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PMP and Uniqueness of Calibrating Solution: A Revision

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Abstract

This paper demonstrates the existence of a unique solution of the PMP problem when both observed output quantities and limiting input prices are taken as calibrating benchmarks. This version of PMP avoids the use of a user-determined small positive number ε originally introduced for guaranteeing that the dual (shadow) price of binding input constraints be positive. Furthermore, the paper shows how to obtain endogenous output supply and input demand elasticities that match available information about them in the form of previously estimated parameters for an entire region or sector. The framework is applied to a sample of farms also for the case that admits no production for some of the crop activities. The calibrating solution is very close to the observed values of output quantities and input prices. The calibrating model does not use the matrix of fixed technical coefficient and reproduces identical calibrating solutions.

Keywords: positive mathematical programming, solution uniqueness, supply elasticities, calibrating model

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1. Introduction

This paper proposes an approach to Positive Mathematical Programming (PMP) that guarantees the uniqueness of the calibrating solution. This result relies upon the use of all the available information, including prices of limiting. To exemplify, a large list of PMP empirical studies has been restricted to one limiting constraint, namely land. Yet, the available price of land (at either a regional or local level) has not been part of a calibrating relation. This is the starting point of the paper. Toward the goal of dealing also with calibrating input prices we discuss first the PMP approach as understood to date.

The original formulation of the PMP methodology (Howitt, 1995a, 1995b) was based upon the estimation of the marginal cost associated with the observed production plan (or the difference between known per-output unit accounting cost and effective economic marginal cost). Phase I of this model took the following specification (Howitt, 1995a, p. 151):

Primal $\max TNR = \mathbf{p}'\mathbf{x} - \mathbf{c}'\mathbf{x}$ (1)

subject to $A\mathbf{x} \leq \mathbf{b}$ structural constraints (2)

$\mathbf{x} \leq \bar{\mathbf{x}} + \boldsymbol{\varepsilon}$ calibration constraints (3)

and $\mathbf{x} \geq \mathbf{0}$, where A is a matrix of technical coefficients of dimensions $(m \times n, m < n)$ and all the other vectors are conformable to it. In particular, $\bar{\mathbf{x}} > \mathbf{0}$ is a vector of realized and observed levels of outputs whose utilization qualifies the positive feature of the PMP model. Vector $\mathbf{b}$ refers to limited input supplies. In the case of input land, the technical coefficients of the A matrix are computed by dividing the number of acres allocated to a crop by the realized and observed crop production, that is, $acres_{ij} / \bar{x}_{ij} = A_{ij}$. Then, $b_i = \sum_{j=1}^{J} acres_{ij}$. Vectors $\mathbf{p}$ and $\mathbf{c}$ represent market output prices and unit accounting costs, respectively. The parameter vector $\boldsymbol{\varepsilon}$ is composed of small, positive (user-determined) numbers whose role is to guarantee that the dual variables of the binding structural constraints achieve a positive value. In Howitt's words (1995a, p. 151): "The $\boldsymbol{\varepsilon}$ perturbation on the calibration constraints decouples the true resource constraints from the calibration constraints and ensures that the dual values on the allocable resources represent the marginal values of the resource constraints." With these stipulations, the dual of model (1)-(3) is stated as

$$\text{Dual} \qquad \min TC = \mathbf{b}'\mathbf{y} + \boldsymbol{\lambda}'[\bar{\mathbf{x}} + \boldsymbol{\varepsilon}] \tag{4}$$

$$\text{subject to} \qquad A'\mathbf{y} + \boldsymbol{\lambda} + \mathbf{c} \geq \mathbf{p} \tag{5}$$

with $\mathbf{y} \geq \mathbf{0}, \boldsymbol{\lambda} \geq \mathbf{0}$, where $\mathbf{y}$ represents the $(m \times 1)$ vector of shadow prices of the structural constraints and the $(n \times 1)$ vector $\boldsymbol{\lambda}$ represents the shadow prices of the calibration constraints. In the dual constraints (5) there are n constraints and $(m + n)$ variables (at the optimal primal solution $\mathbf{x}^*$, relation (5) is satisfied with the equality sign given that $\mathbf{x}^* = \bar{\mathbf{x}} + \boldsymbol{\varepsilon} > \mathbf{0}$ and complementary slackness conditions). Hence, in this specification, the PMP model is underdetermined (ill posed) because it admits an infinite number of $(\mathbf{y}, \boldsymbol{\lambda})$ solutions. This is the reason why the parameter $\boldsymbol{\varepsilon}$ is introduced in model (1)-(3) in order to elicit a dual solution with positive values of the shadow price $\mathbf{y}$ of the binding structural constraints. This means that at least m components of the vector $\boldsymbol{\lambda}$ assume a zero value.

Another criticism of the original PMP approach regards the specification of the calibration constraints. Why is the solution vector $\mathbf{x}$ of model (1)-(3) stated as less-than-or-equal to the observed vector of output levels ($\bar{\mathbf{x}} + \boldsymbol{\varepsilon}$) in the calibration constraints (3)? The answer was (is): to guarantee a nonnegative dual vector of shadow prices $\boldsymbol{\lambda}$. Admittedly, this is an unsatisfactory answer. Given that vector $\bar{\mathbf{x}}$ represents observed (by the econometrician) output levels that are realized by the producer in a previous economic cycle, the measured $\bar{\mathbf{x}}$ may contain some measurement error that either overstates or understates the level of the production plan consistent with the technical and economic information of a given producer. A more plausible specification of the calibration constraints, therefore, may be $\mathbf{x} = \bar{\mathbf{x}} + \mathbf{h}$, where $\mathbf{h}$ is a conformable vector of unrestricted deviations from $\bar{\mathbf{x}}$.

Furthermore, a measure of the limiting input price vector $\bar{\mathbf{y}}$ may be available at either a regional or more local level. For example, the price of agricultural land is surely available, either by region or by area. The estimate may not be fitting every single individual farm but it can be assumed that it will fall within a reasonable range of the actual optimal land value of this farm as obtained by solving model (1)-(3). If the information on land price and other important limiting inputs is available, it should be used in a PMP approach in order to avoid violating the principal tenet of the methodology: all the available information should be used. Also in this case, therefore, it seems plausible to state a calibration constraint for the dual variable vector as $\mathbf{y} = \bar{\mathbf{y}} + \mathbf{u}$, where $\mathbf{u}$ is a conformable vector of unrestricted deviations from $\bar{\mathbf{y}}$.

Within this novel PMP framework, the notion of a calibrating solution assumes a different structure from the original formulation of model (1)-(3). In that model, a calibrating solution achieves the obvious values as $\mathbf{x}^* = \bar{\mathbf{x}} + \boldsymbol{\varepsilon}$. Many critics of PMP have objected that this equation represents a tautology. In fact, the equality between the optimal solution of model (1)-(3) and the vector of observed output levels is achieved because the – presumably – available information on the limiting input-price side is ignored. With the more general specification of the calibration constraints in the form of $\mathbf{x} = \bar{\mathbf{x}} + \mathbf{h}$ and $\mathbf{y} = \bar{\mathbf{y}} + \mathbf{u}$, a calibrating solution $(\mathbf{x}^*, \mathbf{y}^*)$ will not, in general, be tautologically equal to $(\bar{\mathbf{x}}, \bar{\mathbf{y}})$. But it can be arranged to make the solution $(\mathbf{x}^*, \mathbf{y}^*)$ as close as possible to the observed quantities and prices, given the structure of the problem. This approach resembles an econometric estimation where the goal is to minimize the residuals of a system of regressions. An objective of this novel phase I PMP methodology, therefore, is to make deviations $(\mathbf{h}, \mathbf{u})$ as small as possible.

2. The Use of $\bar{\mathbf{x}}$ and $\bar{\mathbf{y}}$ in PMP

To justify the structure of the novel phase I PMP model we begin with two preliminary analyses. First, suppose that a preliminary phase I of the PMP methodology is concerned with solving the following problem

$$\max TNR = \mathbf{p}'\mathbf{x} - \mathbf{c}'\mathbf{x} \tag{6}$$

subject to $\quad A\mathbf{x} \leq \mathbf{b} \quad$ dual variables $\mathbf{y}$ (7)

$\quad \mathbf{x} = \bar{\mathbf{x}} + \mathbf{h} \quad$ dual variables $\boldsymbol{\lambda}$ (8)

with $\mathbf{x} \geq \mathbf{0}$ and $\mathbf{h}$ free. Furthermore, we wish to minimize the sum of squared deviations, $\mathbf{h}'W\mathbf{h}/2$, as in a weighted least-squares approach. The W matrix is diagonal with elements $p_j > 0$ on the main diagonal, $j = 1,...,n$. The effective objective function, therefore, will be expressed as an auxiliary function such as

$\max AUX = \mathbf{p}'\mathbf{x} - \mathbf{c}'\mathbf{x} - \mathbf{h}'W\mathbf{h}/2$. The purpose of the weight matrix W is to measure each component of the auxiliary objective function in dollars. Forming the Lagrange function and assembling the corresponding first-order-necessary conditions (FONC) will give

$$L = \mathbf{p}'\mathbf{x} - \mathbf{c}'\mathbf{x} - \mathbf{h}'W\mathbf{h}/2 + \mathbf{y}'[\mathbf{b} - A\mathbf{x}] + \boldsymbol{\lambda}'[\bar{\mathbf{x}} + \mathbf{h} - \mathbf{x}] \tag{9}$$

$$\frac{\partial L}{\partial \mathbf{x}} = \mathbf{p} - A'\mathbf{y} - \boldsymbol{\lambda} - \mathbf{c} \leq \mathbf{0} \tag{10}$$

$$\frac{\partial L}{\partial \mathbf{h}} = -W\mathbf{h} + \boldsymbol{\lambda} = \mathbf{0}. \tag{11}$$

From relation (11), $\boldsymbol{\lambda} = W\mathbf{h}$ and, thus, relation (10) can be reformulated as

$$A'\mathbf{y} + W\mathbf{h} \geq \mathbf{p} - \mathbf{c}\,. \tag{12}$$

Relation (11) represents a case of self duality, where a dual variable is equal (up to a scalar) to a primal variable.

Analogously, let us consider the following problem

$$\min TC = \mathbf{b}'\mathbf{y} \tag{13}$$

$$\text{subject to} \qquad A'\mathbf{y} \geq \mathbf{p} - \mathbf{c} \qquad \text{dual variables } \mathbf{x} \tag{14}$$

$$\mathbf{y} = \bar{\mathbf{y}} + \mathbf{u} \qquad \text{dual variables } \boldsymbol{\psi} \tag{15}$$

with $\mathbf{y} \geq \mathbf{0}$ and $\mathbf{u}$ free. Again, we wish to minimize the sum of squared deviations, $\mathbf{u}'V\mathbf{u}/2$, as in a weighted least-squares approach. The matrix V is diagonal with elements $b_i / \bar{y}_i > 0$ on the main diagonal, $i = 1,...,m$. The effective objective function, then, will be expressed as an auxiliary function such as $\min AUX2 = \mathbf{b}'\mathbf{y} + \mathbf{u}'V\mathbf{u}'/2$. The purpose of the V matrix is to render homogeneous the units of measurement of all the terms in the objective function and to scale the deviations $\mathbf{u}$ according to the size of the input constraints. Forming the Lagrange function and assembling the corresponding FONCs give

$$L^* = \mathbf{b}'\mathbf{y} + \mathbf{u}'V\mathbf{u}/2 + \mathbf{x}'[A'\mathbf{y} + \mathbf{c} - \mathbf{p}] + \boldsymbol{\psi}'[\mathbf{y} - \bar{\mathbf{y}} - \mathbf{u}] \tag{16}$$

$$\frac{\partial L^*}{\partial \mathbf{y}} = \mathbf{b} - A\mathbf{x} + \boldsymbol{\psi} \geq \mathbf{0} \tag{17}$$

$$\frac{\partial L^*}{\partial \mathbf{u}} = V\mathbf{u} - \boldsymbol{\psi} = \mathbf{0}\,. \tag{18}$$

From the self-dual relation (18), $\boldsymbol{\psi} = V\mathbf{u}$ and, thus, relation (17) can be reformulated as

$$A\mathbf{x} \leq \mathbf{b} + V\mathbf{u}\,. \tag{19}$$

This discussion leads to a specification of a phase I PMP model that combines the duality relations of a LP problem together with the least-squares necessary conditions involving deviations $\mathbf{h}$ and $\mathbf{u}$. Combining constraints (12) and (19) with the calibration relations (8) and (15), we can write the relevant phase I PMP model as the problem of finding nonnegative vectors $\mathbf{x} \geq \mathbf{0}, \mathbf{y} \geq \mathbf{0}$ and unrestricted vectors $\mathbf{h}$ and $\mathbf{u}$ such that

$$A\mathbf{x} \leq \mathbf{b} + V\mathbf{u} \qquad \text{dual variables } \mathbf{y} \tag{20}$$

$$A'\mathbf{y} + W\mathbf{h} + \mathbf{c} \geq \mathbf{p} \qquad \text{dual variables } \mathbf{x} \tag{21}$$

$$\mathbf{x} = \bar{\mathbf{x}} + \mathbf{h} \qquad \text{dual variables } W\mathbf{h} \tag{22}$$

$$\mathbf{y} = \bar{\mathbf{y}} + \mathbf{u} \qquad \text{dual variables } V\mathbf{u} \tag{23}$$

together with the associated complementary slackness conditions. This PMP approach avoids using the user-determined parameter $\boldsymbol{\varepsilon}$.

3. Solution Uniqueness of the phase I PMP model

A least-squares (LS) solution is unique if and only if the matrix of "explanatory" variables has full rank. To verify this crucial condition in relation to model (20)-(23) we assume that vectors $\bar{\mathbf{x}}$ and $\bar{\mathbf{y}}$ have all positive components and that, largely for this reason, $\mathbf{x} > \mathbf{0}$ and $\mathbf{y} > \mathbf{0}$ (this assumption will be relaxed in section 8). This implies – via complementary slackness conditions associated to relations (20)-(23) – that

$$A\mathbf{x} = \mathbf{b} + V\mathbf{u} \tag{24}$$

$$A'\mathbf{y} + W\mathbf{h} = \mathbf{p} - \mathbf{c} . \tag{25}$$

Substituting constraints (22) and (23) into (24) and (25), and rearranging terms, we obtain

$$-V\mathbf{u} + A\mathbf{h} = \mathbf{b} - A\bar{\mathbf{x}} \tag{26}$$

$$A'\mathbf{u} + W\mathbf{h} = \mathbf{p} - A'\bar{\mathbf{y}} - \mathbf{c} \tag{27}$$

and in matrix notation

$$\begin{array}{ccccc} \begin{bmatrix} -V & A \\ A' & W \end{bmatrix} & \begin{bmatrix} \mathbf{u} \\ \mathbf{h} \end{bmatrix} & = & \begin{bmatrix} \mathbf{b} - A\bar{\mathbf{x}} \\ \mathbf{p} - A'\bar{\mathbf{y}} - \mathbf{c} \end{bmatrix} \\ M & \mathbf{z} & = & \mathbf{q}. \end{array} \tag{28}$$

The matrix M is of full rank because the nonsingular weight matrices V and W are on the main diagonal. Hence, the least-squares solution $\hat{\mathbf{u}}$ and $\hat{\mathbf{h}}$ is unique. It follows that the solution $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ of model (20)-(23) is also unique. Given the structure of the M matrix, an inverse of M exists even if the A matrix is not of full rank.

The explicit, least-squares solution of (28) is

$$\begin{bmatrix} \hat{\mathbf{u}} \\ \hat{\mathbf{h}} \end{bmatrix} = \begin{bmatrix} -(V + AW^{-1}A')^{-1} & V^{-1}A(A'V^{-1}A + W)^{-1} \\ W^{-1}A'(V + AW^{-1}A')^{-1} & (A'V^{-1}A + W)^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{b} - A\bar{\mathbf{x}} \\ \mathbf{p} - A'\bar{\mathbf{y}} - \mathbf{c} \end{bmatrix}. \tag{29}$$

The optimal and calibrating LS levels of the primal and dual variables $\mathbf{x}$ and $\mathbf{y}$, then, are obtained as a simple addition according to the specification given in constraints (22) and (23) with $\hat{\mathbf{x}} = \bar{\mathbf{x}} + \hat{\mathbf{h}}$ and $\hat{\mathbf{y}} = \bar{\mathbf{y}} + \hat{\mathbf{u}}$.

4. Phase II: Specification of a General Cost Function

Phase II of a PMP approach deals with the estimation of marginal cost and input demand functions to be used in a calibrating model for the analysis of various policy scenarios. Following economic theory, we postulate that the total cost function of interest takes on the following symmetric and extended Leontief specification:

$$C(\mathbf{x},\mathbf{y}) = (\mathbf{g}'\mathbf{y})(\mathbf{f}'\mathbf{x}) + (\mathbf{g}'\mathbf{y})\mathbf{x}'Q\mathbf{x}/2 + (\mathbf{f}'\mathbf{x})[(\mathbf{y}^{1/2})'G\mathbf{y}^{1/2}] \qquad (30)$$

where the $(n\times n)$ matrix Q is symmetric and positive definite, the $(m\times m)$ matrix G is symmetric and negative semidefinite (a cost function is concave in input prices), the components of vector $\mathbf{f}$ and vector $\mathbf{g}$ are free to take on any value. We require that $\mathbf{f}'\mathbf{x} > 0$ and $\mathbf{g}'\mathbf{y} > 0$. From theory, a cost function is homogeneous of degree one in input prices. This requirement drives to a large extent the specification of the symmetric cost function presented in relation (30). The vector of output marginal costs is stated as

$$MC_{\mathbf{x}} = \frac{\partial C}{\partial \mathbf{x}} = (\mathbf{g}'\mathbf{y})\mathbf{f} + (\mathbf{g}'\mathbf{y})Q\mathbf{x} + \mathbf{f}[(\mathbf{y}^{1/2})'G\mathbf{y}^{1/2}] = A'\mathbf{y} + W\mathbf{h} + \mathbf{c} \qquad (31)$$

while, by Shephard lemma, the vector of demand functions for inputs is stated as

$$\frac{\partial C}{\partial \mathbf{y}} = (\mathbf{f}'\mathbf{x})\mathbf{g} + \mathbf{g}(\mathbf{x}'Q\mathbf{x})/2 + (\mathbf{f}'\mathbf{x})\Delta(\mathbf{y}^{-1/2})'G\mathbf{y}^{1/2} = A\mathbf{x} \qquad (32)$$

where the matrix $\mathbf{\Delta}(\mathbf{y}^{-1/2})$ is diagonal with terms $y_i^{-1/2}$ on the main diagonal.

The vector of output supply functions comes from relation (31) by equating it to the vector of market output prices, $\mathbf{p}$, and inverting the marginal cost function to obtain

$$\mathbf{x} = -Q^{-1}\mathbf{f} - Q^{-1}\mathbf{f}[(\mathbf{y}^{1/2})'G\mathbf{y}^{1/2}]/(\mathbf{g}'\mathbf{y}) + [1/(\mathbf{g}'\mathbf{y})]Q^{-1}\mathbf{p} \qquad (33)$$

that leads to the supply elasticity matrix

$$\Xi \equiv \Delta(\mathbf{p})\left[\frac{\partial \mathbf{x}}{\partial \mathbf{p}}\right]\Delta(\mathbf{x}^{-1}) = \Delta(\mathbf{p})Q^{-1}\Delta(\mathbf{x}^{-1})/(\mathbf{g}'\mathbf{y}) \qquad (34)$$

where matrices $\Delta(\mathbf{p})$ and $\Delta(\mathbf{x}^{-1})$ are diagonal with elements p_j and x_j^{-1}, respectively, on the main diagonals. Relation (34) includes all the own- and cross-price elasticities for all the output commodities admitted in the model.

The demand elasticities of limiting inputs can be easily measured from the input demand functions of relation (32). Suppose two limiting inputs form the structural constraints of the model. Then, the portion of the demand function that involves input prices can be stated as

$$\begin{aligned} b_1 + u_1 &= K_1 + (\mathbf{f}'\mathbf{x})[G_{11} + y_1^{-1/2} G_{12} y_2^{1/2}] \\ b_2 + u_2 &= K_2 + (\mathbf{f}'\mathbf{x})[G_{22} + y_1^{1/2} G_{12} y_2^{-1/2}] \end{aligned} \tag{35}$$

where K_1 and K_2 do not involve input prices. The (2×2) matrix of derivatives of the demand functions results in

$$\begin{bmatrix} \dfrac{\partial(b_1+u_1)}{\partial y_1} & \dfrac{\partial(b_1+u_1)}{\partial y_2} \\ \dfrac{\partial(b_2+u_2)}{\partial y_1} & \dfrac{\partial(b_2+u_2)}{\partial y_2} \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{2} y_1^{-3/2} G_{12} y_2^{1/2} & \dfrac{1}{2} y_1^{-1/2} G_{12} y_2^{-1/2} \\ \dfrac{1}{2} y_1^{-1/2} G_{12} y_2^{-1/2} & -\dfrac{1}{2} y_1^{1/2} G_{12} y_2^{-3/2} \end{bmatrix} (\mathbf{f}'\mathbf{x}) \tag{36}$$

This means that with only one limiting input, its demand elasticity will be equal to zero (as in a Leontief fixed coefficient specification) since the term G_{11} drops out of the derivative in (36). In a Leontief cost function, inputs are substitutes.

5. Exogenous and Disaggregated Output Supply Elasticities

PMP has been applied frequently to analyze farmers' behavior to changes in agricultural policies. A typical empirical setting is to map out several areas, say T areas, in a region (or state) and to assemble a representative farm model for each area (or to treat each area as a large farm).

When supply elasticities are exogenously available (say the own-price elasticities of crops) at the regional (or state) level (via econometric estimation or other means), a connection of all area models with these exogenous elasticities can be specified by establishing a weighted sum of all the areas endogenous own-price elasticities and the given regional elasticities. The weights are the share of each area's revenue over the total revenue of the region.

Let us suppose that exogenous own-price elasticities of supply are available at the regional level for all the J crops, say $\overline{\eta}_j, j = 1,...,J$. Then, the relation among these exogenous own-price elasticities and the corresponding areas' elasticities can be established as a weighted sum such as

$$\overline{\eta}_j = \sum_{t=1}^{T} w_{tj}\eta_{tj} \tag{37}$$

where the weights are the areas' revenue shares in the region (state)

$$w_{tj} = \frac{p_{tj}x_{tj}}{\sum_{s=1}^{T} p_{sj}x_{sj}} \tag{38}$$

and

$$\eta_{tj} = p_{tj}Q_t^{jj}x_t^{-1} / (\mathbf{g}_t'\mathbf{y}_t) \tag{39}$$

where Q_t^{jj} is the jth element on the main diagonal in the inverse of the Q_t matrix.

6. Estimation of the Cost Function Parameters

Using the optimal LS solutions of $\mathbf{x,y,h}$ and $\mathbf{u}$ for each of the T areas, $\hat{\mathbf{x}}_t, \hat{\mathbf{y}}_t, \hat{\mathbf{h}}_t$ and $\hat{\mathbf{u}}_t$, obtained from solving phase I model (20)-(23), it is possible to proceed to the estimation of parameters $Q, G, \mathbf{f}$ and $\mathbf{g}$ of the cost function (30). The programming model that executes the estimation of the marginal cost (31) and input demand (32) functions in the presence of exogenous supply elasticities for a region (state) that is divided into T areas takes on the following specification:

$$\min LS = \sum_{t=1}^{T} (\mathbf{d}_t'\mathbf{d}_t + \mathbf{r}_t'\mathbf{r}_t)/2 \tag{40}$$

subject to

$$(\mathbf{g}_t'\hat{\mathbf{y}}_t)\mathbf{f}_t + (\mathbf{g}_t'\hat{\mathbf{y}})Q_t\hat{\mathbf{x}}_t + \mathbf{f}_t[(\hat{\mathbf{y}}_t^{1/2})'G_t\hat{\mathbf{y}}_t] + \mathbf{d}_t = A_t\hat{\mathbf{y}}_t + W_t\hat{\mathbf{h}}_t + \mathbf{c}_t \qquad \text{marginal cost function}$$

$$(\mathbf{f}_t'\hat{\mathbf{x}}_t)\mathbf{g}_t + \mathbf{g}_t(\hat{\mathbf{x}}_t'Q_t\hat{\mathbf{x}}_t)/2 + (\mathbf{f}_t'\hat{\mathbf{x}}_t)\Delta(\hat{\mathbf{y}}_t^{-1/2})'G_t\hat{\mathbf{y}}_t^{1/2} + \mathbf{r}_t = A_t\hat{\mathbf{x}}_t \qquad \text{input demand function}$$

$$Q_t = L_tD_tL_t' \qquad \text{positive semidefinitenes of } Q_t$$

$$Q_tQ_t^{-1} = I_t \qquad \text{definiteness of } Q_t$$

$$\eta_{t,j,k} = \Delta(p_{tj})Q_t^{jk}\Delta(\hat{x}_{tk}^{-1})/(\mathbf{g}_t'\hat{\mathbf{y}}_t) \qquad \text{endogenous own- and cross-supply elasticities}$$

$$w_{tj} = \frac{p_{tj}\hat{x}_{tj}}{\sum_{s=1}^{T} p_{sj}\hat{x}_{sj}} \qquad \text{revenue shares}$$

$$\eta_{tj} = p_{tj}Q_t^{jj}\hat{x}_{tj}^{-1}/(\mathbf{g}_t'\hat{\mathbf{y}}_t) \qquad \text{endogenous own supply elasticities}$$

$$\bar{\eta}_j = \sum_{t=1}^{T} w_{tj}\eta_{tj} \qquad \text{disaggregation of exogenous elasticities}$$

with $D_t > 0$, $\mathbf{g}_t$, and $\mathbf{f}_t$ free; $\mathbf{f}_t'\mathbf{x}_t \geq 0$ and $\mathbf{g}_t'\mathbf{y}_t \geq 0$, $\mathbf{d}_t \geq \mathbf{0}, \mathbf{r}_t \geq \mathbf{0}$. Vector variables $\mathbf{d}_t \geq \mathbf{0}, \mathbf{r}_t \geq \mathbf{0}$ perform the role of auxiliary slack variables that will equal to zero identically when minimized by the GAMS solver (the GAMS solver requires an explicit objective function). In this way, the system of relations involving the specification of marginal cost and demand functions for inputs will be estimated as they appear in equations (31) and (32).

Model (40) is highly nonlinear in the constraints and a successful solution of it depends crucially on the choice of an initial point that falls in the neighborhood of the equilibrium solution. This specification was applied to three different samples of $T = 14$ farms (areas) each producing four crops (sugar beet, soft wheat, corn and barley) using only land as a limiting input. Using the GAMS software program, an equilibrium solution was achieved in all the three cases. GAMS includes the solver BARON (Branch And Reduce Optimization Navigator) for the global solution of nonlinear problems. The user manual states (2015): "… BARON implements deterministic and global optimization algorithms of the branch-and-bound type that are guaranteed to provide global optima under fairly general assumptions. These assumptions include the existence of finite lower and upper bounds on nonlinear expressions to be solved."

Table 1 presents the observed output levels and the percent deviation obtained from solving model (20)-(23) (alternatively solving model (28)). The primal solution $\hat{\mathbf{x}}$ is almost equal to the observed output levels $\bar{\mathbf{x}}$ for every farm.

The same event characterizes the dual solution. Table 2 presents the deviations from the observed land input prices and the percent deviation of the optimal dual solution, $\hat{\mathbf{y}}$. Also in this case, the percent deviation is minimal in every farm.

Table 1. Observed Output Levels, $\bar{\mathbf{x}}$, and Percent Deviation (dev) of the LS Calibrated Solution, $\hat{\mathbf{x}}$

	Sugar Beet	Soft Wheat	Corn	Barley	Sugar Beet	Soft Wheat	Corn	Barley
Farm	$\bar{\mathbf{x}}$	$\bar{\mathbf{x}}$	$\bar{\mathbf{x}}$	$\bar{\mathbf{x}}$	% dev	% dev	% dev	% dev
1	1133.4240	305.4032	341.3693	18.2398	0.026	0.060	0.157	1.341
2	3103.7830	861.7445	478.4465	59.8025	0.016	0.042	0.052	0.637
3	1547.9780	450.7937	881.9748	7.6887	0.010	-0.003	0.011	0.164
4	3488.3540	821.3934	1493.332	51.1247	0.002	0.019	0.023	0.526
5	959.1102	468.2848	478.9261	28.2406	0.032	0.001	0.091	1.136
6	942.2039	801.1288	1283.591	152.581	0.049	0.059	0.046	0.384
7	1600.7310	695.8293	899.4739	66.9718	0.023	0.068	0.061	0.683
8	3507.5490	1212.8550	1237.584	98.0497	0.006	0.047	0.048	0.388
9	1050.5370	332.3773	498.0150	63.6696	0.043	0.188	0.120	0.846
10	3473.6780	952.5199	774.7402	84.0070	0.010	0.039	0.062	0.444
11	1245.7220	765.1689	501.9673	59.5366	0.030	0.047	0.101	0.718

12	3276.1450	1100.1680	742.9419	177.974	0.014	0.031	0.074	0.326
13	877.0970	380.9171	564.6091	76.2122	0.048	0.055	0.105	0.683
14	1430.9460	768.6901	1309.392	67.7906	0.026	0.038	0.035	0.604

Table 2. Deviations of $\hat{\mathbf{y}}$ from $\bar{\mathbf{y}}$: vector $\hat{\mathbf{u}}$

	Absolute Deviation	Observed Land Prices	Percent Deviation
Farm	$\hat{\mathbf{u}}$	$\bar{\mathbf{y}}$	%
1	0.0053817	4.42	0.122
2	0.0026860	4.38	0.061
3	0.0004449	6.98	0.006
4	0.0018006	5.73	0.031
5	0.0031117	4.40	0.071
6	0.0014600	1.86	0.078
7	0.0032416	3.65	0.089
8	0.0018922	3.36	0.056
9	0.0052767	2.75	0.192
10	0.0027213	4.28	0.064
11	0.0029836	3.28	0.091
12	0.0011904	1.93	0.062
13	0.0028811	2.32	0.124
14	0.0022795	4.03	0.057

The weighted LS minimization of the primal and dual deviations $(\mathbf{h},\mathbf{u})$ has produced a largely satisfactory result in this sample. This goal is accomplished mainly by virtue of the disparate definitions of the diagonal weight matrices W and V. In matrix W, the diagonal terms are defined as output prices, that is p_j, where p_j is the price of the j-th output. In matrix V, the diagonal terms are defined as (quantity divided by price), that is $b_i / \bar{y}_i$, where b_i and $\bar{y}_i$ are the quantity and the observed price of the i-th limiting input. The purpose of the different treatment of the available information on the output and input sides is to guarantee that the components of model (20)-(23) be defined by coherent units of measurement, that is, dollars.

The estimated parameters of the cost function are reported in Tables 3 and 4. For reasons of space, only three Q matrices are reported.

Table 3. Intercepts $\hat{\mathbf{f}}$, $\hat{\mathbf{g}}$ and $\hat{G}$ Matrix of the Marginal Cost and Input demand Functions

Farm	$\hat{\mathbf{f}}$								
	Sugar Beet	Soft Wheat	Corn	Barley		$\hat{\mathbf{g}}$	$\hat{G}$	$\hat{\mathbf{f}}'\hat{\mathbf{x}}$	$\hat{\mathbf{g}}'\hat{\mathbf{y}}$
1	0.1110	0.0912	-0.0727	0.6183		0.00191	-1.2669	140.294	0.00847
2	-0.0112	0.6636	-0.0784	0.7116		0.00110	-0.9190	542.721	0.00484
3	0.5937	0.6745	0.4603	1.0948		0.00222	-0.0313	1637.550	0.01550
4	-0.0549	0.2153	0.2302	1.0018		0.00061	-1.0016	380.591	0.00350
5	0.0174	0.5424	-0.0297	0.6213		0.00361	-0.5182	274.242	0.01589
6	-0.7008	1.6163	6.0416	1.0965		0.01073	-0.2639	8561.452	0.01998
7	0.2155	0.2852	0.2414	0.9854		0.00328	-0.7473	827.362	0.01198
8	-0.0769	0.7406	0.4971	0.9387		0.00791	-0.9633	1336.688	0.02660
9	-0.0559	0.7323	0.3863	0.8988		0.00941	-1.2263	435.449	0.02592
10	0.0300	0.8861	-0.2342	0.9465		0.00055	-0.6090	846.879	0.00234
11	0.2427	0.4817	-0.0555	0.9430		0.00638	-0.7449	699.825	0.02095
12	0.0796	0.8584	0.4385	1.0428		0.00650	-1.4001	1717.901	0.01255
13	0.7831	0.3158	-0.3314	0.8222		0.00711	-0.9164	683.351	0.01651
14	0.1802	0.6635	0.1982	0.9977		0.00925	-1.2669	1095.888	0.03732

Table 4. Matrices $\hat{Q}$ and $\hat{D}$ for Three Farms

	Matrix $\hat{Q}$					Matrix $\hat{D}$			
Farm 1	Sugar Beet	Soft Wheat	Corn	Barley		Sugar Beet	Soft Wheat	Corn	Barley
S. Beet	0.90363	-1.97461	-0.88227	0.06447		0.90363			
S.Wheat	-1.97461	5.83223	2.23097	0.35591			1.51732		
Corn	-0.88227	2.23097	1.49261	0.17779				0.57068	
Barley	0.06447	0.35591	0.17779	21.75286					21.55051
Farm 2									
S. Beet	0.71517	-2.04607	-0.76495	-0.00159		0.71517			
S.Wheat	-2.04607	7.25493	2.41519	-0.05663			1.40123		
Corn	-0.76495	2.41519	1.53268	-0.03759				0.67780	
Barley	-0.00159	-0.05663	-0.03759	18.98344					18.97949
Farm 3									
S. Beet	1.24597	0.24597	-2.27223	-0.42147		1.24597			
S.Wheat	0.24597	1.95471	-1.25809	-0.02225			1.90615		
Corn	-2.27223	-1.25809	4.76858	0.85444				0.28099	
Barley	-0.42147	-0.02225	0.85444	6.78018					6.59126

All 14 farms achieved a nonsingular $\hat{Q}$ matrix. This feature is instrumental in defining the matrix of endogenous supply elasticities. Table 5 presents the endogenous own- and cross-price supply elasticities for three farms.

Table 5. Endogenous Own- and Cross-Supply Elasticities for Three Farms

Farm 1	Sugar Beet	Soft Wheat	Corn	Barley
S. Beet	0.2001	0.1952	0.1321	-0.1091
S. Wheat	0.2815	0.6056	-0.2563	-0.1763
Corn	0.2052	-0.2760	1.2485	-0.1514
Barley	-0.0089	-0.0100	-0.0079	0.5927
Farm 2				
S. Beet	0.2487	0.2182	0.1855	0.0133
S. Wheat	0.3081	0.4399	-0.2513	0.0162
Corn	0.1454	-0.1395	1.5539	0.0191
Barley	0.0012	0.0011	0.0023	0.4196
Farm 3				
S. Beet	0.1839	0.1379	0.1727	-0.1676
S. Wheat	0.2347	0.3725	0.2474	-0.5665
Corn	0.4893	0.4121	0.4952	-0.9548
Barley	-0.0044	-0.0087	-0.0088	2.5417

We stipulated that regional, exogenous own-price supply elasticities were available in the magnitude of 0.5 for sugar beet, 0.4 for soft wheat, 0.6 for corn and 0.3 for barley. The endogenous own-price elasticities of all farms were aggregated to be consistent with the regional exogenous elasticities according to relation (37). Table 6 presents the farms' own-price supply elasticities and the revenue weights used in the aggregation relation.

Table 6. Disaggregation/Aggregation of the Regional, Exogenous Supply Elasticities.

	Exogenous Own-Supply Elasticities					Revenue Weights			
Farms	Sugar Beet:0.5	Soft Wheat:0.4	Corn: 0.6	Barley: 0.3		Sugar Beet	Soft Wheat	Corn	Barley
1	0.2001	0.6056	1.2485	0.5927		0.0406	0.0291	0.0295	0.0165
2	0.2487	0.4399	1.5539	0.4196		0.1334	0.0937	0.0489	0.0628
3	0.1839	0.3725	0.4952	2.5417		0.0527	0.0446	0.0699	0.0070
4	0.2225	0.4774	0.5665	0.9868		0.1000	0.0893	0.1383	0.0536
5	0.1599	0.4512	0.8430	0.5691		0.0326	0.0413	0.0385	0.0256
6	8.6932	0.9332	0.5011	0.1080		0.0371	0.0828	0.1151	0.1601
7	0.0990	0.2906	0.3522	0.1918		0.0502	0.0688	0.0769	0.0606
8	0.1347	0.2714	0.2307	0.0823		0.1288	0.1292	0.1022	0.0931
9	0.1303	0.2670	0.3384	0.1544		0.0376	0.0335	0.0426	0.0576
10	0.2954	0.3940	1.9745	0.9603		0.1027	0.0930	0.0649	0.0825
11	0.1085	0.3682	0.2755	0.2195		0.0424	0.0737	0.0417	0.0539
12	0.1843	0.2692	0.2407	0.1197		0.1555	0.1079	0.0685	0.1868
13	0.0947	0.2861	0.4050	0.1486		0.0299	0.0336	0.0454	0.0689
14	0.0883	0.2455	0.3772	0.1349		0.0564	0.0795	0.1175	0.0711

7. Calibrating Equilibrium Model

With the estimates of the cost function parameters $\hat{f}, \hat{g}, \hat{Q}, \hat{G}$ it is possible to formulate a calibrating equilibrium model for each farm (sector, area) of the following structure

$$\min CSC = \mathbf{z}_{pt}{}'\mathbf{y}_t + \mathbf{z}_{dt}{}'\mathbf{x}_t = 0 \tag{41}$$

subject to

$$(\hat{\mathbf{f}}_t'\mathbf{x}_t)\hat{\mathbf{g}}_t + \hat{\mathbf{g}}_t(\mathbf{x}_t'\hat{Q}_t\mathbf{x}_t)/2 + (\hat{\mathbf{f}}_t'\mathbf{x}_t)\Delta(\mathbf{y}_t^{-1/2})'\hat{G}_t\mathbf{y}_t^{1/2} + \mathbf{z}_{pt} = \mathbf{b}_t + V\hat{\mathbf{u}}_t$$

$$(\hat{\mathbf{g}}_t'\mathbf{y}_t)\hat{\mathbf{f}}_t + (\hat{\mathbf{g}}_t'\mathbf{y}_t)\hat{Q}_t\mathbf{x}_t + \hat{\mathbf{f}}_t[(\mathbf{y}_t^{1/2})'\hat{G}_t\mathbf{y}_t^{1/2}] = \mathbf{p}_t + \mathbf{z}_{dt}$$

with $\mathbf{x}_t \geq \mathbf{0}, \mathbf{y}_t \geq \mathbf{0}, \mathbf{z}_{pt} \geq \mathbf{0}, \mathbf{z}_{dt} \geq \mathbf{0}$. The variables $\mathbf{z}_{pt}$ and $\mathbf{z}_{dt}$ are slack-surplus variables of the primal and dual constraints, respectively. The solution of the equilibrium model (41) produces optimal values of the primal and dual variables, $\mathbf{x}_t$ and $\mathbf{y}_t$ that are identical to the solution values of model (20)-(23). Notice that the matrix of constant technical coefficients, A_t, no longer appears in the calibrating equilibrium model (41). This elimination removes the last vestige of a linear structure that has been considered too rigid for representing the choices of a producer. The objective function (*CSC*) of model (41) combines all the complementary slackness conditions of the farm (region, area) sample. Hence, its optimal value must be equal to zero. Model (41) can be used to perform response analysis to variations in prices, subsidies, quotas, input quantities, and other parameters for a variety of policy scenarios.

8. PMP Uniqueness With Missing Observations

Empirical reality compels a further consideration of the above methodology in order to deal with farm samples where not all farms produce all commodities. It turns out that very little must be changed for obtaining a unique and calibrating solution in the presence of missing commodities, their prices and the corresponding technical coefficients.

To exemplify, suppose that the farm sample displays the following Table 7 of observed crop levels.

Table 7. Observed Output Levels, $\bar{\mathbf{x}}$, with non produced commodities

	Sugar Beet	Soft Wheat	Corn	Barley
Farm	$\bar{\mathbf{x}}$	$\bar{\mathbf{x}}$	$\bar{\mathbf{x}}$	$\bar{\mathbf{x}}$
1	1133.4240	0.0	341.3693	18.2398
2	3103.7830	861.7445	0.0	59.8025
3	0.0	450.7937	881.9748	0.0
4	3488.3540	821.3934	1493.332	51.1247
5	959.1102	468.2848	0.0	28.2406
6	942.2039	801.1288	1283.591	152.581
7	1600.7310	0.0	899.4739	66.9718
8	0.0	1212.8550	1237.584	98.0497
9	1050.5370	332.3773	0.0	63.6696
10	3473.6780	952.5199	774.7402	0.0
11	0.0	765.1689	501.9673	59.5366
12	3276.1450	1100.1680	0.0	177.974
13	877.0970	380.9171	564.6091	76.2122
14	1430.9460	0.0	1309.392	0.0

Other missing information deals with prices and unit accounting costs associated with the zero-levels of crops. Furthermore, the technical coefficients of the farms not producing the observed crops also equal to zero. Hence, we can state that, for $t = 1,...,T$, the number of farms, and $j = 1,...,J$, the number of crops, if $\bar{x}_{tj} = 0$, also $p_{tj} = 0$, $c_{tj} = 0$ and $A_{tij} = 0$. Furthermore, suppose that only one input, land, is involved in this farm sample. Then, the land price is observed for all farms.

As to the solution of the Phase I PMP specification, we expect that $x_{tj} = \overline{x}_{tj} + h_{tj}$ for $\overline{x}_{tj} > 0$, and $h_{tj} = x_{tj} = 0$ for $\overline{x}_{tj} = 0$. It turns out that the least-squares computation of the deviations u_{ti} and h_{tj} expressed by equation (29) produces the desired estimates of the deviations h_{tj} and crop levels x_{tj} when the observed level of those crops equals zero, $\overline{x}_{tj} = 0$. This is so because the first term on the RHS of (29) is equal to zero by construction, $b_i - \sum_{j=1}^{J} A_{ij}\overline{x}_j = b_i - \sum_{j=1}^{J} (acres_{ij} / \overline{x}_j)\overline{x}_j = 0$. The second term on the RHS of (29) reduces to zero because of the zero information about non-produced crops, $p_j - \sum_{i=1}^{I} A_{ij}\overline{y}_i - c_j = 0 - 0\overline{y}_i - 0 = 0$. Therefore, $\hat{h}_{tj} = \hat{x}_{tj} = 0$ for $\overline{x}_{tj} = 0$ and the least-squares PMP solution is unique also in this more elaborate case.

The estimation of the cost function carries through as in section 6 without modification. Also the Phase III calibrating model expressed in (41) needs no adjustment.

9. Results for a farm sample with missing production of some crops

The observed crop production of a 14-farm sample is given in Table 7. Also the corresponding output prices, $p_{tj} = 0$, and accounting costs, $c_{tj} = 0$, are part of the data sample for the no-production levels $\overline{x}_{tj} = 0$, as reported in Table 7. Furthermore, $A_{tij} = 0$ for the same activities of no-production.

Table 8 presents the unique least-squares estimates of the crop levels and the corresponding percentage deviation from the observed sample data.

Table 8. Estimated Output Levels, $\hat{\mathbf{x}}$, and Percent Deviation (dev) for the sample with missing crop production (compare with Table 7)

	Sugar Beet	Soft Wheat	Corn	Barley	Sugar Beet	Soft Wheat	Corn	Barley
Farm	$\hat{\mathbf{x}}$	$\hat{\mathbf{x}}$	$\hat{\mathbf{x}}$	$\hat{\mathbf{x}}$	% dev	% dev	% dev	% dev
1	1133.7140	0	341.9053	18.4843	0.0256	0	0.1570	1.3400
2	3104.2820	862.1098	0.0000	60.1834	0.0161	0.0424	0	0.6369
3	0	450.7820	882.0680	0	0	-0.0026	0.0106	0
4	3488.4150	821.5529	1493.6830	51.3938	0.0017	0.0194	0.0235	0.5264
5	959.4208	468.2891	0	28.5614	0.0324	0.0009	0	1.1360
6	942.6667	801.6001	1284.1790	153.1671	0.0491	0.0588	0.0458	0.3840
7	1601.1000	0	900.0223	67.4290	0.0231	0	0.0610	0.6825
8	0	1213.4210	1238.1750	98.4298	0	0.0466	0.0478	0.3876
9	1050.9910	333.0022	0	64.2084	0.0433	0.1880	0	0.8463
10	3474.0410	952.8955	775.2208	0	0.0105	0.0394	0.0620	0
11	0	765.5305	502.4727	59.9640	0	0.0473	0.1007	0.7179
12	3276.6110	1100.5140	0	178.5547	0.0142	0.0314	0	0.3260
13	877.5201	381.1268	565.2019	76.7330	0.0482	0.0550	0.1050	0.6833
14	1431.3200	0	1309.8500	0	0.0261	0	0.0350	0

Except for two cells, the percent deviations of the estimated crop levels from the observed production quantities are below 1 percent. The cells with a zero estimated quantity level correspond to the cells with observed zero level of production, as in Table 7. Table 9 presents the estimated land price and the percent deviation from the observed input price.

Table 9. Deviations of $\hat{\mathbf{y}}$ from $\bar{\mathbf{y}}$

	Estimated Land Prices	Observed Land Prices	Percent Deviation
Farm	$\hat{\mathbf{y}}$	$\bar{\mathbf{y}}$	%
1	4.428035	4.42	0.1818
2	4.382827	4.38	0.0645
3	6.980315	6.98	0.0045
4	5.731801	5.73	0.0314
5	4.402587	4.40	0.0588
6	1.861460	1.86	0.0785
7	3.653809	3.65	0.1044
8	3.362198	3.36	0.0654
9	2.756308	2.75	0.2294
10	4.281756	4.28	0.0410
11	3.283229	3.28	0.0984
12	1.931129	1.93	0.0585
13	2.322881	2.32	0.1242
14	4.031362	4.03	0.0338

The deviations of the estimated land prices from the observed prices are all below one percent. Table 10 presents the estimates of the parameters of the cost function under the condition of zero production for some crops in various farms.

Table 10. Intercepts $\hat{\mathbf{f}}$, $\hat{\mathbf{g}}$ and $\hat{G}$ Matrix of the Marginal Cost and Input Demand Functions for the case of zero production of some crop in various farms

Farm	$\hat{\mathbf{f}}$								
	Sugar Beet	Soft Wheat	Corn	Barley		$\hat{\mathbf{g}}$	$\hat{G}$	$\hat{\mathbf{f}}'\hat{\mathbf{x}}$	$\hat{\mathbf{g}}'\hat{\mathbf{y}}$
1	-0.15426	0.00274	0.73961	-0.11144		0.00686	-1.9508	75.930	0.03038
2	0.07435	-0.14574	0.03714	0.32875		0.00495	-3.1350	124.945	0.02171
3	0.03532	0.27086	-0.11507	0.03964		0.00052	-1.0871	20.595	0.00363
4	-0.02920	0.07372	0.10372	0.85513		0.00441	-2.4222	157.570	0.02530
5	0.02132	0.01858	0.11481	0.06645		0.00754	-2.3602	31.051	0.03318
6	0.22974	0.22787	-0.02587	0.26590		0.01186	-5.3411	406.732	0.02208
7	0.13824	-0.00074	-0.14506	0.29086		0.00273	-3.6725	110.382	0.00997
8	0.01525	0.40319	-0.12078	-0.17109		0.01373	-3.0222	322.849	0.04616
9	0.11620	-0.08339	0.00722	-0.03553		0.00499	-2.9037	92.071	0.01375
10	-0.00636	0.36252	-0.14708	0.00406		0.00076	-2.4324	209.320	0.00325
11	0.00236	0.30162	-0.12984	-0.24600		0.00764	-2.8158	150.906	0.02507
12	0.10700	0.16788	0.00002	0.57873		0.00004	-2.6811	638.676	0.00001
13	0.24139	0.42193	-0.25400	-0.19708		0.01046	-2.9149	213.946	0.02431
14	0.05358	0.06649	0.07745	0.01622		0.00686	-2.6086	178.140	0.03038

Table 11 presents the own price elasticities of the 14 farms that correspond to the observed and exogenous price elasticities of the four crops.

Table 11. Disaggregation/Aggregation of the Regional, Exogenous Supply Elasticities when some crops are not produced in various farms

	Exogenous Own-Supply Elasticities					Revenue Weights			
Farms	Sugar Beet:0.5	Soft Wheat:0.4	Corn: 0.6	Barley: 0.3		Sugar Beet	Soft Wheat	Corn	Barley
1	0.257	0	0.385	0.722		0.0523	0	0.0368	0.0198
2	0.289	0.409	0	0.251		0.1719	0.1139	0	0.0748
3	0	0.515	1.740	0.000		0	0.0542	0.0871	0
4	1.873	0.262	0.428	0.333		0.1288	0.1086	0.1726	0.0639
5	0.381	0.577	0	0.445		0.0421	0.0502	0	0.0306
6	0.052	0.221	0.322	0.120		0.0479	0.1007	0.1437	0.1904
7	0.149	0	0.656	0.225		0.0647	0	0.0960	0.0723
8	0	0.329	0.294	0.122		0	0.1571	0.1275	0.1108
9	0.212	0.407	0	0.335		0.0485	0.0408	0	0.0688
10	0.241	0.309	0.794	0		0.1323	0.1130	0.0810	0
11	0	0.399	0.313	0.268		0	0.0896	0.0520	0.0643
12	0.487	0.714	0	0.531		0.2004	0.1311	0	0.2220
13	0.142	0.325	0.837	0.259		0.0385	0.0409	0.0567	0.0822
14	0.305	0	0.583	0		0.0727	0	0.1465	0

The calibrating model (41) applies also to this data sample without any modification.

10. Conclusion

We have achieved the objective of using all the available information about output quantities and input prices, and the formulation of a calibrating PMP model that is free of the rigidities of a linear programming structure. In the process, we dispense with the necessity of dealing with the user-determined vector of small and arbitrary positive numbers ε which is required by the traditional PMP methodology. We also demonstrated the uniqueness of the calibrating solution. Two empirical examples were presented. In the first sample of 14 farms and 4 crops, all farms produce every commodity. In the second sample, some of the farms do not produce all the commodities. This is the typical case. It is shown that the uniqueness of the calibrating solution is maintained also in this more elaborate case.

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