal elements of H_t —defined as $h_{iit} = \sigma_{it}^2 > 0$ for all i and t —are given by specifications similar to (8). In general ρ_{ij} could be time varying; but if ρ_{ij} is constant for all t , considerable simplifications arise in estimation and inference.

To see this, rewrite the conditional covariance matrix, H_t , as $H_t = D_t \Gamma D_t$, where D_t denotes an $(N \times N)$ diagonal matrix with elements $\sigma_{1t}, \dots, \sigma_{Nt}$, and Γ is an $(N \times N)$ time-invariant, symmetric, positive definite matrix, where $\{\Gamma_{ij}\} = \rho_{ij}$. Assuming conditional normality, the log likelihood function is

$$(10) L(\varrho) = -\frac{TN}{2} \log 2\pi + \sum_{t=1}^T \log \|J_t\| - \frac{T}{2} \log |\Gamma| - \sum_{t=1}^T \log |D_t| - \frac{1}{2} \sum_{t=1}^T \hat{\epsilon}_t' \Gamma^{-1} \hat{\epsilon}_t,$$

where $\hat{\epsilon}_t = D_t^{-1} \epsilon_t$ is an $(N \times 1)$ vector of standardized residuals, T is sample size, J_t is the Jacobian of the system, and ϱ represents all unknown parameters in ϵ_t and H_t . Under standard regularity conditions the Maximum Likelihood (ML) estimate of ϱ is asymptotically normal. Unlike other multivariate GARCH (ARCH) setups though, only one $(N \times N)$ matrix inversion is called for during each evaluation of (10). Also, $\log |D_t| = \sum_{i=1}^N \log \sigma_{it}$. We use Broyden's algorithm, along with numerical derivatives, in the maximization of (10) to estimate a multivariate GARCH-M model for the beef marketing channel.

V. Data and Model Specification

The application is with a three-equation model of the beef market, including equations for retail demand, (expected) farm-retail margin, and short-run beef supply. Salient model features include: the rational-expectation of retail price risk in the margin equation; the expected margin determined by the REH; farm price risk in the beef production equation; and risk response estimated endogenously by using a multivariate GARCH-M model.

Although our primary focus is on determining risk response in the beef marketing channel, there is substantial evidence that short-run cattle supplies react to current farm price and, perhaps, to current (farm) price risk (Jarvis; Antonovitz and Green). In a short-run model, it is therefore necessary to endogenize beef supply. By including a measure of farm price risk in the supply equation, the relative importance of risk in short-run beef supply vis-a-vis the marketing margin also can be assessed.

The data analyzed are monthly for 1970-89. Using monthly data facilitates

estimation of risk effects, inasmuch as beef inventories are seldom held for extended periods of time, and because it is often easier to identify conditional heteroskedasticity with high-frequency data; using monthly data, however, also adds dynamic complexities (Heien). Appropriate lag distributions can only be identified largely through preliminary analysis.

Retail beef demand is specified in price dependent form as

$$\begin{aligned}
 (11) \quad \Delta RPB_t = & \gamma_0 + \gamma_1 \Delta QB_t + \gamma_2 \Delta QB_{t-1} + \gamma_3 \Delta QB_{t-2} + \gamma_4 \Delta RPP_t + \gamma_5 \Delta INC_t \\
 & + \gamma_6 \Delta INC_{t-1} + \gamma_7 \Delta INC_{t-2} + \gamma_8 \Delta INC_{t-3} + \gamma_9 \text{SIN1}_t + \gamma_{10} \text{COS1}_t \\
 & + \gamma_{11} \text{SIN2}_t + \gamma_{12} \text{COS2}_t + \gamma_{13} \Delta RPB_{t-1} + \gamma_{14} \Delta RPB_{t-2} + \gamma_{15} \Delta RPB_{t-3} \\
 & + \gamma_{16} \Delta RPB_{t-10} + \gamma_{17} \Delta RPB_{t-11} + \epsilon_{1t},
 \end{aligned}$$

where Δ is a first-difference operator; RPB_t is retail beef price, cents per lb.; QB_t is commercial beef production, million pounds; RPP_t is retail pork price, cents per lb.; INC_t is personal disposable income, billion dollars; SIN1_t - COS2_t are harmonic variables for six- and twelve-month cycles; and $\gamma_0, \dots, \gamma_{17}$ are parameters.^{10/} All beef prices were obtained from White et al. Retail pork prices and beef production were collected from Livestock and Meat Statistics. Income data were collected from various issues of the Survey of Current Business. All prices and income are deflated by the Consumer Price Index (CPI) (1967 = 1.0), collected from the Survey of Current Business.^{11/} Retail demand is specified in first-difference form because Phillips-Perron tests fail to reject the null hypothesis of a unit root in (real) retail beef prices (table 1).

The (inverse) retail demand equation includes current and lagged changes in beef production. Lagged production is relevant if retail beef prices respond

gradually to quantity changes. The change in retail pork price is included because pork is a substitute for beef. Current and lagged changes in disposable income capture income effects in beef demand. The lag distribution on changes in retail beef price was determined largely by examining the autocorrelation and partial autocorrelation functions for ϵ_{1t} .

The specification of the (expected) farm-retail margin equation consistent with (7), where $\Pi(\underline{x}, Q)$ is approximated with a linear form, is

$$(12) \quad \bar{M} = \theta_0 + \delta_1 \sigma_{1t} + \theta_1 QB_t + \theta_2 PR_t + \theta_3 WR_t + \theta_4 SIN1_t + \theta_5 COS1_t + \theta_6 SIN2_t \\ + \theta_7 COS2_t + \theta_8 MRG_{t-1} + \theta_9 MRG_{t-2} + \theta_{10} MRG_{t-3} + \theta_{11} MRG_{t-4} \\ + \theta_{12} MRG_{t-6} + \theta_{13} MRG_{t-11} + \theta_{14} MRG_{t-12} + \epsilon_{2t},$$

where,

$$\bar{M} = E(RPB_t | \psi_{t-1}) - FPB_t,$$

and where $E(RPB_t | \psi_{t-1})$ denotes the rational-expectation of retail beef price, cents per lb.; FPB_t is farm price of beef in retail equivalents (net of by-product value), cents per lb.; σ_{1t} is the rational-expectation of the standard deviation of (real) retail beef price; PE_t is the price of energy, index; WR_t is the meat packing wage rate, dollars per hour; MRG_{t-j} denotes the lagged (realized) farm-retail margin; and $\theta_0, \dots, \theta_{14}$ and δ_1 are parameters. Phillips-Perron tests indicate the null hypothesis of a unit root can be rejected for the (real) farm-retail margin (table 1); equation (12) is thus specified in the levels of the data. The energy price index was acquired from the Survey of Current Business and wage rate data were obtained from Employment and Earnings.

As before, all prices are deflated by the CPI.

The rational-expectation of the time-varying standard deviation of retail price, σ_{1t} , measures the effect of output price risk on the farm-retail margin. Beef production, QB_t , is included because, as suggested by theory, production levels should impact the expected margin. Prices for energy and labor reflect important input costs in beefpacking and processing (Schroeter), and harmonic variables are included to capture seasonality. Lastly, guided largely by preliminary analysis, the lag distribution on observed margins was specified.

Short-run beef production is specified as

$$\begin{aligned}
 (13) \quad QB_t = & \nu_0 + \nu_1 \text{FPB}_t + \delta_2 \sqrt{h_{22t}} + \nu_2 \text{PCO}_t + \nu_3 \text{OFD}_{t-1} + \nu_4 \text{SIN1}_t \\
 & + \nu_5 \text{COS1}_t + \nu_6 \text{SIN2}_t + \nu_7 \text{COS2}_t + \nu_8 \text{QB}_{t-1} + \nu_9 \text{QB}_{t-2} + \nu_{10} \text{QB}_{t-3} \\
 & + \nu_{11} \text{QB}_{t-4} + \nu_{12} \text{QB}_{t-5} + \nu_{13} \text{QB}_{t-6} + \nu_{14} \text{QB}_{t-7} + \nu_{15} \text{QB}_{t-8} \\
 & + \nu_{16} \text{QB}_{t-9} + \nu_{17} \text{QB}_{t-10} + \nu_{18} \text{QB}_{t-11} + \nu_{19} \text{QB}_{t-12} + \epsilon_{3t},
 \end{aligned}$$

where $\sqrt{h_{22t}}$ is the time-varying conditional standard deviation of (real) farm beef price; PCO_t is the price of corn paid by farmers, dollars per bushel; OFD_t is cattle on feed in seven states, thousand head; and ν_0, \dots, ν_{19} and δ_2 are parameters. Corn prices were obtained from Agricultural Prices and cattle on feed data were collected from Livestock and Meat Statistics. All prices are deflated by the CPI.

Beef supply is specified in the levels of the data because Phillips-Perron tests reject the null hypothesis of a unit root in QB_t (table 1). Both current farm price and price risk can influence short-run beef supply (Jarvis), and corn is an important input cost in fed beef production. The available stock of

marketable (fed) cattle is reflected by last period's cattle on feed numbers.^{12/} The twelfth-order lag specification for QB_t captures short-run supply dynamics.

The expected farm-retail margin, obtained according to the REH, is the left-hand-side variable in (12). Because short-run beef supply depends on farm price, the rational-expectations reduced forms for the mean and standard deviation of retail price are complicated beyond those outlined in preceding sections. Details on obtaining the model's final form are outlined in the appendix. In general though, $E(RPB_t | \psi_{t-1})$ will depend on—in addition to model parameters—expectations of retail pork price, disposable income, price of energy, meatpacking wage rate, and price of corn.

As is common practice in rational-expectations modelling (see, e.g., Hoffman; Diebold and Pauly), expectations of exogenous variables are obtained by using univariate autoregressions. Because Phillips-Perron tests indicate the null hypothesis of a unit root cannot be rejected for any contemporaneously exogenous variable (table 1), the auxiliary autoregressions are specified as

$$(14) \quad A_i(L)\Delta X_{it} = \epsilon_{it}, \quad i = 4, \dots, 8,$$

where $X_{4t} = RPP_t$, $X_{5t} = INC_t$, $X_{6t} = PE_t$, $X_{7t} = WR_t$, $X_{8t} = PCO_t$; $A_i(L)$, $i = 4, \dots, 8$, is an eleventh-order polynomial in L , the backshift operator; and ϵ_{it} , $i = 4, \dots, 8$, are random error terms. With predictions generated from OLS estimates of the autoregressions in (14), $E(RPB_t | \psi_{t-1})$ can be evaluated.^{13/}

Finally, preliminary analysis indicated GARCH(1,1) processes for h_{11t} , h_{22t} , and h_{33t} were adequate for specifying H_t . The conditional variance-covariance structure for the three-equation GARCH-M model in (11)–(13) is then

$$(15) \quad h_{iit} = \omega_i + \alpha_{i1} \epsilon_{it-1}^2 + \beta_{i1} h_{iit-1},$$

$$h_{ijt} = \rho_{ij} (h_{iit} h_{jjt})^{1/2}, \quad i, j = 1 \text{ (RPB}_t\text{)}, 2 \text{ (}\bar{M}_t\text{)}, 3 \text{ (QB}_t\text{)}, \quad i \neq j.$$

Under conditional normality, ML parameter estimates are obtained by using (10).

VI. Estimation Results

ML estimates for the rational-expectations GARCH-M model of the beef marketing channel are reported in table 2. Short-run flexibilities and elasticities for key exogenous variables, at the data means, are recorded in table 3. Several model diagnostics are presented in table 4.

Turning to the results, the point estimates for α_{i1} and β_{i1} , $i=1,2,3$, are positive and individually significant (table 2), indicating the presence of conditional heteroskedasticity in the error terms of the structural equations. Further evidence of conditional heteroskedasticity is obtained by restricting $\delta_1 = \delta_2 = 0$ and estimating the model that nests the homoskedastic specification. Conditional on $\delta_1 = \delta_2 = 0$, the resulting Likelihood Ratio (LR) test statistic for $\alpha_{i1} = \beta_{i1} = 0$, $i=1,2,3$, is 134.384, a value of an asymptotic $\chi^2(6)$ distribution under the null hypothesis. The homoskedastic model is thus rejected at any reasonable level.^{14/} In all cases, the unconditional variances, $\hat{\omega}_i / (1 - \hat{\alpha}_{i1} - \hat{\beta}_{i1})$, are defined because $\hat{\alpha}_{i1} + \hat{\beta}_{i1} < 1$ for all i .

Estimates for the conditional correlation parameters also are individually significant. The LR test statistic for $\rho_{ij} = 0$ for all $i \neq j$ is 86.582, which asymptotically under the null hypothesis is the realization of a $\chi^2(3)$ distribution. This overwhelming rejection of independence indicates short-run beef prices and production are significantly correlated, the conditional

correlation with farm and retail prices (-0.634) being the strongest.^{15/}

Conditional variances and covariances are plotted for the sample period in figure 1. Of interest is that the conditional variance for the expected margin generally exceeds that of retail price, indicating more volatility in farm prices than retail prices. Furthermore, figure 1 shows variances and covariance were generally much more volatile during the 1970s, and were especially large during the mid-1970s. Although the model provides no structural explanation for the extreme price volatility observed in the beef market during the mid-1970s, this period was associated with wage and price controls, unstable grain and energy prices, and high and variable rates of inflation.

Retail demand equation results show that all coefficients for current and lagged beef production are significant (table 2). The short-run retail price flexibility for beef production is, as might be expected, small (-0.09) (table 3). The effect of retail pork price on retail beef price is positive and significant, with a short-run flexibility of 0.101. Disposable income also has a positive and significant relationship with retail beef price, the short-run flexibility being about 0.17 (table 3).

Turning to the margin equation, note that all economic variables are significant at usual levels (table 2). Of interest is the estimate for δ_1 , the marginal effect of (expected) retail price risk on farm-retail margins, is positive and significant. This result is consistent with theory and provides evidence that beefpacker-processors react adversely to output price risk. The corresponding short-run (expected) farm-retail price spread flexibility for retail price risk is 0.064 (table 3). Overall, these results are consistent with those of Brorsen et al.; Brorsen, Chavas, and Grant; and Schroeter and Azzam. Other economic variables in the margin equation have plausible signs and

magnitudes. For instance, beef production has a positive, albeit insignificant, relationship with the expected margin. Input prices also have a positive and significant effect on expected short-run farm-retail margins (table 2).

Estimates for beef supply equation parameters also are plausible. For example, short-run beef supply has a significant, negative relationship with current farm price and a significant, positive relationship with farm price risk. These results are consistent with Jarvis' theory of cattle supply, where cattle are viewed as both a consumption and investment good. Short-run own price and risk elasticities (-0.33 and 0.08, respectively) compare favorably with previous estimates (e.g., Antonovitz and Green). Short-run beef supply has a significant, positive relationship with corn price and (lagged) cattle on feed (table 2).

Several diagnostic tests are reported in table 4. First, skewness and kurtosis estimates for each standardized residual series do not indicate significant departures from normality. Ljung-Box tests for up to 24'th order serial correlation in the standardized residuals signify that, with the exception of the production equation, autocorrelation is not present.^{16/} Similarly, Ljung-Box tests for up to 24'th order serial correlation in the squared standardized residuals are, with the exception of retail price, insignificant in the $\chi^2(24)$ distribution at the 5% level.

As an added check, Pagan-Sabau consistency tests are employed. These tests determine if the conditional variances are consistent with the second-moment pattern of the residuals, and require estimating OLS regressions of the type

$$\hat{\epsilon}_{it} \hat{\epsilon}_{jt} = b_{ij0} + b_{ij1} \hat{h}_{ijt}, \quad i, j=1, 2, 3,$$

where under the null hypothesis of model consistency b_{ij1} should not differ

significantly from unity. T-statistics for the null hypothesis $b_{ij1} = 1$, obtained by using White's correction for heteroskedasticity, are reported in the lower panel of table 4. In all cases, the t-statistics are insignificant at the 5% level, indicating the conditional variance process is consistent.

In summary, the constant conditional correlations model with a GARCH(1,1) conditional variance structure provides a reasonable representation of the conditional variance dynamics in the beef marketing channel.

VII. Assessment of Risk

In this section the estimated GARCH-M model is used to determine the role and relative importance of risk in the beef marketing channel. This is accomplished by performing several additional tests, and by simulating the model to infer time-varying risk premia in the beef marketing channel for 1971-89.

First, the LR test statistic of 29.652 for $\delta_1 = \delta_2 = 0$ is significant in the $\chi^2(2)$ distribution at all usual levels, thus indicating risk terms are jointly significant in beef margin and supply equations. Next, the hypothesis that short-run risk effects in the margin and supply equations are identical is tested by restricting (locally) the risk elasticity in the margin equation to equal that of the supply equation. The resulting LR test statistic is 1.862, a value well below 3.841, the critical value from the $\chi^2(1)$ distribution at the 0.05 level. This result is meaningful because it provides strong evidence that risk impacts, as gauged by elasticities, are of equal importance in short-run beef margin and supply equations.

The preceding results show that price risk is significant at several levels in the beef market; they say nothing, however, about how risk has influenced market performance. The relative importance of risk in the margin equation is

evaluated by computing $RRP_t = \hat{\delta}_1 \sigma_{1t} / [E(RPB_t | \psi_{t-1}) - FPB_t]$, the implied relative risk premium. The results, graphed in figure 2 (both original data and smoothed), range from a peak of 17.7% to a low of 3.1%, the average being 6.4%. Furthermore, a break in the ratio occurred between 1979 and 1981, with the ratio stabilizing around 4% after 1981. On balance, the variable risk premium in the farm-retail margin for beef, although not of overwhelming importance, is certainly not negligible.

That the break in RRP occurs in the late 1970s is of interest because this period also was associated with sharp increases in (four-firm) concentration in beefpacking (Ward). Consequently, a relevant question is: has increased market concentration influenced the relative importance of risk in the marketing channel? To examine this issue, a dynamic regression equation linking annual average RRP's and concentration ratios for 1972-89 is estimated. The results are:

$$\ln RRP_t = 3.097 - 0.569 \ln CR_t + 0.434 \ln RRP_{t-1} + e_t,$$

(2.088) (-1.968) (1.798)

$$e_t = 0.681 e_{t-1} - 0.580 e_{t-2}, \quad R^2 = 0.875, \quad D-W = 2.087,$$

(3.542) (-3.021)

where \ln is the natural logarithm, CR_t is the four-firm concentration ratio, e_t is white noise; and T-ratios are in parentheses.^{17/} The elasticity for CR_t is negative and significant, indicating higher concentration levels accompany lower relative risk premiums. Although these results are clearly tentative, they do suggest that reorganization in the beefpacking industry has had a significant dampening effect on the relative price risk faced by remaining firms.

The foregoing results bracket the role of risk in the beef marketing

margin. But they do not indicate how risk impacts equilibrium prices and quantity in the beef market. To assess the combined effects of retail and farm price risk on prices and production, the model is simulated stochastically after setting $\delta_1 = \delta_2 = 0$. The results, summarized in table 5, show that for all years average farm beef price would have increased in the absence of risk. The biggest impacts also were recorded for the 1970s when beef prices were generally more volatile (figure 1). During 1971-89, for example, average farm price would have been higher by 2.66 cents per retail lb., or 5.28% above observed levels. Accordingly, farm-retail price spreads would have been below observed levels by, on average, 6.05%, or 2.11 cents per retail lb. Moreover, the average farm-retail margin would have been lower each year, with the implied market equilibrium risk premium ranging from 1.6% to 11.8%.

The average retail beef price would have been higher during all but three years (1971, 1973, and 1978) under a "no-risk" scenario. The average increase, however, is only 0.7%, or 0.56 cents per lb. above observed levels. Thus, the (equilibrium) farm-retail margin declines even though retail prices rise. Higher retail prices result from setting δ_2 to zero in the supply equation which, coupled with higher farm prices, drives short-run beef production down on average by 9.6% (table 5). Clearly, market participant response to price risk has, at times, had large impacts on equilibrium prices and production in the beef market.

VIII. Conclusions

This paper has sought to determine the role of risk in the farm-retail price spread for beef. Although previous research has found significant risk effects in price linkage equations for wheat, rice, and pork, similar results have not been reported for beef. To address this issue, the model developed here

makes use of recent econometric advances, and is a type of multivariate GARCH-M model under rational-expectations.

By using monthly data for 1970-89, and Bollerslev's (1990) constant conditional correlations model, ML estimates of retail demand, farm-retail margin, and beef production equations were obtained. The estimated GARCH-M model provides a good fit to the data; the parameter estimates have plausible signs and magnitudes; the estimated conditional variance structure indicates substantial GARCH effects; and implied flexibilities (elasticities) are reasonable.

Of particular interest is that price risk, as measured by the rational-expectation of the standard deviation of retail price, is significant in the price spread equation. An LR test also revealed that short-run risk effects in margin and supply equations, as measured by elasticities, are essentially identical. The implication is that price risk is no more or less important for packer-processors than for primary producers in the beef market. The impact of risk was further evaluated by computing the relative risk premium in the margin equation, and by simulating the model after setting risk terms to zero. The results indicate that price risk has, from time-to-time, had a significant impact on equilibrium beef prices and production. Consequently, as Johnson et al. argue, risk sharing arrangements, such as a futures contract for boxed beef, could improve pricing efficiency and performance in beef marketing.

Footnotes

- 1/ Schroeter and Azzam do include expected output price in their conceptual model, but do not use expected prices in their empirical analysis.
- 2/ GARCH-in-Mean simply implies the model's time-varying conditional variance-covariance terms are inputs in the conditional mean equations.
- 3/ Rather, most boxed beef transactions occur on an "offer-acceptance" basis.
- 4/ Conversely, following Schroeter and Azzam, we could assume packer-processors exert market power. Schroeter estimated the incidence of market power in the U.S. beefpacking industry, however, and concluded that although monopoly/monopsony price distortions were often statistically significant during 1951-83, these distortions have been relatively small (respectively, 1 % and 3%) and stable since 1970. Consequently, even though concentration in beefpacking has increased steadily in recent years, there is not strong evidence that noncompetitive behavior has resulted in large price distortions.
- 5/ We do not distinguish between wholesale and retail functions in the beef marketing channel. This assumption is not overly restrictive because trade in carcass beef has declined in importance, and because large retailers often buy directly from packer-processors (Johnson et al.). The result is that wholesaling operations in the beef market have declined in importance over time. It is understood, however, that it is packer-processors, as opposed to retailers, that face undiversifiable price risk.
- 6/ For the implied risk premium $R = \delta^* \gamma$ in (6), the term δ^* will also vary with γ . Throughout the remainder of the paper we therefore use a first-order approximation to the risk premium of the form $R = \bar{R} + \delta_1 \gamma$, where \bar{R} is an unspecified constant term.

- 7/ This follows by solving (3) for \bar{p} and substituting the result into (7).
- 8/ That retail demand and margin equations would possess an identical error process follows from the assumption that the error term in the demand equation is linear and additive. If, for example, the demand equation's error process were multiplicative, then the error term in the margin equation would be functionally related to that of the demand equation, although they would no longer be identical.
- 9/ In addition to Bollerslev's (1990) method, the multivariate GARCH-M model also was estimated by using Bollerslev, Engle, and Wooldridge's diagonal vech model and Baillie and Myers' positive definite model; Bollerslev's (1990) setup, however, provided the most satisfactory representation of the model's time-varying conditional covariance structure.
- 10/ The harmonic variables are $SIN1_t = \sin(2\pi t/6)$, $COS1_t = \cos(2\pi t/6)$, $SIN2_t = \sin(2\pi t/12)$, and $COS2_t = \cos(2\pi t/12)$.
- 11/ The choice of deflator, or whether not to deflate at all, is not obvious in models with risk. To begin, we follow Pope, who warns against estimating risk effects in models that use nominal prices. The choice of deflator is less clear because no single price index is likely appropriate for all levels of the marketing channel; we follow Chavas and Holt, however, in using the consumer price index.
- 12/ More fitting, and in keeping with the rational-expectations strategy, forward expectations of farm price and price risk, along with current farm price and risk variables, should appear in the beef supply equation. Complexities associated with including intertemporal expectations, however, render such a specification beyond the scope of this study.
- 13/ As Hoffman indicates, two-step estimation procedures of the type employed

here yield biased standard errors, even in large samples. It is beyond the reach of this study though to deal appropriately with this issue.

14/ The LR test statistic for the null hypothesis $\delta_1 = \delta_2 = 0$ and $\alpha_{i1} = \beta_{i1} = 0$, $i=1,2,3$, is 164.486, a value well above any reasonable critical value in the asymptotic $\chi^2(8)$ distribution under the null hypothesis.

15/ A negative correlation with retail price and the expected margin seems counterintuitive. But the only stochastic variable on the left-hand-side of margin equation (12) is $-FPB_t$, the negative farm price of beef. Consequently, a negative correlation coefficient simply reflects a high and positive correlation with RPB_t and FPB_t , as expected. This hypothesis was confirmed by re-estimating the model after normalizing the margin equation on farm price, in which case an identically large and positive estimate for $\hat{\rho}_{12}$ was obtained.

16/ It is hardly surprising that the supply equation's standardized residuals are autocorrelated. Given that an 8-12 year cattle cycle has been identified previously, and that non-fed beef production is highly cyclical, it is difficult to estimate a monthly beef supply equation associated with a white-noise innovation series.

17/ The four-firm concentration ratio data for the beefpacking industry were provided by Bruce Marion.

References

- Antonovitz, F., and R. Green. "Alternative Estimates of Fed Beef Supply Response to Risk." Amer. J. Agr. Econ. 72(1990):475-87.
- Aradhyula, S.V., and M.T. Holt. "Risk Behavior and Rational Expectations in the U.S. Broiler Market." Amer. J. Agr. Econ. 71(1989):892-902.
- Baron, D.P. "Price Uncertainty, Utility, and Industry Equilibrium in Pure Competition." Int. Econ. Rev. 11(1970):463-80.
- Baillie, R.T., and R.J. Myers. "Bivariate GARCH Estimation of the Optimal Commodity Futures Hedge." J. Appl. Econometrics 6(1991):109-24.
- Behrman, J.R. Supply Response in Underdeveloped Agriculture: A Case Study of Four Major Annual Crops in Thailand, 1937-1963. Amsterdam: North-Holland Publishing Co., 1968.
- Bollerslev, T. "Generalized Autoregressive Conditional Heteroskedasticity." J. Econometrics 31(1986):307-27.
- _____. "Modelling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH Model." Rev. Econ. Statist. 72(1990):498-505.
- Bollerslev, T., R.F. Engle, and J.M. Wooldridge. "A Capital Asset Pricing Model with Time Varying Covariances." J. Polit. Econ. 96(1988):116-131.
- Brorsen, B.W., J.-P. Chavas, W.R. Grant, and L.D. Schnake. "Marketing Margins and Price Uncertainty: The Case of the U.S. Wheat Market." Amer. J. Agr. Econ. 67(1985):733-39.
- Brorsen, B.W., J.-P. Chavas, and W.R. Grant. "A Market Equilibrium Analysis of the Impact of Risk on the U.S. Rice Industry." Amer. J. Agr. Econ. 69(1987):733-739
- Chavas, J.-P., and M.T. Holt. "Acreage Decisions under Risk: The Case of Corn and Soybeans." Amer. J. Agr. Econ. 72(1990):529-38.

- Diebold, F.X., and P. Pauly. "Endogenous Risk in a Portfolio-Balance Rational-Expectations Model of the Deutschemark-Dollar Rate." Eur. Econ. Rev. 32(1988):27-53.
- Engle, R.F. "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of UK Inflation." Econometrica 50(1982):987-1008.
- Engle, R.F., D.M. Lilien, and R.P. Robins. "Estimating Time Varying Risk Premia in the Term Structure: The ARCH-M Model." Econometrica 55(1987):391-407.
- Gardner, B.L. "The Farm-Retail Price Spread in a Competitive Food Industry." Amer. J. Agr. Econ. 57(1975):399-409.
- Hayenga, M.L., and L.F. Schrader. "Formula Pricing in Five Commodity Marketing Systems." Amer. J. Agr. Econ. 62(1980):753-59.
- Heien, D. "Markup Pricing in a Dynamic Model of the Food Industry." Amer. J. Agr. Econ. 62(1980):10-18.
- Hoffman, D.L. "Two-Step and Related Estimators in Contemporary Rational-Expectations Models: An Analysis of Small Sample Properties." J. Bus. and Econ. Statist. 9(1991):51-61.
- Jarvis, L. "Cattle as Capital Goods and Ranchers as Portfolio Managers: An Application to the Argentine Cattle Sector." J. Polit. Econ. 82(1974):489-520.
- Johnson, D.G., et al. Competitive Issues in the U.S. Beef Sector: Can Beef Compete in the 1990s. Minneapolis: Hubert Humphrey Institute of Public Affairs, University of Minnesota, 1989.
- Just, R.E. "An Investigation of the Importance of Risk in Farmers' Decisions." Amer. J. Agr. Econ. 56(1974):14-25.
- Ljung, G.M., and G.E.P. Box. "On a Measure of Lack of Fit in Time-Series Models." Biometrika 65(1978):297-303.
- Pagan, A.R., and H.C.L. Sabau. "Consistency Tests for Heteroskedastic and Risk Models." Australian National University working paper, 1987.

- Pagan, A., and A. Ullah. "The Econometric Analysis of Models with Risk Terms." J. Appl. Econometrics 3(1988):87-105.
- Phillips. P.C.B., and P. Perron. "Testing for a Unit Root in Time Series Regressions." Biometrika 75(1988):335-46.
- Pope, R. "Empirical Estimation and Use of Risk Preferences: An Appraisal of Estimation Methods that Use Actual Economic Data." Amer. J. Agr. Econ. 64(1982):376-83
- Sandmo, A. "On the Theory of the Competitive Firm Under Price Uncertainty." Amer. Econ. Rev. 61(1971):64-73
- Schroeter, J.R. "Estimating the Degree of Market Power in the Beef Packing Industry." Rev. Econ. Statist. 70(1988):158-62.
- Schroeter, J., and A. Azzam. "Marketing Margins, Market Power, and Price Uncertainty." Amer. J. Agr. Econ. 73(1991):990-99.
- U.S. Department of Agriculture. Livestock and Meat Statistics. Washington D.C., various issues.
- U.S. Department of Commerce, Bureau of Economic Analysis. Survey of Current Business. Washington, D.C., various issues.
- U.S. Department of Labor. Employment and Earnings of the United States. Washington D.C., various issues.
- Ward, C.E. Meatpacking Competition and Pricing. Blaksburg VA: Research Institute on Livestock Pricing, 1988.
- White, F.T. Jr., et al. Choice Beef Prices and Price Spread Series: Methodology and Revisions. U.S. Department of Agriculture, ERS, Staff Report AGES 9106, February, 1991.
- White, H. "A Heteroskedasticity Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity," Econometrica 48(1980), 817-38.
- Wohlgenant, M.K. "Demand for Farm Output in a Complete System of Demand Functions." Amer. J. of Agr. Econ. 71(1989):241-52.

Table 1. Phillips-Perron Tests for Unit Roots in Beef Model Data.

$$Y_t = \hat{\alpha}Y_{t-1} + \hat{e}_t$$

$$Y_t = \mu^* + \alpha^*Y_{t-1} + e_t^*$$

$$Y_t = \bar{\mu} + \bar{\beta}(t - T/2) + \bar{\alpha}Y_{t-1} + \bar{e}_t$$

	Statistic					
	$Z(\tau_{\hat{\alpha}})$	$Z(\tau_{\alpha^*})$	$Z(\phi_1)$	$Z(\tau_{\bar{\alpha}})$	$Z(\phi_2)$	$Z(\phi_3)$
Null Hypothesis:	$\hat{\alpha} = 1$	$\alpha^* = 1$	$\alpha^* = 1;$ $\mu^* = 0$	$\bar{\alpha} = 1$	$\bar{\alpha} = 1;$ $\bar{\beta} = 0$	$\bar{\alpha} = 1;$ $\bar{\beta} = \bar{\mu} = 0$
RPB _t	-0.48	-1.36	0.92	-3.08	-3.39	4.00
M _t	-0.54	-5.49*	15.02*	-5.83*	-11.38*	11.61*
QB _t	-0.25	-12.16*	56.59*	-12.36*	-51.12*	70.07*
RPP _t	-0.46	-1.71	1.33	-3.04	3.30	4.02
INC _t	4.48*	0.51	110.72*	-1.38	5.63	5.37*
PE _t	0.04	-1.42	0.81	-0.70	1.23	1.32
WR _t	-2.28*	0.88	3.73	-1.94	5.17	4.85
PCO _t	-0.90	-1.49	1.13	-2.54	2.28	2.72
95% cv's	-1.95	-2.86	4.59	-3.41	6.25	4.68

Note: An asterisk denotes significance at the 95% level. 95% cv's are tabled critical values at the 95% level.

Table 2. Maximum Likelihood Estimates of a Rational-Expectations Multivariate GARCH-M Model of the U.S. Beef Sector, 1971-89.

Equation	Parameter	Variable	Coefficient	Standard Error
Retail Demand: ^{a/}	γ_0	constant	-0.267	0.087
	γ_1	ΔQB_t	-0.378	0.096
	γ_2	ΔQB_{t-1}	-0.393	0.108
	γ_3	ΔQB_{t-2}	-0.271	0.079
	γ_4	ΔRPP_t	0.137	0.049
	γ_5	ΔINC_t	0.018	0.008
	γ_6	ΔINC_{t-1}	0.013	0.009
	γ_7	ΔINC_{t-2}	0.015	0.004
	γ_8	ΔINC_{t-3}	0.017	0.009
	γ_9	$SIN1_t$	-0.065	0.097
	γ_{10}	$COS1_t$	0.157	0.100
	γ_{11}	$SIN2_t$	-0.062	0.108
	γ_{12}	$COS2_t$	-0.305	0.128
	$\gamma_{13} - \gamma_{17}$	$\Sigma \Delta RPB_{t-j}$	0.173	
Farm-Retail Margin:	θ_0	constant	8.038	2.861
	δ_1	σ_{1t}	1.529	0.565
	θ_1	QB_t	0.032	0.082
	θ_2	PE_t	2.044	0.520
	θ_3	WR_t	1.878	0.532
	θ_4	SIN_{1t}	0.288	0.151
	θ_5	COS_{1t}	0.117	0.156
	θ_6	SIN_{2t}	-0.184	0.174

Table 2. (Continued).

Equation	Parameter	Variable	Coefficient	Standard Error
	θ_7	COS_{2t}	0.192	0.169
	$\theta_8 - \theta_{14}$	ΣMRG_{t-j}	0.321	
Commercial Production:	ν_0	constant	14.266	2.315
	ν_1	FPB_t	-0.123	0.176
	δ_2	$\sqrt{h_{22t}}$	0.602	0.158
	ν_2	PCO_t	0.862	0.298
	ν_3	OFD_{t-1}	0.398	0.114
	ν_4	SIN1_t	-0.119	0.066
	ν_5	COS1_t	-0.099	0.060
	ν_6	SIN2_t	-0.628	0.112
	ν_7	COS2_t	-0.337	0.095
	$\nu_8 - \nu_{19}$	ΣQB_{t-j}	0.306	
Retail Price Variance:	ω_1	constant	0.049	0.027
	α_{11}	ϵ_{1t-1}^2	0.134	0.043
	β_{11}	h_{1t-1}	0.832	0.046
Margin Variance:	ω_2	constant	0.057	0.046
	α_{21}	ϵ_{2t-1}^2	0.066	0.021
	β_{21}	h_{2t-1}	0.918	0.024
Production Variance:	ω_3	constant	0.031	0.019
	α_{31}	ϵ_{3t-1}^2	0.163	0.070
	β_{31}	h_{3t-1}	0.779	0.079

Table 2. (Continued).

Equation	Parameter	Variable	Coefficient	Standard Error
Conditional Correlations:	ρ_{12}	constant	-0.634	0.053
	ρ_{13}	constant	0.357	0.091
	ρ_{23}	constant	-0.312	0.108
Log Likelihood:			-1052.055	

Note: For retail demand, $\Sigma \Delta RP B_{t-j}$ denotes the sum of the estimated coefficients on (differenced) retail beef prices at lags 1-3 and 10-11. For the margin equation, ΣMRG_{t-j} denotes the sum of the estimated coefficients on observed farm-retail margins at lags 1-4, 6, and 11-12. For the production equation, ΣQB_{t-j} denotes the sum of the estimated coefficients on commercial beef production at lags 1-12.

a/ The squared simple correlations between actual and simulated one-step-ahead predictions of retail and farm beef prices (in levels), the (actual) farm-retail price spread, and commercial beef production are 0.983, 0.919, 0.638, and 0.777, respectively.

Table 3. Key Short-Run Elasticities and Flexibilities.

Equation	Variable	Elasticity/ Flexibility
Retail Demand:	QB_t	-0.086
	RPP_t	0.101
	INC_t	0.171
Farm-Retail Margin:	σ_{1t}	0.064
	QB_t	0.019
	PE_t	0.177
	WR_t	0.181
Commercial Production:	FPB_t	-0.333
	$\sqrt{h_{22t}}$	0.076
	PCO_t	0.047
	OFD_{t-1}	0.158

Note: All elasticities and flexibilities are evaluated at the sample means.

Table 4. Diagnostic Tests for the Estimated Multivariate GARCH-M Model.

Statistic	RPB_t	MRG_t	QB_t
Residual Skewness and Kurtosis:			
m_3	0.055	-0.411	-0.002
m_4	3.242	4.128	0.032
Ljung-Box Q Tests:			
$Q(24)$	32.185	24.089	69.850
$Q^2(24)$	55.700	35.285	20.388
T-Statistics for Pagan-Sabau Consistency Tests:			
RPB_t	0.552 (0.581)	-	-
MRG_t	0.939 (0.348)	0.165 (0.837)	-
QB_t	0.807 (0.421)	1.475 (0.142)	0.452 (0.652)

Note: The statistics m_3 and m_4 denote the standardized residual skewness and kurtosis. $Q(24)$ and $Q^2(24)$ denote Ljung-Box statistics for up to 24'th order serial correlation in the standardized residuals and squared standardized residuals, respectively. T-statistics for Pagan-Sabau consistency tests were obtained by using White's correction for heteroskedasticity. Asymptotic p-values are in parentheses.

Table 5. Average Monthly Simulated Market Equilibrium Impacts of Risk on the Beef Market by Year, 1971-89.

Year	RPB _t			FPB _t			QB _t		
	Actual	Siml.	Percent	Actual	Siml.	Percent	Actual	Siml.	Percent
1971	87.51	87.48	-0.04	58.26	58.70	0.74	1808	1632	-9.69
1972	93.06	93.74	0.75	60.62	63.43	4.77	1852	1629	-11.97
1973	104.94	104.91	-0.02	71.23	74.30	4.74	1757	1566	-10.72
1974	97.47	99.01	1.65	62.23	67.63	9.10	1904	1637	-13.95
1975	94.27	94.70	0.47	61.55	63.73	3.88	1973	1618	-17.88
1976	85.43	87.36	2.23	49.56	55.42	11.88	2139	1781	-16.67
1977	80.27	81.04	0.96	47.31	49.60	4.96	2082	1854	-10.91
1978	91.36	91.27	-0.11	57.08	59.50	4.27	2001	1746	-12.71
1979	102.31	102.48	0.19	65.19	69.76	7.12	1772	1580	-10.63
1980	94.61	95.54	0.98	59.00	62.00	5.16	1789	1599	-10.56
1981	86.13	87.12	1.13	51.10	54.17	6.08	1855	1688	-9.00
1982	82.41	82.97	0.70	48.82	50.77	4.16	1864	1724	-7.49
1983	78.42	79.09	0.86	45.86	48.03	4.83	1922	1792	-6.64
1984	75.63	76.01	0.50	45.24	46.40	2.61	1952	1824	-6.49
1985	70.92	71.46	0.75	39.54	41.84	6.02	1963	1877	-4.31
1986	69.02	69.63	0.89	38.03	41.00	7.90	2018	1860	-7.70
1987	69.98	70.15	0.25	40.72	42.22	3.77	1950	1853	-4.97
1988	70.61	70.78	0.24	41.82	43.39	3.81	1952	1842	-5.59
1989	71.49	71.68	0.28	42.41	44.29	4.52	1915	1831	-4.32
Avg.	84.52	85.08	0.67	51.87	54.54	5.28	1919	1733	-9.59

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Note: Percent denotes the average percentage increase (decrease) in the simulated value relative to the respective observed value.

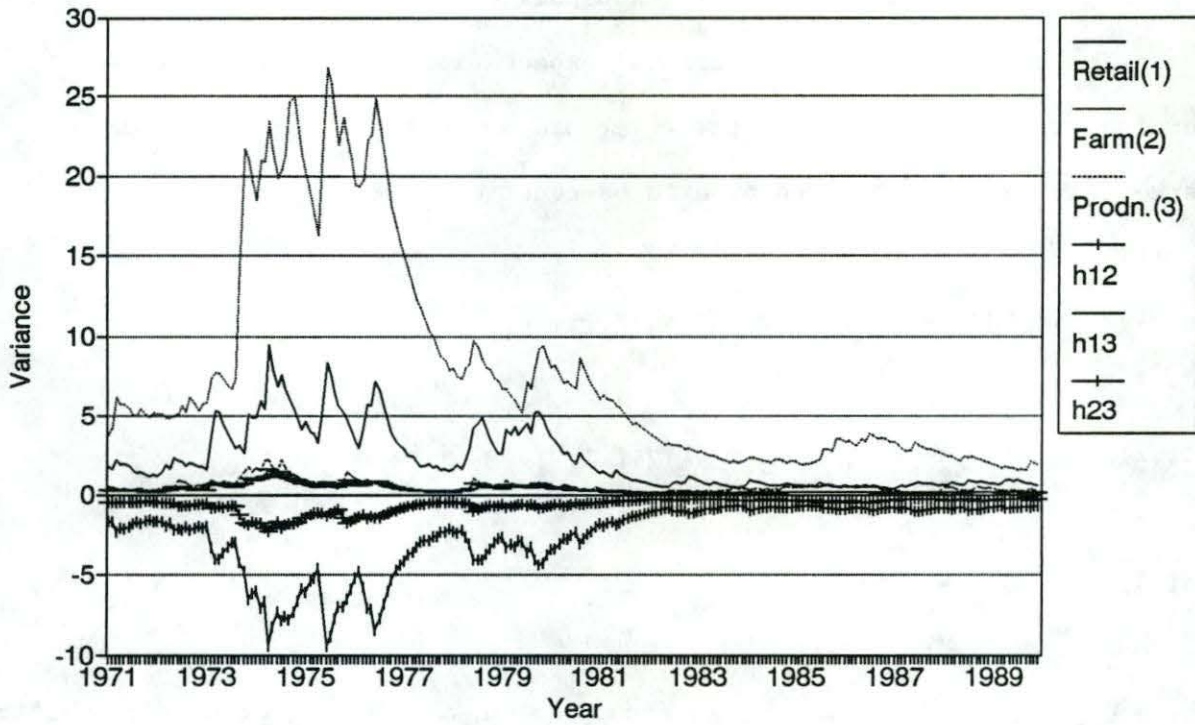


Figure 1. Time-Varying Conditional Variance Structure for the Estimated GARCH-M Model, 1971-89.

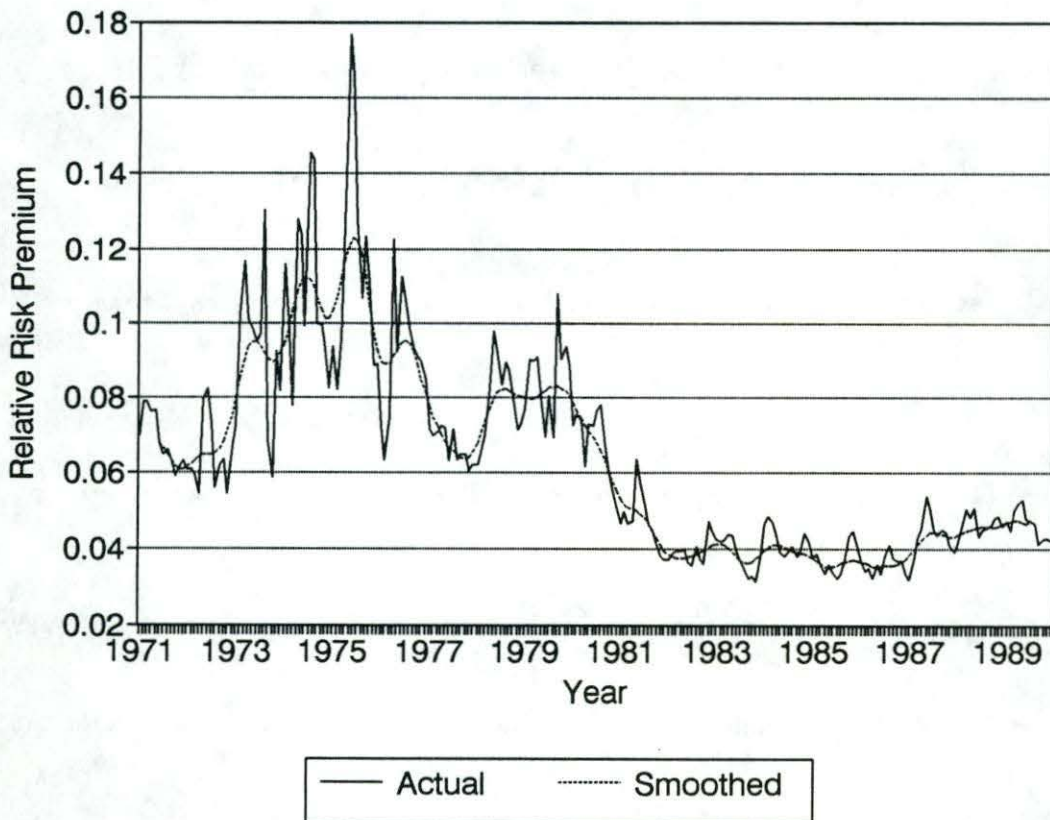


Figure 2. Time-Varying Relative Risk Premium in the Farm-Retail Margin.

Appendix

The derivation of the rational-expectations equilibrium for a stylized version of the market model presented in the text follows. Assume the beef market can be characterized by a three-equation model as

$$(a1.1) \quad RPB_t = \gamma_0 + \gamma_1 QB_t + \gamma_2 X_{1t} + \epsilon_{1t}$$

$$(a1.2) \quad RPB_t^e - FPB_t = \theta_0 + \delta_1 \sigma_{1t} + \theta_1 QB_t + \theta_2 X_{2t} + \epsilon_{2t}$$

$$(a1.3) \quad QB_t = \nu_0 + \nu_1 FPB_t + \delta_2 \sqrt{h_{22t}} + \nu_2 X_{3t} + \epsilon_{3t},$$

where X_{1t} , X_{2t} , and X_{3t} denote exogenous demand, margin, and production shifters, respectively, and all remaining variables are as defined in the text. Solving the preceding three-equation system for the reduced form of RPB_t yields

$$(a2) \quad RPB_t = \gamma_0 + \gamma_1 (1 + \nu_1 \theta_1)^{-1} (\nu_0 + \nu_1 [RPB_t^e - \theta_0 - \delta_1 \sigma_{1t} - \theta_2 X_{2t}] + \delta_2 \sqrt{h_{22t}} + \nu_2 X_{3t}) + \gamma_2 X_{1t} + \epsilon_{1t} + \gamma_1 (1 + \nu_1 \theta_1)^{-1} (\epsilon_{3t} - \nu_1 \epsilon_{2t}),$$

and taking expectations of RPB_t , conditional on information available at $t-1$, ψ_{t-1} , gives

$$(a3) \quad RPB_t^e = [1 - \gamma_1 \nu_1 / (1 + \nu_1 \theta_1)]^{-1} (\gamma_0 + \gamma_1 (1 + \nu_1 \theta_1)^{-1} [\nu_0 - \nu_1 (\theta_0 + \delta_1 \sigma_{1t} + \theta_2 X_{2t}^e) + \delta_2 \sqrt{h_{22t}} + \nu_2 X_{3t}^e] + \gamma_2 X_{1t}^e),$$

where $RPB_t^e = E(RPB_t | \psi_{t-1})$, and X_{1t}^e , X_{2t}^e , and X_{3t}^e are conditional expectations

of exogenous variables.

In the present case (a.3) is not the final-form for the rational-expectation of retail price. This is because the rational-expectation of the standard error of retail price, σ_{1t} , has not been specified. To that end, evaluate $RPB_t - RPB_t^e$, the rational-expectation forecast error, to obtain

$$(a4) \quad RPB_t - RPB_t^e = \epsilon_{1t} + \gamma_1(1 + \nu_1\theta_1)^{-1}(\epsilon_{3t} - \nu_1\epsilon_{2t}),$$

where forecast error variances for exogenous variables have been omitted. (It is, of course, preferable to include conditional variances and covariances of exogenous variables in (a4). As Diebold and Pauly indicate (p. 37), however, including auxiliary autoregressions for exogenous variables, along with the associated variance-covariance structure, in a GARCH-M rational-expectations model of the type considered here is, at present, not feasible.)

Taking the conditional expectation of the square of (a.4) gives

$$(a5) \quad \sigma_{1t}^2 = h_{11t} + [\gamma_1/(1 + \nu_1\theta_1)]^{-2}[\nu_1^2 h_{22t} + h_{33t}] \\ - 2\gamma_1(1 + \nu_1\theta_1)^{-1}[\gamma_1 h_{12t} - h_{13t} + \gamma_1\nu_1 h_{23t}]$$

and taking the square root of (a5) (and assuming first-order certainty equivalence), the rational-expectation for the forecast standard error of retail price obtains. Substituting (a5) into (a3) yields

$$(a6) \quad RPB_t^e = [1 - \gamma_1\nu_1/(1 + \nu_1\theta_1)]^{-1}(\gamma_0 + \gamma_1(1 + \nu_1\theta_1)^{-1}[\nu_0 + \delta_2 \sqrt{h_{22t}}] \\ - \nu_1[\theta_0 + \delta_1(h_{11t} + [\gamma_1/(1 + \nu_1\theta_1)]^{-2}[\nu_1^2 h_{22t} + h_{33t}])]$$

$$\begin{aligned}
& - 2\gamma_1(1 + \nu_1\theta_1)^{-1}[\gamma_1 h_{12t} - h_{13t} + \gamma_1\nu_1 h_{23t}]^{1/2} + \theta_2 x_{2t}^e) \\
& + \nu_2 x_{3t}^e] + \gamma_2 x_{1t}^e),
\end{aligned}$$

an expression for the final form of RPB_t^e . Substituting (a6) and the positive square root of (a5) into margin equation (a1.2) for, respectively, RPB_t^e and σ_{1t} , yields the estimable model form. Because conditional variance and covariance terms are arguments in (a1.3), (a5), and (a6), the resulting model is a type of GARCH-M model under rational expectations.