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ENDOGENOUS RISK IN A RATIONAL-EXPECTATION MODEL OF THE U.S. BROILER MARKET: A MULTIVARIATE ARCH-M APPROACH

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I. Introduction

In recent years the rational expectations hypothesis (REH) has emerged as an important alternative for modelling price expectations in commodity markets. Indeed, Goodwin and Sheffrin (1982), Shonkwiler and Maddala (1985), Holt and Johnson (1989), and others have presented compelling evidence that the REH is often superior to more traditional extrapolative methods for generating expectations in commodity models. More recently, attempts have been made to extend the rational expectations approach to more general settings. One such extension, based on the hypothesis that agents are risk averse, expands the REH to include endogenous risk.

One problem with using the REH to infer risk response lies in obtaining time-varying measures for conditional expectations of variances and covariances of exogenous variables. Diebold and Pauly (1988) and Aradhyula and Holt (1989) used univariate ARCH (Engle, 1982) and GARCH (Bollerslev, 1986) models to estimate time-varying conditional variances.[1/] By virtue of the rational expectations restrictions, the resulting estimable form is similar to the ARCH-in-mean (ARCH-M) model proposed originally by Engle, Lilien, and Robins (1987) since both conditional means and variances enter the structural equations.

While previous research has illustrated the potential for modelling risk effects under the REH using an ARCH (GARCH) approach, several problems remain. To begin, neither Diebold and Pauly (1988) or Aradhyula and Holt (1989) imposed cross-equation constraints between the structural model's reduced form and the autoregressions for exogenous variables, arguing that estimation of a multivariate ARCH-M model with rational expectations was infeasible. Consequently, due to the "generated regressors" problem, the empirical results of both studies are somewhat clouded (Pagan, 1984). Of equal importance is that the univariate ARCH and GARCH processes employed in these studies do not allow for time-varying conditional covariances among exogenous variables, a potentially important consideration in rational-expectations models with risk terms.

This paper examines the feasibility of modelling risk response under the REH using a multivariate ARCH-M approach. Unlike previous research, a nonlinear maximum likelihood estimator (MLE) is used to estimate parameters of the auxiliary autoregressions and the time-varying conditional covariance matrix simultaneously with those of the structural equations. A direct implication is that inference problems associated with standard two-step estimators are avoided--a problem infrequently dealt with in applied work.[2/] Furthermore, the multivariate ARCH-M approach allows for time-varying variances and covariances in the solution of the rational-expectations model, something not accounted for previously. Finally, although similar setups have been alluded to by Engle and Bollerslev (1986) and Baillie (1989), this study represents the first known attempt to actually estimate a rational-expectations model with endogenous risk using a multivariate ARCH-M framework.

In section II, a general framework for modelling rational expectations of first and second moments of endogenous variables is developed. In section III, we discuss the methods used to implement the model, focusing on the multivariate ARCH model with constant conditional correlations proposed by Cecchetti, Cumby, and Figlewski (1988) and Bollerslev (1990). In section IV a market model for broilers is specified that includes rational expectations of first and second moments of price in the supply equation. We focus on broilers because previous research has found strong evidence in favor of the REH in this market (e.g., Huntzinger, 1979; Goodwin and Sheffrin, 1982), and because price risk also appears important in supply decisions (Aradhyula and Holt, 1989). In section V empirical results are presented and assessed. Section VI concludes the study.

II. General Framework

The general framework used to assess the implications of risk in a rational expectations setup closely parallels the linear rational-expectations models considered by Wallis (1980), Mishkin (1983), and others in that parameters of the original structure are estimated simultaneously with those of the auxiliary autoregressions for exogenous variables. Our model is more general, however, in that rational expectations of forecast error variances and covariances also enter the specification.

Consider a "classical" static market model consisting of G equations where agents form expectations about means, variances, and, possibly, covariances of J endogenous variables $(G \geq J)$:

$$B\underline{y}_t + A_1\underline{y}_t^e + A_2\text{vech}(y_t^v) + \Gamma_1\underline{x}_{1t} + \Gamma_2\underline{x}_{2t} = \underline{u}_{1t}. \quad (1)$$

Here B, A_1, and y_t^v are (G x G) parameter matrices; A_2 is a G x G(G + 1)/2 parameter matrix; Γ_1 and Γ_2 are respectively (G x K_1) and (G x K_2) parameter matrices; $\underline{y}_t$ and $\underline{y}_t^e$ are (G x 1) vectors; $\underline{x}_{1t}$ is a K_1-dimensional vector of exogenous and predetermined variables whose one-period-ahead values are known with certainty; $\underline{x}_{2t}$ is a K_2-dimensional vector of exogenous variables whose values in period t are not known at time t − 1; and vech(·) is the vectorization operator.[3/] Also, in the spirit of the conditional heteroskedasticity model, let $\underline{u}_{1t}$ denote a (G x 1) vector of joint normally distributed error terms where $E(\underline{u}_{1t}|\Omega_{t-1}) = \underline{0}$ and $\text{var}(\underline{u}_{1t}|\Omega_{t-1}) = \psi_{1t}$. Here Ω_{t-1} is the σ-field (information set) generated by all available information through period t − 1 and ψ_{1t} is the (G x G) (possibly) time-varying positive definite conditional covariance matrix.

In model (1) vector $\underline{y}_t^e$ denotes unobservable expectations, formed in period t − 1, about unknown values of J endogenous variables, and y_t^v denotes unobservable expectations, also formed in period t − 1, about forecast error variances and covariances of J endogenous variables. The model in (1) is closed by assuming agents form expectations about $\underline{y}_t^e$ and y_t^v in accordance with the REH. In other words, expectations formed by agents about means, variances, and covariances of relevant endogenous variables are consistent with the underlying model structure depicted in (1). Thus, expectations of mean vector $\underline{y}_t^e$ and covariance matrix y_t^v, conditional on Ω_{t-1}, are given by

$$\underline{y}_t^e = E_{t-1}(\underline{y}_t|\Omega_{t-1}), \tag{2}$$

$$y_t^v = E_{t-1}[(y_t - E_{t-1}(y_t|\Omega_{t-1}))(y_t - E_{t-1}(y_t|\Omega_{t-1}))']. \tag{3}$$

The econometric implications of (2) and (3) are deduced as follows. The standard reduced form of (1) is

$$\underline{y}_t = -B^{-1}A_1\underline{y}_t^e - B^{-1}A_2\text{vech}(y_t^v) - B^{-1}\Gamma_1\underline{x}_{1t} - B^{-1}\Gamma_2\underline{x}_{2t}^e + B^{-1}\underline{u}_{1t}, \tag{4}$$

and taking conditional expectations of (4) yields

$$\underline{y}_t^e = -B^{-1}A_1\underline{y}_t^e - B^{-1}A_2\text{vech}(y_t^v) - B^{-1}\Gamma_1\underline{x}_{1t} - B^{-1}\Gamma_2\underline{x}_{2t}^e. \tag{5}$$

Here $\underline{x}_{2t}^e$ represents the expectation vector of exogenous variables whose values are "unknown" at time t − 1. Note that equation (5) yields only a partial reduced form for $\underline{y}_t^e$ since rational expectations of forecast error variances and covariances also appear on the right-hand side. The model can be closed only by obtaining an expression for the rational expectation of y_t^v.

To complete the model specification, it is necessary to specify how expectations about $\underline{x}_{2t}$ are formed. We assume $\underline{x}_{2t}$ follows a vector autoregressive (VAR) process of the form[4/]

$$\Phi(L)\underline{x}_{2t} = \underline{u}_{2t}, \tag{6}$$

where $\Phi(L)$ is a polynomial in the lag operator L of order p such that $\Phi(L) = I + \Phi_1 L + \ldots + \Phi_p L^p$ and all roots of $|\Phi(L)| = 0$ lie outside the unit circle. Properties of error vector $\underline{u}_{2t}$ include $E(\underline{u}_{2t}|\Omega_{t-1}) = \underline{0}$ and $\text{var}(\underline{u}_{2t}|\Omega_{t-1}) = \psi_{2t}$, ψ_{2t} being a $(K_2 \times K_2)$ (possibly) time-varying positive definite covariance matrix. Also, $\underline{u}_{2t}$ is possibly correlated with $\underline{u}_{1t}$, implying $E(\underline{u}_{1t}\underline{u}_{2t}'|\Omega_{t-1}) = \psi_{12t}$, a $(G \times K_2)$ time-varying matrix of conditional covariances. Letting $\Psi(L) = -\Phi_1 L - \ldots - \Phi_p L^p$, the conditional expectation of $\underline{x}_{2t}$ is then

$$\underline{x}_{2t}^e = E_{t-1}(\underline{x}_{2t}|\Omega_{t-1}) = \Psi(L)\underline{x}_{2t} = -\Phi_1\underline{x}_{2t-1} - \ldots - \Phi_p\underline{x}_{2t-p}. \tag{7}$$

To obtain the final form for $\underline{y}_t^e$, we derive the rational expectations counterpart for y_t^v. The error in the rational expectation can be obtained by subtracting $\underline{y}_t^e$ in (5) from $\underline{y}_t$ in (4), giving:

$$\underline{y}_t - \underline{y}_t^e = -B^{-1}\Gamma_2(\underline{x}_{2t} - \underline{x}_{2t}^e) + B^{-1}\underline{u}_{1t} = -B^{-1}\Gamma_2\underline{u}_{2t} + B^{-1}\underline{u}_{1t}. \tag{8}$$

Taking the conditional expectation of the outer product of (8) gives

$$y_t^v = E_{t-1}[(\underline{y}_t - \underline{y}_t^e)(\underline{y}_t - \underline{y}_t^e)'|\Omega_{t-1}] = B^{-1}\psi_{1t}B^{-1'} + B^{-1}\Gamma_2\psi_{2t}\Gamma_2'B^{-1'}$$

$$- B^{-1}(\Gamma_2\psi_{12t}' + \psi_{12t}\Gamma_2')B^{-1'}. \quad (9)$$

Matrix equation (9) shows rational expectations of forecast error variances and covariances of endogenous variables are a function of model parameters and elements of the (possibly) time-varying covariance matrices, ψ_{1t} , ψ_{2t}, and ψ_{12t}.

The formulation in (9) differs from that considered by Diebold and Pauly (1988) and Aradhyula and Holt (1989) in several fundamental ways. For example, Diebold and Pauly (1988) assumed all elements in ψ_{2t} and ψ_{12t} were zero, although they did allow the (scalar) covariance matrix ψ_{1t} to vary over time. Aradhyula and Holt (1989) allowed diagonal elements of ψ_{2t} to be nonzero and time-varying, but constrained off-diagonal elements of ψ_{2t} and all elements of ψ_{12t} to zero. Constraining conditional covariances among exogenous variables (ψ_{2t}) and between endogenous and exogenous variables (ψ_{12t}) to zero may be especially problematic in a rational expectations context. This is because nonzero time-varying covariances can provide important information about overall forecast variances of endogenous variables.[5/] Moreover, if ψ_{2t} and ψ_{12t} are not zero, then the additional restrictions associated with (9) must be imposed in the estimation.

To derive the model's final form, first substitute matrix equation (9) for y_t^v and expression (7) for $\underline{x}_{2t}^e$ in equation (5). Assuming $(B + A_1)$ is nonsingular and letting $\Pi_{.1} = -(B + A_1)^{-1}$, the resulting final form expression for $\underline{y}_t^e$ is:

$$\underline{y}_t^e = \Pi_{.1}\Gamma_1\underline{x}_{1t} + \Pi_{.1}\Gamma_2\Psi(L)\underline{x}_{2t} + \Pi_{.1}A_2\text{vech}[B^{-1}\psi_{1t}B^{-1'} + B^{-1}\Gamma_2\psi_{2t}\Gamma_2'B^{-1'}$$

$$- B^{-1}(\Gamma_2\psi_{12t}' + \psi_{12t}\Gamma_2')B^{-1'}]. \quad (10)$$

Substitution of (9) and (10) for $\underline{y}_t^e$ and y_t^v, respectively, in the observable reduced form equations (4) yields the final-form relations for $\underline{y}_t$:

$$\underline{y}_t = \Pi_{.2}\underline{x}_{1t} + \Pi_{.3}\Psi(L)\underline{x}_{2t} + \Pi_{.4}\text{vech}[B^{-1}\psi_{1t}B^{-1'} + B^{-1}\Gamma_2\psi_{2t}\Gamma_2'B^{-1'}$$

$$- B^{-1}(\Gamma_2\psi_{12t}' + \psi_{12t}\Gamma_2')B^{-1'}] + \underline{v}_t, \tag{11}$$

where

$$\Pi_{.2} = -B^{-1}(A_1\Pi_{.1} + I)\Gamma_1, \qquad \Pi_{.3} = -B^{-1}(A_1\Pi_{.1} + I)\Gamma_2,$$

$$\Pi_{.4} = -B^{-1}(A_1\Pi_{.1} + I)A_2, \text{ and } \qquad \underline{v}_t = -B^{-1}(\Gamma_2\underline{u}_{2t} - \underline{u}_{1t}).$$

As is typical in rational-expectations models, the final-form equations depend on lagged values of exogenous variables, $\underline{x}_{2t}$. However, the final forms in (11) differ from those in previous rational-expectations models in that endogenous variables also depend on conditional variances and covariances of forecast errors associated with structural equations and auxiliary autoregressions. As indicated in (11), endogenizing risk in a rational-expectations model gives rise to additional nonlinear cross-equation and covariance restrictions which are imposed in the estimation.

To finalize the model specification, the process governing the time-varying behavior of covariance matrix

$$H_t = \begin{bmatrix} \psi_{1t} & \psi_{12t} \\ \psi_{12t}' & \psi_{2t} \end{bmatrix}$$

must be defined. The multivariate ARCH model is considered ideal for this purpose. An interesting feature of ARCH processes is they can be viewed as a parsimonious time-series analogue for modelling time-varying conditional variances and covariances, an analogue that parallels the autoregressive model in (6) generating expectations of exogenous variables. Furthermore, with an ARCH specification the final forms in (11) are a multivariate restricted reduced-form version of Engle et al.'s (1987) single-equation ARCH-M model.

III. Estimation Framework

An unrestricted ARCH specification for the covariance structure of even moderately sized models involves an unmanageable number of parameters. It is necessary, then, to consider parsimonious specifications of the covariance matrix H_t. Several simplifying parameterizations have been suggested, including the linear diagonal GARCH model in Bollerslev, Engle, and Wooldridge (1988); the latent factor ARCH model in Diebold and Nerlove (1989); the factor ARCH model in Engle, Ng, and Rothschild (1990); and the multivariate generalized ARCH model with constant conditional correlations considered first by Cecchetti, Cumby, and Figlewski (1988) and generalized by Bollerslev (1990). We focus on the latter method because it represents a major reduction in computational complexity.

Bollerslev's (1990) model allows for time-varying conditional variances and covariances, but assumes constant conditional correlations. Let h_{ijt} denote the ij^{th} element of H_t. Then the conditional correlation, evaluated at time t - 1, between the i^{th} and j^{th} elements of $\underline{y}_t^*$, y_{it}^* and y_{jt}^*, is defined in the usual way by $\rho_{ijt} = h_{ijt}(h_{iit}h_{jjt})^{-1/2}$ where $\rho_{ijt} \in [-1,1]$ for all t (to simplify notation, we define $N = G + K_2$ and $\underline{y}_t^* = (\underline{y}_t', \underline{x}_{2t}')'$). Although ρ_{ijt} can in general be time-varying, it may be useful to assume $\rho_{ijt} = \rho_{ij}$ for all t; i.e., that the conditional correlations are constant. It then follows that

$$h_{ijt} = \rho_{ij}(h_{iit}h_{jjt})^{1/2}, \quad j = 1,\ldots,N, \quad i = j+1,\ldots,N.$$

An appealing feature of the constant conditional correlations model is the simplified estimation and inference procedures. Specifically, write the conditional variances as $h_{iit} = \sigma_{it}^2 > 0$ for all i and t. Conditional covariance matrix H_t can then be partitioned as $H_t = D_t \varsigma D_t$ where D_t is an N x N stochastic diagonal matrix with elements $\sigma_{1t}, \ldots, \sigma_{Nt}$ and ς is an N x N time invariant symmetric positive definite matrix with typical element ρ_{ij}.

Assuming conditional normality, the log likelihood function for the rational-expectations model with time-varying covariance matrix H_t is

$$L(\underline{\theta}) = -\frac{TN}{2}(2\pi) - \frac{T}{2}\log|\varsigma| - \sum_{t=1}^{T}\log|D_t| - \frac{1}{2}\sum_{t=1}^{T}\tilde{\underline{u}}_t'\varsigma^{-1}\tilde{\underline{u}}_t, \quad (12)$$

where $\tilde{\underline{u}}_t = D_t^{-1}\underline{u}_t$ is a N x 1 vector of standardized residuals and $\underline{\theta}$ is a parameter vector. Although the likelihood function in (12) is still highly nonlinear in the parameters, only one N x N matrix inversion is called for during each function evaluation as compared to T inversions for the vector ARMA-type parameterizations.[6/] Also, $\log|D_t| = \sum_{i=1}^{N}\log\sigma_{it}$. See Bollerslev (1990) for additional details. We use the Davidon-Fletcher-Powell (DFP) algorithm along with numerical first derivatives in the maximization of (12).

IV. Model Specification

The framework outlined in previous sections is used to estimate a quarterly broiler model. Our starting point is a two-equation structural model similar in some respects to Goodwin and Sheffrin's (1982). The main differences are that the present model includes rational expectations of first <u>and</u> second moments of price in the supply equation, and the conditional covariance matrix is time-varying.

The structural portion of the model consists of two behavioral equations for broiler price and production. Broiler demand is specified in price-dependent form as

$$PC_t = a_0 + \sum_{j=1}^{3} a_j Qj_t + a_4 BD_t + a_5 PB_t + a_6 PP_t + a_7 PT_t + a_8 t + u_{1t}, \qquad (13)$$

where PC_t is the real wholesale price of broilers; Qj_t is a quarterly intercept shifter, j = 1,2,3; BD_t is the quantity of broilers consumed, ready to cook; PB_t is the real retail price of beef; PP_t is the real retail price of pork; PT_t is the real wholesale price of turkey; t is a linear time trend; and u_{1t} is a random error term. Wholesale broiler price is used because there is ample evidence to suggest that price determination occurs at the wholesale level (e.g., Goodwin and Sheffrin, 1982). Prices of other meats, including beef, pork, and turkey, are included because these items are likely substitutes for broilers. A trend term is also specified to capture potential omitted factors such as income, population growth, and changes in tastes and preferences. All prices are deflated by the wholesale price index (1967 = 100).

Broiler supply is specified as:

$$BP_t = b_0 + \sum_{j=1}^{3} b_j Qj_t + b_4 PC_t^e + b_5 PC_t^v + b_6 FC_{t-1} + b_7 t + b_8 BP_{t-1} + u_{2t}, \qquad (14)$$

where BP_t is broiler production; PC_t^e is the rational expectation of the real wholesale price of broilers in time t, viewed from period t − 1; PC_t^v is the rational expectation of the variance of real wholesale broiler price, also viewed at time t − 1; FC_{t-1} and t are, respectively, the lagged real price of broiler feed and a linear time trend; and u_{2t} is a random error.[7/]

Several observations are in order regarding the specification in (14).

First, there is approximately an eight-week production cycle for broilers. Thus on a quarterly basis, expectations formed in the previous period are relevant for explaining current-period production. Second, potential production response to price risk is captured by including the ex ante expectation of price variance, PC^{v}_{t}. Also, lagged feed costs reflect the major component of purchased input items in broiler production (Lasley, 1983). The broiler industry has also experienced substantial technological change. To account for this change, a linear time trend is included in (14). Finally, lagged production is included since producers may not adjust fully to desired output levels in the short run.

The model is closed by relating production to consumption with the identity

$$BP_t = BD_t + OD_t, \tag{15}$$

where OD_t denotes "other demand," the sum of net export demand and net stock demand. Other demand, OD_t, is subsequently treated as exogenous in the model.[8/]

To implement the rational-expectations model, it is necessary to specify how one-step-ahead predictions of conditional means and variances of exogenous variables PB_t, PP_t, PT_t, and OD_t are formed. These exogenous variables are represented by univariate autoregressive models of the form

$$\beta_i(L)X_{it} = \beta_{i0} + \beta_{i\tau}t + u_{it}, \quad i = 3,\ldots,6, \tag{16}$$

where $X_{3t} = PB_t$, $X_{4t} = PP_t$, $X_{5t} = PT_t$, $X_{6t} = OD_t$; $\beta_i(L)$ is a polynomial lag operator; and u_{it}, i=3,...,6, are random error terms. For purposes of solving for the rational price predictor in (10), mean expectations of exogenous variables are obtained from (16) in a manner consistent with (7).

All that remains is to specify the structure generating conditional variances and covariances. Preparatory analysis suggested the model's conditional variance structure could be reasonably represented by sixth-order ARCH processes, where

$$h_{iit} = \alpha_{i0} + \alpha_{i1}(1/21)\left[\sum_{j=1}^{6}(7-j)u_{it-j}^{2}\right],$$

$$h_{ijt} = \rho_{ij}(h_{iit}h_{jjt})^{1/2}, \quad i,j = 1,\ldots,6,\ i \neq j, \tag{17}$$

and where ρ_{ij} denotes the ij^{th} constant conditional correlation. This preliminary analysis also revealed that conditional variances associated with broiler production, BP_t, and price, PC_t, were time invariant. Hence, $h_{11t} = \alpha_{10}$ and $h_{22} = \alpha_{21}$ for all t. The implication is that the conditional covariance between broiler production and price is constant, but that remaining conditional covariances are time varying. The monotonically declining weight structure in (17) is similar to the lag structure used by Engle (1982), Engle et al. (1987), and Diebold and Nerlove (1989). The declining lag structure implies squared innovations from the distant past have a smaller effect on current conditional variances and covariances than do recent squared innovations.

The specifications in (13)-(17) provide a basis for imposing, estimating, and testing the REH with endogenous price risk in a multivariate ARCH-M model of the U.S. broiler industry. Assuming $\underline{u}_t \sim N(\underline{0}, H_t)$, $\{H_t\}_{ij} = h_{ijt}$, maximum likelihood estimates of the ARCH-M model's parameters can be obtained; these tes embody <u>all</u> restrictions implied by rational expectations of the first and second moments of price.

V. Estimation Results

A. Autoregressive Equations and ARCH Effects

The analysis begins with an investigation of the auxiliary autoregressions for exogenous variables in the rational-expectations model. Using quarterly data from 1960-1989, SUR estimates of the four-equation model in (16) with a homoskedastic error process were obtained for PB_t, PP_t, PT_t, and OD_t.[9/] Each variable was found to be well-represented by an AR(6) process. Results of the Ljung-Box portmanteau test for each residual series, reported in the second row of Table 1, indicate that with the possible exception of pork price, there does not appear to be significant serial correlation in the estimated residuals. However, the application of the McLeod-Li test to each series of squared residuals indicates that, again with the possible exception of pork, the assumption of homoskedasticity can be soundly rejected. This conclusion is also verified by the results of Lagrange Multiplier tests for up to sixth-order ARCH effects in each autoregressive equation (Table 1). Furthermore, the portmanteau statistics for cross-products of squared residuals indicate possible ARCH effects in the covariances as well.

A different picture emerges when the four-equation system is estimated using the multivariate ARCH process in (17). As indicated in the lower panel of Table 1—again with the exception of the retail price of pork—there appears to be little evidence of serial correlation in the residuals of the estimated equations in the multivariate ARCH model. Importantly, the McLeod-Li $Q^2(10)$ statistics associated with the normalized squared residuals and cross-products of normalized squared residuals for the multivariate ARCH model are generally smaller than for the homoskedastic model.

Additional evidence in support of the ARCH specification is obtained by testing for the absence of conditional heteroskedasticity. The Likelihood Ratio (LR) test statistic for homoskedasticity—that is, $\alpha_{31} = \ldots = \alpha_{61} = 0$—equals 64.024, which asymptotically under the null hypothesis is the realization of a $\chi^2(4)$ distribution. Thus, the hypothesis of no conditional heteroskedasticity in the innovations of the autoregressive models for PB_t, PP_t, PT_t and OD_t, is soundly rejected.

As a final check on the specification of the multivariate ARCH model with constant conditional correlations, a consistency test due to Pagan and Sabau (1987) was also computed. The Pagan-Sabau test essentially requires the time varying conditional variances to be unbiased predictors of the second moments of the associated residuals. The test involves estimating regressions of the type

$$\hat{u}_{it}\hat{u}_{jt} = \gamma_{ij0} + \gamma_{ij1}\hat{h}_{ijt}, \quad i,j = 1,\ldots,4.$$

Under the null hypothesis of model consistency, the OLS estimates of γ_{ij1} should not differ significantly from unity. The t-statistics for the consistency tests—obtained using White's (1980) correction for heteroskedasticity—are reported in Table 2. In each case, the asymptotic p-values exceed 15%, implying the null hypothesis of model consistency in the second moments cannot be rejected.

Overall, there does not appear to be any serious misspecifications in the ARCH component of the model. On balance the results suggest exogenous variables in the broiler model can be reasonably represented by sixth-order autoregressions, sixth-order ARCH models, and constant conditional correlations.

B. A Multivariate ARCH-M Model with Rational Expectations

Using the above specifications and quarterly data for 1960 through 1989, maximum likelihood estimates are obtained for the multivariate ARCH-M broiler model with rational expectations and risk effects in the supply equation. Rational predictors for first and second moments of broiler price are derived using the framework in section II. The result is <u>all</u> cross-equation and covariance restrictions implied by rationality are incorporated in the estimation. FIML estimates of structural and autoregressive equations (13)-(17) are reported in Table 3, and corresponding estimates of ARCH parameters and conditional correlation coefficients are reported in Table 4.[10]

Table 3 indicates all parameters associated with economic variables in the price and production equations have theoretically acceptable signs and are statistically significant. High simulation R^2's for both structural equations also suggest the model does a reasonable job of explaining historical movements in the broiler industry. In the demand equation, the implied own-price elasticity is -0.74, and implied elasticities with respect to PB_t, PP_t, and PT_t are 0.61, 0.28, and 0.25, respectively.[11] Hence, broiler demand is inelastic in the short-run and beef appears to be the most important substitute for broilers. Broiler consumption has also trended steadily upward over time.

Turning to the supply equation, the results are consistent with the hypothesis that broiler producers are risk averse. Specifically, coefficients associated with PB_t^e and PB_t^v are positive and negative, respectively, and both are significant at usual levels. Lagged feed cost, FC_{t-1}, also has a negative and significant effect on broiler production. The respective expected price and variance elasticities are 0.15 and -0.027, and the lagged feed cost elasticity is -0.022, all being well within the range of previous estimates (e.g., Chavas and Johnson, 1982; Aradhyula and Holt, 1989).

While less interpretation can be attached to the estimated autoregressive models in Table 3, each model provides a good fit to the data as indicated by the high R^2's. Each autoregression is also dynamically stable, with the modulus of the dominant root from the companion matrix for each univariate autoregression ranging from a low of 0.80 for turkey to a high of 0.96 for pork. Regarding the variance structure of the estimated model, note in Table 4 that all estimated α_{i1} parameters, i=3,...,6, in the ARCH conditional variance equations are significantly greater than zero. The unconditional variances are also well defined; i.e., $\alpha_{i1} < 1$, i=3,...,6.

Results in Table 4 also show that five estimated conditional correlation coefficients are significant at the 0.10 level. More important, however, the LR test statistic for $\rho_{ij} = 0$, $i \neq j$, equals 45.516, which is well above the value for the $\chi^2(15)$ distribution at any reasonable level. In addition, the assumptions used by Aradhyula and Holt (1989) that $\psi_{12t} = 0$ and all off-diagonal elements of ψ_{2t} equal zero can be tested by restricting $\rho_{ij} = 0$ for i = 1,...,6, j = 3,...,6. The resulting LR test statistic has a p-value well below 0.001 for the $\chi^2(14)$ distribution. Additional LR test statistics of the independent hypotheses that off-diagonal elements of $\psi_{2t} = 0$ or $\psi_{12t} = 0$ are also well above the 95% fractiles for the appropriate χ^2 distributions. On balance, the evidence confirms that conditional covariances are nonzero and are time-varying, a possibility not allowed for in previous rational-expectations models with risk.

C. Further Model Validation

To determine how the multivariate ARCH-M model with rational expectations compares to more traditional approaches for modelling ex ante price and variance expectations, several additional tests and comparisons were performed. To begin, an alternative method is used to generate ex ante means and variances of broiler price. Specifically, price and variance expectations are generated by fixed weighted moving average processes of the type

$$PB_t^e = \sum_{i=1}^{6} [(7-i)PB_{t-i}]/21,$$

$$PB_t^v = \sum_{i=1}^{6} [(7-i)(PB_{t-i} - PB_{t-i}^e)^2]/21. \tag{18}$$

Extrapolative parameterizations similar to (18) have been used extensively to investigate effects of risk in agricultural supply analysis. See, for exapmple, Lin (1977), Brorsen, Chavas, and Grant (1987), or Chavas and Holt (1990).

The estimated log likelihood for the weighted moving average model is -1478.28, which compares with -1470.93 for the rational-expectations model. The ARCH-M model with rational expectations provides a better fit to the data than the less informationally efficient alternative. This result has practical significance since, for example, price and risk supply elasticities for the extrapolative predictors model are 0.074 and -0.004, respectively, estimates that are correspondingly 50% and 83% smaller in absolute terms than those of the rational-expectations model (Table 3).

A mechanical nesting procedure is also used to allow the data to discriminate between the two methods. The procedure is to estimate an augmented model using expectations of the form $PC_t^e = \lambda_1 PC_{\alpha t}^e + (1 - \lambda_1)PC_{\beta t}^e$ and $PC_t^v = \lambda_2 PC_{\alpha t}^v + (1 - \lambda_2)PC_{\beta t}^v$, where "$\alpha$" denotes rational expectations, "β" denotes extrapolative expectations, and λ_1 and λ_2 are "mixing" parameters. Note that the above specifications for price and variance expectations nests the rational-expectations model. The point estimates for λ_1 and λ_2 are 0.722 and 0.947, respectively. The LR test statistic for $\lambda_1 = \lambda_2 = 1$ is 3.14, which has an asymptotic p-value of 0.79. These results provide strong evidence that broiler producers form price and risk expectations in a manner more consistent with the REH than with extrapolative methods.

As a final check on the validity of the ARCH-M rational-expectations model, we estimated a regression of the form

$$PC_t = \delta_0 + \delta_1 PC^e_{\alpha t} + \delta_2 PC^e_{\beta t} + e_t. \tag{20}$$

If the rational expectations approach is valid, then $\delta_0 = \delta_2 = 0$ and $\delta_1 = 1$. Results obtained using White's (1980) correction for heteroskedasticity are reported in the first column of Table 5. Extrapolative expectations apparently add no predictive power relative to the rationally expected price; however, while the predictive power of the rationally expected price is superior, this does not guarantee that producers actually respond to rationally determined prices. To explore this issue, the regression in (20) was repeated using predicted supplies from the alternative models. Results reported in the second column of Table 5 verify the superior predictive capability of the supply equation with rational expectations. Overall, the predictive performance tests provide further compelling evidence in favor of the multivariate ARCH-M model with rational expectations and endogenous risk.

VI Conclusions

This paper extends Wallis's (1980) linear rational expectations framework to include variance and covariance terms. The resulting model is a type of restricted multivariate ARCH-M model where the ARCH-in-mean effects are due to the rational expectations restrictions. The multivariate ARCH-M framework was used to obtain FIML estimates of a quarterly model of the U.S. broiler industry that included rational expectations of price and price variance in the supply equation. The time-varying conditional covariance structure of the model was specified using Bollerslev's (1990) constant conditional correlations setup. And to avoid the generated regressors problem, structural equations and auxiliary autoregressions were estimated simultaneously.

The empirical results were encouraging, indicating, among other things, that broiler producers do apparently react adversely to price risk. The FIML estimates of the multivariate ARCH-M rational-expectations model also provided a good fit to the data. Several comparisons between the rational-expectations model and a model based on extrapolative price and variance predictors were also made. Specifically, the extrapolative predictors are of the type frequently used to investigate risk effects in agricultural supply analysis. The collective evidence indicates the rational expectations paradigm is clearly superior to the extrapolative approach in the broiler market. Consequently, it may prove useful to model price and risk expectations in other markets using a multivariate ARCH-M rational expectations approach.

References

Aradhyula, Satheesh V., and Matthew T. Holt, "Risk Behavior and Rational Expectations in the U.S. Broiler Market," American Journal of Agricultural Economics 79 (Nov. 1989), 892-902.

Baillie, Richard T., "Econometric Tests of Rationality and Market Efficiency," Econometric Reviews 8 (1989), 151-186.

Bollerslev, Tim, "Generalized Autoregressive Conditional Heteroskedasticity," Journal of Econometrics 31 (June 1986), 307-327.

_______, "Modelling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH Model," Review of Economics and Statistics 72 (August 1990), 498-505.

Bollerslev, Tim, Robert F. Engle, and Jeffrey M. Wooldridge, "A Capital Asset Pricing Model with Time Varying Covariances," Journal of Political Economy 96 (Feb. 1988), 116-131.

Brorsen, B. Wade, Jean-Paul Chavas, and Warren R. Grant, "A Market Equilibrium Analysis of the Impact of Risk on the U.S. Rice Industry," American Journal of Agricultural Economics 69 (Nov. 1987), 733-739.

Cecchetti, Stephen G., Robert E. Cumby, and Stephen Figlewski, "Estimation of the Optimal Futures Hedge," Review of Economics and Statistics 70 (Nov. 1988), 623-630.

Chavas, Jean-Paul, and Stanley R. Johnson, "Supply Dynamics: the Case of U.S. Broilers and Turkeys," American Journal of Agricultural Economics 64 (Aug. 1982), 558-564.

Chavas, Jean-Paul, and Matthew T. Holt, "Acreage Decisions under Risk: The Case of Corn and Soybeans," American Journal of Agricultural Economics 72 (Aug. 1990), 529-538.

Diebold, Francis X., and Peter Pauly, "Endogenous Risk in a Portfolio-Balance Rational-Expectations Model of the Deutschemark-Dollar Rate," European Economic Review 32 (Jan. 1988), 27-53.

Diebold, Francis X., and Marc Nerlove, "The Dynamics of Exchange Rate Volatility: A Multivariate Latent Factor ARCH Model," Journal of Applied Econometrics 4 (Jan. 1989), 1-21.

Engle, Robert F., "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of UK Inflation," Econometrica 50 (July 1982), 987-1008.

Engle, Robert F., and Tim Bollerslev, "Modelling the Persistence of Conditional Variances," Econometric Reviews 5 (1986), 1-50.

Engle, Robert F., David M. Lilien, and Russell P. Robins, "Estimating Time Varying Risk Premia in the Term Structure: The ARCH-M Model," Econometrica 55 (March 1987), 391-407.

Engle, Robert F., Victor Ng, and Michael Rothschild, "Asset Pricing with a Factor ARCH Covariance Structure: Empirical Estimates for Treasury Bills," Journal of Econometrics 45 (1990), 213-237.

Goodwin, Thomas H., and Steven M. Sheffrin, "Testing the Rational Expectations Hypothesis in an Agricultural Market," Review of Economics and Statistics 64 (Nov. 1982), 658-667.

Hoffman, Dennis L., "Two-Step and Related Estimators in Contemporary Rational-Expectations Models: An Analysis of Small Sample Properties," Journal of Business and Economic Statistics 9 (Jan. 1991), 51-61.

Holt, Matthew T., and Stanley R. Johnson, "Bounded Price Variation and Rational Expectations in an Endogenous Switching Model of the U.S. Corn Market," Review of Economics and Statistics 71 (Nov. 1989), 605-613.

Huntzinger, R. LeVar, "Market Analysis with Rational Expectations: Theory and Estimation," Journal of Econometrics, 10 (June 1979), 127-145.

Lasley, Floyd A., The U.S. Poultry Industry: Changing Economics and Structure, USDA Economic Research Report No. 502 (1983).

Lin, William, "Measuring Aggregate Supply Response under Instability," American Journal of Agricultural Economics 59 (Dec. 1977), 903-907.

Ljung, Greta M., and George E. P. Box, "On a Measure of Lack of Fit in Time-Series Models," Biometrika 65 (1978), 297-303.

McLeod, A. I., and W. K. Li, "Diagnostic Checking ARMA Time Series Models Using Squared Residual Autocorrelations," Journal of Time Series Analysis 4 (1983), 269-273.

Mishkin, Frederic S., A Rational Expectations Approach to Macroeconomics (Chicago: The University of Chicago Press, 1983).

Pagan, Adrian, "Econometric Issues in the Analysis of Regressions with Generated Regressors," International Economic Review 25 (1984), 221-247.

Pagan, Adrian R., and H. C. L. Sabau. "Consistency Tests for Heteroskedastoc and Risk Models," Australian National University working paper, 1987.

Shonkwiler, J. Scott, and G. S. Maddala, "Modeling Expectations of Bounded Prices: An Application to the Market for Corn," Review of Economics and Statistics 67 (Nov. 1985), 697-702.

Wallis, Kenneth F., "Econometric Implications of the Rational Expectations Hypothesis," Econometrica 48 (Jan. 1980) 49-74.

White, Halbert, "A Heteroskedasticity Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticty," Econometrica 48 (May 1980), 817-838.

Table 1. Summary Statistics for Preliminary Estimates of the Homoskedastic SUR and Multivariate ARCH Models for Exogenous Variables

		PB	PP	PT	OD
Homoskedastic SUR Model					
Q(10)		9.663 (0.471)	18.876 (0.042)	16.259 (0.092)	6.999 (0.726)
$Q^2(10)$	PB	36.478 (0.000)	–	–	–
	PP	18.235 (0.051)	16.786 (0.079)	–	–
	PT	14.727 (0.142)	18.017 (0.055)	27.298 (0.002)	–
	OD	18.190 (0.052)	12.432 (0.257)	11.291 (0.335)	29.957 (0.001)
Multivariate ARCH Model					
Q(10)		10.164 (0.426)	25.505 (0.004)	17.068 (0.073)	9.512 (0.484)
$Q^2(10)$	PB	5.106 (0.884)	–	–	–
	PP	13.007 (0.223)	4.321 (0.932)	–	–
	PT	7.641 (0.664)	13.026 (0.222)	24.107 (0.007)	–
	OD	8.593 (0.571)	12.116 (0.277)	10.205 (0.423)	4.562 (0.918)
ARCH(6)[a/]		16.310 (0.012)	11.839 (0.066)	15.831 (0.015)	12.468 (0.052)

Note: Q(10) and $Q^2(10)$ denote the Ljung-Box and McLeod-Li tests, respectively, for up to 10th-order serial correlation in $\hat{u}_{it}$ and $\hat{u}_{it}\hat{u}_{jt}$ in the homoskedastic SUR model and $\hat{u}_{it}\hat{h}_{iit}^{-1/2}$ and $\hat{u}_{it}\hat{h}_{iit}^{-1/2}\hat{u}_{jt}\hat{h}_{jjt}^{-1/2}$ in the multivariate ARCH model. Asymptotic p-values are in parentheses.

a/ Denotes the Lagrange Multiplier test for sixth-order ARCH.

Table 2. T-Statistics for Pagan-Sabau Consistency Tests

	PB	PP	PT	OD
PB	1.021 (0.309)	–	–	–
PP	0.043 (0.966)	0.993 (0.323)	–	–
PT	0.332 (0.741)	1.362 (0.176)	0.496 (0.621)	–
OD	0.419 (0.676)	1.131 (0.261)	0.905 (0.368)	0.989 (0.800)

Note: Asymptotic p-values are in parantheses.

Table 3. Maximum Likelihood Estimates of Structural and Autoregressive Equations

Broiler Price

$$PC_t = \underset{(7.623)}{-3.901} + \underset{(0.261)}{2.046}\, Q1_t + \underset{(0.608)}{4.537}\, Q2_t + \underset{(0.542)}{4.275}\, Q3_t - \underset{\substack{(0.030)\\ [-0.741]}}{0.127}\,(BP_t - OD_t) + \underset{\substack{(0.063)\\ [0.607]}}{0.216}\, PB_t + \underset{\substack{(0.060)\\ [0.284]}}{0.137}\, PP_t + \underset{\substack{(0.072)\\ [0.246]}}{0.257}\, PT_t + \underset{(0.460)}{1.354}\, t + u_{1t} \qquad R^2 = 0.867$$

Broiler Supply

$$BP_t = \underset{(2.497)}{-44.717} + \underset{(2.667)}{11.267}\, Q1_t + \underset{(2.909)}{29.303}\, Q2_t + \underset{(2.543)}{10.847}\, Q3_t + \underset{\substack{(0.237)\\ [0.146]}}{1.604}\, PC^e_t - \underset{\substack{(0.955)\\ [-0.027]}}{2.109}\, PC^v_t - \underset{\substack{(0.395)\\ [-0.022]}}{0.879}\, FC_{t-1} + \underset{(0.358)}{1.211}\, t + \underset{(0.028)}{0.970}\, BP_{t-1} + u_{2t} \qquad R^2 = 0.997$$

Beef Price

$$(1 - \underset{(0.098)}{1.110}\, L + \underset{(0.144)}{0.493}\, L^2 - \underset{(0.157)}{0.625}\, L^3 + \underset{(0.156)}{0.237}\, L^4 + \underset{(0.147)}{0.229}\, L^5 - \underset{(0.100)}{0.112}\, L^6)\, PB_t = \underset{(3.178)}{9.808} - \underset{(0.024)}{0.033}\, t + u_{3t} \qquad R^2 = 0.983$$

Pork Price

$$(1 - \underset{(0.083)}{1.105}\, L + \underset{(0.121)}{0.127}\, L^2 + \underset{(0.122)}{0.271}\, L^3 - \underset{(0.114)}{0.469}\, L^4 + \underset{(0.111)}{0.733}\, L^5 - \underset{(0.077)}{0.421}\, L^6)\, PP_t = \underset{(1.765)}{9.196} - \underset{(0.031)}{0.071}\, t + u_{4t} \qquad R^2 = 0.986$$

Turkey Price

$$(1 - \underset{(0.092)}{0.771}\, L + \underset{(0.136)}{0.154}\, L^2 - \underset{(0.123)}{0.080}\, L^3 - \underset{(0.123)}{0.179}\, L^4 + \underset{(0.123)}{0.343}\, L^5 - \underset{(0.081)}{0.128}\, L^6)\, PT_t = \underset{(1.153)}{13.434} - \underset{(0.030)}{0.253}\, t + u_{5t} \qquad R^2 = 0.981$$

Other Demand

$$(1 - \underset{(0.099)}{0.705}\, L - \underset{(0.119)}{0.171}\, L^2 + \underset{(0.111)}{0.037}\, L^3 - \underset{(0.106)}{0.437}\, L^4 + \underset{(0.096)}{0.303}\, L^5 + \underset{(0.091)}{0.017}\, L^6)\, OD_t = -\underset{(0.291)}{0.373} + \underset{(0.037)}{0.059}\, t + u_{6t} \qquad R^2 = 0.978$$

Note: Values in parentheses are asymptotic standard errors. Values in brackets are elasticities evaluated at the data means. R^2 denotes the square of the simple correlation coefficient between actual and simulated values of the variable.

Table 4. Maximum Likelihood Estimates of ARCH Components and Conditional Correlations

i	PC	BP	PB	PP	PT	OD
α_{i0}	1.452 (0.209)	23.910 (2.894)	1.866 (0.908)	3.784 (1.373)	4.269 (1.068)	0.809 (0.265)
α_{i1}	–	–	0.818 (0.235)	0.590 (0.230)	0.447 (0.169)	0.741 (0.209)
$\rho_{PC,i}$	1.000 (–)	–	–	–	–	–
$\rho_{BP,i}$	0.015 (0.167)	1.000 (–)	–	–	–	–
$\rho_{PB,i}$	−0.221 (0.164)	−0.095 (0.107)	1.000 (–)	–	–	–
$\rho_{PP,i}$	−0.146 (0.180)	−0.274 (0.102)	0.355 (0.092)	1.000 (–)	–	–
$\rho_{PT,i}$	−0.156 (0.190)	−0.312 (0.108)	0.146 (0.101)	0.355 (0.092)	1.000 (–)	–
$\rho_{OD,i}$	−0.123 (0.105)	0.219 (0.114)	−0.064 (0.103)	0.036 (0.106)	0.047 (0.111)	1.000 (–)

Note: Asymptotic standard errors are in parentheses.

Table 5. Predictive Performance Tests

	Expected Broiler Price $PC_t = \delta_0 + \delta_1 PC^e_{\alpha t} + \delta_2 PC^e_{\beta t}$	Broiler Production $BP_t = \delta_0 + \delta_1 BP_{\alpha t} + \delta_2 BP_{\beta t}$
δ_0	0.324 (0.324)	−0.657 (1.400)
δ_1	1.005 (6.994)	0.913 (6.254)
δ_2	−0.013 (0.105)	0.090 (0.608)
R^2	0.867	0.997
DW	1.734	1.737
Test Results		
	H_0: $\delta_0 = \delta_2 = 0$, $\delta_1 = 1$	
F-statistics	0.251	0.246
Asymptotic p-values	0.139	0.136

Note: Estimates obtained using White's (1980) correction for heteroskedasticity. T-ratios, in absolute values, are in parentheses. Asymptotic p-values are from the F(3, 107) distribution.

Footnotes

1/ ARCH denotes Autoregressive Conditional Heteroskedasticity and GARCH denotes Generalized Autoregressive Conditional Heteroskedasticity.

2/ Hoffman (1991) concluded that among the several ways of dealing with the generated regressors problem in rational-expectations models, the double-length estimator (DLE) performed best for moderate and large sample sizes. The DLE is esentially equivalent to the first iteration of a steepest-ascent nonlinear MLE algorithm, and hence is asymptotically equivalent to MLE.

3/ The structural equations can always be re-ordered so the first J equations are associated with the J endogenous variables for which expectations are sought. Matrices A_1 and A_2 can then be augmented with rows of zeros corresponding to equations in which expectations do not enter as inputs.

4/ More generally, $\underline{x}_{2t}$ could follow a vector autoregressive moving average (VARMA) process. However, any "invertible" VARMA model can always be recast as an infinite autoregressive representation. In practice, it is typically more convenient to use either univariate or multivariate autoregressions for exogenous variables.

5/ It is not possible to state a priori whether omitting covariances among exogenous variables would increase or decrease the rational expectations forecast variance. This is because covariance effects will depend on the signs of structural parameters, as well as whether or not the exogenous variables considered exhibit positive or negative correlation.

6/ This simplification can be important in practice. In the model considered presently, there are 80 parameters to be estimated. With 110 observations, and using two-sided numerical derivatives, the number of matrix inversions per iteration are reduced from 17,820 to 162 using the constant conditional correlations model.

7/ Broiler feed is measured as the simple average of the per pound price of corn and soybean meal, the major feed ingredients in broiler production.

8/ Over the sample period, other chicken demand accounted on average for 2.5% of total chicken disappearance.

9/ The first six observations were used to generate sixth-order lags for the autoregressive models of exogenous variables. Four additional observations are used subsequently in the structural model to initialize a fourth-order autoregressive error process. All empirical results are thus for the period 1962(3)-1989(4), a total of 110 observations.

10/ Initial results indicated the presence of first-order and fourth-order serial correlation in the residuals of the inverse demand and supply equations, respectively. AR(1) and AR(4) error processes are subsequently specified for these equations. In the estimation the structural equations were quasi-first-differenced, and the autocorrelation parameters are estimated jointly with the other parameters.

11/ While the implied demand elasticities are slightly higher than previous estimates (Goodwin and Sheffrin, 1982), they are still plausible. The estimates reported by Goodwin and Sheffrin (1982) were obtained using data for the period 1968-1977. Consequently, differences in the period of analysis may largely explain discrepancies in elasticity estimates.