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# BOUNDED PRICE VARIATION MODELS WITH RATIONAL EXPECTATIONS AND PRICE RISK

By

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Abstract: The bounded price variation model under rational expectations is extended to include higher moments of price in the supply equation. The resulting framework is then used to estimate a disequilibrium model of the U.S. corn market that includes price risk.

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## 1.  Introduction

There are many instances where the market clearing assumption is appropriate only part of the time.  For example, the U.S. government supports prices for many domestic agricultural products.  If the market price falls below the established support price, producers can sell to the government at the support price and the market is in disequilibrium.  Methods for estimating models of markets which exhibit occasional disequilibrium (i.e., bounded price variation) have been investigated by Maddala and Nelson (1975) and Maddala (1983a, 1983b).

Chanda and Maddala (1983, 1984) and Shonkwiler and Maddala (1985) have recently explored ways in which the bounded price variation model can be extended to include rational expectations in the supply equation.  But because the probability of market disequilibrium must be determined endogenously, the resulting model specifications are highly nonlinear and closed-form expressions can not be obtained.  Consequently, the estimation procedures proposed are limited either to an instrumental variables approach or to specifying a priori the probability of equilibrium.  Most recently, Holt and Johnson (1989) show that the Fair-Taylor (1983) method can be used to obtain maximum likelihood (ML) estimates of a bounded prices model under rational expectations without making restrictive assumptions about the probability of equilibrium.

While progress has been made, it follows that previous studies have not considered the possibility that price supports also affect higher moments of the price distribution.  This is particularly important if producers exhibit risk averse behavior (Eeckhoudt and Hansen, 1980).  In this paper, the bounded prices model under rational expectations is extended to include price risk.

Specifically, expressions for the second and third central moments of truncated prices are developed. The resulting framework is then used in combination with the Fair-Taylor method (1983) to estimate a bounded prices model of the U.S. corn market.

## 2. The Model

Consider the following market model with an exogenously set lower price limit, $\bar{P}_t$:

$$D_t = \alpha_1' X_{1t} + \alpha^* P_t + e_{1t} \tag{1}$$

$$S_t = \beta_1' X_{2t} + \beta_1^* P_t^e + \beta_2^* \sigma_{vt}^2 + \beta_3^* \sigma_{vt}^3 + e_{2t} \tag{2}$$

$$Q_t = D_t = S_t \quad \text{if} \quad \bar{P}_t \leq P_t \tag{3}$$

$$Q_t = D_t < S_t \quad \text{if} \quad P_t > \bar{P}_t \tag{4}$$

where $D_t$ is quantity demanded, $S_t$ is quantity supplied, $Q_t$ denotes quantity transacted, $P_t$ is market price, $P_t^e$ is the rational expectation of price formed when production decisions are made, $\sigma_{vt}^2$ is the rational expectation of the variance of price, $\sigma_{vt}^3$ is the rational expectation of the third central moment of price, also formed at the time production decisions are made, $X_{1t}$ and $X_{2t}$ are vectors of supply and demand shifters, respectively, and $e_{1t}$ and $e_{2t}$ are are joint normally distributed random variables with mean zero and variance-covariance matrix $\Sigma$. With observations on $\bar{P}_t$ and $P_t$, we can classify the data points belonging to equilibrium ($\Psi_1$) and those belonging to excess supply ($\Psi_2$). The above model differs from previous bounded price variation models

with rational expectations in that rational expectations of higher moments are also included in the supply equation.

The rational predictors for price and price risk are derived as follows. The reduced form price equation from (1)-(3) is:

$$P_t = (\alpha^*)^{-1}(\beta_1' X_{2t} + \beta_1^* P_t^e + \beta_2^* \sigma_{vt}^2 + \beta_3^* \sigma_{vt}^3 - \alpha_1' X_{1t} + e_{2t} - e_{1t}). \quad (5)$$

Taking expectations of both sides of (5) gives the rational predictor:

$$P_t^* = (\alpha^*)^{-1}(\beta_1' X_{2t}^e + \beta_1^* P_t^e + \beta_2^* \sigma_{vt}^2 + \beta_3^* \sigma_{vt}^3 - \alpha_1' X_{1t}^e) \quad (6)$$

where $X_{1t}^e$ and $X_{2t}^e$ denote the expectations of demand and supply shifters, respectively. Likewise, taking the variance operator through equation (5) gives a similar expression for the (untruncated) rational expectation of price variance:

$$\sigma_p^2 = (\alpha^*)^{-2}(\beta_1' \psi_1 \beta_1 + \alpha_1' \psi_2 \alpha_1 + \alpha_1' \psi_3 \beta_1 + \sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) \quad (7)$$

where $\psi_1$ is the variance-covariance matrix associated with (unknown) supply shifters, $\psi_2$ is similarly defined for demand shifters, and $\psi_3$ is the variance-covariance matrix between $X_{1t}$ and $X_{2t}$. Here $\sigma_1^2$, $\sigma_2^2$, and $\sigma_{12}^2$ denote the structural variance-covariance terms from the matrix $\Sigma$.

The expressions in (6)-(7) are not the appropriate supply inducing expectations in the bounded prices model, however. This is because the truncation effects of the price support (i.e., the probabilities of

equilibrium) have not yet been incorporated. Using standard results for truncated normal distributions, it can be shown that the expectations relating (6) and (7) to the truncated expectations of the mean, variance, and third central moment of price are:[1]

$$P_t^e = \bar{P}_t \Phi(K_t) + \sigma_p \phi(K_t) + P_t^*[1 - \Phi(K_t)], \tag{8}$$

$$\sigma_{vt}^2 = \bar{P}_t^2 \Phi(K_t) + \sigma_p^2 K_t \phi(K_t) + 2P_t^* \sigma_p \phi(K_t) + [P_t^{*2} + \sigma_p^2][1 - \Phi(K_t)] - P_t^{e2}, \tag{9}$$

$$\sigma_{vt}^3 = \bar{P}_t^2 [\bar{P}_t - 3P_t^e] \Phi(K_t) + \sigma_p^3 K_t^2 \phi(K_t) + 3\sigma_p^2 (P_t^* - P_t^e) K_t \phi(K_t)$$

$$+ [2\sigma_p^3 + 3P_t^{*2}\sigma_p - 6P_t^e P_t^* \sigma_p]\phi(K_t) + [P_t^{*3} - 3P_t^e P_t^* \tag{10}$$

$$+ 3\sigma_p^2(P_t^* - P_t^e)][1 - \Phi(K_t)] + 2P_t^{e3},$$

where,

$$K_t = [\bar{P}_t - (\alpha^*)^{-1}(\beta_1 X_{2t}^e + \beta_1^* P_t^e + \beta_2^* \sigma_{vt}^2 + \beta_3^* \sigma_{vt}^3 - \alpha_1' X_{1t}^e]/\sigma_p \tag{11}$$

and

$$1 - \Phi(K_t) = Prob[(\alpha^*)^{-1}(e_{2t} - e_{1t}) > \bar{P}_t - (\alpha^*)^{-1}(\beta_1' X_{2t} + \beta_1^* P_t^e \tag{12}$$

$$+ \beta_2^* \sigma_{vt}^2 + \beta_3^* \sigma_{vt}^3 - \alpha_1' X_{1t})].$$

Here $\Phi(\cdot)$ and $\phi(\cdot)$ denote respectively the density and distribution function of the standard normal and $1 - \Phi(K_t)$ is the probability of market equilibrium. The simultaneous solution of equations (6)-(12) gives the rational predictors for the truncated mean, variance, and third central moment of price. Clearly,

closed-form expressions for these expectations can not be obtained. However, given some initial parameter estimates, the model can be solved numerically using the Fair-Taylor (1983) algorithm.

## 3. Some Empirical Results

The above framework was used to estimate a bounded prices model of the U.S. corn market. Data on corn price, production, disappearance, stocks, and exports were obtained form various USDA publications for the 1950-85 period. Thirteen years were identified in the sample as disequilibrium periods ($\Psi_2$) while seventeen years were associated with market clearing ($\Psi_1$). Demand shifters used include corn exports ($EXP_t$), total disposable income ($INC_t$), and domestic livestock production ($LPRO_t$). Likewise, a time trend (Time), lagged production ($S_{t-1}$), and an intercept shifter for 1983 ($D83_t$) were specified in the supply equation. Finally, first-order AR models were used to predict values for the unknown exogenous variables and the parameters of these models were estimated simultaneously with the supply-demand system.[2]

The ML estimation results are presented in table 1.[3] The estimated model fits the data well as indicated by the Baxter and Cragg (1970) generalized $R^2$ (0.986). Moreover, all estimated parameters have plausible signs and magnitudes and, with the exception of $LPRO_t$ and $S_{t-1}$, all variables are statistically significant. Importantly, the results confirm that price risk, as measured by $\sigma_{vt}^2$ and $\sigma_{vt}^3$, is a meaningful component of corn supply response. The supply elasticities are 0.429 for expected price, -0.046 for price variance, and 0.054 for the third central moment of price; hence, price risk is quantitatively important but its impact on production is proportionally less than the mean price effect. Finally, the estimated values

for the third moment are positive at all data points, thus confirming that government farm programs tend to result in right-skewed price distributions. Moreover, the results indicate that the mean effect and the skew effect tend to work in the same direction.

## 4. Conclusions

Previous studies have not explored the linkages between price support programs and producers' risk perceptions. This paper has developed a framework for evaluating price risk in a bounded prices model under rational expectations. Unlike previous studies, price risk was defined to include both the second and and third central moment of the price distribution. The resulting framework was used to estimate a supply-demand system for the U.S. corn market. The empirical results suggest that price risk is an important component of corn supply response and that government price support programs can affect higher moments of the equilibrium price distribution as well.

Table 1. Maximum likelihood estimates of a bounded price variation model of the U.S. corn market, 1950-85.

| Equation | Parameter | Variable | Coefficient | Standard Error |
|---|---|---|---|---|
| Demand | $\alpha_0$ | Intercept | 46.476 | 1.940 |
| | $\alpha_1$ | $P_t$ | - 17.072 | 1.128 |
| | $\alpha_2$ | $EXP_t$ | 3.194 | 0.256 |
| | $\alpha_3$ | $INC_t$ | 0.111 | 0.025 |
| | $\alpha_4$ | $LPRO_t$ | 0.030 | 0.041 |
| Supply | $\beta_0$ | Intercept | 3.989 | 1.194 |
| | $\beta_1$ | $P_t^e$ | 10.561 | 1.539 |
| | $\beta_2$ | $\sigma_{vt}^2$ | - 17.972 | 2.014 |
| | $\beta_3$ | $\sigma_{vt}^3$ | 49.888 | 1.107 |
| | $\beta_4$ | Time | 1.030 | 0.170 |
| | $\beta_5$ | $S_{t-1}$ | 0.116 | 0.110 |
| | $\beta_6$ | $D83_t$ | - 44.024 | 4.939 |
| Exports | $\eta_0$ | Intercept | 0.993 | 0.460 |
| | $\eta_1$ | $EXP_{t-1}$ | 0.924 | 0.041 |
| Income | $\gamma_0$ | Intercept | - 0.029 | 0.303 |
| | $\gamma_1$ | $INC_{t-1}$ | 1.086 | 0.005 |
| Livestock | $\theta_0$ | Intercept | 4.252 | 1.019 |
| | $\theta_1$ | $LPRO_{t-1}$ | 0.966 | 0.012 |

Log of Likelihood                     -207.045

## Footnotes

1/ A detailed derivation of the results reported in equations (8)-(12) can be obtained from the author upon request.

2/ As suggested by Hoffman (1987), this approach results in consistent estimates of the parameters' standard errors.

3/ See Maddala (1983b) and Holt and Johnson (1989) for details on the appropriate likelihood function to be maximized.

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