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A DEMAND SYSTEMS APPROACH TO THE ANALYSIS OF COMMODITY PROMOTION PROGRAMS: THE CASE OF CANADIAN FATS AND OILS

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A DEMAND SYSTEMS APPROACH TO THE ANALYSIS OF COMMODITY PROMOTION PROGRAMS: THE CASE OF CANADIAN FATS AND OILS

This paper presents a Rotterdam type demand systems approach to measuring the impacts of advertising on commodity demand. A household production framework motivates an advertising induced augmentation model of structural change in Canadian fats and oils consumption, 1973-86. This framework generates restrictions on the advertising parameters (slopes) that are implied by demand theory under the augmentation hypotheses where advertising expenditures are assumed "to augment" the transformation of consumption goods into consumer utility. As with the demand systems framework generally, these restrictions can increase degrees of freedom, increase the precision of parameter estimates (i.e., if the restrictions are true), and provide a theoretically consistent means of specifying own and cross commodity advertising as well as price effects. In addition, these procedures provide an inductive basis for testing alternative hypotheses concerning advertising induced change in consumption.

Rigorous and thorough incorporation of factors generally considered to exhibit dynamic impacts (such as advertising) into demand systems can be quite problematical. All of the single equation subtleties in measuring the shape and length of the lag distribution as well as choosing an appropriate functional form for the advertising/sales response are compounded in a demand system with non-linear cross equation restrictions on the advertising slopes as well as the familiar demand theory (TRIAD) restrictions. Considerable research effort is required to more fully understand these issues. In particular, a more systematic and concerted effort on the incorporation of cross-commodity, dynamic advertising sales response in the presence of strong seasonal consumption patterns is warranted. What follows is one approach to addressing these issues.

The paper proceeds with a brief literature review of demand systems incorporating advertising to motivate the Rotterdam specification used. Next, a conceptual framework for investigating explicit hypotheses concerning the impacts of advertising on commodity consumption is presented. The empirical specification follows with a discussion of the data, the advertising stocks/dynamic specification, the seasonally adjusted Rotterdam model, the implied restrictions, and the econometric specification. Results of the estimations and hypotheses tests are then presented and discussed, followed by a summary and conclusions.

Demand Systems with Advertising

The single equation advertising sales response literature is both diverse and in depth.[1/] The extant literature on demand systems incorporating advertising, however, is relatively sparse. Two general conceptual approaches characterize this advertising in demand systems literature: adding advertising effects to duality based demand systems using the translog indirect utility or Almost Ideal Demand System (AIDS); or incorporating advertising into the Rotterdam demand system following Theil

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1/ There are several excellent references on this vast literature. In particular, see the published proceedings from the Farm Foundation/USDA 1985 seminar, Research On Effectiveness Of Agricultural Commodity Promotion as well as the 1983 NC-117 Monograph, Advertising and the Food System (Connor and Ward).

(1980a). Examples of the first approach using translog indirect utility specification include: Amuah; Goddard (1988a, 1988b, 1989); Goddard and Amuah; Goddard and Tielu. AIDS specifications incorporating advertising are found in the work of Green. Examples of the second approach using a Rotterdam specification include Aviphant, Lee, and Brown, Clements and Salavanthan, and Duffy. An alternative Rotterdam type approach will be explored in what follows.

It should be emphasized that both approaches impose a priori structure on the preferences and yield implicit or explicit restrictions required to "solve" the problem of identification of structural change (Diamond, McFadden, and Rodriguez). In contrast to the highly non-linear demand functions that derive from the non-homothetic translog indirect utility specification, however, the commonly estimated demand functions that derive from AIDS and (absolute price) Rotterdam frameworks are linear in parameters. This greatly facilitates the incorporation of non-linear dynamics and structural change hypotheses in the demand systems with advertising/sales response. Work by Wohlgenant suggests, moreover, that Rotterdam-type models are fairly robust in their ability to flexibly model demand systems relative to AIDS and translog specifications. Since Rotterdam demand systems have been shown to provide reasonable and robust approximations (Barnett; Mountain; Theil (1980b); Wohlgenant), and are easy to implement empirically (particularly for incorporating the impacts of preference shift variables such as generic advertising), this is the approach taken. In a sense, the fairly robust, linear in demand parameters Rotterdam specification allows the analyst to concentrate the available non-linear estimation capacity on unraveling the dynamics of the advertising/sales response.[2]

The Conceptual Framework

The conceptual framework for this analysis assumes that advertising potentially augments (i.e., can increase or decrease) the marginal utility of commodity consumption through informational and/or persuasive messages. Thus, assume that consumers behave as if they optimize a well behaved utility function represented as

$$\max_{Z>0} U(Z_1, \ldots, Z_Q) \tag{1.a}$$

$$\text{subject to:} \quad Z_i = Z_i(X_i, A, B), \ i = 1, \ldots, Q \tag{1.b}$$

$$M = \Sigma_i \ P_i X_i \tag{1.c}$$

where the P_i are the prices of market goods, M is income (or, group expenditure in the context of the second stage of a two stage budgeting process under weak separability), and where the household production functions (Z_i) specify how purchased market goods (X_i) are affected by the vector of advertising measures (A) and other exogenous factors (B) such as seasonality, trends and demographics.

2/ While the Rotterdam model has been criticized for a lack of flexibility in modeling demand relations (e.g., Phlips), this demand system is flexible in the sense of not imposing a priori restrictions on the local Allen elasticities of substitution (AES). The apparent lack of flexibility arises from the local consistency with the theory; or somewhat equivalently, Rotterdam models become inflexible if they are forced to be globally consistent. Note, however, that Mountain has shown the discrete Rotterdam formulation can be derived from the individual consumer as valid linear approximation in variable space with an order of approximation no lower than other popular flexible functional forms such as the translog, AIDS, etc.

The Z_i functions are interpreted as "effective" quantity levels, which are functions of the observed quantities X_i "augmented" in a positive or negative manner by the advertising vector (A). This is exactly the structural change hypothesis, where advertising is hypothesized to shift the marginal rates of substitution between commodities (hopefully) in favor of the commodity being advertised. To make this more explicit, substitute the functions (1b) into the utility function. The optimization problem in (1) becomes

$$\max_{X>0} U\{ Z_1(X_1,A,B), \ldots, Z_Q(X_Q,A,B) \} \quad \text{s.t. (1.c)} \qquad (2.a)$$

or, in "reduced form",

$$\max_{X>0} U(X_1, \ldots, X_Q; A,B) \quad \text{s.t. (1.c)} \qquad (2.b)$$

where the vectors A and B are interpreted as exogenous demand (preference) shifters (Deaton and Muellbauer, Chap. 10). Assuming usual regularity conditions (i.e., well-behaved preferences, an interior solution, etc.), the demand functions associated with the maximization of (2.b) subject to the budget constraint (1.c) can be represented as

$$X_i = f(P_1, \ldots, P_Q, M, A, B), \quad i = 1, \ldots, Q. \qquad (3)$$

To further focus this conceptual model, more explicit formulation of the functions Z_i in (1b) is required. Pollak and Wales summarize two popular special cases of this approach as translating and scaling. Both of these special cases of the augmentation hypothesis generate implied restrictions of the demand system impacts of advertising. These restrictions are derived next, and then used below to analyze the impacts of advertising in a demand systems context.

Additive Augmentations (Translating)

In the case of translating, the functions (1b) (or, equivalently, the additively augmented, "effective" quantities) are specified as

$$Z_i = X_i - T_i(A) \qquad (4)$$

where $T_i(A)$ represents some function of the advertising variables that additively augment the observed X_i. Noting that (4) implies $X_i = Z_i + T_i(A)$ and substituting this relation into the budget constraint (1c) yields the associated commodity demand functions under the translating (additive augmentation) hypothesis as

$$X_i(P_1, \ldots, P_Q, M, A, B) =$$

$$Z_i(\mathbf{P}/(M - \Sigma_j P_jT_j(A)), B) + T_i(A) \qquad (5)$$

where $\mathbf{P} = (P_1,\ldots,P_Q)$, $\mathbf{P}/(M - \Sigma_j P_jT_j(A))$ is the vector of (adjusted income) normalized prices,[3/] $X_i(\cdot)$ represents the classical, Marshallian demand specification of observed quantities, and $Z_i(\cdot)$ represents the Marshallian demand for "effective" or additively augmented (translated) quantities.

Note that this additive form of the augmentation hypothesis implies that the demand impacts of the advertising induced preference shifts basically act like income effects and generates moderately non-linear specifications that are empirically convenient. To further clarify this point, assume that the augmentations are of the form $T_i(A) = T_i(A_i)$, that is are determined by the "own" (versus cross) advertising factors.[4/] The advertising slopes implied by this translating hypothesis then take the form

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3/ Note that this normalized price vector basically imposes homogeneity on the price and income effects of the Marshallian demand functions.

4/ Note that this specification does not imply that cross-commodity advertising effects are zero. This will be clarified further below.

$$\frac{\partial X_i}{\partial A_j} = (\delta_{ij} - P_j \frac{\partial X_i}{\partial M}) \frac{\partial T_j}{\partial A_j} \qquad (6)$$

where δ_{ij} is the Kronecker delta and $(\partial X_i/\partial M)$ is the income effect. This implication of the additive augmentation (translating) hypothesis will be used below to generate empirical restrictions for hypotheses testing within a demand systems framework.

Multiplicative Augmentations (Scaling)

In the case of scaling, the functions (1b) (or, equivalently, the multiplicatively augmented, "effective" quantities) are specified as

$$Z_i = X_i/S_i(A). \qquad (7)$$

Noting that (7) implies $X_i = Z_i*S_i(A)$ and substituting this relation into the budget constraint (1c) yields the associated commodity demand functions under the scaling (multiplicative augmentation) hypothesis as

$$X_i(P_1, \ldots, P_Q, M, A, B) =$$

$$Z_i(PS/M, B)*S_i(A) \qquad (8)$$

where $PS/M = \{P_i*S_i(A)/M\}$ is the vector of scaled and normalized prices, $X_i(\cdot)$ represents the classical Marshallian demand specification for the observed quantities, and $Z_i(\cdot)$ represents the Marshallian demand for "effective" or multiplicatively augmented (scaled) quantities.

Note that this multiplicative form of the augmentation hypothesis implies that the demand impacts of advertising induced preference shifts basically act like price effects and generates multiplicative specifications. To further clarify this point, assume that the multiplicative augmentations are of the form $S_i(A) = S_i(A_i)$, that is are determined by the "own" (versus cross) advertising factors. The advertising slopes implied by this scaling hypothesis then take the form

$$\frac{\partial X_i}{\partial A_j} = \frac{\partial S_j(A)}{\partial A_j} \frac{1}{S_j(A)} \left[\frac{\partial X_i^c}{\partial P_j} P_j - P_j X_j \frac{\partial X_i}{\partial M} + \delta_{ij} X_i \right] \tag{9}$$

where δ_{ij} is the Kronecker delta, $\partial X_i^c/\partial P_j$ is the (Slutsky) compensated price slope, and $(\partial X_i/\partial M)$ is income effect. This implication of the multiplicative augmentation (scaling) hypothesis will also be used below to generate empirical restrictions for testing within a demand systems framework.

Examples of this type of scaling approach (either explicitly or implicitly) are numerous in the advertising/sales response literature. The "pure repackaging" model of Fisher and Shell explicitly formulates this type of advertising augmentation as disembodied taste change; that is, a change in tastes independent of any change in commodity quality but due to an information gain concerning the product (also see Deaton and Muellbauer (Chap. 10)). Further extensions and refinements of this model to advertising/sales analysis include Dixit and Norman, and Nichols.

Empirical Specification

The Data

Quarterly data on the Canadian fats and oils sector from 1973 to 1986 were used for the empirical analysis. Total retail packaged sales (metric tonnes, catalogue 32-006) and the corresponding price indices (1981 = 100, catalogue 62-010) of butter, margarine, shortening and salad oils were obtained from Statistics Canada. Monthly data were aggregated to quarters where necessary. The consumption data were then converted to kilos per capita using population data from Statistics Canada (catalogue 91-201). Nominal prices are derived by rescaling the price indices to the first quarter of 1981 = 100 and applying the following base period prices: butter

($3.94/kilogram), margarine ($2.387/kilogram), shortening ($2.11/kilogram), and salad oils ($2.67/kilogram).5/

The advertising data are from two sources. Monthly media advertising expenditures (aggregated across T.V., newspaper, and magazine) for margarine, shortening and salad oils are from Elliot Research and Media Measurement Institute, Toronto. These aggregate advertising expenditures are primarily brand specific in nature. While Elliot advertising measures for butter are available, generic butter advertising purchased by the Dairy Bureau of Canada (DBC) from 1978 to present are used in this analysis as they are felt to be more appropriate. All advertising expenditures are converted to a $/100,000 person, quarterly basis using the Canadian population figures above.

Advertising Dynamics

The research task is to partial out the effects of prices, expenditures, and commodity advertising in order to isolate the **ceteris paribus** impacts of the own- and cross-advertising effects. While the research analyst is generally forced to impose some prior structure on the advertising sales response surface, there does exist some latitude in how one proceeds. For example, considerable conceptual and empirical results support the existence of delayed peak in the advertising/sales response (Bass and Clark; Clark;

5/ The butter and shortening base period prices are weighted averages from Table 11 of Statistics Canada Catalogue 62-010. Following Statistics Canada suggestions, the industry leader price for margarine and salad oils was utilized. In 1981, Kraft's Parkay brand of margarine was the market leader and sold for $2.17 per two pound tubs, or $1.085/pound * 2.206 pounds/kilogram = $2.387/kilogram. Proctor and Gamble's Crisco was the leading salad oil in 1981, and sold for $2.43/liter (Ambler Food Pricing Services of Toronto). Converting salad oil density at 0.91kg/100cm^3 yields $2.43/liter * 1 liter/0.93kg = $2.67/kg. I wish to express thanks to Ellen Goddard and Brian Cozzarin of the University of Guelph for providing these data.

Jastram; Little; Simon and Arndt). The presence of a lagged peak or geometric decay in the advertising/sales response surface can be evaluated with dynamic stocks specifications that "flexibly" allow for either type of response to be supported by the data. Several members from the family of restricted infinite distributed lag (rational, pascal, gamma, geometric polynomial, and exponential lag) specifications are among the available options that can be used to evaluate these hypotheses (Judge, et al.).

The carryover effects of advertising on commodity demand are commonly viewed as a stock (versus flow) concept in the agricultural economics, advertising/sales response literature (e.g., Kinnucan and Forker). Advertising stocks of "good-will" are then treated as capital stocks with delayed response, carryover effects. Popular specifications of these carryover effects include low order (usually linear or quadratic) polynomial distributed lags (Thompson and Eiler; Ward and Davis; Ward and Myers), lagged quantity variables using the partial adjustment or habit formation hypothesis (Amuah; Goddard (1988b, 1989); Goddard and Amuah; Goddard and Tielu), moving average or moving sums of advertising (Aviphant, Lee and Brown), and the Pascal distribution (Kinnucan and Forker).

In contrast to the extant literature, a second order (quadratic) exponential lag specification of advertising stocks was chosen for its flexibility and parameter parsimony. Given the monthly, bi-monthly and quarterly based econometric evidence (e.g., Clarke) that most of the cumulative effects of advertising are likely to occur within 9 months for mature, frequently purchased, and low-priced, products like butter, margarine, shortening and salad oils, five lags are used to specify advertising stock of the i^{th} commodity as:

$$K_i = \sum_{j=0}^{5} W_{i,t-j} A_{i,t-j} \tag{10}$$

$$W_{i,t-j} = [\exp(D_{0i} + D_{1i}*j + D_{2i}*j^2)]. \tag{11}$$

The weight on the last lag (j=5) is restricted to be zero (following Thompson and Eiler; Ward and Davis; Ward and Myers), i.e., $W_{i,t-5} = 0$. Similarly, the weight on current period (j=0) advertising expenditures is restricted to be one, i.e., $W_{i,t} = 1$, a normalization which fixes the scale of measurement for the advertising stock K_i. These end-point restrictions reduce the dynamic specification to a single parameter. Incorporating five lags, noting that exp(0) = 1, and taking exp(-20) $\cong$ 0 for purposes of approximation, yields the following specification of the lag weights:

$$D_{0i} = 0$$

$$D_{1i} = -4.0 - D_{2i}*5$$

$$\begin{aligned} W_{i,t-j} &= [\exp(D_{0i} + D_{1i}*j + D_{2i}*j^2)] \\ &= [\exp(-4.0*j + D_{2i}*(j^2 - 5*j)]. \end{aligned} \tag{12}$$

While this specification is essentially arbitrary, as are all specifications of unobserved dynamic response functions, it does allow for either geometric decay or a lagged peak in the response surface of interest.[6/] Figure 1 demonstrates the flexibility of this specification for various levels of the quadratic term.

The Rotterdam Specification with Seasonal Adjustment

Given the pronounced seasonality and consumption trends characterizing the data to be analyzed, important issues arise concerning the proper

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6/ It should also be noted that several issues concerning truncation remainder terms are essentially ignored in this treatment. See Judge et al. (pp. 694-696) for more details.

handling of seasonality. A general approach is to use quarterly variables (e.g. Ward and Myers; Kinnucan, 1983) and a trend term to capture the other exogenous shift factors (B) in (3) above.[7/] Taking the advertising variables as the stock measures defined above, the directly specified demand functions in (3) could then be represented as

$$X_i = f(P_1, \ldots, P_Q, M, K_1, \ldots, K_Q, D_1, D_2, D_3, D_4, TREND). \quad (13)$$

where D_j is a dummy variable for the j^{th} quarter, j=1,2,3,4).

In quarterly or monthly data, the dependent variable of a distributed lag formulation may exhibit seasonality that is not captured by seasonal changes in the regressors (Judge, et al.). Seasonal adjustments of the data or the inclusion of seasonal dummy or seasonal harmonic variables are commonly used in the agricultural economics literature (Kinnucan and Forker; Thompson and Eiler; Ward and Davis; Ward and Myers). Deseasonalizing the data can generate high order moving average error terms and generally distort the lag function estimates in distributed lag models. Wallis, however, demonstrates that this prospect is less likely if one deseasonalizes both the dependent and independent variables with the same seasonal adjustment. Alternatively, one can specify a variable lag formulation using seasonal (dummy) interaction terms in the distributed lag formulation following

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7/ It is unclear how best to proceed in specifying the Rotterdam model with quarterly data. Duffy (p. 1055, footnote 5) acknowledges these difficulties and proceeded to estimate an annual model (without dynamics) after the **ad hoc** addition of quarterly dummy variables to Theil's Rotterdam with advertising specification resulted in those dummy variables being almost the only significant variables in the estimated equations. If most of the advertising/sales response literature is correct concerning the importance of dynamic advertising effects, then aggregation to an annual level is clearly offensive.

Tinsley. This is seldom done in the extant agricultural economics literature.

Following Wallis, both the dependent variables and the regressors are de-seasonalized by taking fourth differences. Note that the quarterly/-seasonal dummy variables disappear (are zeroed out) by this seasonal adjustment and the intercepts become the trend parameters. A de-seasonalized Rotterdam differential approximation to this reduced form demand curve incorporating the trend and quarterly/seasonal dummy variables can be represented as

$$w_i^* dlnX_i = \alpha_i + \Sigma_q G_{iq}*dlnP_q + B_i*(dlnX) + \Sigma_q A_{iq}*dK_q \tag{14}$$

where: $w_i^* = (w_{i,t} + w_{i,t-4})/2$ are the average budget shares; w_i are the budget shares of the i^{th} commodity at time t; $dlnX_i = (lnX_{i,t} - lnX_{i,t-4})$; $dlnP_q = (lnP_{q,t} - lnP_{q,t-4})$; $dlnX = \Sigma_i w_i^* dlnX_i$ is the Divisia quantity index of consumption; $G_{iq} = (\partial X_i^c/\partial P_q)*(P_q P_i/M) = E_{ij}^* * w_i$ (where E_{ij}^* are the compensated price elasticities); $B_i = P_i*(\partial X_i/\partial M) = E_{iM}*w_i$ (where $P_i*(\partial X_i/\partial M)$ is the marginal budget share and E_{iM} is income (or expenditure) elasticity); $dK_q = (K_{q,t} - K_{q,t-4})$ is the fourth difference in advertising stocks; and $A_{iq} = (\partial X_i/\partial K_q)*(P_i/M)$ are the unrestricted, advertising stocks induced demand responses. Given this fourth order log differential approximation to the reduced form seasonal demand function in (13), the "intercept" terms α_i are interpreted as measures of structural change not accounted for by the rest of the model.

Note that the advertising stocks specification is in difference rather than log differential form. This specification is chosen for a variety of reasons. First, given the presence of zero levels of butter advertising by the Canadian Dairy Bureau prior to the third quarter of 1978, the log differential specification creates problems since the log of zero does not exist. Second, de-seasonalized, log differential stocks add considerable non-linearity to an already non-linear specification.[8/] Given the convergence difficulties encountered in the econometric estimation of (14), to be discussed below, and the potential log of zero problems, the use of differences rather than log differentials was considered a reasonable way to proceed.

The Restrictions Implied By The Theory

The familiar TRIAD restrictions from demand theory for this specification are: $\Sigma_i B_i = 1$ (adding up); $\Sigma_q G_{iq} = 0$ (homogeneity of compensated price slopes); and $G_{iq} = G_{qi}$ (Slutsky symmetry). In order for the system to add up, the following must also hold: $\Sigma_i \alpha_i = 0$, and $\Sigma_i A_{iq} = 0$. In addition, demand theory implies the Tintner-Ichimura relations on the advertising stock slopes (Phlips, pp. 181-83; Theil 1980a):

$$\frac{\partial X_i}{\partial K_q} = \{ -(1/\lambda)*(\Sigma_q \frac{\partial X_i^c}{\partial P_q} *V_{qr}) \} \qquad (17)$$

where λ is the marginal utility of income, and $V_{qr} = \partial^2 U/(\partial X_q \partial K_r)$.

8/ To see this note that

$$dlnK_q = \ln(K_{q,t}) - \ln(K_{q,t-4})$$

$$= \ln\{ \sum_{j=0}^{5} W_{i,t-j} A_{i,t-j} \} - \ln\{ \sum_{j=0}^{5} W_{i,t-j} A_{i,t-j-4} \} \qquad (15)$$

In contrast, the de-seasonalized, fourth difference stock specification in (14) derives as

$$dK_q = (K_{q,t} - K_{q,t-4})$$

$$= (\sum_{j=0}^{5} W_{i,t-j} A_{i,t-j}) - (\sum_{j=0}^{5} W_{i,t-j} A_{i,t-j-4})$$

$$= \sum_{j=0}^{5} W_{i,t-j} (A_{i,t-j} - A_{i,t-j-4}) \qquad (16)$$

which is considerably less non-linear in than (15).

Note that (17) states the demand system restrictions on the effects of preference shift factors (such as advertising) that are implied by the theory. While these restrictions are usually not developed nor utilized in most of the applied literature, they can be tested for and, if found true, aid in the estimation of demand systems incorporating these shift factors. The Rotterdam/advertising work exploits this relation (see Theil (1980a), Clements and Salavanthan, Duffy, and Aviphant, Lee, and Brown). On the other hand, (17) also indicates that the demand analytics of these shift factors require the additional knowledge of how these shift factors influence the curvature of the preference function (i.e., via the V_{qr} elements, the second order cross-partial derivatives of $U(\cdot)$). That is, knowledge or assumptions about these second order curvature properties imposes the structure necessary to identify the nature of these preference shifts. This information is either **implicitly or explicitly** imposed in the specification and estimation of commodity demand functions incorporating preference shift factors. The translating and scaling augmentation hypotheses explicitly provide some of this **a priori** structure.

To further clarify this point in the present context for the translating hypothesis, denote $T_q^* = (\partial T_q/\partial K_q)*(P_q/M)$, $(q = 1, \ldots, Q)$ as the translated advertising/stock parameters to be estimated, and multiply (6) by P_i/M to yield the following restrictions on the advertising/stock parameters A_{iq} in (14):

$$A_{iq} = (\delta_{iq} - B_i)*T_q^* \qquad (18)$$

where δ_{iq} is the Kronecker delta, and B_i is the estimated marginal budget share from (14). Hence, (18) yields a set of a modestly nonlinear, cross equation restrictions amenable to empirical hypothesis testing against the unrestricted specification of the A_{iq}.[9/] Similarly for scaling, denote $S_q^* = (\partial S_j(A)/\partial K_q)/K_q$, $(q = 1, \ldots, Q)$ as the scaled advertising stock parameters to be estimated, and multiply (9) by P_i/M to yield the following restrictions on the advertising/stock parameters A_{iq} in (14):

$$A_{iq} = \{ G_{iq} - w_q*B_i + \delta_{iq}*w_i \}*S_q^* \qquad (19)$$

where, as above in (18), δ_{iq} is the Kronecker delta, B_i is the estimated marginal budget share from (14), and G_{iq} is the estimated compensated price response from (14). In contrast to (18), note that the scaling augmentation hypothesis in (19) yields a set of more nonlinear, cross-equation restrictions involving the G_{iq} terms. These restrictions are likewise amenable to empirical hypothesis testing against the unrestricted specification of the A_{iq}.[10/]

Equations (18) and (19) summarize the empirical restrictions jointly implied by demand theory and the translating or scaling augmentation hypothesis in the context of the de-seasonalized Rotterdam demand system incorporating advertising stocks in (14). These restrictions are empirically testable and, noting that the derivative of the budget constraint with

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9/ Note that for all $i \neq q$, $T_q^* > 0$ (i.e., positive translating parameters) and $B_i > 0$ (i.e., normal goods) implies $A_{iq} < 0$ (i.e., negative cross commodity advertising effects). Conversely, for all $i = q$, positive own advertising effects ($A_{ii} > 0$) implies that either $T_i^* > 0$ and $B_i < 1$, or, if $B_i > 1$, that $T_i^* < 0$.

10/ Note that $(G_{iq} - w_q*B_i) = w_i*(E_{iq}^* - w_q*E_{iM}) = w_i*E_{iq}$ where E_{iq} is the uncompensated, Marshallian price elasticity. Thus, for all $i \neq q$ and $S_q^* > 0$, $A_{iq} > 0$ if $E_{iq} > 0$ (gross complements) and $A_{iq} < 0$ if $E_{iq} < 0$ (gross substitutes). Hence, the direction of cross commodity advertising effects parallels the uncompensated price effects for positive scaling/stock estimates, an intuitively appealing result. Conversely, the direction is opposite for negative scale/stock estimates. Similarly, assuming all $E_{ii} < 0$ (i.e., no Giffen goods), then $A_{ii} > 0$ if $S_i^* > 0$ and $|E_{ii}| < 1$ (i.e., inelastic Marshallian own-price elasticity). Conversely, if $S_i^* < 0$, then $A_{ii} > 0$ implies $|E_{ii}| > 1$ (i.e., elastic Marshallian own price elasticity).

respect to K_q implies that $\Sigma_i A_{iq} = 0$ if adding-up is to be satisfied, generate Q versus $Q^2 - Q$ advertising response parameters to estimate in the restricted demand system. Hence, at the cost of adding non-linearity to the demand system, these cross-equation restrictions can provide an inductive basis for evaluating alternative hypotheses on the nature of advertising induced structural change, reduce the parameter space and, if true, increase the precision of the resulting estimates.

Econometric Specification

Substitution of (10), (11), and (12) into (14) yields a seemingly unrelated, non-linear demand system with an "unrestricted" advertising/stocks specification.11/ The non-linear restrictions to impose negative semi-definiteness on the matrix of Slutsky price effects (following Barten) and Geyskens12/ are also evaluated with this specification and Geyskens. The additional substitution of (18) or (19) for the advertising/stock effects, A_{iq}, yields non-linear nested specifications of the additive (translating) or

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11/ In addition to the second order exponential specification of advertising stocks (with end point restrictions), the restrictions from demand theory (adding up, homogeneity and Slutsky symmetry) are imposed throughout as maintained hypotheses. Hence, "unrestricted" in this context relates to (17) in the sense of no explicit hypotheses on the impact of the advertising stocks on consumption (or marginal utility).

12/ Cholesky decomposition of a symmetric, negative definite matrix yields $G_p^* = -P'*P$ where $P = (P_{iq})$ is upper triangular. For Q = 4 (and, assuming homogeneity, i.e., $\Sigma_q G_{iq} = 0$) this yields the following nonlinear restrictions on the G_{iq} price terms of (14):

$$G_{11} = -P_{11}*P_{11}; \quad G_{12} = -P_{11}*P_{12}; \quad G_{13} = -P_{11}*P_{13}$$

$$G_{22} = -P_{12}*P_{12} - P_{22}*P_{22}; \quad G_{23} = -P_{12}*P_{13} - P_{22}*P_{23}$$

$$G_{33} = -P_{13}*P_{13} - P_{23}*P_{23} - P_{33}*P_{33}.$$

Note that the same number of price related parameters are required, but they enter the estimation in considerably more non-linear fashion.

multiplicative (scaling) advertising/stock augmentation hypotheses, respectively. Non-linear seemingly unrelated regression (SUR) was performed using SAS's SYSNLIN software. Following Chavas and Segerson, consistent estimates of the unrestricted cross-equation residual covariance matrix were obtained and used in the estimation of all nested models.13/ Gallant-Jorgenson test procedures are used to test all hypotheses.14/

Results

Table 1 summarizes the parameter estimates for the unrestricted, the scaled and the translated advertising/stocks specification. All results presented have negative semi-definite compensated price effects imposed.15/ Also, each equation is corrected for the presence of first order

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13/ These procedures yield estimates that are consistent, asymptotically efficient, and invariant to the omitted equation using non-iterated SUR, i.e., the results are asymptotically equivalent to those from maximum likelihood. This facilitates empirical estimation as iterated SUR (for example ITSUR in SAS's SYSNLIN) can be expensive in very non-linear demand systems such as those incorporating cross-commodity advertising dynamics and explicit augmentation hypotheses.

14/ Basically this test involves using the same consistent estimate of the cross equation covariance matrix of residuals to estimate both the "unrestricted" and restricted models. The Gallant and Jorgensen test statistic is computed as $T_0 = nS^{**} - nS^{*}$ where S^{**} and S^{*} are the weighted error sum of squares evaluated at convergence for the restricted and unrestricted models, respectively. This test statistic is asymptotically distributed as chi-square with degrees of freedom equal to the difference in the number of parameters in the unrestricted and restricted models.

15/ The unrestricted advertising results are not presented to conserve space and are available upon request from the author. With few exceptions, the imposition of the negative semi-definiteness restrictions also resulted in the imposition of Pss=Pmm=Pms=0 (see the unrestricted and translated advertising results in Table 1) or Pss=0 (scaled advertising) in order to get convergence in the estimation routines. Note that the corresponding price effects are not zero under these additional restrictions. Thus, for example, Pss=Pmm=Pms=0 implies that Gss = -Pbs*Pbs, Gmm = -Pbm*Pbm, and Gms = -Pbm*Pbs.

autocorrelation following Berndt and Savin. Table 2 summarizes various hypotheses tests of alternative specifications using Gallant-Jorgenson procedures. Second stage expenditure, price (compensated and uncompensated), and advertising elasticities for each model, evaluated at the 1978-86 means (the period during which the DBC was explicitly advertising butter), are summarized in Table 3.

An "unrestricted" advertising model was estimated first with the 1973-86 data as a standard, second stage Rotterdam specification using first rather than fourth differences. Positive, compensated own-price effects for margarine and shortening (salad oils is the omitted equation throughout), negative (and significant) own advertising stocks effects for butter and margarine resulted, and were considered unacceptable. The somewhat ad hoc addition of trend terms, with and without quarterly dummy variables, yielded essentially the same results. Similar to the Duffy, the addition of quarterly dummy variables resulted in the loss of almost all significance in the price and advertising effects. Imposing negative semi-definiteness on the matrix of compensated price effects yielded theoretically plausible price effects, but failed to change the negative own commodity advertising/stocks effects. The Gallant-Jorgenson test of the curvature restrictions given the unrestricted advertising model is 8.88 (Table 2). As indicated in Table 2, the null hypotheses of negative semi-definite compensated price effects is marginally rejected at the $\alpha = 0.05$ level for 3 degrees of freedom. At any other degrees of freedom and/or α levels evaluated, the null would fail to reject.[16/] Hence, given the importance of theoretically consistent compensated price effects for the specification, and hence, testing of the augmentation hypotheses (particularly scaling, see equation (17)), the results that follow have negative semi-definiteness imposed as a maintained hypothesis.

The summary statistics on the second page of Table 1 indicate the relative predictive power of the alternative specifications. The minimum root mean square errors (RMSE) across specifications and commodities indicate that the unrestricted specification is marginally superior in terms of RMSE criterion (i.e., due to the shortening RMSE). Given that the dependent variables are "divisia share" weighted fourth differences of logarithms, the individual equation R-Squares are quite good. Durbin-Watson "D" statistics (not presented) indicate that all equations do not have significant first order autocorrelation problems after the Berndt and Savin corrections. The corresponding Gallant-Jorgenson tests for $\rho = 0$ across all equations (see Table 2) indicate that the null hypotheses of no first order autocorrelation is rejected at the 5 percent level for each advertising stocks specification. The parameter estimates and associated elasticities are virtually identical with or without the correction for autocorrelation, and efficiency gains (evidenced by higher asymptotic "T-values") are minimal. Note, moreover, that the autocorrelation parameters (RHO in Table 1) are also quite similar across the alternative advertising stocks specifications (i.e., RHO equals

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16/ It is unclear how many degrees of freedom (d.f.) to evaluate the chi-square critical value at: there are 6 non-linear restrictions imposed by negative semi-definiteness, and three additional restrictions (Pss=Pmm=Pms=0) were required to obtain convergence. Strictly speaking, there is no difference in the number of price related parameters (i.e., the G_{iq} (without curvature) or the P_{iq} (with curvature)); 3 degrees of freedom result from the Pss=Pmm=Pms=0 restriction required to obtain convergence.

0.215. 0.236, or 0.220 for the unrestricted, scaled and translated models, respectively).

The calculated Gallant-Jorgensen test statistics for the null hypotheses of separate scaled or translated versus the unrestricted advertising stocks specification (see Table 2) indicate that these hypotheses cannot be rejected at the $\alpha = 0.05$ level. The further assumption of identical (versus separate) dynamic stocks (i.e., $D_{2i} = D_2$ for all i via equations (12), (10) and (14)), when tested against the unrestricted, separate stocks specification marginally rejects for scaling (15.72 computed test statistic versus 15.5 critical value, $\alpha = 0.05$ level) and fails to reject for translating at the $\alpha = 0.05$ level of significance (see Table 2).[17/] These results suggest that more parsimonious and simpler stock specifications also fit these data.[18/]

Moving on to the parameter estimates and associated elasticities, recall that the intercept terms i_0 are basically trend variables in difference models. Thus, the results on the first page of Table 1 indicate negative (positive) ceteris paribus trends in butter (margarine) consumption as suggested by the plots of the consumption data. These parameters, however are not statistically different from zero. In contrast, the estimated

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17/ These identical quadratic stocks terms were estimated as -1.41, -1.38, and -1.33 for the unrestricted, scaled and translated stocks specifications, respectively. Each of these parameters was significant at the 0.0001 level. While the identical stocks results are not presented here to conserve space, they are available from the author on request. Parameter estimates and the associated elasticities are quite similar to the separate stocks results presented here.

18/ It should be noted that the identical stocks specifications are much less computationally intensive than separate stocks (particularly under nonlinear parameter structure such as scaling, translating and/or negative semi-definite compensated price effects). These simpler specifications also generate useful starting values for more complex specifications, an important, practical insight when estimating highly non-linear demand systems.

conditional income effects are quite statistically significant and similar across the alternative specifications. The corresponding conditional expenditure elasticities (see Table 3) are likewise quite similar across the estimated advertising stocks specifications with butter (shortening) being the most elastic (inelastic).

The conditional, compensated price effects (with negative semi-definiteness imposed) in Table 1 are generally not statistically significant with the exception of P_{bb} (all models), P_{bm} and P_{ms} (scaled stocks). The estimated price parameters and associated compensated price elasticities (see Table 3) from the unrestricted and translated specifications are quite similar: compensated own price responses are inelastic (with butter twice as own price responsive as margarine); and, all goods are net substitutes for butter while margarine, salad oils, and shortening are all net complements for each other. The associated uncompensated (Marshallian) price elasticities (Table 3) exhibit similar patterns with the exceptions that all goods are found to be gross complements. While the gross complementarity of butter and margarine may seem counter-intuitive, similar results are reported by Pitts and Herlihy, and by Goddard (1988b).

The price elasticities associated with the scaled stocks specification are marginally different than the unrestricted and translated results (see Table 3). This reflects the fact that advertising is explicitly hypothesized to rescale the price effects under this specification. Despite these differences, results very similar to those from the unrestricted and translated specifications are found: that is, compensated own price effects remain inelastic (with butter more responsive than margarine); all goods are net substitutes for butter; shortening and margarine are net complements;

and, all goods (except salad oils for shortening), and butter/margarine in particular, exhibit gross complementarity.

Moving to the estimated advertising parameters in Table 1, note that all advertising parameters except those associated with margarine (i.e., A_{mm} (unrestricted), S_m (scaled), T_b (translated)) are not statistically different from zero. Moreover, the associated own commodity advertising elasticities for margarine (as well as butter and shortening, see Table 3) are negative and the butter/margarine cross-promotion effects are positive, results that are exactly opposite to prior expectations.[19/] The advertising elasticities are moderately robust with respect to magnitudes across equations (with some exceptions), but given the general lack of parameter significance on the advertising stocks effects (except the margarine own effect), further discussion of these results is probably not warranted. It is worth noting, however, that similar results were obtained using 1978-86 data (i.e., that period for which the DBC directly promoted butter), first (versus fourth) difference specifications with/without trend terms, with/without quarterly dummy variables, and with/without imposing the curvature restrictions.[20/] In addition, a similar lack of significance of the advertising effects (but with more intuitive signs (i.e., positive own- and negative cross-advertising

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19/ The stocks elasticity reflects the percentage changes in current consumption due to a one percent change in the stock of advertising "good will", **ceteris paribus**. These stocks are computed from equations (10), (11), and (12) using the dynamic stocks quadratic terms from Table 1. In contrast, the current period elasticity (which via (12) has a weight of one) reflects the percentage change in current consumption due to a one percent change in current advertising, **ceteris paribus**. Hence, these latter measures are not "discounted" for future consumption induced by current advertising.

20/ Using the Canadian CPI to deflate commodity advertising (in the absence of a more appropriate cost of advertising index), also yielded essentially the same results.

effects for butter and margarine)) have been found Goddard (1988b) using nearly identical data with an indirect translog specification.[21/]

Aside from these somewhat negative (or certainly counter-intuitive) results, the estimated dynamics in the advertising stocks are quite interesting. The estimated magnitudes of the quadratic exponential stocks, D_{2i}, are fairly robust across the alternative specifications: butter -1.2 to -1.3; margarine 0.0; shortening -2.8 to -2.9 (exception: translated stocks, -0.9); and, salad oils -1.4 to -1.5. Referring to Figure 1, these results indicate the dynamic specification used is capable of identifying geometric decay (i.e., when the quadratic term is 0.0 as in the case of margarine) as well as lagged peaks in the advertising/sales response. Thus, for example, butter advertising was found to attain it's peak impact within 2-3 months, to decay and cease by the third quarter. A similar pattern is found for salad oils. Margarine, decays immediately and virtually ceases within one quarter. Shortening, in contrast, has very small initial advertising impacts which increase rapidly, peak within 4-6 months, then decay quite rapidly and cease by the third quarter.

While the assumed functional form for the advertising stocks is unquestionably imposing quite a bit of structure on the shape of these estimated advertising/sales response surfaces, this type of approach appears promising. Given the competitive response by the margarine industry during much of the period analyzed, researchable questions concerning the advertising media mix arise. Quick hitting competitive messages and/or

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21/ The biggest difference in data reflects the use of actual advertising expenditures supplied by the DBC in contrast to Goddard's use of the Elliot Research estimates of buffer media expenditures. Goddard uses an indirect translog utility specification with lagged dependent variables; hence, the advertising specifications evaluated here are quite different.

vigorous price/promotions via print media might yield relatively rapid advertising/sales decay. Also, the degree of price competition between brands could induce relatively quick advertising decay; i.e., if there was little brand loyalty and lots of price induced switching between brands. In contrast, if there was a clearly dominant brand in a category such as Crisco in shortening (and perhaps salad oils), then one might expect this stronger brand loyalty (and perhaps a different media mix and message) to be manifest as a relatively longer delayed peak in the advertising sales response. Further research might explore the feasibility of disentangling competitive, cross-commodity, advertising sales response by type of media. The demand systems approach, such as demonstrated here, provides a rigorous ceteris paribus framework for such analysis.

Summary and Conclusions

Summary

A household production framework is used to motivate an advertising induced, commodity augmentation model of structural change which is then applied to Canadian fats and oils consumption. Dynamic cross-commodity advertising effects are estimated with quarterly data (1973-86) under unrestricted, additive (translating) and multiplicative (scaling) augmentation hypotheses using a fourth difference (seasonally adjusted) Rotterdam demand specification. Both of the latter hypotheses are non-linear nested within the unrestricted model, hence amenable to nonlinear hypotheses tests. The data analyzed fail to reject both hypotheses.

A second order exponential with five lags is used for the specification of advertising stocks of butter, margarine, shortening, and salad oils. This dynamic stock specification flexibly allows for a geometric or lagged peak decay function, an issue of considerable conceptual and empirical interest. The fairly robust results indicate a lagged peak advertising sales response for butter (2-3 months), salad oils (3-4 months), and shortening (4-6 months) but a rapid (within one quarter) geometric decay for margarine. Each of these dynamic stock measures is estimated in each equation to allow for theoretically consistent cross-commodity advertising effects. As well, the hypotheses of identical versus separate stocks specifications failed to reject for both scaling and translating.

The procedures used are unable to find a significant positive impact of butter advertising on butter consumption over the time periods analyzed. Similar results were found across several alternative specifications of log-differential Rotterdam type demand systems (with second order exponential stocks). Given similar difficulties with these data using translog (Goddard 1988b) and AIDS (Chang and Kinnucan) specifications, these demand system results appear to be relatively robust. Considering that a key **ceteris paribus** factor, health/diet induced concerns for dietary cholesterol from animal fats such as butter, is not explicitly accounted for in this research (the intercept/trend terms for butter (margarine) were found to be negative (positive), however), these results may not be unrealistic. <u>If</u> a large stock of negative messages or "bad will" based on moderately reputable sources (correctly or incorrectly, such as medical journals and family physicians) is accumulating over time, <u>and if</u> the "good will" messages are unable to overcome this counter veiling "bad will" on a sustained basis, <u>then</u> the butter messages could be swamped out with the net effect of no significant influence on consumption.

Conclusions

A primary motivation for the systems approach concerns correct specification of the **ceteris paribus** demand context within which to isolate the impacts of advertising. In particular, the demand systems approach provides a theoretically consistent way to specify own and cross commodity advertising effects in addition to the more familiar own and cross price and income effects. While the familiar homogeneity restriction can be imposed in a single equation framework, adding up and, more importantly, Slutsky symmetry require a systems context. Since the Slutsky restrictions are required for demand functions to be "integrable" back to an expenditure function, which in turn can be used to construct a consistent utility function, the cross-equation Slutsky relationships give a complete list of the restrictions imposed by the utility maximization hypothesis (Varian, pp. 100-1). Thus, Slutsky symmetry is **the** crucial implication of demand theory. This suggests that disciplinary progress through inductively based research on the issues of advertising's impacts on commodity demand requires a systems framework.

Aside from explicitly imposing necessary a priori structure on the objective (utility, indirect utility, expenditure) or demand functions, the benefits of deriving structural inferences on advertising effects which derive from a theoretically consistent conceptual model are primarily to generate inductive insights concerning a complex measurement problem. Thus, the derivation of additional structure (implied restrictions) about advertising effects using a theoretically consistent demand system can reduce data needs, facilitate hypothesis testing, and, hopefully, increase statistical precision concerning these effects. This dimension of advertising/consumption response methodology is largely unexplored in the extant empirical literature. Hence, at the cost of being more explicit about the structure imposed on a problem to make it empirically tractable, we may be able to gain inductive insights and more precision in our estimates of advertising/sales response by exploring these types of restrictions more fully. Hopefully the conceptual framework and empirical analysis presented here will illustrate these benefits and stimulate additional research in this area.

FIGURE 1: HYPOTHETICAL ADVERTISING STOCKS.

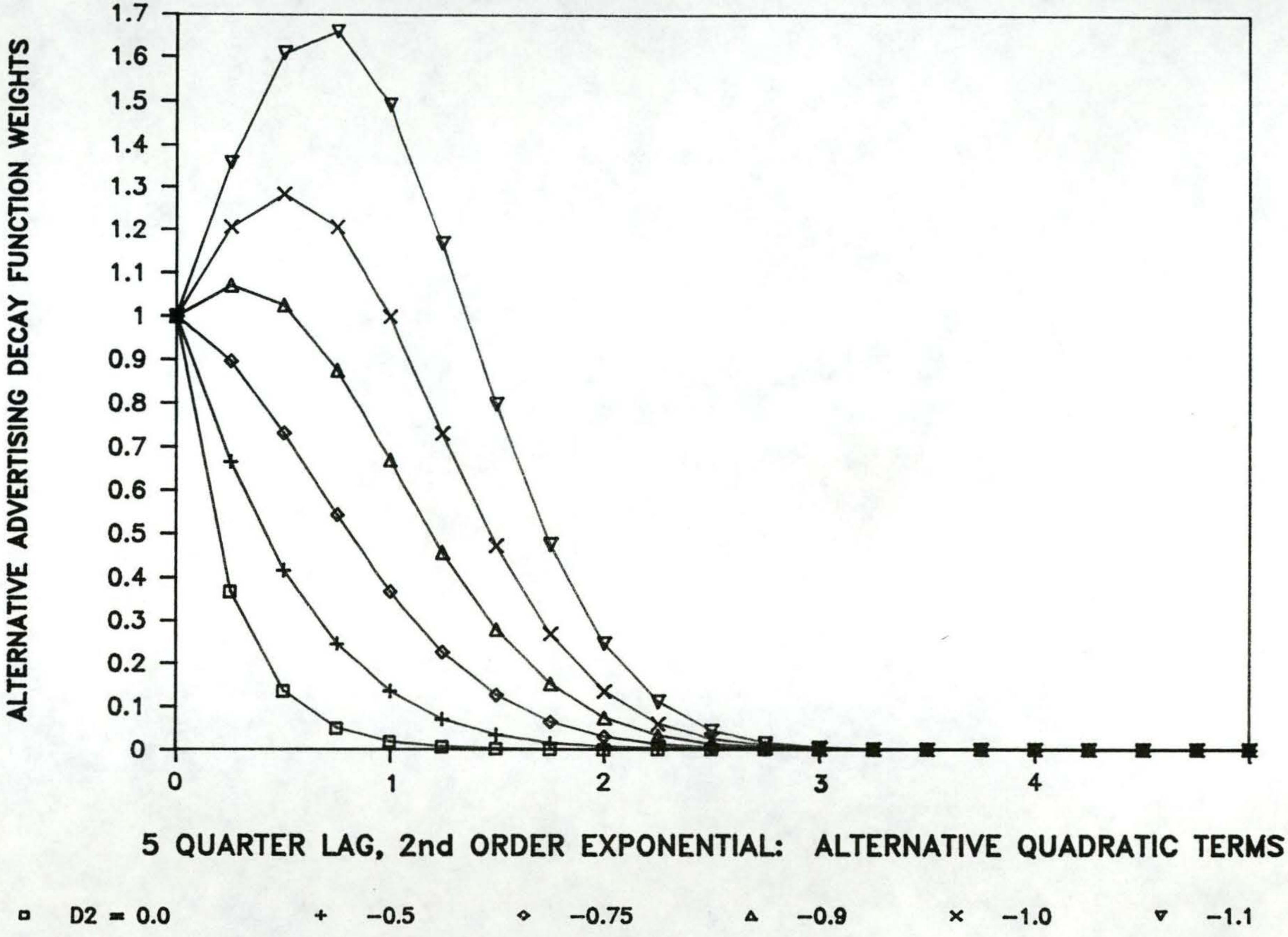

Table 1. Alternative Parameter Estimates of the Canadian Fats and Oils Sector, 1973 - 1986 Using Deseasonalized (4th Difference) Rotterdam Specification.[1]

	UNRESTRICTED ADVERTISING		SCALED ADVERTISING		TRANSLATED ADVERTISING
			INTERCEPTS:		
b0	-0.001		-0.000		-0.002
m0	0.002		-0.000		-0.000
s0	-0.005		-0.003		-0.004
			CONDITIONAL INCOME EFFECTS:		
Bb	0.504 ****		0.501 ****		0.497 ****
Bm	0.275 ****		0.268 ****		0.264 ****
Bs	0.085 **		0.102 ***		0.096 ***
			CONDITIONAL, COMPENSATED PRICE EFFECTS:[2]		
Pbb	0.319 ***		0.332 ****		0.311 ***
Pbm	-0.171		-0.233 ****		-0.185
Pbs	-0.145		-0.056		-0.124
Pmm	-		-0.075		-
Pms	-		-0.168*		-
Pss	-		-		-
			ADVERTISING STOCK EFFECTS (Aiq, Si, or Ti):		
Abb	-0.00017	Sb	-0.00289	Tb	-0.00041
Abm	0.00004	Sm	-0.00283 **	Tm	-0.00048 *
Abs	-0.00000	Ss	0.00000	Ts	0.00049
Abo	0.00010	So	-0.00223	To	-0.00012
Amb	0.00003		-		-
Amm	-0.00040 **		-		-
Ams	-0.00000		-		-
Amo	0.00004		-		-
Asb	0.00009		-		-
Asm	0.00018		-		-
Ass	0.00000		-		-
Aso	-0.00007		-		-
			DYNAMIC STOCKS QUADRATIC TERM:		
D2b	-1.322 ****		-1.229 ****		-1.247 ****
D2m[3]	-		-		-
D2s	-2.808 ***		-2.876 **		-0.866 *
D2o	-1.494 ****		-1.437 ****		-1.431 ****

Table 1 (Continued).

	UNRESTRICTED ADVERTISING	SCALED ADVERTISING	TRANSLATED ADVERTISING
	AUTO-CORRELATION TERM:		
RHO	0.215 **	0.236 **	0.220 **
	ROOT MEAN SQUARE ERRORS:		
BUTTER	0.027	0.027	0.027
MARGARINE	0.023	0.023	0.023
SHORTENING	0.015	0.016	0.017
	R-SQUARES:		
BUTTER	0.627	0.616	0.605
MARGARINE	0.494	0.484	0.478
SHORTENING	0.391	0.331	0.217
	OBJECTIVE FUNCTION (WEIGHTED SUM OF SQUARED ERRORS) FOR SYSTEM:		
OBJ*N:	113.094	120.386	125.878

1) These results have adding-up, homogeneity, Slutsky symmetry and negative semi-definiteness imposed. The ****, ***, **, and * indicate statistical significance at the alpha = 0.001, 0.01, 0.05 and 0.10 level, respectively.

2) These parameters are the non-zero estimates of the Cholesky decomposition of $\{G_{iq}\}$ which imposes the restriction that this matrix of compensated price slopes in symmetric, negative semi-definite (see Barten and Geyskens).

3) The additional restriction that Dm2 = 0 was imposed when the model failed to converge after iterating to Dm2 = 0. This implies that the lag response in margarine advertising occurs within one quarter (see Figure 1 for the implied lag shape due to this restriction).

SOURCE: Computations by the authors using SAS PROC SYSNLIN, Marquardt algorithm with convergence criterion of 0.001.

Table 2. Gallant-Jorgenson (G-J) Hypotheses Test Results for Alternative Specifications of the Deseasonalized (4th Difference) Rotterdam Model of the Canadian Fats and Oils Sector, 1973-86.*

Null Hypotheses/ Model:	G-J Test	Degrees Freedom	Critical Chi-Square: $\alpha = 0.05$	Critical Chi-Square: $\alpha = 0.025$	$\alpha = 0.05$ Inference
Negative Semi-Definiteness	8.88	3	7.81	9.35	Reject
		6	12.6	14.4	FTR
		9	16.9	19.0	FTR
No Autocorrelation:					
Unrestricted	4.11	1	3.84	5.02	Reject
Scaled	5.58	1	3.84	5.02	Reject
Translated	5.47	1	3.84	5.02	Reject
Separate Scaled Stocks versus Unrestricted	7.29	6	12.6	14.4	FTR
Identical Scaled Stocks versus Unrestricted	15.72	8	15.5	17.5	Reject
Separate Translated Stocks versus Unrestricted	12.78	8	15.5	17.5	FTR
Identical Translated Stocks versus Unrestricted	17.65	10	18.3	20.5	FTR

* These test statistics are computed as $T_0 = nS^{**} - nS^{*}$ where S^{**} and S^{*} are the weighted error sum of squares evaluated at convergence for the restricted and unrestricted models, respectively. This test statistic is asymptotically distributed chi-square with degrees of freedom equal to the difference in the number of parameters in the unrestricted and restricted models. FTR indicates "failure to reject" the corresponding null hypothesis at the $\alpha = 0.05$ level.

Source: Computations by the author using the results from SAS's SYSNLIN package.

Table 3. Demand and Advertising Elasticity Estimates for the Canadian Fats and Oils Sector Evaluated at the 1978:3 - 1986:4 Means.

	Expenditure Elasticities:			
	BUTTER	MARGARINE	SHORTENING	SALAD OILS
UNRESTRICTED ADVERTISING:	1.3358	0.9466	0.4363	0.9895
SCALED ADVERTISING:	1.3297	0.9222	0.5257	0.9315
TRANSLATED ADVERTISING:	1.3184	0.9061	0.4922	1.0440

Model:	Compensated Price Elasticities:			
QUANTITY	BUTTER	MARGARINE	SHORTENING	SALAD OILS
UNRESTRICTED ADVERTISING:				
BUTTER	-0.2701	0.1874	0.2374	0.0084
MARGARINE	0.1446	-0.1003	-0.1271	-0.0045
SHORTENING	0.1225	-0.0850	-0.1076	-0.0038
SALAD OILS	0.0031	-0.0021	-0.0027	-0.0001
SCALED ADVERTISING:				
BUTTER	-0.2928	0.2662	0.0950	0.1054
MARGARINE	0.2054	-0.2060	-0.1314	0.0586
SHORTENING	0.0490	-0.0878	-0.1612	0.2797
SALAD OILS	0.0384	0.0277	0.1976	-0.4436
TRANSLATED ADVERTISING:				
BUTTER	-0.2558	0.1975	0.1984	0.0027
MARGARINE	0.1524	-0.1177	-0.1183	-0.0016
SHORTENING	0.1024	-0.0791	-0.0794	-0.0011
SALAD OILS	0.0010	-0.0007	-0.0008	-0.0000

Model:	Uncompensated Price Elasticities:			
QUANTITY	BUTTER	MARGARINE	SHORTENING	SALAD OILS
UNRESTRICTED ADVERTISING:				
BUTTER	-0.7738	-0.2013	-0.0225	-0.1752
MARGARINE	-0.2123	-0.3758	-0.3112	-0.1346
SHORTENING	-0.0420	-0.2119	-0.1925	-0.0638
SALAD OILS	-0.3701	-0.2901	-0.1952	-0.1361
SCALED ADVERTISING:				
BUTTER	-0.7942	-0.1208	-0.1637	-0.0774
MARGARINE	-0.1423	-0.4743	-0.3108	-0.0682
SHORTENING	-0.1492	-0.2408	-0.2635	0.2074
SALAD OILS	-0.3128	-0.2434	0.0164	-0.5716
TRANSLATED ADVERTISING:				
BUTTER	-0.7529	-0.1861	-0.0581	-0.1785
MARGARINE	-0.1892	-0.3814	-0.2945	-0.1261
SHORTENING	-0.0832	-0.2223	-0.1752	-0.0687
SALAD OILS	-0.3927	-0.3045	-0.2039	-0.1435

Table 3 (Continued).

Model: QUANTITY	Advertising Stocks Elasticities: BUTTER	MARGARINE	SHORTENING	SALAD OILS
UNRESTRICTED ADVERTISING:				
BUTTER	-0.1200	0.0062	-0.0139	0.0653
MARGARINE	0.0234	-0.0754	-0.0271	0.0343
SHORTENING	0.1166	0.0507	0.0623	-0.0851
SALAD OILS	0.1145	0.0709	0.0073	-0.1313
SCALED ADVERTISING:				
BUTTER	-0.1221	0.0276	-0.0167	0.0579
MARGARINE	0.0508	-0.0803	-0.0213	0.0397
SHORTENING	0.0604	0.0437	0.0588	-0.0572
SALAD OILS	0.1419	0.0325	0.0078	-0.1619
TRANSLATED ADVERTISING:				
BUTTER	-0.1116	0.0341	-0.0115	0.0307
MARGARINE	0.0758	-0.0655	-0.0079	0.0211
SHORTENING	0.0412	0.0127	0.0406	0.0115
SALAD OILS	0.0873	0.0270	-0.0091	-0.1452

Model: QUANTITY	Current Period Advertising Elasticities: BUTTER	MARGARINE	SHORTENING	SALAD OILS
UNRESTRICTED ADVERTISING:				
BUTTER	-0.0226	0.0061	-0.0000	0.0060
MARGARINE	0.0044	-0.0740	-0.0000	0.0031
SHORTENING	0.0220	0.0498	0.0000	-0.0078
SALAD OILS	0.0216	0.0696	0.0000	-0.0120
SCALED ADVERTISING:				
BUTTER	-0.0316	0.0271	-0.0000	0.0067
MARGARINE	0.0132	-0.0788	-0.0000	0.0046
SHORTENING	0.0156	0.0429	0.0000	-0.0066
SALAD OILS	0.0368	0.0319	0.0000	-0.0187
TRANSLATED ADVERTISING:				
BUTTER	-0.0272	0.0335	-0.0072	0.0036
MARGARINE	0.0185	-0.0643	-0.0050	0.0025
SHORTENING	0.0100	0.0125	0.0255	0.0014
SALAD OILS	0.0213	0.0265	-0.0057	-0.0172

SOURCE: Computations by the authors.

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