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HETEROSCEDASTIC TOBIT MODELS: THE HOUSEHOLD DEMAND FOR FRESH POTATOES REVISITED

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Heteroscedastic Tobit Models: The Household Demand for Fresh Potatoes Revisited

The use of limited dependent variable models to estimate food demand and/or Engel functions using cross-sectional data has become standard practice in the agricultural economics literature (e.g., Lane; Thraen, Hammond, and Buxton; Huang, et al. (1981); Cox, Ziemer, and Chavas (CZC); Senauer and Young; Cox and Wohlgenant; McCracken and Brandt; Haines, Guilkey, and Popkin). As is well known, limited dependent variable models such as tobit correct for the censoring (or truncation) of cross-sectional food consumption data due to non-consumption during the survey's observation period. Most applications, however, assume a homoscedastic variance specification despite the fact that cross-sectional data are frequently heteroscedastic (Prais and Houthakker; Fomby, Hill, and Johnson). Heteroscedastic tobit specifications in the agricultural economics literature include Huang, Raunikar, and Tynan (HRT) and Lee.

In contrast to the general linear model where heteroscedasticity results in a loss of efficiency but does not bias the parameter estimates, non-homoscedastic variances in the tobit model are much more problematical. The maximum likelihood (ML) tobit estimator has been shown to be inconsistent and biased in the context of truncated samples (i.e., only the non-limit observations are available) by Hurd, and in censored samples by Arabmazar and Schmidt, and Maddala and Nelson. While the direction of bias resulting from this type of mis-specification is not clear analytically (Maddala and Nelson; Maddala), there is some empirical evidence that "... estimated parameters, marginal effects, and elasticity measures are underestimated when homoscedasticity is assumed" (HRT, p. 203). Given that cross-sectional demand models are often heteroscedastic, applied economists are interested in the magnitudes and significance of the differences in these response measures estimated under alternative variance specifications. While HRT found that total elasticities are similar under both specifications (i.e., homoscedastic versus heteroscedastic), the component (conditional and market entry) elasticities vary much more between the alternative specifications. However, HRT provide no measures of the statistical significance of these differences.

The purpose of this paper is to provide further insight into the impacts of alternative heteroscedastic variance specifications in the tobit model. We generalize the previous work of HRT in three directions: (1) we allow for three common heteroscedasticity specifications following Fomby, Hill, and Johnson; (2), we compute approximate, asymptotic standard errors on the total, conditional and market entry elasticities following CZC; and (3), we statistically test for the significance of the differences in these elasticity estimates from alternative variance specifications. These specifications and their associated tobit likelihood functions are presented first, followed by an extension of the McDonald and Moffitt tobit decompositions to the general heteroscedastic context. These results show that heteroscedastic tobit specifications are straightforward generalizations of the homoscedastic case. Next, the impacts of alternative tobit variance specifications are examined using the fresh potato model and data of CZC. Estimation results are then compared in the following order: parameter estimates, marginal effects (elasticities), and tests of the significance of the differences in elasticities for the alternative specifications. Conclusions and summary are then provided.

Alternative Variance Specifications of the Tobit Model

A general tobit model allowing for either heteroscedastic or homoscedastic variances can be specified as

$$(1)\qquad Y_i = \begin{cases} X_i\beta+\varepsilon_i & \text{if} \quad X_i\beta+\varepsilon_i > 0 \\ 0 & \text{if} \quad X_i\beta+\varepsilon_i \le 0 \end{cases}$$

where $\varepsilon_i \sim IN(0,\sigma_i^2)$, X_i is a 1xK vector of regressors, and β is a Kx1 vector of parameters to be estimated. Letting $\sigma_i = \sigma_i(\delta)$ denote the dependence of each variance specification on the parameter vector δ, the sample likelihood function associated with (1) is

$$(2)\qquad L(\beta,\delta) = \prod_{0}(1-\Phi(\beta,\sigma_i))\prod_{1}(2\pi\sigma_i^2)^{-1/2} \exp((-1/2) ((Y_i-X_i\beta)/\sigma_i)^2)$$

where $\Phi(\beta,\sigma_i) = \Phi_i(X_i\beta/\sigma_i) = \Phi_i$ is the standard normal cumulative distribution function (with the associated density function $\phi(\beta,\sigma_i) = \phi_i = (2\pi)^{-1/2} \exp((-1/2)\ (X_i\beta/\sigma_i)^2))$ and $\mathbf{0} = \{Yi: Yi = 0\}$, $\mathbf{1} = \{Yi: Yi > 0\}$. The associated log likelihood function is

$$(3)\qquad \log L(\beta,\delta) = \sum_{0} \log(1-\Phi_i)-1/2 \sum_{1}\{\log 2\pi + \log \sigma_i^2 + (\frac{Y_i-X_i\beta}{\sigma_i})^2\}.$$

We refer to the variance σ_i^2 under four specifications: homoscedastic, and following Fomby, Hill and Johnson (pp. 177-187), three heteroscedastic cases:

(4a) VAR0: $\sigma_i^2 = \sigma^2 = \delta_0$ (Homoscedastic)

(4b) VAR1: $\sigma_i^2 = Z_i\delta$ (Linear Variance)

(4c) VAR2: $\sigma_i^2 = (Z_i\delta)^2$ (Linear Standard Deviation or Squared Variance)

(4d) VAR3: $\sigma_i^2 = \exp(Z_i\delta)$ (Multiplicative or Exponential Variance) where, in each case other than (4a), Z_i is a $1xK_1$ vector of some subset of the regressors X_i (including an intercept term corresponding to δ_o), δ is a K_1x1 vector of parameters to be estimated, and δ_0 is a scalar variance parameter in (4a). Note that (4b)-(4d) nest the homoscedastic case (4a) when $\delta_j = 0$ for all $j > 0$ (i.e., $\delta_o = \{\sigma^2, \sigma^2, \sigma, \log \sigma\}$ for VAR0-VAR3). Following Maddala and HRT, likelihood ratio tests are appropriate for testing the null hypothesis of homoscedasticity ($\delta_j = 0$ for all $j > 0$) against each alternative variance specification.

The first partial derivatives of the log likelihood with respect to β (regardless of the variance specification) are

$$(5) \qquad \frac{\partial \log L}{\partial \beta} = -\sum_0 \frac{1}{\sigma_i} \frac{\phi_i}{1-\Phi_i} X_i' + \sum_1 \frac{1}{\sigma_i^2} (Y_i - X_i\beta)X_i'.$$

The general form of the first partial derivatives of the log likelihood with respect to δ, via the chain rule, are

$$(6) \qquad \frac{\partial \log L}{\partial \delta} = \frac{\partial \log L}{\partial \sigma_i^2} \frac{\partial \sigma_i^2}{\partial \delta},$$

where

$$(7) \qquad \frac{\partial \log L}{\partial \sigma_i^2} = \sum_0 \frac{1}{2\sigma_i^3} \frac{\phi_i}{1-\Phi_i} X_i\beta + \sum_1 \frac{1}{2\sigma_i^4} [(Y_i - X_i\beta)^2 - \sigma_i^2],$$

and the alternative variance specifications (4a)-(4d) imply

$$(8a) \qquad \frac{\partial \sigma_i^2}{\partial \delta} = 1 \qquad \text{(VAR0)}$$

$$(8b) \qquad \frac{\partial \sigma_i^2}{\partial \delta} = Z_i' \qquad \text{(VAR1)}$$

$$(8c) \qquad \frac{\partial \sigma_i^2}{\partial \delta} = 2\sigma_i Z_i' \qquad \text{(VAR2)}$$

$$(8d) \qquad \frac{\partial \sigma_i^2}{\partial \delta} = \sigma_i^2 Z_i' \qquad \text{(VAR3).}$$

Note that (6), (7), and (8c) correspond to the heteroscedasticity formulation found in Maddala (p.180) and HRT. Similarly, (6), (7) and (8a) correspond to the traditional homoscedastic tobit results. Thus, analytical expressions for these first derivatives with respect to δ are straight forward generalizations of the homoscedastic results.1/

Given the ease of implementation and accuracy of numerically approximating the matrix of second partial derivatives with software such as GAUSS, the analytical expressions for the second derivatives of the alternative log likelihood functions are perhaps of less interest. In the present context, however, again note that the chain rule yields the following result similar to (6) above:

$$(9) \qquad \frac{\partial^2 \log L}{\partial\delta\partial\delta'} = \frac{\partial^2 \log L}{\partial\sigma_i^2\partial\sigma_i^2}\,\frac{\partial\sigma_i^2}{\partial\delta}\,\frac{\partial\sigma_i^2}{\partial\delta'} + \frac{\partial \log L}{\partial\sigma_i^2}\,\frac{\partial^2\sigma_i^2}{\partial\delta\partial\delta'}$$

$$(10) \qquad \frac{\partial^2 \log L}{\partial\beta\partial\delta'} = \frac{\partial^2 \log L}{\partial\beta\partial\sigma_i^2}\,\frac{\partial\sigma_i^2}{\partial\delta'}$$

where $\partial\sigma_i^2/\partial\delta$ is defined in (8a)-(8d), $\partial^2\sigma_i^2/\partial\delta\partial\delta' = \{0,\ 0,\ 2Z_i'Z_i,\ \sigma_i^4 Z_i'Z_i\}$ for VAR0-VAR3, respectively, and where $\partial^2\log L/\partial\sigma_i^2\partial\sigma_i^2$ and $\partial^2\log L/\partial\beta\partial\sigma_i^2$ are the second derivatives of the homoscedastic tobit likelihood function as found in Amemiya (pp. 1000-1) or Maddala (pp. 154-155).2/ The second derivatives with respect to β are identical to the homoscedastic results regardless of the variance specification (see Amemiya; Maddala). Hence, analytical second derivatives for these alternative variance specifications are also straight forward extensions of the homoscedastic results.3/

Alternative Expectations, Marginal Effects and Elasticities

McDonald and Moffitt's decomposition for the homoscedastic Tobit model can also be generalized to include heteroscedastic formulations. These results are simplified considerably through use of the chain rule similar to (6), (9), and (10) above. Following McDonald and Moffitt, the Tobit conditional (EY^*) and unconditional (EY) expectations generalize under heteroscedasticity as

$$\text{(11)} \qquad EY_i^* = E(Y_i | Y_i > 0)$$
$$= X_i\beta + \sigma_i\phi(z_i)/\Phi(z_i)$$

$$\text{(12a)} \qquad EY_i = X_i\beta\Phi_i(z_i) + \sigma_i\phi(z_i)$$

$$\text{(12b)} \qquad = \Phi(z_i)*EY_i^*$$

where $z_i = X_i\beta/\sigma_i$ and (11), (12a) and (12b) correspond to (3), (2), and (4) of McDonald and Moffitt (p. 318), respectively.

From (12b), the marginal effects of the regressors of EY yield the identity

$$\text{(13)} \qquad \frac{\partial EY_i}{\partial x_k} = \Phi(z_i)\,\frac{\partial EY_i^*}{\partial x_k} + EY_i^*\,\frac{\partial \Phi(z_i)}{\partial x_k}.$$

Hence, analytical expressions for $\partial EY_i^*/\partial x_k$ and $\partial\Phi(z_i)/\partial x_k$ under the alternative variance specifications are required to compute this total effect decomposition. The marginal effect of the regressors on the probability of being above the limit (i.e., $Y > 0$) is

$$(14) \qquad \frac{\partial \Phi_i}{\partial x_k} = \phi_i \frac{\partial z_i}{\partial x_k}$$

$$= \left.\frac{\partial \Phi_i}{\partial x_k}\right|_{\sigma_i^2} + \frac{\partial \Phi_i}{\partial \sigma_i^2} \frac{\partial \sigma_i^2}{\partial x_k}$$

$$= \Phi\_X_{i0k} - \frac{\phi_i z_i}{2\sigma_i^2} \frac{\partial \sigma_i^2}{\partial x_k}$$

where $\Phi_i = \Phi(z_i)$, $\phi_i = \phi(z_i)$, $\partial\sigma_i^2/\partial x_k = \{0, \delta_k, \partial\sigma_i\delta_k, \sigma_i^2\delta_k\}$ for VAR0-VAR3, respectively, and $\Phi\_X_{i0k} = \phi_i\beta_k/\sigma_i$ corresponds to VAR0, the homoscedastic case in McDonald and Moffitt ((6), p.319).4/ Also, $\partial\sigma_i^2/\partial x_k = 0$ for all $x_k \notin Z$, i.e. any regressor not part of the variance specification. Hence, the change in the probability of being above the limit has two components under VAR1-VAR3: a direct effect similar to the homoscedastic response (i.e., Φ_X_{i0k}) and an indirect effect via the heteroscedastic variance. The direction of the bias introduced by omitting this indirect effect is unclear as the signs of z_i and δ_k are unknown a priori.

Similarly, note that heteroscedastic variances imply direct and indirect marginal effects on the conditional expectation EY^* where the indirect effects originate in the $\sigma_i\phi_i/\Phi_i$ component of (11). To see this, note that

$$(15) \qquad \frac{\partial EY_i^*}{\partial x_k} = \left.\frac{\partial EY_i^*}{\partial x_k}\right|_{\sigma_i^2} + \frac{\partial EY_i^*}{\partial \sigma_i^2} \frac{\partial \sigma_i^2}{\partial x_k}$$

$$= EY\underline{*}X_{i0k} + (1/2\sigma_i)(\phi_i/\Phi_i)[1 + z_i^2 + z_i\phi_i/\Phi_i] \frac{\partial \sigma_i^2}{\partial x_k}$$

where $EY^*\_X_{i0k} = \beta_k[1 - z_i\phi_i/\Phi_i - (\phi_i/\Phi_i)^2]$ corresponds to the homoscedastic (constant variance) marginal response of EY^* to the regressor x_k as in McDonald and Moffitt ((7), p.319), and $\partial(\sigma_i^2)/\partial x_k = 0$ for all $x_k \notin Z$.[5/]

Last, substitution of (14) and (15) into (13) yields the so called total effect (actually, the marginal response of unconditional expectation) due to the $k^{\underline{th}}$ regressor as

$$(16) \qquad \frac{\partial EY_i}{\partial x_k} = \left.\frac{\partial EY_i}{\partial x_k}\right|_{\sigma_i^2} + \frac{\partial EY_i}{\partial \sigma_i^2}\frac{\partial \sigma_i^2}{\partial x_k}$$

$$= EY\_X_{i0k} + \frac{\phi_i}{2\sigma_i}\frac{\partial \sigma_i^2}{\partial x_k}$$

where $EY\_X_{i0k} = \beta_k\Phi_i$ is the constant variance total response (McDonald and Moffitt, p.319).[6/] Elasticity conversion of the marginal effects (13)-(16) is straightforward (HRT, p.202, footnote 3). As noted by HRT in an elasticity context (p.202, footnote 3), any two estimates from (14), (15), and (16) will yield the third given the decomposition in (13). Hence, it may be convenient to compute (15) from (13), (14), and (16) as do HRT.

The preceeding discussion demonstrates that heteroscedastic variances are straightforward chain rule extensions of the homoscedastic Tobit model. Given the potential bias in estimating marginal responses suggested by (14), (15), and (16) (i.e. through the heteroscedastic variance induced components which augment the constant variance effects), applied researchers may wish to evaluate the magnitude and significance of the differences in marginal responses computed from heteroscedastic versus homoscedastic specifications.

While HRT provide some insight concerning magnitude differences, statistical tests for significant differences are not provided.

The Data and Empirical Model

Since our intent is to implement and show the impact alternative variance specifications on homoscedastic Tobit models commonly estimated in the literature, we analyze the fresh potato model and data used in CZC. We utilize data from the USDA 1977-78 Nationwide Food Consumption Survey (NFCS) to estimate the demand for at home fresh potato consumption in the Western region as a function of prices (fresh potatoes (PFRESH); frozen potatoes (PFROZEN); and dehydrated potatoes (PDEHYDRA)), family size (family size squared (FSIZE2)); 21 meal size (21MEALSIZE); and age/sex categories (number of household members: children less than 5 years old (KIDS<5); children age 6-15 (KIDS>5); adult females (FADULT); and adult males (MADULT)), the logarithm of income (LOGINCOME), and dummy variables for sex of meal planner MALEPLAN (male = 1), education of meal planner (ELEMED, elementary school = 1; COLLED, college education = 1), urbanization (SUBURB, suburban = 1; NMETRO, non-metro = 1), and geographic subregion (PACIFIC, pacific = 1). The omitted dummy variable categories are: female meal planner with high school education, metro, and mountain region. See CZC for more details concerning these data and the demand specification.

We follow CZC in computing asymptotic standard errors for elasticities associated with the total, conditional and market entry components of (13).[7/] Similarly, we follow CZC and use these standard errors to compute asymptotic "t-tests" of the significance of the differences between elasticity estimates generated under alternative variance assumptions.[8/] The results are provided in Table 2.

Results

The four alternative variance specifications of the tobit model are estimated by maximum likelihood (ML) techniques.[9/] The resultant parameter estimates and model summary statistics are compared in Table 1. As the parameters of Table 1 reflect the marginal effects of the regressors on the unobserved, latent variable (Maddala), it is frequently more insightful to contrast the marginal effects (13). Table 2 summarizes these contrasts in elasticity format as the decomposition of the tobit elasticities associated with the parameter estimates of Table 1. Table 3 then compares the magnitude and significance of the differences between these alternative elasticity estimates.

We evaluated several heteroscedastic variance specifications as subsets of CZC's original model. While economic theory does not provide much guidance concerning specification of the Z matrix in (4b)-(4d), we found little evidence nor justification for including prices; hence our variance specifications are restricted to household characteristics (income, family size and composition). While other potential sources of heteroscedasticity could be explored more fully, the model results we present performed quite well across all heteroscedastic specifications.[10/] Hence, we hold this Z specification constant across the heteroscedastic models for comparison purposes.

Similar to the findings of HRT (p.200 and their Table 2), we find that the relative standard errors of all heteroscedastic specifications in Table 1 are generally smaller than the comparable homoscedastic results. Hence, the significance levels of the heteroscedastic parameter estimates are generally higher than the homoscedastic results. Exceptions include the parameters for income (LOGINCOME), male meal planners (MALEPLAN), non-metro areas (NMETRO), and pacific subregion (PACIFIC). Note, however, that income is also a highly significant parameter in all heteroscedastic specifications (DLOGINCOME).

All parameters of the heteroscedastic specifications are significant at standard α levels (with the exception of 21-meal-size (21MEALSIZE)) and negative (with the exceptions of DINTERCEPT (as expected), family size squared (FSIZE2) and 21-meal-size (21MEAL)). The absolute value of the parameter estimates is smaller under all heteroscedastic specifications with the exceptions of price of frozen potatoes (PFROZEN), family size squared (FSIZE2), older children (KIDS>5), adult males (MADULT) and adult females (FADULT). However, as FMSIZE2, KIDS>5, MADULT, and FADULT all have large and highly significant δ estimates, the total impact of these regressors cannot be assessed from the parameter estimates in Table 1.

Comparison of the regression summary statistics from Table 1 indicates that all heteroscedastic specifications have greater log likelihood function values than does the homoscedastic specification. As the heteroscedastic specifications also have more parameters, this result is not unexpected. The magnitude ranking of the log likelihood function values, (VAR1, VAR2, VAR3, VAR0), while not a statistical test, indicates that VAR1 and VAR2 outperform VAR0, as does VAR3 to a lesser extent. Of the three heteroscedastic specifications, VAR3 appears to offer the smallest marginal improvement over VAR0 using this informal criterion.

Following Maddala (p. 180) and HRT, the heteroscedastic specifications nest the homoscedastic case (VAR0) when $\delta_i = 0$ for all i other than the intercept, δ_0. The likelihood ratio tests of the null hypothesis of homoscedastic variances versus the alternative heteroscedastic specifications are 248.12, 282.34 and 266.38 for VAR1-VAR3, respectively. These tests are asymptotically distributed as chi-square with 7 degrees of freedom (i.e., the difference in the number of parameters for VAR1-VAR3 versus VAR0). Given the corresponding critical Chi-Square value (18.5 at the 0.01 percent level), it follows that the null hypotheses of homoscedastic variance are soundly rejected in favor of heteroscedastic specifications. Similar results were found by HRT with respect to VAR2 in the context of broiler expenditures.

The four remaining performance measures of Table 1 are prediction criteria summarized across the sample used for estimation. CORR(Y,EY) and RMSE(EY) are the correlation of the observed dependent variable with the unconditional expectation from (12a) and the associated root mean square error computed over the full sample. CORR(Y1,EY*) and RMSE(EY*) are similarly defined for the conditional expectation from (11) computed over the conditional or non-limit sample (i.e., Y > 0). The magnitude ranking of the unconditional prediction based summary measures is {VAR0, VAR1, VAR2, and VAR3}. VAR1 and VAR2 are quite similar with VAR3 marginally worse in predictive power in these data. The magnitude ranking of the conditional sample relative predictive power is {VAR1/VAR2, VAR0, VAR3} for the CORR(Y1,EY*) measure and {VAR1, VAR2, VAR3, VAR0} for the RMSE(EY*) measure. Based on these predictive criteria, it follows that the homoscedastic model predicts better over the full sample while VAR1 and VAR2 predict better over the conditional sample; hence no clear preference among these estimators emerges based on these prediction criteria.11/

As indicated in the discussion of (13)-(16) above, the regressors contained in Z have both direct (i.e., via the β's) and indirect (i.e., via the δ's in the variance specification) impacts. Hence, it is more meaningful to compare the estimated "total" (i.e., direct and indirect) marginal effects across models rather than the estimated parameters from Table 1. We summarize these effects as elasticities. In order to demonstrate impacts of the alternative variance specifications and to facilitate comparison with CZC results, we limit our discussion to the elasticities presented by CZC, that is, the elasticity effects of PFRESH, PFROZEN, PDEHYDRA, LOGINCOME, and 21MEALSIZE. These elasticities and their associated significance levels are summarized in Table 2.12/

The price elasticities of Table 2 summarize direct effects (since these variables are not used in the variance specification) and demonstrate the impacts of alternative variance specifications on factors not inducing heteroscedasticity. The PFRESH elasticities are smaller (in absolute value) and of equal statistical significance under VAR1-VAR3 relative to VAR0 for all elasticities (i.e., total, market entry, and conditional). In contrast, the PFROZEN elasticities are larger and statistically significant at higher alpha levels under all heteroscedastic specifications relative to the homoscedastic. The PDEHYDRA elasticities are not statistically significant for any elasticity measure nor specification.

Table 3 indicates, however, that none of these differences among the alternative price elasticity estimates are statistically significant. Hence, the decomposition of total price effects into market entry and conditional components also do not differ significantly by variance specification. These results suggests that, for the model and data estimated, the variance specification <u>does not</u> have statistically significant impacts on response measures for variables that are not part of the variance specification.

Similar to the HRT results, however, variables with indirect effects can generate statistically significant differences in response measures. The total elasticities for both LOGINCOME and 21MEALSIZE are smaller in magnitude (absolute value for LOGINCOME) under VAR0 than VAR1-VAR3 a result similar to HRT (p.202). Despite the facts of modest improvement in parameter significance under VAR1-VAR3, total elasticities which are statistically significant at the 5 percent level or better, and some relatively large magnitude differences (e.g., LOGINCOME for VAR0 versus VAR3), Table 3 indicates that none of these differences in total elasticities are statistically significant.

Similar to HRT, and in contrast to the price elasticities and total elasticities for LOGINCOME and 2IMEALSIZE discussed above, the variations in the component elasticities can be quite different. For example, while there are no significant differences among any of the market entry or conditional elasticities for 21MEALSIZE (Table 3), all of the heteroscedastic models indicate small and statistically insignificant market entry elasticities but sizable and statistically significant conditional elasticities associated with LOGINCOME (Table 2). As indicated in Table 3, the heteroscedastic estimates of market entry and conditional LOGINCOME elasticities are statistically different from the homoscedastic estimates. Given that the heteroscedastic LOGINCOME conditional elasticities are 2-5 times as large as the homoscedastic estimate, the alternative variance specifications can have further demonstrated in Table 3 where the VAR1 conditional elasticity estimates for LOGINCOME are statistically different from both the VAR2 and VAR3 results. Given the likelihood ratio tests of the homoscedastic versus the alternative variance specification above, these results suggest that some of the elasticity estimates in CZC are quite questionable and that more attention to the variance specifications in tobit models is warranted.

Summary and Conclusions

The objectives of this article were to generalize the work of HRT in analyzing the impacts of alternative variance specifications for the commonly estimated homoscedastic tobit model. The results show three heteroscedastic specifications that are chain-rule extensions of the homoscedastic case when the sources of heteroscedasticity are a subset of the original independent variables. These generalizations are computationally easy to implement using ML procedures (such as found in GAUSS or TSP) and are amenable to testing heteroscedastic versus homoscedastic null hypotheses using likelihood ratio tests. Tobit elasticity decomposition procedures (following McDonald and Moffitt) also generalize in an intuitive manner using a chain-rule approach. Analytical expressions for these decompositions are provided in the hope that applied economists will use them to approximate asymptotic standard errors for the point elasticity estimates that usually are presented (alone) in the agricultural economics literature (see CZC).

The empirical application to the model and data used by CZC demonstrates the variety of impacts alternative variance specifications have on tobit models commonly found in the literature. In contrast to the HRT results, the computation of standard errors for the tobit elasticities allows comparison of the statistical significance of the differences in alternative elasticity estimates. Thus, while magnitude results for total elasticities similar to HRT were found (i.e., generally smaller under homoscedasticity for variables that are part of the heteroscedastic specification), none of the differences were statistically significant. Similar to the HRT results, significantly different market entry and conditional elasticity estimates for income were found under heteroscedasticity, while no significant differences were found for 21-meal-size. Both of these variables were used in the heteroscedastic specification. Last, the alternative variance specifications were found to have no significant impacts on the elasticity effects of regressors which were not part of the variance specification.

Since most cross-sectional food consumption data can be expected to manifest heteroscedastic variances, appropriate research procedures should be to test for its presence and perform corrections where necessary. In contrast to the general linear model, the costs of failure to do so in a limited dependent variable context are loss of parameter unbiasedness and consistency. In the context of the tobit models discussed, alternative heteroscedastic specifications are straight forward generalizations of extant, ML homoscedastic estimation routines. Hopefully this paper will make these procedures easier to implement and more widely used.

REFERENCES

Amemiya, T. "Regression Analysis When the Dependent Variable is Truncated Normal." Econometrica 41(1973):997-1016.

Arabmazar, A. and P. Schmidt. "Further Evidence on the Robustness of the Tobit Estimator to Heteroscedasticity." Journal of Econometrics 7(1981):253-258.

Cox, T. L. and R. F. Ziemer. "An Empirical Comparison of Alternative Tobit Estimates." Selected paper presented at the Western Agricultural Economics Association Meetings, July 7-9, 1985 Saskatoon, Canada. Abstract WJAE 10(1985):427.

Cox, T. L., R. F. Ziemer and J-P. Chavas. "Household Demand for Fresh Potatoes: A Disaggregated Cross-Sectional Analysis." Western Journal of Agricultural Economics 9(1984):41-57.

Cox, T. L. and M. K. Wohlgenant. "Prices and Quality Effects in Cross-Sectional Demand Analysis." American Journal of Agricultural Economics 68(1986):908-19.

Fomby, T., R. C. Hill and S. R. Johnson. Advanced Econometric Methods. New York: Springer-Verlag, New York, Inc., 1984.

Haines, P., D. K. Guilkey and B. M. Popkin. "Modeling Food Consumption Decision as a Two-Step Process." American Journal of Agricultural Economics 70(1988):541-552.

Huang, C. L., S. M. Fletcher, and R. Raunikar. "Modeling the Effects of the Food Stamp Program on Participating Household's Purchases: An Empirical Application." Southern Journal of Agricultural Economics 15(1981):21-28.

Huang, C. L., R. Raunikar, and H. L. Tynan. "Heteroscedasticity in Broiler Meat Expenditures Pattern Estimation." Western Journal of Agricultural Economics 11(1986):195-203.

Hurd, M. "Estimation in Truncated Samples When There is Heteroscedasticity." Journal of Econometrics 41(1979):247-58.

Lane, S. "Food Distribution and Food Stamp Effects on Food Consumption and Nutritional 'Achievement' of Low Income Persons in Kern County, California." American Journal of Agricultural Economics 60(1978):108-16.

Lee, J. Y. "Imputed Missing Incomes and Marginal Propensity to Consume Food." Western Journal of Agricultural Economics 11(1986):115-22.

Maddala, G. S. Limited Dependent and Qualitative Variables in Econometrics. Cambridge: Cambridge University Press, 1983.

Maddala, G. S. and F. D. Nelson. "Specification Errors in Limited Dependent Variable Models." New York: National Bureau of Economic Research Working Paper 96, 1975.

McCracken, V. A. and J. A. Brandt. "Household Consumption of Food Away From Home: Total Expenditure and by Type of Food Facility." American Journal of Agricultural Economics 69(1987):274-84.

McDonald, John F. and Robert A. Moffitt, "The Uses of Tobit Analysis." The Review of Economics and Statistics 62(1980):318-21.

Prais, S. J. and H. S. Houthakker. The Analysis of Family Budgets. Cambridge: Cambridge University Press, 1983.

Senauer, B. and N. Young. "The Impact of Food Stamps on Food Expenditures: Rejection of the Traditional Model." American Journal of Agricultural Economics 68(1986):37-43.

Thraen, C. S., J. W. Hammond and B. M. Buxton. "Estimating Components of Demand Elasticities from Cross-Sectional Data." American Journal of Agricultural Economics 60(1978):674-77.

ENDNOTES

1/ Note that (7) is scalar at each observation; hence, evaluating this scalar and the associated (8a)-(8d) via (6) yields the derivatives of interest.

2/ Notational correspondence between these cites and that used in this paper follows by noting that $\sigma_i f_i = \phi_i$, $F_i = \Phi_i$.

3/ These analytical expressions are not presented here to conserve space, but are available from the authors on request.

4/ Note that for VAR2, (14) yields $(\phi_i/\sigma_i)[\beta_k - z_i\delta_k]$ as found in HRT (eq (7), footnote 2, p. 200).

5/ Again note that for VAR2 (i.e., $\partial\sigma_i^2/\partial x_k = \partial\sigma_i\delta_k$), (15) yields (6) of HRT (p.200, footnote 2).

6/ For VAR2, (16) yields (8) of HRT (p. 200, footnote 2). Note the typo where their $\delta_j F(Z)$ in (8) should be $\delta_j f(Z)$ using $F(Z) = \Phi_i$ and $f(Z) = \phi_i$ to convert the HRT notation to that used here.

7/ We wish to thank Tom Cox for supplying his SAS PROC IML code (as well as the data used) for computation of these standard errors using analytical expressions for the homoskedastic case. We compared these results with those using numerical derivatives and found virtually identical results. As the analytical expressions required for the heteroscedastic specifications are extremely tedious, this comparison suggests that it is quite reasonable to use numerical derivatives for this purpose. Given the relative ease of using these numerical procedures with software such as GAUSS or SAS PROC IML, it follows that statistical bounds on these point estimates are relatively easy to obtain.

8/ In contrast to CZC, however, note that the parameter estimates (β, δ) (and hence, the associated elasticity response measures) from the alternative variance specifications are independent. Thus, the variance of the difference between two elasticities from alternative specifications has no covariance term (see CZC, p.50, footnote 12).

ENDNOTES

1/ Note that (7) is scalar at each observation; hence, evaluating this scalar and the associated (8a)-(8d) via (6) yields the derivatives of interest.

2/ Notational correspondence between these cites and that used in this paper follows by noting that $\sigma_i f_i = \phi_i$, $F_i = \Phi_i$.

3/ These analytical expressions are not presented here to conserve space, but are available from the authors on request.

4/ Note that for VAR2, (14) yields $(\phi_i/\sigma_i)[\beta_k - z_i\delta_k]$ as found in HRT (eq (7), footnote 2, p. 200).

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9/ All ML results are computed using analytical derivatives and the method of Newton (Fomby, Hill and Johnson, pp.610-611) with a log likelihood convergence criterion of 0.001. All specifications were verified using numerical derivatives, as well. As expected, the use of analytical derivatives converged more quickly (but took longer for each iteration), compared to the use of numerical derivatives. The estimated parameter variance-covariance matrices reported here, however, are computed from the analytical hessians of the alternative likelihood functions via (6), (7), (8a)-(8d), (9), and (10) following Amemiya and Maddala.

10/ Note that nested hypotheses concerning the specification of Z are easily generated. More exhaustive analyses of the variance specifications could thus use likelihood ratio tests to guide and motivate the final specification. However, this would entail additional computation expense. Moreover, possible pre-test bias remains a concern. Finally, these issues, while potentially interesting, are not the focus of this paper.

11/ It should be noted that if relative prediction accuracy is the research objective, then all of these Tobit models are likely to be found inferior to censored (full sample) or truncated (conditional sample) ordinary least squares (OLS). See Cox and Ziemer for discussion of this issue in the context of the homoskedastic Tobit model. In contrast, research interest in unbiased estimates of the marginal impacts of regressors in the context of limited dependent variables generally motivates the use of tobit-type models.

12/ The approximate, asymptotic standard errors for these measures as well as the continuous regressors not discussed by CZC (and not presented here), are available upon request.

Table 1. Comparison of Maximum Likelihood Tobit Parameter Estimates for Alternative Variance Specifications.

REGRESSORS	VAR0	VAR1	VAR2	VAR3
	BETA'S			
INTERCPT	-0.408 (1.350)	-2.305* (1.271)	-2.127* (1.262)	-1.675 (1.269)
FRESH	-4.927*** (1.159)	-3.244*** (0.957)	-3.130*** (0.973)	-3.289*** (0.995)
FROZEN	1.828* (0.996)	2.208*** (0.818)	2.135*** (0.819)	1.938** (0.835)
DEHYDRA	0.768 (0.496)	0.571 (0.403)	0.570 (0.408)	0.521 (0.418)
PSIZE2	-0.077*** (0.028)	-0.151*** (0.046)	-0.157*** (0.049)	-0.145*** (0.050)
21MEALSIZE	0.015*** (0.002)	0.015*** (0.001)	0.015*** (0.001)	0.015*** (0.001)
LOGINCOME	-0.255** (0.129)	-0.097 (0.127)	-0.124 (0.121)	-0.160 (0.122)
MALEPLAN	-1.188*** (0.418)	-0.812** (0.353)	-0.773** (0.351)	-0.772** (0.357)
ELEMED	1.136*** (0.346)	0.959*** (0.321)	0.927*** (0.325)	0.916*** (0.327)
COLLED	-0.966*** (0.214)	-0.859*** (0.178)	-0.859*** (0.177)	-0.852*** (0.181)
SUBURB	0.059 (0.229)	-0.042 (0.188)	-0.008 (0.188)	0.031 (0.193)
NMETRO	0.917*** (0.273)	0.543** (0.233)	0.578** (0.234)	0.639*** (0.237)
PACIFIC	-0.580** (0.271)	-0.329 (0.228)	-0.376* (0.228)	-0.406* (0.233)
KIDS<5	-0.429 (0.299)	-0.021 (0.353)	0.036 (0.365)	0.029 (0.367)
KIDS>5	0.497* (0.288)	0.849** (0.358)	0.858** (0.376)	0.757** (0.380)

Table 1. Continued.

REGRESSORS	VAR0	VAR1	VAR2	VAR3
MADULTS	1.010***	1.294***	1.323***	1.274***
	(0.284)	(0.327)	(0.334)	(0.337)
FADULTS	0.053	0.636	0.700*	0.696*
	(0.332)	(0.394)	(0.395)	(0.391)
	------------ DELTA'S (VARIANCE ELEMENTS) ------------			
DINTERCEPT	18.736***	49.298***	9.350***	5.364***
	(0.700)	(3.625)	(0.890)	(0.460)
DLOGINCOME	-	-3.190***	-0.574***	-0.296***
		(0.365)	(0.112)	(0.057)
DFSIZE2	-	2.833***	0.244***	0.068***
		(0.477)	(0.043)	(0.014)
D21MEALSIZE	-	0.009	0.002*	0.001
		(0.006)	(0.001)	(0.001)
DKIDS<5	-	-15.006***	-1.409***	-0.467***
		(2.709)	(0.294)	(0.121)
DKIDS>5	-	-14.472***	-1.131***	-0.223*
		(3.245)	(0.324)	(0.128)
DMADULTS	-	-12.892***	-1.216***	-0.367***
		(2.423)	(0.283)	(0.123)
DFADULTS	-	-9.030***	-0.734**	-0.184
		(2.922)	(0.314)	(0.133)
PREDICTED VARIANCE	18.74	17.99	16.22	14.95
REGRESSION SUMMARY STATISTICS:				
LOGL	-4980.870	-4838.810	-4839.700	-4847.680
CORR(Y,EY)	0.497	0.483	0.482	0.421
RMSE(EY)	3.393	3.416	3.418	3.571
CORR(Y1,EY*)	0.236	0.317	0.317	0.247
RMSE(EY*)	3.722	3.679	3.700	4.357

NOTE: Standard errors are in parentheses. Significance levels for alpha = 0.10, 0.05, and 0.01 are indicated by *, **, and ***, respectively. The variance specifications are denoted as: VAR0 = Homoskedastic Variance; VAR1 = Linear Variance; VAR2 = Linear Standard Deviation; and, VAR3 = Multiplicative Variance.

SOURCE: Computations by the authors.

Table 2. Comparison of McDonald and Moffitt Tobit Decompositions for the Alternative Variance Specifications: Selected Regressors as Elasticities.

REGRESSOR	VAR0	VAR1	VAR2	VAR3
		Total Effect Elasticities		
PFRESH	-.170***	-.112***	-.113***	-.122***
PFROZEN	.210*	.253***	.255***	.239**
PDEHYDRA	.183	.135	.141	.133
LNINCOME	-.567**	-.657**	-.981***	-1.085***
21MEALSIZE	.929***	.918***	.982***	1.009***
		Market Entry Elasticities		
PFRESH	-.091***	-.059***	-.059***	-.063***
PFROZEN	.113*	.132***	.133***	.123**
PDEHYDRA	.098	.071	.073	.069
LNINCOME	-.305**	.088	.181	.152
21MEALSIZE	.499***	.450***	.461***	.473***
		Conditional Elasticities		
PFRESH	-.079***	-.053***	-.054***	-.059***
PFROZEN	.097*	.120***	.122***	.116**
PDEHYDRA	.085	.064	.068	.065
LNINCOME	-.262**	-.746***	-1.162***	-1.558***
21MEALSIZE	.430***	.468***	.521***	.557***

NOTE: These elasticities are evaluated at the sample means. Significance levels are indicated by ***, **, and * for $\alpha = 0.01$, 0.05, and 0.10, respectively. Associated approximate, asymptotic standard errors and omitted continuous variable elasticities are available upon request.

SOURCE: Computations by the authors.

Table 3. Comparison of Magnitude and Significance of Differences in Tobit Elasticities for Selected Continuous Regressors.

REGRESSOR	VAR0 WITH VAR1			VAR0 WITH VAR2			VAR0 WITH VAR3		
	TOTAL	MKT ENTRY	COND'L	TOTAL	MKT ENTRY	COND'L	TOTAL	MKT ENTRY	COND'L
PFRESH	.059	.033	.026	.058	.033	.025	.048	.029	.019
	(.052)	(.028)	(.024)	(.053)	(.028)	(.025)	(.055)	(.029)	(.026)
PFROZEN	.043	.020	.023	.045	.020	.025	.029	.011	.019
	(.148)	(.079)	(.069)	(.150)	(.080)	(.071)	(.154)	(.081)	(.073)
PDEHYDRA	-.047	-.027	-.020	-.042	-.025	-.017	-.049	-.029	-.020
	(.152)	(.081)	(.071)	(.155)	(.082)	(.073)	(.159)	(.084)	(.075)
LNINCOME	-.091	.393*	-.484***	-.414	.485**	-.899***	-.518	.457**	-1.296***
	(.388)	(.223)	(.179)	(.390)	(.236)	(.237)	(.411)	(.234)	(.302)
21MEALSIZE	-.011	-.049	.039	.053	-.038	.092	.080	-.026	.128
	(.133)	(.071)	(.070)	(.135)	(.073)	(.074)	(.137)	(.075)	(.085)

REGRESSOR	VAR1 WITH VAR2			VAR1 WITH VAR3			VAR2 WITH VAR3		
PFRESH	-.001	0.000	-.001	-.011	-.004	-.006	-.010	-.005	-.005
	.048	.025	.023	.050	.026	.024	.051	.026	.025
PFROZEN	.003	.000	.002	-.013	-.009	-.004	-.016	-.009	-.007
	(.136)	(.071)	(.065)	(.139)	(.072)	(.067)	(.142)	(.074)	(.069)
PDEHYDRA	.006	.002	.003	-.002	-.002	.000	-.008	-.005	-.003
	(.139)	(.073)	(.067)	(.144)	(.075)	(.069)	(.147)	(.076)	(.071)
LNINCOME	-.324	.092	-.416*	-.428	.064	-.812***	-.104	-.028	-.396
	(.371)	(.240)	(.230)	(.393)	(.237)	(.297)	(.395)	(.250)	(.335)
21MEALSIZE	.064	.011	.053	.091	.023	.089	.027	.012	.036
	(.130)	(.068)	(.080)	(.132)	(.070)	(.089)	(.134)	(.072)	(.093)

NOTE: These differences are computed from the elasticity estimates in Table 2. Approximate asymptotic standard errors are in parentheses. Significance levels for the null hypotheses that the differences are not statistically different than zero are indicated by ***, **, and * for the alpha = 0.01, 0.05 and 0.10 levels, respectively. Similar results for the omitted continuous regressors are available upon request.

SOURCE: Computations by the authors.