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RISK, RATIONAL EXPECTATIONS, AND PRICE STABILIZATION IN THE U.S. CORN MARKET

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Abstract

The bounded price variation model is extended to include second, third, and fourth central moments of the truncated price distribution in the supply equation. The framework is used, in conjunction with Fair and Taylor's procedure for estimating nonlinear rational expectations models, to obtain maximum likelihood estimates of a model of the U.S. corn market. The model is used to simulate the market equilibrium effects of minimum-maximum price bands on the corn industry. The results show that price stabilization can affect both the shape and position of the truncated price distribution and that risk effects are potentially important.

Key Words: Bounded prices, corn market, higher moments, price stabilization, rational expectations, risk.

I. Introduction

Government programs have played an important role in U.S. agriculture for over fifty years (Cochrane and Ryan; Gardner). Although various policy instruments have been used, price support loan programs and target price-deficiency payment schemes have emerged as the cornerstone of most U.S. agricultural policies (Schmitz and Chambers). Consequently, much research has focused on specifying and estimating the effects of government price support programs on aggregate agricultural supply relationships (Houck and Ryan; Houck et al.; Gallagher; Lee and Helmberger).

While progress has been made, relatively little is known about the relationship between price supports and the subjective price and risk expectations of agricultural producers. This informational void is surprising given that (1) risk variables are often important in agricultural supply models (Behrman; Just; Traill; Hurt and Garcia), and that (2) the presence of price supports will modify the subjective price and risk expectations of producers (Boehlje and Griffin; Featherstone et al.). The implication is that the supply response often attributed to government price support activities may not be adequately characterized. An inadequate supply response characterization can, in turn, have important implications for effectively measuring the potential costs and benefits associated with government intervention (Newbery and Stiglitz) and for determining the market equilibrium response to modifications in the risk environment faced by producers (Brorsen, Chavas, and Grant; Myers).

Most recently, the linkages between government price supports and the subjective expectations of producers have been explored using bounded price variation models (Shonkwiler and Maddala; Holt and Johnson; Holt). In short, this approach assumes that producers form rational price expectations while explicitly recognizing that price supports truncate the density function of price. However, with the exception of Holt's study, to date bounded price variation models have not been extended to include price risk.

Given the above, the objectives of this paper are to (a) present a framework for examining the effects of government price support programs on producers' rational price and risk expectations, and (b) to investigate empirically the potential effects of price stabilization on the U.S. corn market. In so doing, this paper builds on recent research by Aradhyula and Holt, Antonovitz and Green, and others by including rational expectations of the first four central moments of the truncated price distribution in the supply equation. Third and fourth central moments can be important because even if the underlying price distribution is symmetric, truncation can result in price distributions which are skewed and/or leptokurtic or platykurtic (Johnson and Kotz). Previous research has found higher-order moments to be important at the micro level (e.g., Antle and Goodger; Buccola; Antle; Nelson and Preckel). Yet this study is the first known attempt to include such a detailed specification of price risk in an aggregate supply model.

The remainder of the paper is organized as follows. The next section presents a conceptual framework for examining the effects of government intervention in a market equilibrium setting. A bounded price variation model that includes rational expectations and price risk is then introduced and its econometric implications explored. Next, a supply-demand model for the U.S. corn market which includes price risk, rational expectations, and truncation is specified and estimated. The model is then used to examine the effects of price stabilization in a market equilibrium framework. The final section reviews the results and concludes the study.

II. Conceptual Framework

This section develops a market model that includes rational expectations, price uncertainty, and risk-averse producers. The conceptual framework developed here provides the foundations for the empirical work reported in following sections.

Consider a competitive industry consisting of N identical firms, each producing a homogeneous commodity and facing a random output price, p. Although free entry and exit is permitted, the number of firms N is assumed fixed in the short run. Furthermore, the output price is not observed at the time production decisions are made. Production technology is represented by the production function $q = f(x)$ where x is a vector of inputs. To simplify the discussion, production is assumed to be nonstochastic. Random market price p is characterized by the stochastic inverse market demand function

$$p = D(Q, u), \ \partial p/\partial Q < 0, \tag{1}$$

where u is a random variable with distribution function H(u) and $Q = Nq$ is industry output. A given output value, say Q^e, determines the conditional price distribution $F(p|Q^e)$. Consequently, expected market price is

$$\bar{p}(Q^e) = \int_0^\infty p dF(p|Q^e) = \int_0^\infty D(Q, u) dH(u) \tag{2}$$

and second- and higher-order central moments of the price distribution are given by

$$\sigma_p^k(Q^e) = \int_0^\infty (p - \bar{p})^k dF(p|Q^e) = \int_0^\infty [D(Q, u) - \bar{D}]^k dH(u), \; k \geq 2. \tag{3}$$

Government intervention occurs through a system of minimum and maximum prices that are used to support and stabilize prices received by producers.[1/] The minimum price is denoted by p_m and the maximum price is given by p_M. Following Quiggin and Anderson, this minimum-maximum price system is enforced by a tax-subsidy or buffer-fund scheme. Producers receive a direct per-unit subsidy equal to $(p_m - p)$ if the realized market price falls below the minimum price. Likewise, if the market price exceeds p_M, producers pay a direct per-unit tax equal to $(p - p_M)$. Otherwise, the stabilization agency takes no action.

Minimum and maximum price bands will truncate the probability density function of price, as perceived by producers, from below at p_m and above at p_M (Eeckhoudt and Hansen). The truncated random price v is equal to p_m when $p \leq p_m$ and p_M when $p \geq p_M$. Stochastic terms v and p coincide if $p_m < p < p_M$. Hence, the truncated conditional density $g(v|Q)$ takes a zero value in the intervals $[0, p_m]$ and $[p_M, \infty]$. At $p = p_m$, $g(v|Q)$ assumes a value equal to the probability $F(p_m|Q)$ and at $p = p_M$, $g(v|Q)$ takes a value equal to the probability $1 - F(p_M|Q)$. For values of p strictly between p_m and p_M, the densities $f(p|Q)$ and $g(v|Q)$ are identical.

Under risk aversion, firms attempt to maximize expected utility of profit. Assuming that each firm possesses a von Neumann-Morgenstern utility function $u(\pi)$ which is increasing $(du/d\pi > 0)$ and concave $(d^2u/d\pi^2 < 0)$ under risk aversion, the problem is to

$$\max_{\{x\}} Eu(\pi) = \int_{p_m}^{p_M} u[vf(x) - r'x]dG(v|Q^e) \tag{4}$$

where $\pi = pf(x) - r'x$ is random profit (r' being a vector of known input prices) and $G(v|Q^e)$ represents the decision maker's subjective beliefs about the truncated price distribution $G(v|Q)$. The first-order conditions associated with (4) are

$$\frac{\partial Eu(\pi)}{\partial x} = \int_{p_m}^{p_M} u'[vf_x(x) - r']dG(v|Q^e) = 0. \tag{5}$$

Assuming that the sufficient second-order conditions are satisfied, (5) can be solved, in principle, for the optimal choice functions $x^*(r, \bar{v}, \underset{\sim}{\sigma}_v^k; Q^e)$ and $q^*(r, \bar{v}, \underset{\sim}{\sigma}_v^k; Q^e) = f(x^*(r, \bar{v}, \underset{\sim}{\sigma}_v^k; Q^e)$. Here $\bar{v}$ denotes the mean of the truncated density $g(v|Q^e)$ and $\underset{\sim}{\sigma}_v^k$ denotes a vector of second- and (possibly) higher-order central moments associated with $g(v|Q^e)$. The notation used indicates that the firm's optimal decisions depend on the subjective estimate, Q^e, of industry output.[2/]

Industry supply is obtained by summing firm-level supply across all producers and is given by

$$Q = Nq^*(r, \bar{v}, \underset{\sim}{\sigma}_v^k; Q^e) = Q(r, v, \underset{\sim}{\sigma}_v^k; Q^e) \tag{6}$$

To close the model, it is necessary to relate Q^e to Q . This correspondence is obtained by assuming that agents form rational expectations about the truncated price distribution, $g(v|Q)$ (Newbery and Stiglitz). This implies that if industry output Q in (6) differs from expected output, Q^e, that agents will revise their output estimates. Hence, a short-run rational expectations equilibrium is characterized by the condition $Q^e = Q$, the closing identity in a rational expectations model with price uncertainty and risk-averse agents.

Under a competitive rational expectations equilibrium, industry output represents the fixed-point of the mapping from the right-hand side to the left-hand side of (6). The equilibrium n-tuple of industry output and expected moments of price solves the system of m + 1 equations

$$Q^* = Q(r, \bar{v}, \sigma_v^k; Q^*),$$

$$\bar{v} = p_m F(p_m | Q^*) + \int_{p_m}^{p_M} p dF(p|Q^*) + p_M[1 - F(p_M|Q^*)] \tag{7}$$

$$\sigma_v^k = p_m^k F(p_m|Q^*) + \int_{p_m}^{p_M} [p - \bar{p}]^k dF(p|Q^*) + p_M^k[1 - F(p_M|Q^*)], \; k = 2, \ldots, m.$$

Eeckhoudt and Hansen have investigated the comparative statics associated with a mean-preserving price squeeze implemented through a minimum-maximum price system. While their results show that a mean-preserving price squeeze unambiguously increases output of risk-averse firms, they did not consider market feedback.[3/] In the present case, a positive supply response induced by a minimum-maximum price squeeze will result in lower expected market prices. But expected market prices determine the values of the moments for the truncated price distribution in (7). The net result is that changes in minimum and maximum prices will have both a direct (i.e., truncation) and indirect (i.e., expected market price) impact on the moments of the truncated price distribution.

As illustrated in subsequent sections, the indirect effect may actually dominate the direct effect. This means that the expected producer price, $\bar{v}$, would fall as the minimum-maximum price band is squeezed.[4/] Consequently, the effects of price stabilization on production, expected producer price, and other variables of interest must be determined empirically when market feedback is incorporated.

II. Estimation Framework

In this section, an empirical framework is developed which maintains key elements of the stylized model including rational expectations, exogenous price limits, stochastic demand, and risk-averse producers. In particular, the bounded prices model under rational expectations considered by Chanda and Maddala (1983, 1984), Shonkwiler and Maddala, and Holt and Johnson is extended to include higher-order moments of the (truncated) producer price distribution

Consider the following market model with an exogenously set lower price limit, $\bar{P}_t$:[5/]

Demand

$$D_t = \alpha_1' X_{1t} + \alpha^* P_t + u_{1t} \tag{8}$$

Supply

$$S_t = \beta_1' X_{2t} + \beta_1^* P_t^e + \beta_2^* \sigma_{vt}^2 + \beta_3^* \sigma_{vt}^3 + \beta_4^* \sigma_{vt}^4 + u_{2t} \tag{9}$$

$$Q_t = D_t = S_t \quad \text{if} \quad \bar{P}_t \leq P_t \tag{10}$$

$$Q_t = D_t < S_t \quad \text{if} \quad P_t > \bar{P}_t \tag{11}$$

where D_t is quantity demanded, S_t is quantity supplied, Q_t is quantity transacted, P_t is the market clearing price, and P_t^e is the rational expectation of price formed at the time production decisions are made. Likewise, σ_{vt}^2 is the rational expectation of price variance, σ_{vt}^3 is the rational expectations of the third central moment of price, and σ_{vt}^4 is the rational expectation of the fourth central moment of price, also formed at the time production decisions are made. Terms X_{1t} and X_{2t} denote vectors of supply and demand shifters, respectively, and u_{1t} and u_{2t} are joint normally distributed random variables with mean zero and variance-covariance matrix Σ. With observations on P_t and $\bar{P}_t$, the data points belonging to equilibrium (Ψ_1) and those belonging to excess supply (Ψ_2) can be classified.

The model in (8)-(11) represents a market for a commodity where price supports truncate the equilibrium price distribution and where agents form rational expectations. The model differs from previous specifications of bounded price variation models in that rational expectations of the second through fourth central moments of price have also been included in the supply equation. The maintained hypothesis is that producers are risk averse and that government price support operations will explicitly modify producers' perceptions about the stochastic environment in which prices are determined. Estimation can proceed only after the model is closed by incorporating the rational expectations assumption.

The expressions for the rational expectations of price and price uncertainty are derived as follows. The restricted reduced form price equation derived from (8)-(10) is:

$$P_t = (\alpha^*)^{-1}(\beta_1' X_{2t} + \beta_1^* P_t^e + \beta_2^* \sigma_{vt}^2 + \beta_3^* \sigma_{vt}^3 + \beta_4^* \sigma_{vt}^4 - \alpha_1' X_{1t} + u_{2t} - u_{1t}). \quad (12)$$

Taking the expectation of both sides of (12) conditional on Ω_{t-1}, the information set available at the time production decisions are made, gives the rational price predictor:

$$P_t^* = (\alpha^*)^{-1}(\beta_1' X_{2t}^e + \beta_1^* P_t^e + \beta_2^* \sigma_{vt}^2 + \beta_3^* \sigma_{vt}^3 + \beta_4^* \sigma_{vt}^4 - \alpha_1' X_{1t}^e) \tag{13}$$

where X_{1t}^e and X_{2t}^e denote respectively the expectations of (unknown) demand and supply shifters. Likewise, taking the conditional variance operator through (12) gives an expression for the rational expectation of price variance (Aradhyula and Holt):

$$\sigma_p^2 = (\alpha^*)^{-2}(\beta_1'\psi_1\beta_1 + \alpha_1'\psi_2\alpha_1 + \alpha_1'\psi_3\beta_1 + \sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) \tag{14}$$

where ψ_1 is the variance-covariance matrix associated with (unknown) supply shifters , ψ_2 is similarly defined for demand shifters, and ψ_3 is the variance-covariance matrix between X_{1t} and X_{2t}. Likewise, σ_1, σ_2, and σ_{12} are the structural variance-covariance terms from the matrix Σ.

The truncation effects of the price support program are incorporated by accounting for the probability that the support price will be effective. Given the joint normality of the error terms u_{1t} and u_{2t}, it follows that the underlying (e.g., untruncated) price distribution is also normal. It can then be shown that the expressions relating the untruncated expectations in (13) and (14) to the expectations of the first four central moments of truncated price are:[6/]

$$P_t^e = \bar{P}_t\Phi(K_t) + \sigma_p\phi(K_t) + P_t^*[1 - \Phi(K_t)], \tag{15}$$

$$\sigma_{vt}^2 = \bar{P}_t^2\Phi(K_t) + \sigma_p^2 K_t\phi(K_t) + 2P_t^*\sigma_p\phi(K_t) + [P_t^{*2} + \sigma_p^2][1 - \Phi(K_t)] - P_t^{e2}, \quad (16)$$

$$\sigma_{vt}^3 = \bar{P}_t^2[\bar{P}_t - 3P_t^e]\Phi(K_t) + \sigma_p^3 K_t^2 \phi(K_t) + 3\sigma_p^2 (P_t^* - P_t^e)K_t\phi(K_t)$$

$$+ [2\sigma_p^3 + 3P_t^{*2}\sigma_p - 6P_t^eP_t^*\sigma_p]\phi(K_t) + [P_t^{*3} - 3P_t^eP_t^* \quad (17)$$

$$+ 3\sigma_p^2(P_t^* - P_t^e)][1 - \Phi(K_t)] + 2P_t^{e3},$$

$$\sigma_{vt}^4 = \bar{P}_t^2[\bar{P}_t^2 - 4P_t^e\bar{P}_t + 6P_t^{e2}]\Phi(K_t) + \sigma_p^4 K_t^3 \phi(K_t) + 4\sigma_p^3 [P_t^* - P_t^e] K_t^2 \phi(K_t)$$

$$+ 3[\sigma_p^4 + 2\sigma_p^2(P_t^* - P_t^e)^2] K_t \phi(K_t) + 4\sigma_p[2\sigma_p^2(P_t^* - P_t^e) \quad (18)$$

$$+ P_t^*(P_t^{*2} - 3P_t^eP_t^* + 3P_t^{e2})] \phi(K_t) + [P_t^{*4} - 4P_t^eP_t^{*3} + 6P_t^{e2}P_t^{*2}$$

$$+ 6\sigma_p^2(P_t^* - P_t^e)^2 + 3\sigma_p^4][1 - \Phi(K_t)] - 3P_t^{e4},$$

where,

$$K_t = [\bar{P}_t - (\alpha^*)^{-1}(\beta_1 X_{2t}^e + \beta_1^* P_t^e + \beta_2^* \sigma_{vt}^2 + \beta_3^* \sigma_{vt}^3 + \beta_4^* \sigma_{vt}^4 - \alpha_1' X_{1t}^e]/\sigma_p \quad (19)$$

and

$$1 - \Phi(K_t) = \text{Prob}[(\alpha^*)^{-1}(u_{2t} - u_{1t}) > \bar{P}_t - (\alpha^*)^{-1}(\beta_1' X_{2t} + \beta_1^* P_t^e \quad (20)$$

$$+ \beta_2^* \sigma_{vt}^2 + \beta_3^* \sigma_{vt}^3 + \beta_4^* \sigma_{vt}^4 - \alpha_1' X_{1t})].$$

Here $\Phi(\cdot)$ and $\phi(\cdot)$ denote respectively the distribution and density functions of the standard normal. Likewise, $1 - \Phi(K_t)$ is the probability that the support price is not effective. The result is that the expressions for the truncated moments in equations (15)-(18) are functions of the price support level, the probability of market equilibrium, and the mean and variance of the underlying (untruncated) price distribution.

The simultaneous solution of equations (13)-(20) yields the rational expectations for the first four central moments of the truncated price distribution in the bounded prices model. Clearly, an analytical solution cannot be obtained; but given some initial estimates of the structural parameters, it is possible to solve the system numerically. Following Fair and Taylor, a Gauss-Seidel simulation algorithm can be embedded in an iterative nonlinear maximum likelihood estimation routine.[7/] The resulting numerical solutions for the rational predictors contain implicitly all structural information implied by the rational expectations hypothesis just as they would if analytical reduced forms could be obtained.

III. Empirical Model and Estimation Results

The above procedures are used to estimate a bounded price variation model of the U.S. corn market. The model consists of two structural equations which identify the total demand and supply of corn, as well as three autoregressive models for predicting the values of the unknown exogenous variables used in the rational expectations simulations.

Model Specification

The specific form of the demand equation is

$$D_t = \alpha_o + \alpha_1 P_t + \alpha_2 EXP_t + \alpha_3 INC_t + \alpha_4 LPRO_t + u_{1t}$$

where EXP_t denotes corn exports, INC_t is total disposable income, and $LPRO_t$ is a livestock production index. Exports are included to account for the largely exogenous growth in livestock herds in importing countries (Arzac and

Wilkinson). Income reflects shifts in the derived demand for corn due to increased consumer demand for livestock products. Likewise, higher levels of livestock production should increase the demand for corn. For observations belonging to Ψ_1, the market is in equilibrium, $D_t = S_t + STK_{t-1}$ where STK_{t-1} represents carryover stocks, and P_t is freely determined. Likewise, for observations in Ψ_2 the market is in disequilibrium, $D_t = S_t + STK_{t-1} - CCC_t$ where CCC_t is government removals, and P_t is set equal to the loan rate.

The supply equation is specified as

$$S_t = \beta_o + \beta_1 P_t^e + \beta_2 \sigma_{vt}^2 + \beta_3 \sigma_{vt}^3 + \beta_4 \sigma_{vt}^4 + \beta_5 t + \beta_6 S_{t-1} + \beta_7 PIK_t + u_{2t}.$$

Here P_t^e is the rational expectation of the truncated mean price and σ_{vt}^k is the rational expectation of the k'th truncated central moment of price, $k = 2, \ldots, 4$. A linear trend t is included to capture technology shifts and lagged production S_{t-1} is included because complete adjustments to desired production levels may not occur within one year. Lastly, a dummy variable PIK_t is used to discount the effects of the PIK program on production in 1983.[8]

The model is estimated using data for 1950-1985. As a consequence, thirteen years in the sample are identified as disequilibrium periods (Ψ_2) while seventeen years are classified as market clearing (Ψ_1). Furthermore, beginning in 1962 the target price is used in place of the loan rate when solving the rational expectations model; however, the loan rate is still used in the demand equation during periods of disequilibrium. This approach is adopted because the target price is more indicative of the overall price support being offered to producers through government programs while the loan rate still serves as a market floor. Finally, univariate second-order autoregressive models are specified for the exogenous variables EXP_t, INC_t, and $LPRO_t$ since their values are unknown at the time expectations are formed.[9]

Estimation Results

Table 1 shows maximum likelihood results, along with several measures of model fit and performance. The Baxter and Cragg generalized R^2 for the estimated system is 0.989, indicating the model fits the data well. All estimated supply and demand parameters have theoretically correct signs and all coefficients associated with economic variables, with the exception of livestock production and lagged production, are significant at usual α levels. Importantly, the sign on the estimated coefficients for the expected price P_t^e and the expected third central moment of price σ_{vt}^3 in the supply equation are positive and significant. At the same time, the estimated coefficients for the expected second and fourth central moments of price (σ_{vt}^2 and σ_{vt}^4, respectively) are negative and significant.[10]

The estimation results reveal that the truncated price distribution is right-skewed for all observations with the average value of Pearson's skewness parameter being 2.407.[11] Positive skewness indicates the mean of the truncated price distribution is above the mode, implying that prices are more likely to be below the truncated mean. The truncated price distribution is also leptokurtic for all but five observations in the sample with an average value of 11.913 for Pearson's kurtosis coefficient. Hence, even though the underlying price distribution is normal, it follows that government price support programs result in producer price distributions which are no longer symmetric.

Given the above results, the signs on the coefficients associated with higher-order moments in the supply equation are plausible. For instance, the negative sign associated with σ^2_{vt} is justified under the assumption of risk aversion since risk-averse agents prefer decreasing variance (Meyer). The positive sign for σ^3_{vt} is also reasonable because agents that exhibit decreasing or constant absolute risk aversion prefer positive skewness (Tsiang). While similar reasoning cannot be used to rationalize the negative coefficient for σ^4_{vt}, this result is also plausible if producers prefer peaked distributions to flatter distributions (i.e., thinner tails to fatter tails).

The short-run elasticities of supply with respect to the truncated mean, variance, third central moment, and fourth central moment of price at the means of the data are 0.451, -0.027, 0.047, and -0.032, respectively. The short-run own-price elasticity of demand is -0.534. All elasticity estimates are reasonable and compare favorably with those reported elsewhere (e.g., Shonkwiler and Maddala; Brorsen, Chavas, and Grant; Holt and Johnson).

IV. Price Stabilization Experiments

The estimated rational expectations model provides a rich framework within which to examine alternative price support and stabilization strategies in a market equilibrium context. One advantage of the approach used is that the expressions for the truncated moments in (15)-(20) can be readily modified to allow for a maximum price (e.g., truncation of the upper tail). These modified expressions are reported in the Appendix. Operationally, a price stabilization program that uses a system of minimum and maximum prices could be administered in a fashion similar to that described in previous sections. That is, producers receive a subsidy if the market price falls below the minimum price and pay a tax if the realized price is above the maximum price.

The adjustments in production, expected market and producer prices, and higher-order moments of price are examined by simulating the model at the data means over a wide range of minimum and maximum prices.[12/] Selected results are reported in Table 2. Observe that low levels of the minimum price accompanied with high levels of the maximum price have little impact on expected prices, risk variables, or production. This implies that the market price distribution and the truncated price nearly coincide for relatively wide minimum-maximum price bands.

As the range over which prices can adjust freely is reduced, the second and fourth central moments of producer price decline and the third central moment increases initially, thus providing a direct incentive to expand production. However, higher production levels translate into lower expected market prices (Figure 1), thus resulting in lower expected producer (truncated) prices. In other words, the market price effect initially dominates the truncation effect. But for minimum price levels exceeding $1.40 and maximum price levels below $2.90, the truncation effect outweighs the market price effect, and the expected producer price increases (Figure 1).

Interestingly, production levels increase over the entire range of minimum and maximum prices (Figure 2). This occurs even though the expected producer price initially declines as the width of the price bands is narrowed (Figure 1). The implication is that the risk reduction induced by the system of minimum and maximum prices more than offsets the mean price response over a certain range. This result highlights the importance of risk in a market equilibrium framework analysis of price support or stabilization policies since production would clearly decline in the absence of risk effects.

The plots in Figures 3, 4, and 5 illustrate the effects of alternative price bands on the second, third, and forth central moments of producer price, respectively. The truncated variance tends to increase with the maximum price and decreases with the minimum price. The third central moment increases initially with the minimum price and then declines at the $1.40-$1.50 level. This response is similar to but opposite that of the truncated mean. Finally, the fourth central moment increases with the maximum price and decreases with the minimum price; a response similar to that observed for the truncated variance.

Pearson's skewness and kurtosis coefficients for the truncated price distribution can also be inferred from Table 2. As expected, skewness parameters are near zero for low levels of the minimum price ($0.30-$1.10) and high levels of the maximum price ($4.00-$3.20). At price bands below these levels, the producer price distribution becomes increasingly right-skewed. The values for the kurtosis parameters are also near zero initially, but become increasingly negative (e.g., platykurtic) as the band width is reduced initially. The truncated price distribution displays leptokurtic behavior only when the minimum price reaches $1.50 and the maximum price reaches $2.80. These results confirm that price stabilization can affect both the shape and the position of the producer price distribution.

Table 2 also illustrates the level of government involvement at alternative minimum and maximum price levels. The expected government subsidy, or the difference between the expected producer price and the expected market price, is initially low but increases to $0.51 per bushel when producers are allowed to face only a $0.30 price band. Interestingly, expected gross farm revenues actually decline initially as the minimum-maximum price band is reduced from the $0.30-$4.00 level; however, beyond the $1.10-$3.20 minimum-maximum price levels, expected gross farm revenues increase.

Expected gross farm revenues from government sources increase from zero initially to nearly 25% as the minimum-maximum band widths are reduced (Table 2).

Short-run flexibilities and elasticities with respect to minimum and maximum prices are reported in Table 3. The results show that the expected market price flexibilities with respect to the minimum price are small and negative for wide price bands and increase in magnitude as the band width is narrowed. Hence, minimum price increases will decrease the expected market price due to the production response induced by lower risk. Likewise, flexibilities for the expected producer price with respect to the minimum price are small and negative initially (the market price effect outweighs the truncation effect), increase in magnitude, and eventually become positive (the truncation effect outweighs the market price effect). Moreover, the biggest impact on expected market and producer prices occur with relatively narrow band widths.

Table 3 also contains elasticities for the second, third, and fourth central moments with respect to the minimum price. The elasticities for the second and fourth central moments are initially negative and small and increase in magnitude to nearly -10.0 when the band width is narrowed to \$0.30. Alternatively, elasticities for the third central moment are large (greater than 1.0) and positive for wide price bands and steadily decline and become negative as the band widths are narrowed.

Production elasticities with respect to the minimum price are also reported in Table 3. Production elasticities are positive at all levels and increase in magnitude to 0.371 when the band width is \$0.30. Finally, elasticities for $\Phi(K_{1t})$ (the probability of the minimum price being effective) and for 1 - $\Phi(K_{2t})$ (the probability of the maximum price being effective) with respect to the minimum price are also reported (Table 3). The results show that elasticities for $\Phi(K_{1t})$ are positive and elastic at all band widths, but decline in magnitude with band widths below \$1.50. A similar but opposite response is noted for the elasticities of 1 - $\Phi(K_{2t})$ with respect to the minimum price.

Similar results are reported in Table 3 with respect to the maximum price. Although space prohibits a complete description, it is interesting that small increases in the maximum price result in modestly higher production levels for most band widths. This result cannot be predicted from the theory since a ceteris paribus increase in the maximum price is associated with offsetting spread and location effects (Eeckhoudt and Hansen, p. 1066).

V. Conclusions

Previous research has not adequately addressed the relationship between government price support programs and the subjective price and risk expectations of producers. To explore this issue, conceptual and empirical models were developed that included minimum and maximum price limits, risk-averse producers, and rational expectations. Following Eeckhoudt and Hansen, the system of minimum and maximum prices was assumed to truncate producers' subjective density function of price. The empirical analysis was based on a bounded price variation model that included rational expectations of the first four central moments of the truncated price distribution. The result is that the empirical definition of risk used here is more detailed than in anyprevious aggregate-level study.

The empirical framework was used to obtain maximum likelihood estimates of a model of the U.S. corn market. The results are encouraging because truncation, rational expectations, and price risk (including third and fourth central moments) are all relevant in the estimated model. The model was then used to investigate the equilibrium (reduced form) impacts of alternative minimum and maximum price levels on the U.S. corn market. It was found, for instance, that because of market price feedback, expected producer prices actually decline for a wide range of minimum and maximum prices. However, production levels always increased as minimum-maximum price bands were narrowed. This result is entirely due to the risk reduction arising from the truncation of the producer price distribution and highlights the importance of viewing government price support and stabilization programs in an equilibrium setting.

More work is required to extend the results of this study. For instance, cross-commodity effects were not included in the supply equation. While this facilitated model specification and estimation, other research has shown that such relationships are potentially important (Chavas and Holt). It may also be desirable to extend the framework to include supply response for participants and nonparticipants in government programs since this would allow for a more complete characterization of the role of commodity programs in producer decision making. Finally, more work is required to extend the model to include different sources of risk.

Table 1. Maximum likelihood estimates of a bounded price variation model of the U.S. corn market, 1950-85.

Equation	Parameter	Variable	Coefficient	Standard Error
Demand	α_0	Intercept	49.474	10.289
	α_1	P_t	- 17.501	2.159
	α_2	EXP_t	3.131	0.324
	α_3	INC_t	0.119	0.030
	α_4	$LPRO_t$	0.004	0.146
Supply	β_0	Intercept	2.578	4.013
	β_1	P^e_t	11.146	2.227
	β_2	σ^2_{vt}	- 12.030	6.257
	β_3	σ^3_{vt}	55.298	1.169
	β_4	σ^4_{vt}	- 21.485	5.408
	β_4	t	1.067	0.213
	β_5	S_{t-1}	0.130	0.106
	β_6	PIK_t	- 44.777	3.520
Exports	η_0	Intercept	0.968	0.488
	η_1	EXP_{t-1}	0.836	0.122
	η_2	EXP_{t-2}	0.096	0.122
Income	γ_0	Intercept	0.252	0.736
	γ_1	INC_{t-1}	1.512	0.206
	γ_2	INC_{t-2}	- 0.467	0.228
Livestock	θ_0	Intercept	5.157	1.985
	θ_1	$LPRO_{t-1}$	0.725	0.164
	θ_2	$LPRO_{t-2}$	0.234	0.161
Log of Likelihood			-201.755	
Generalized R^2			0.989	

Table 2. Simulations of Corn Supply/Demand Model with Alternative Minimum and Maximum Price Levels.

Prices				Higher Moments				Government Subsidy		Gross Farm Revenue		
										With	Without	
Minimum	Maximum	Market	Producer	Second a/	Third	Fourth	Production	Unit	Total	Subsidy	Subsidy	Percent b/
0.30	4.00	2.10	2.10	0.327	0.001	0.317	3.987	0.00	0.00	8.39	8.39	0.00
0.50	3.80	2.09	2.09	0.326	0.002	0.307	4.005	0.00	0.00	8.39	8.39	0.00
0.70	3.60	2.07	2.07	0.321	0.005	0.285	4.048	0.00	0.00	8.38	8.38	0.00
0.90	3.40	2.01	2.02	0.309	0.015	0.246	4.145	0.00	0.02	8.37	8.35	0.00
1.10	3.20	1.90	1.92	0.298	0.039	0.184	4.339	0.02	0.08	8.34	8.26	0.01
1.30	3.00	1.76	1.83	0.215	0.062	0.114	4.585	0.07	0.30	8.38	8.08	0.04
1.40	2.90	1.71	1.81	0.177	0.061	0.085	4.677	0.10	0.48	8.48	8.00	0.06
1.50	2.80	1.67	1.82	0.141	0.054	0.059	4.743	0.15	0.70	8.63	7.93	0.08
1.60	2.70	1.64	1.84	0.107	0.043	0.038	4.793	0.20	0.96	8.84	7.88	0.11
1.70	2.60	1.62	1.88	0.075	0.029	0.021	4.834	0.26	1.26	9.09	7.83	0.14
1.80	2.50	1.60	1.93	0.048	0.017	0.010	4.875	0.33	1.61	9.39	7.78	0.17
1.90	2.40	1.57	1.98	0.025	0.007	0.003	4.924	0.41	2.03	9.76	7.73	0.21
2.00	2.30	1.53	2.04	0.009	0.002	0.001	4.987	0.51	2.55	10.19	7.64	0.25

Note: All results were obtained by simulating the rational expectations model at the means of the sample data. All prices are expressed in dollars per bushel and production is in billion bushels. Gross farm revenues are expressed in billions of dollars.

a/ The variance of the untruncated (e.g., market) price distribution is 0.328.

b/ Percent denotes the proportion of gross farm revenue (including subsidies) that is attributed to government support.

Table 3. Elasticities for Selected Variables with Respect to Minimum and Maximum Prices.

Elasticity w.r.t.	Minimum Price	Maximum Price	Prices: Market	Prices: Producer	Higher Moments: Second[a]	Higher Moments: Third	Higher Moments: Fourth	Production	$\Phi(K_{1t})$	$1-\Phi(K_{2t})$
Minimum Price:										
	0.50	3.80 :	-0.021	-0.020	-0.016	7.033	- 0.080	0.019	3.365	-0.245
	0.80	3.50 :	-0.125	-0.115	-0.136	8.497	- 0.442	0.110	5.708	-1.214
	1.10	3.20 :	-0.436	-0.317	-1.033	5.311	- 1.642	0.334	7.816	-3.214
	1.40	2.90 :	-0.334	0.078	-2.607	-0.677	- 2.846	0.214	4.342	-2.199
	1.70	2.60 :	-0.336	0.474	-4.718	-4.460	- 5.529	0.197	2.646	-1.855
	2.00	2.30 :	-0.690	0.806	-9.305	-9.617	-10.396	0.371	1.498	-2.933
Maximum Price:										
	0.50	3.80 :	0.002	0.003	0.025	12.417	0.176	-0.002	-0.026	-9.099
	0.80	3.50 :	0.001	0.005	0.089	6.710	0.522	-0.001	-0.008	-8.553
	1.10	3.20 :	-0.014	-0.003	0.181	2.572	1.110	0.011	0.085	-8.023
	1.40	2.90 :	-0.044	-0.012	0.315	1.904	2.325	0.028	0.155	-7.515
	1.70	2.60 :	-0.092	0.004	1.205	4.162	6.838	0.054	0.185	-6.729
	2.00	2.30 :	-0.087	0.060	10.151	25.983	51.367	0.047	0.083	-5.691

Note: All results were obtained by simulating the rational expectations model at the means of the sample data. Variable $\Phi(K_{1t})$ denotes the probability that expected market price will be below the minimum price and $1-\Phi(K_{2t})$ is the probability that the expected market price will be above the maximum price.

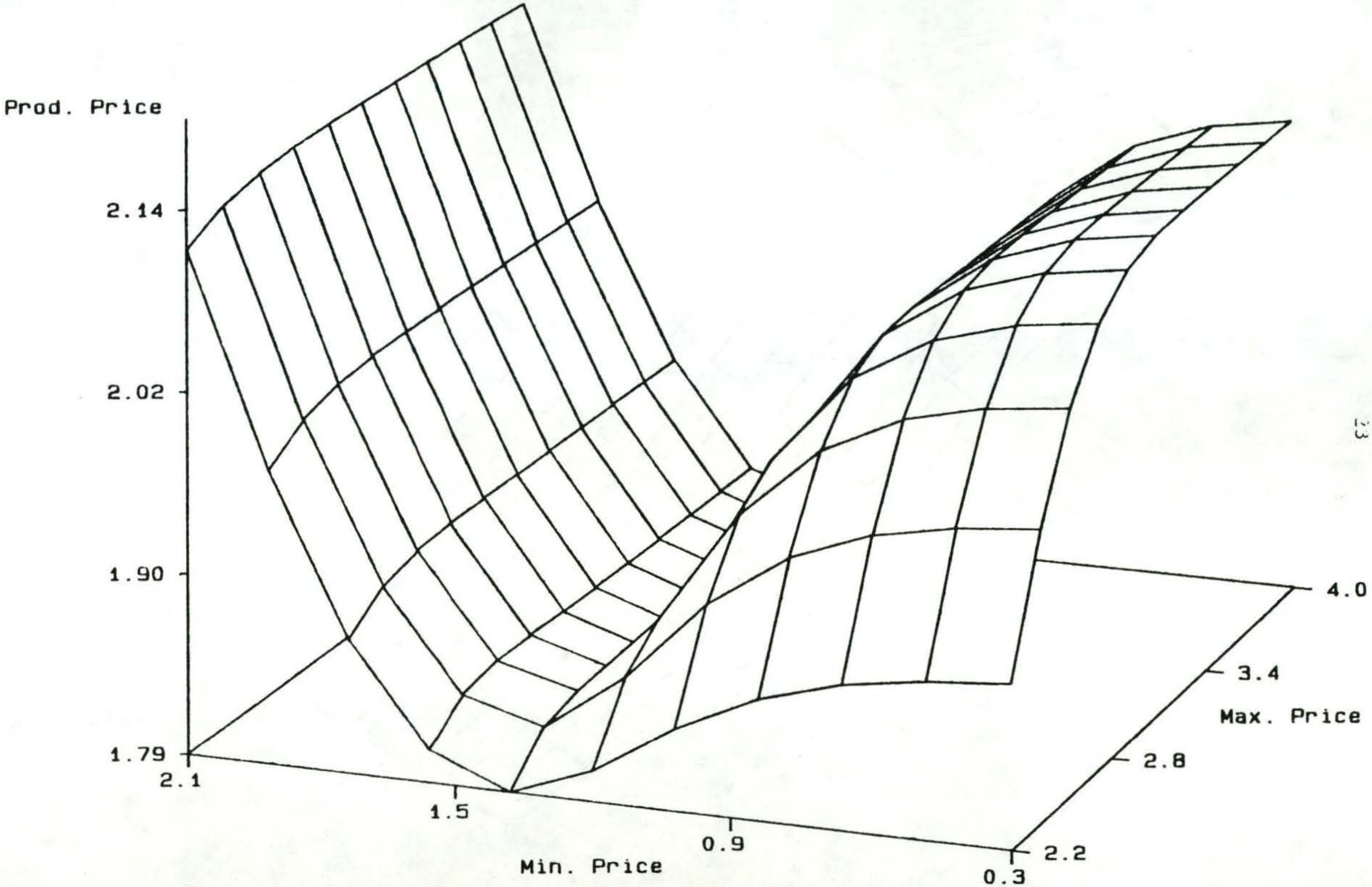

Figure 1. Relationship between expected producer price and minimum and maximum prices

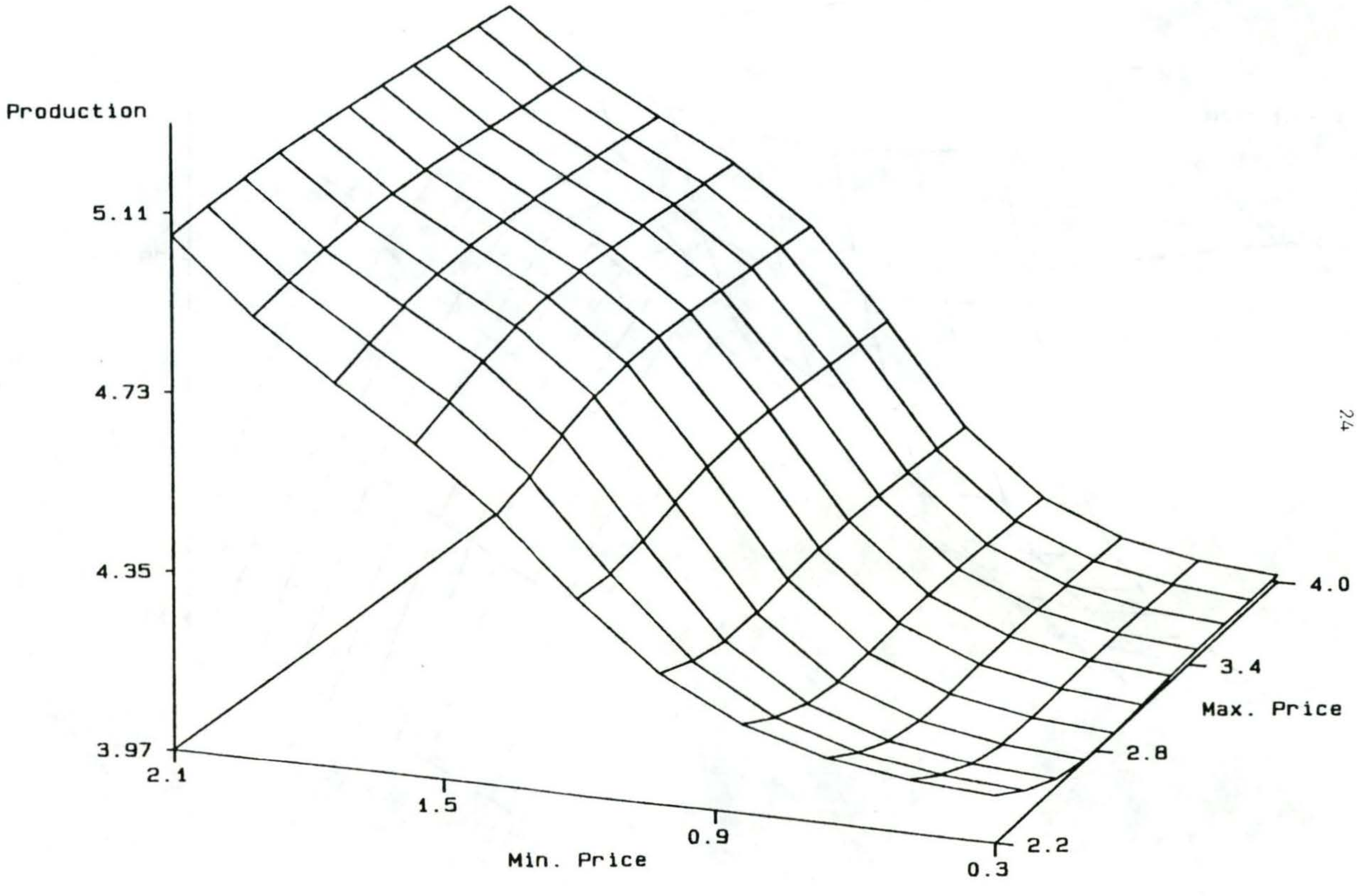

Figure 2. Relationship between expected production and minimum and maximum prices

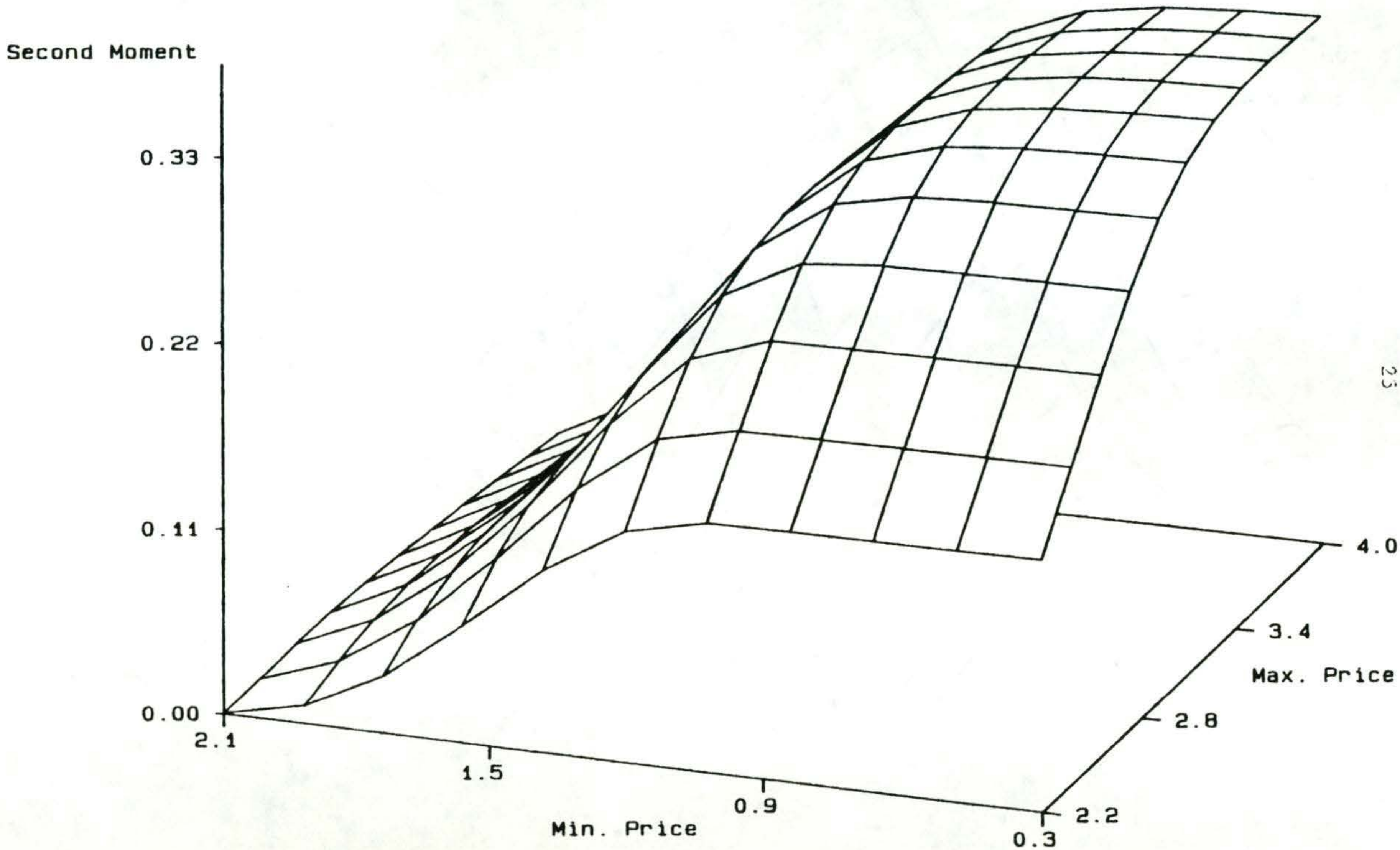

Figure 3. Relationship between expected variance of truncated price and minimum and maximum prices

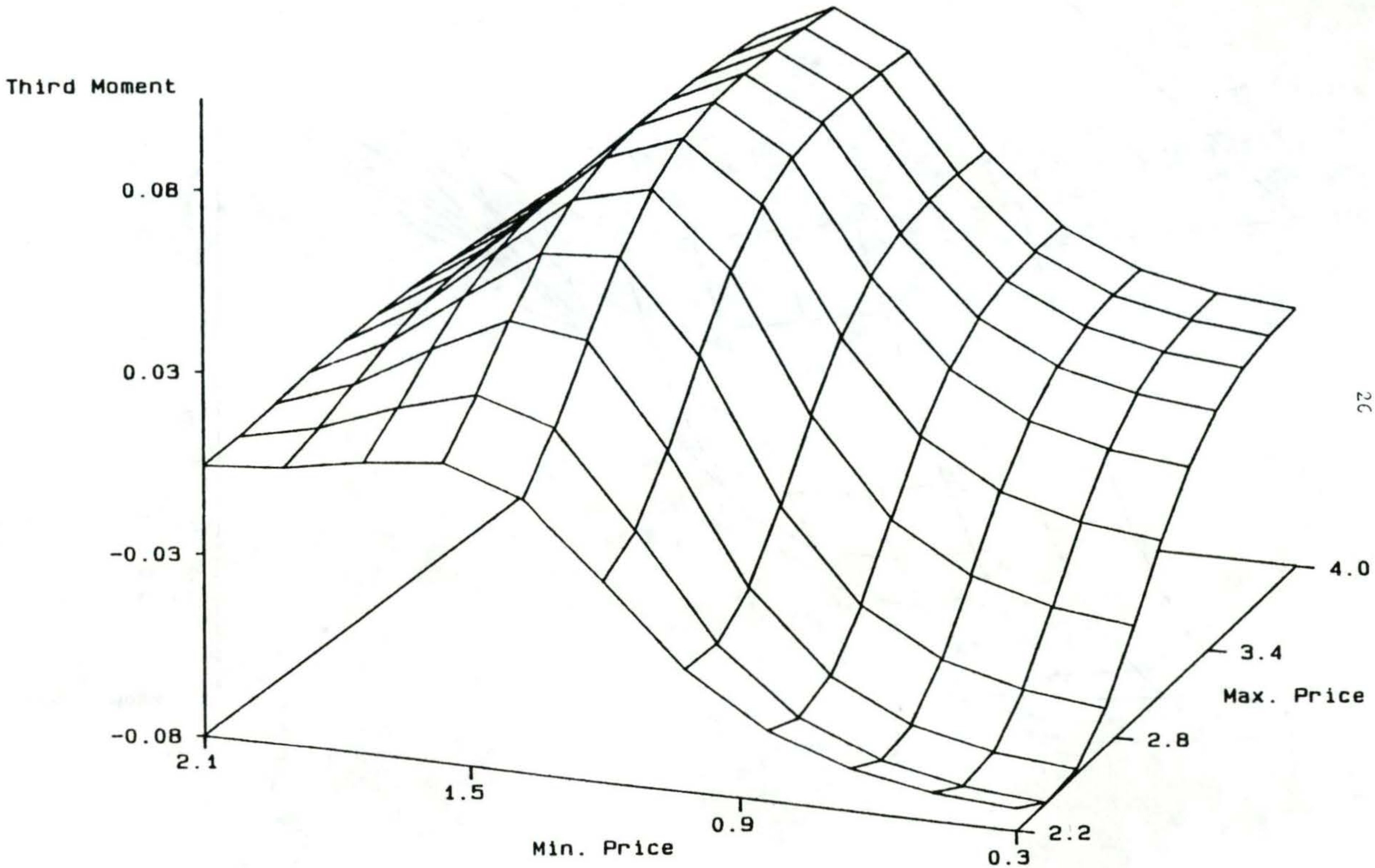

Figure 4. Relationship between expected third central moment of truncated price and minimum and maximum prices

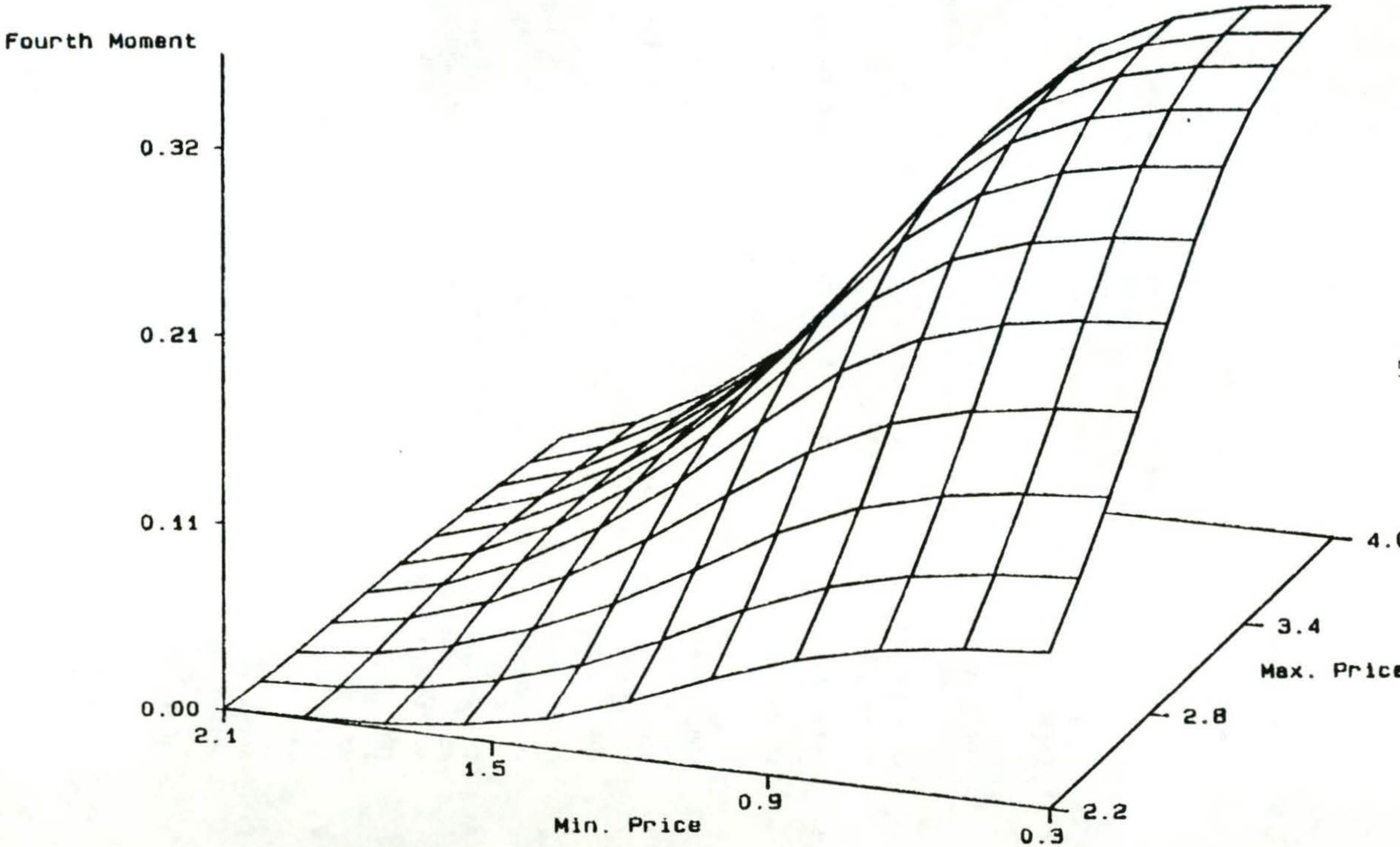

Figure 5. Relationship between expected fourth central moment of truncated price and minimum and maximum prices

Endnotes

1/ This hypothetical system of minimum and maximum price bands is similar to the target price-deficiency payment scheme used in the U.S. and elsewhere. The policy considered here is more general though in that an upper price bound is also included.

2/ The notation used for firm-level decisions implies that the truncated price distribution can be adequately characterized by a finite number of its central moments. While this assumption may not always be theoretically valid, Kendall and Stuart show that a probability distribution can be approximated to the n'th degree by an n'th degree polynomial whose coefficients are functions of the first n moments of the distribution. This moment-based approach is also consistent with the procedure used in many empirical studies where expected utility is approximated by a Taylor's series (Anderson et al.; Antle and Goodger).

3/ Quiggin and Anderson obtain essentially the same results using stochastic dominance concepts.

4/ This can occur, for instance, if the minimum and maximum prices are positioned in the tails of the distribution $f(p|Q)$. In this case, small changes in p_m and p_M would have little direct affect since the probabilities $F(p_m|Q)$ and $1 - F(p_M|Q)$ would not change much.

5/ The estimation framework focuses on a situation where only minimum prices apply. This is because government programs in the U.S. have historically relied only on minimum price supports. As illustrated in the Appendix, the model is easily modified to accommodate a situation where maximum prices also apply.

6/ Detailed derivations of the results in equations (15) through (20), as well as those in the Appendix, can be obtained upon request.

7/ The Fair and Taylor procedure is similar to the methods used by Lowry et al., Miranda and Helmberger, and Glauber, Helmberger, and Miranda for solving nonlinear rational expectations models. However, the Fair-Taylor approach differs in that the iterative solution algorithm is included directly in a maximum likelihood estimation procedure. See Holt and Johnson for further details on the use of Fair and Taylor's procedure to estimate bounded price variation models with rational expectations.

8/ Note that even though only price risk arguments explicitly enter the supply equation, output uncertainty is also incorporated since the structural variance associated with the supply equation appears directly in the price variance equation (14).

9/ These equations are estimated jointly with the supply and demand equations. As suggested by Hoffman, simultaneous estimation of the structural equations and the autoregressive processes used to forecast values for the unknown exogenous variables will yield consistent estimates of the parameters' standard errors in rational expectations models.

10/ A likelihood ratio test of the restrictions implied by rational expectations, risk, and truncation yielded a χ^2 statistic of 7.484 with three degrees of freedom. As a result, these restrictions may not be rejected at the 0.05 level of significance. Thus, the assumptions of rational expectations, truncation, and risk--including higher-order moments--are not rejected by the estimated model.

11/ Pearson's skewness coefficient is given by $\gamma_1 = \sigma_v^3 / (\sigma_v^2)^{1.5}$ and

Pearson's kurtosis coefficient is given by $\gamma_2 = \sigma_v^4 / (\sigma_v^2)^2 - 3$.

Positive values for γ_1 imply the distribution is right-skewed and negative values indicate that it is left-skewed. Likewise, positive values for γ_2 indicate a leptokurtic distribution and negative values indicate a platykurtic distribution.

12/ The range used is \$4.00 to \$2.20 per bushel for the maximum price and \$0.30 to \$2.10 per bushel for the minimum price. The prices were incremented in \$0.05 intervals, resulting in 1369 model simulations.

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Appendix

Using an approach similar to that described in the text, it can be shown that the expressions relating the expectations of the first four central moments of the truncated normal price distribution to both minimum and maximum prices are:

$$P^e_t = \bar{P}_{1t}\Phi(K_{1t}) + \sigma_p[\phi(K_{1t}) - \phi(K_{2t})] + P^*_t[\Phi(K_{2t}) - \Phi(K_{1t})] + \bar{P}_{2t}[1 - \Phi(K_{2t})], \tag{A1}$$

$$\sigma^2_{vt} = \bar{P}^2_{1t}\Phi(K_{1t}) + \sigma^2_p[K_{1t}\phi(K_{1t}) - K_{2t}\phi(K_{2t})] + 2P^*_t\sigma_p[\phi(K_{1t}) - \phi(K_{2t})] + [P^{*2}_t + \sigma^2_p][\Phi(K_{2t}) - \Phi(K_{1t})] + \bar{P}_{2t}[1 - \Phi(K_{2t})] - P^{e2}_t, \tag{A2}$$

$$\sigma^3_{vt} = \bar{P}^2_{1t}[\bar{P}_{1t} - 3P^e_t]\Phi(K_{1t}) + \sigma^3_p\,[K^2_{1t}\phi(K_t) - K^2_{2t}\phi(K_{2t})] + 3\sigma^2_p\,(P^*_t - P^e_t)[K_{1t}\phi(K_{1t}) - K_{2t}\phi(K_{2t})] + [2\sigma^3_p + 3P^{*2}_t\sigma_p - 6P^e_tP^*_t\sigma_p][\phi(K_{1t}) - \phi(K_{2t})] + [P^{*3}_t - 3P^e_tP^{*2}_t + 3\sigma^2_p(P^*_t - P^e_t)] \cdot [\Phi(K_{1t}) - \Phi(K_{2t})] + \bar{P}^2_{2t}[\bar{P}_{2t} - 3P^e_t][1 - \Phi(K_{2t})] + 2P^{e3}_t, \tag{A3}$$

$$\sigma^4_{vt} = \bar{P}^2_{1t}[\bar{P}^2_{1t} - 4P^e_t\bar{P}_{1t} + 6P^{e2}_t]\Phi(K_{1t}) + \sigma^4_p\,[K^3_{1t}\phi(K_{1t}) - K^3_{2t}\phi(K_{2t})] + 4\sigma^3_p\,[P^*_t - P^e_t][K^2_{1t}\phi(K_{1t}) - K^2_{2t}\phi(K_{2t})] + 3[\sigma^4_p + 2\sigma^2_p(P^*_t - P^e_t)^2] \cdot [K_{1t}\phi(K_{1t}) - K_{2t}\phi(K_{2t})] + 4\sigma_p[2\sigma^2_p(P^*_t - P^e_t) + P^*_t(P^{*2}_t - 3P^e_tP^*_t + 3P^{e2}_t)][\phi(K_{1t}) - \phi(K_{2t})] + (P^{*4}_t - 4P^e_tP^{*3}_t + 6P^{e2}_tP^{*2}_t + 6\sigma^2_p(P^*_t - P^e_t)^2 \tag{A4}$$

$$+ 3\sigma_p^4][\Phi(K_{2t}) - \phi(K_{1t})] + \bar{P}_{2t}^2[\bar{P}_{2t}^2 - 4P_t^eP_{1t} + 6P_t^{e2}][1 - \Phi(K_{2t})]$$

$$- 3P_t^{e4},$$

where,

$$K_{1t} = [\bar{P}_{1t} - (\alpha^*)^{-1}(\beta_1 X_{2t}^e + \beta_1^* P_t^e + \beta_2^* \sigma_{vt}^2 + \beta_3^* \sigma_{vt}^3 + \beta_4^* \sigma_{vt}^4 - \alpha_1' X_{1t}^e]/\sigma_p, \quad \text{(A5)}$$

$$K_{2t} = [\bar{P}_{2t} - (\alpha^*)^{-1}(\beta_1 X_{2t}^e + \beta_1^* P_t^e + \beta_2^* \sigma_{vt}^2 + \beta_3^* \sigma_{vt}^3 + \beta_4^* \sigma_{vt}^4 - \alpha_1' X_{1t}^e]/\sigma_p, \quad \text{(A6)}$$

$$\begin{aligned} 1 - \Phi(K_{1t}) = \text{Prob}[(\alpha^*)^{-1}(u_{2t} - u_{1t}) > \bar{P}_{1t} - (\alpha^*)^{-1}(\beta_1' X_{2t} + \beta_1^* P_t^e \\ + \beta_2^* \sigma_{vt}^2 + \beta_3^* \sigma_{vt}^3 + \beta_4^* \sigma_{vt}^4 - \alpha_1' X_{1t})], \end{aligned} \quad \text{(A7)}$$

and

$$\begin{aligned} 1 - \Phi(K_{2t}) = \text{Prob}[(\alpha^*)^{-1}(u_{2t} - u_{1t}) > \bar{P}_{2t} - (\alpha^*)^{-1}(\beta_1' X_{2t} + \beta_1^* P_t^e \\ + \beta_2^* \sigma_{vt}^2 + \beta_3^* \sigma_{vt}^3 + \beta_4^* \sigma_{vt}^4 - \alpha_1' X_{1t})]. \end{aligned} \quad \text{(A8)}$$

Here $\bar{P}_{1t}$ denotes the minimum guaranteed price and $\bar{P}_{2t}$ denotes the maximum guaranteed price. All other variables are as defined in the text. For purposes of simulating the model with both minimum and maximum prices, the expressions in (A1)-(A8) are substituted for those in (15)-(20).