WI

S.P. no. 302

April 1989 Staff papers No. 302

BOUNDED PRICE VARIATION MODELS WITH RATIONAL EXPECTATIONS AND PRICE RISK

By

Matthew T. Holt[*]
University of Wisconsin-Madison

Abstract: The bounded price variation model under rational expectations is extended to include higher moments of price in the supply equation. The resulting framework is then used to estimate a disequilibrium model of the U.S. corn market that includes price risk.

---

[*] Matthew T. Holt is an assistant professor, Department of Agricultural Economics, University of Wisconsin-Madison.

1. Introduction

There are many instances where the market clearing assumption is appropriate only part of the time. For example, the U.S. government supports prices for many domestic agricultural products. If the market price falls below the established support price, producers can sell to the government at the support price and the market is in disequilibrium. Methods for estimating models of markets which exhibit occasional disequilibrium (i.e., bounded price variation) have been investigated by Maddala and Nelson (1975) and Maddala (1983a, 1983b).

Chanda and Maddala (1983, 1984) and Shonkwiler and Maddala (1985) have recently explored ways in which the bounded price variation model can be extended to include rational expectations in the supply equation. But because the probability of market disequilibrium must be determined endogenously, the resulting model specifications are highly nonlinear and closed-form expressions can not be obtained. Consequently, the estimation procedures proposed are limited either to an instrumental variables approach or to specifying a priori the probability of equilibrium. Most recently, Holt and Johnson (1989) show that the Fair-Taylor (1983) method can be used to obtain maximum likelihood (ML) estimates of a bounded prices model under rational expectations without making restrictive assumptions about the probability of equilibrium.

While progress has been made, it follows that previous studies have not considered the possibility that price supports also affect higher moments of the price distribution. This is particularly important if producers exhibit risk averse behavior (Eeckhoudt and Hansen, 1980). In this paper, the bounded prices model under rational expectations is extended to include price risk.

Specifically, expressions for the second and third central moments of truncated prices are developed. The resulting framework is then used in combination with the Fair-Taylor method (1983) to estimate a bounded prices model of the U.S. corn market.

2. The Model

Consider the following market model with an exogenously set lower price limit, $\bar{P}_t$:

$$D_t = \alpha_1' X_{1t} + \alpha^* P_t + e_{1t} \tag{1}$$

$$S_t = \beta_1' X_{2t} + \beta_1^* P_t^e + \beta_2^* \sigma_{vt}^2 + \beta_3^* \sigma_{vt}^3 + e_{2t} \tag{2}$$

$$Q_t = D_t = S_t \quad \text{if} \quad \bar{P}_t \leq P_t \tag{3}$$

$$Q_t = D_t < S_t \quad \text{if} \quad P_t > \bar{P}_t \tag{4}$$

where D_t is quantity demanded, S_t is quantity supplied, Q_t denotes quantity transacted, P_t is market price, P_t^e is the rational expectation of price formed when production decisions are made, σ_{vt}^2 is the rational expectation of the variance of price, σ_{vt}^3 is the rational expectation of the third central moment of price, also formed at the time production decisions are made, X_{1t} and X_{2t} are vectors of supply and demand shifters, respectively, and e_{1t} and e_{2t} are are joint normally distributed random variables with mean zero and variance-covariance matrix Σ. With observations on $\bar{P}_t$ and P_t, we can classify the data points belonging to equilibrium (Ψ_1) and those belonging to excess supply (Ψ_2). The above model differs from previous bounded price variation models with rational expectations in that rational expectations of higher moments are also included in the supply equation.

The rational predictors for price and price risk are derived as follows. The reduced form price equation from (1)-(3) is:

$$P_t = (\alpha^*)^{-1}(\beta_1' X_{2t} + \beta_1^* P_t^e + \beta_2^* \sigma_{vt}^2 + \beta_3^* \sigma_{vt}^3 - \alpha_1' X_{1t} + e_{2t} - e_{1t}). \quad (5)$$

Taking expectations of both sides of (5) gives the rational predictor:

$$P_t^* = (\alpha^*)^{-1}(\beta_1' X_{2t}^e + \beta_1^* P_t^e + \beta_2^* \sigma_{vt}^2 + \beta_3^* \sigma_{vt}^3 - \alpha_1' X_{1t}^e) \quad (6)$$

where X_{1t}^e and X_{2t}^e denote the expectations of demand and supply shifters, respectively. Likewise, taking the variance operator through equation (5) gives a similar expression for the (untruncated) rational expectation of price variance:

$$\sigma_p^2 = (\alpha^*)^{-2}(\beta_1' \psi_1 \beta_1 + \alpha_1' \psi_2 \alpha_1 + \alpha_1' \psi_3 \beta_1 + \sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) \quad (7)$$

where ψ_1 is the variance-covariance matrix associated with (unknown) supply shifters, ψ_2 is similarly defined for demand shifters, and ψ_3 is the variance-covariance matrix between X_{1t} and X_{2t}. Here σ_1^2, σ_2^2, and σ_{12}^2 denote the structural variance-covariance terms from the matrix Σ.

The expressions in (6)-(7) are not the appropriate supply inducing expectations in the bounded prices model, however. This is because the truncation effects of the price support (i.e., the probabilities of equilibrium) have not yet been incorporated. Using standard results for truncated normal distributions, it can be shown that the expectations relating (6) and (7) to the truncated expectations of the mean, variance, and third central moment of price are:[1]

$$P_t^e = \bar{P}_t\Phi(K_t) + \sigma_p\phi(K_t) + P_t^*[1 - \Phi(K_t)], \tag{8}$$

$$\sigma_{vt}^2 = \bar{P}_t^2\Phi(K_t) + \sigma_p^2 K_t\phi(K_t) + 2P_t^*\sigma_p\phi(K_t) + [P_t^{*2} + \sigma_p^2][1 - \Phi(K_t)] - P_t^{e2}, \tag{9}$$

$$\begin{aligned}\sigma_{vt}^3 = {} & \bar{P}_t^2[\bar{P}_t - 3P_t^e]\Phi(K_t) + \sigma_p^3 K_t^2\phi(K_t) + 3\sigma_p^2(P_t^* - P_t^e)K_t\phi(K_t) \\ & + [2\sigma_p^3 + 3P_t^{*2}\sigma_p - 6P_t^e P_t^*\sigma_p]\phi(K_t) + [P_t^{*3} - 3P_t^e P_t^* \\ & + 3\sigma_p^2(P_t^* - P_t^e)][1 - \Phi(K_t)] + 2P_t^{e3},\end{aligned} \tag{10}$$

where,

$$K_t = [\bar{P}_t - (\alpha^*)^{-1}(\beta_1 X_{2t}^e + \beta_1^* P_t^e + \beta_2^* \sigma_{vt}^2 + \beta_3^* \sigma_{vt}^3 - \alpha_1' X_{1t}^e]/\sigma_p \tag{11}$$

and

$$\begin{aligned}1 - \Phi(K_t) = \mathrm{Prob}[(\alpha^*)^{-1}(e_{2t} - e_{1t}) > \bar{P}_t - (\alpha^*)^{-1}(\beta_1' X_{2t} + \beta_1^* P_t^e \\ + \beta_2^* \sigma_{vt}^2 + \beta_3^* \sigma_{vt}^3 - \alpha_1' X_{1t})].\end{aligned} \tag{12}$$

Here $\Phi(\cdot)$ and $\phi(\cdot)$ denote respectively the density and distribution function of the standard normal and $1 - \Phi(K_t)$ is the probability of market equilibrium. The simultaneous solution of equations (6)-(12) gives the rational predictors for the truncated mean, variance, and third central moment of price. Clearly, closed-form expressions for these expectations can not be obtained. However, given some initial parameter estimates, the model can be solved numerically using the Fair-Taylor (1983) algorithm.

3. Some Empirical Results

The above framework was used to estimate a bounded prices model of the U.S. corn market. Data on corn price, production, disappearance, stocks, and exports were obtained form various USDA publications for the 1950-85 period. Thirteen years were identified in the sample as disequilibrium periods (Ψ_2) while seventeen years were associated with market clearing (Ψ_1). Demand shifters used include corn exports (EXP_t), total disposable income (INC_t), and domestic livestock production ($LPRO_t$). Likewise, a time trend (Time), lagged production (S_{t-1}), and an intercept shifter for 1983 ($D83_t$) were specified in the supply equation. Finally, first-order AR models were used to predict values for the unknown exogenous variables and the parameters of these models were estimated simultaneously with the supply-demand system.[2/]

The ML estimation results are presented in table 1.[3/] The estimated model fits the data well as indicated by the Baxter and Cragg (1970) generalized R^2 (0.986). Moreover, all estimated parameters have plausible signs and magnitudes and, with the exception of $LPRO_t$ and S_{t-1}, all variables are statistically significant. Importantly, the results confirm that price risk, as measured by σ^2_{vt} and σ^3_{vt}, is a meaningful component of corn supply response. The supply elasticities are 0.429 for expected price, -0.046 for price variance, and 0.054 for the third central moment of price; hence, price risk is quantitatively important but its impact on production is proportionally less than the mean price effect. Finally, the estimated values for the third moment are positive at all data points, thus confirming that government farm programs tend to result in right-skewed price distributions. Moreover, the results indicate that the mean effect and the skew effect tend to work in the same direction.

4. Conclusions

Previous studies have not explored the linkages between price support programs and producers' risk perceptions. This paper has developed a framework for evaluating price risk in a bounded prices model under rational expectations. Unlike previous studies, price risk was defined to include both the second and and third central moment of the price distribution. The resulting framework was used to estimate a supply-demand system for the U.S. corn market. The empirical results suggest that price risk is an important component of corn supply response and that government price support programs can affect higher moments of the equilibrium price distribution as well.

Table 1. Maximum likelihood estimates of a bounded price variation model of the U.S. corn market, 1950-85.

Equation	Parameter	Variable	Coefficient	Standard Error
Demand	α_0	Intercept	46.476	1.940
	α_1	P_t	- 17.072	1.128
	α_2	EXP_t	3.194	0.256
	α_3	INC_t	0.111	0.025
	α_4	$LPRO_t$	0.030	0.041
Supply	β_0	Intercept	3.989	1.194
	β_1	P^e_t	10.561	1.539
	β_2	σ^2_{vt}	- 17.972	2.014
	β_3	σ^3_{vt}	49.888	1.107
	β_4	Time	1.030	0.170
	β_5	S_{t-1}	0.116	0.110
	β_6	$D83_t$	- 44.024	4.939
Exports	η_0	Intercept	0.993	0.460
	η_1	EXP_{t-1}	0.924	0.041
Income	γ_0	Intercept	- 0.029	0.303
	γ_1	INC_{t-1}	1.086	0.005
Livestock	θ_0	Intercept	4.252	1.019
	θ_1	$LPRO_{t-1}$	0.966	0.012
Log of Likelihood			-207.045	

Footnotes

1/ A detailed derivation of the results reported in equations (8)-(12) can be obtained from the author upon request.

2/ As suggested by Hoffman (1987), this approach results in consistent estimates of the parameters' standard errors.

3/ See Maddala (1983b) and Holt and Johnson (1989) for details on the appropriate likelihood function to be maximized.

References

Baxter, N.D. and J.L. Cragg, 1970, Corporate choice among long-term financing instruments, Review of Economics and Statistics 68, 105-114.

Chanda, A. and G.S. Maddala, 1983, Methods of estimation for models of markets with bounded price variation under rational expectations, Economics Letters 13, 181-184, 1984, Erratum in Economics Letters 15, 195-196.

Eeckhoudt, L. and P. Hansen, 1980, Minimum and maximum prices, uncertainty, and the theory of the competitive firm, American Economic Review 70, 1064-1068.

Fair, R.C. and J.B. Taylor, 1983, Solution and maximum likelihood estimation of dynamic nonlinear rational expectations models, Econometrica 51, 1169-1185.

Hoffman, D.L., 1987, Two-step generalized least squares estimators in multi-equation generated regressor models, Review of Economics and Statistics 59, 336-346.

Holt, M.T. and S.R. Johnson, 1989, Bounded price variation and rational expectations in an endogenous switching model of the U.S. corn market, Review of Economics and Statistics, forthcoming.

Maddala, G.S., 1983a, Limited dependent and qualitative variables in econometrics, (Cambridge University Press, Cambridge).

Maddala, G.S., 1983b, Methods of estimation for models of markets with bounded price variation, International Economic Review 24, 361-387.

Maddala, G.S., and F.D. Nelson, 1975, Switching regression models with exogenous and endogenous switching, Proceedings of the Business and Economics Statistics Section, American Statistical Association, 423-426.

Shonkwiler, J.S. and G.S. Maddala, 1985, Modeling expectations of bounded prices: an application to the market for corn, Review of Economics and Statistics 67, 697-702.