



AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
<http://ageconsearch.umn.edu>
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

WI

S-P. no. 299

GIANNINI FOUNDATION OF
AGRICULTURAL ECONOMICS
LIBRARY

MAY 10 1989

WITHDRAWN

A Nonparametric Analysis of the Structure
and Stability of Preferences:
U.S. Food Consumption 1964-83

by

Thomas L. Cox*

and

Jean-Paul Chavas

* Respectively, Assistant Professor and Professor of Agricultural Economics, University of Wisconsin-Madison. The seniority of authorship is shared by the authors, Department of Agricultural Economics Staff Paper #299, December 1988.

A Nonparametric Analysis of the Structure
and Stability of Preferences:
U.S. Food Consumption 1964-83

I - Introduction:

Over the last few decades, a considerable amount of economic research has focused on the analysis of consumer demand. In the typical approach to demand system modelling with time-series data, demands are parametrized as functions of prices and income and then estimated using observable data. However, this well defined procedure suffers from the defect that one is always facing a joint hypothesis: the implications of consumer theory plus the maintained hypotheses concerning the specification of consumer preferences, without being able to differentiate between the two. This can make the investigation of the structure and stability of consumer preferences rather difficult.

The typical parametric approach to the stability problem is to specify demand functions in some form, estimate them and interpret any significant shift in the parameters as evidence of structural change (Braschler; Chavas; Dahlgran; Eales and Unnevehr; Moschini and Miller; Nyankori and Miller; Thurman; Wohlgenant). Although this approach to structural change can be easily implemented empirically, it may not be very useful. First, the structural change results may differ depending on the parametric specification used. For example, in the investigation of meat demand, different model specifications have led to different conclusions: Chavas, Dahlgran, Eales and Unnevehr, Nyankori and Miller, and Thurman found evidence of parameter shifts, while Moschini and Meilke and Wohlgenant argued that meat demand can be explained in terms

of prices and income effects alone. Second, maintained hypotheses concerning the separability structure imposed by the parametric specification can cloud the stability of preferences issue. For example, Eales and Unnevehr found that the implicit separability structure imposed by commonly analyzed meat aggregates (e.g., beef and veal, pork and poultry) is often inappropriate and can distort parametric measurements of structural changes in preferences. Last, changing demand parameters may reflect a deterioration of the local approximation of consumer preferences as consumption deviates from the expansion point of a flexible functional form. Given these parametric difficulties, it is desirable to consider alternative methods for the structure and stability analyses that do not depend on the parametrization of the demand or underlying preference functions.

Nonparametric methods have been proposed for the analysis of consumer behavior (see Afriat; Diewert, 1973; Diewert and Parkan; Varian, 1982, 1983, 1985; Chiappori and Rochet; Bronars). These nonparametric methods do not require ad hoc specification of a functional form for demand equations, while still allowing tests of data for consistency with consumer theory. These methods can complement the more traditional parametric approach to demand analysis (Barnhart and Whitney; Chalfant and Alston; Chiappori and Rochet; McMillan and Amoako-Tuffour; Swofford and Whitney).

The objective of this paper is to propose a nonparametric approach to the structure and stability of consumer preferences and to investigate its usefulness. This approach, presented in section II, extends the work of Afriat, Diewert and Varian by exploiting the

existence of duality relationships in consumer theory. In this context, a nonparametric revealed preference test is derived under differentiability assumptions typically made in parametric demand analysis. Assuming that consumers optimize over preferences subject to a budget constraint, we show how a finite number of consumption observations (involving prices, quantities consumed and total expenditures) can be used to test the consistency of the data with a dual formulation of consumer theory under differentiability. Our proposed test has the advantage of increasing the power of previous nonparametric tests in demand analysis (Bronars). Although these tests are not statistical tests with associated probability statements (Varian, 1985; Bronars), they do provide evidence whether or not a set of consumption data is consistent with some stable and separable representation of consumer preferences. The empirical implementation of these tests is discussed in section III. Their interpretation in terms of testing for weak separability and the stability of preferences is presented in section IV. A number of results obtained from a nonparametric analysis of U.S. food consumption from 1964 to 1983 illustrate the usefulness of the proposed procedures (sections V and VI).

II - Nonparametric Demand Analysis:

The nonparametric approach to demand analysis consists in analyzing a finite body of consumption data with no ad hoc specification of the functional forms for utility or demand functions (see Afriat; Diewert, 1973; Varian, 1982, 1983; Diewert and Parkan). Consider a set of T observations on n -dimensional consumption vectors $x_t = (x_{t1}, \dots, x_{tn})' \geq$

0, corresponding price vectors $p_t = (p_{t1}, \dots, p_{tn})' \geq 0$ and consumer expenditures $I_t > 0$, $t = 1, \dots, T$. The nonparametric method presents a series of tests that these data must satisfy if they are to be consistent with consumer theory (i.e. with the optimization of an objective function subject to constraints). Throughout this paper, we take the optimization hypothesis as maintained, i.e. we assume that the consumer makes consumption choices in an optimizing way. As such, the nonparametric tests consist in checking the existence of a utility function (representing consumer preferences) that is consistent with observed consumption behavior. Our discussion will be limited to the case where the price vectors (p_t) and the quantity vectors (x_t) are positive and finite for all $t=1, \dots, T$. Also, we will make use of normalized prices defined as $v_t = p_t/I_t$, implying that the budget constraint takes the form $v_t'x_t \leq 1$.

One of the main results of nonparametric demand analysis is presented next (Afriat; Diewert, 1973; Varian, 1982, 1983; Diewert and Parkan).

Lemma 1: The following statements are equivalent:

a/ there exists a utility function $u(x)$ such that

$$x_t \text{ solves } \max \{u(x) : v_t'x \leq 1\}, \quad t=1, \dots, T,$$

where $u(x)$ is a continuous and strictly increasing direct utility function.

b/ there exist V_t, λ_t , such that

$$\lambda_t > 0 \tag{1a}$$

$$V_s \leq V_t + \lambda_t [v_t'x_s - 1], \quad s, t=1, \dots, T. \tag{1b}$$

c/ there exists a concave and strictly increasing direct utility function $f(x)$ that rationalizes the data in the sense that

$$x_t \text{ solves } \max \{f(x) : v_t'x \leq 1\}, t=1, \dots, T.$$

These inequalities (1a,1b), first proposed by Afriat, provide directly testable conditions that the data must satisfy in order to be consistent with utility maximization. Implementation of this test consists in checking whether there exists a solution to the linear inequalities (1), where the V_t 's can be interpreted as utility levels and the λ_t ' as marginal utilities of (normalized) income. Alternative ways of investigating the empirical existence of a solution to (1a,1b) have been proposed in the literature (see Diewert, 1973; Varian, 1982, 1983).

Note that Lemma 1 is concerned with the maximization of a direct utility function $u(x)$. This maximization can be used to define the indirect utility function, as stated in the following primal problem:

$$g(v) = \max_x \{u(x) : v'x \leq 1\} \quad (2)$$

where $g(v) = u(x^*(v))$, $x^*(v)$ being the quantity dependent Marshallian demand correspondence.

It can be shown that, if $u(x)$ is a continuous and strictly increasing function, then $g(v)$ is a continuous, strictly decreasing and quasi-convex function for $v > 0$ (see Diewert, 1974, 1982). Using $g(v)$ in (2), consider the dual problem

$$\bar{u}(x) = \min_v \{g(v) : v'x \leq 1\}. \quad (3)$$

where $\bar{u}(x) = g(v^*(x))$, $v^*(x)$ being the price dependent or inverse Marshallian demand correspondence. The conditions under which $\bar{u}(x)$ in

(3) is equal to $u(x)$ establish the existence of duality between the direct utility function $u(x)$ and the indirect utility function $g(v)$. For positive prices and quantities, Diewert (1974, 1982) has shown that $u(x) = \bar{u}(x)$ if the direct utility function is continuous, increasing and quasi-concave. In such a case, the indirect utility function completely characterizes the direct utility function (and vice versa), thus providing an alternative but equivalent representation of consumer preferences. Such duality relationships have been investigated in detail in the literature (e.g. Lau; Samuelson; Diewert, 1974, 1982; Blackorby et al.). By suggesting alternative formulations of consumer theory, duality relationships have stimulated much research on the characterization of consumer behavior (e.g. Anderson; Weymark; Christensen et al.).

What are the nonparametric implications of the dual problem (3)? Noting that $\min_v \{g(v): v'x \leq 1\} = -\max_v \{-g(v): v'x \leq 1\}$, the following result is obtained from lemma 1.

Lemma 2: The following statements are equivalent:

a/ there exists an indirect utility function $g(v)$ such that

$$v_t \text{ solves } \min \{g(v): v'x_t \leq 1\}, \quad t=1, \dots, T,$$

where $g(v)$ is a continuous and strictly decreasing indirect utility function.

b/ there exist $\bar{v}_t, \bar{\lambda}$, such that

$$\bar{\lambda}_t > 0 \quad (4a)$$

$$\bar{v}_t \leq \bar{v}_s + \bar{\lambda}_t [v_s' x_t - 1], \quad s, t=1, \dots, T. \quad (4b)$$

c/ there exists a convex and strictly decreasing indirect utility function $h(v)$ that rationalizes the data in the sense that

$$v_t \text{ solves } \min \{h(v) : v' x_t \leq 1\}, \quad t=1, \dots, T.$$

Equations (4) present necessary and sufficient conditions for the data (v_t, x_t) , $t=1, \dots, T$, to be consistent with the dual (indirect) utility minimization problem (3), where the \bar{v}_t 's can be interpreted as utility levels and the $\bar{\lambda}_t$'s as marginal utilities of (normalized) income. Again a consistency test would involve checking whether there exists a solution to the linear inequalities (4).

Note that the inequalities (1) of the primal problem (2) differ from the inequalities (4) of the dual problem (3). This reflects differences in the domain of definition of the variables. Lemma 1 involves any quantities $x \geq 0$ but only a finite number of prices. In contrast, lemma 2 involves any prices $v \geq 0$ but only a finite number of quantities.

If the direct utility function $u(x)$ is not strictly quasi-concave (as assumed in lemma 1), it is well known that existence of demand functions is not guaranteed. For example, the maximization of a quasi-concave direct utility function generates only a demand correspondence (Chiappori and Rochet). Since parametric demand analysis typically assumes the existence of differentiable demand functions, it is of interest to examine stronger conditions for the representation of

consumer preferences.^{1/} In particular, we consider the case where $u(x)$ is differentiable, strictly increasing and strictly quasi-concave.^{2/} This guarantees the existence of demand functions typically used in parametric demand analysis. This also implies the existence of duality relationships between the primal problem (2) and the dual problem (3): that is, the two problems provide equivalent representations of consumer preferences where $V_t = \bar{V}_t$ (see Diewert, 1974, 1982; Weymark). Moreover, $u(x)$ and $g(v)$ are connected by Legendre transformation where, at equilibrium, $\lambda_t = \bar{\lambda}_t = \frac{\partial U(x_t)}{\partial x_t} \cdot x_t = - \frac{\partial g(v_t)}{\partial v_t} \cdot v_t$ (see Lau; Samuelson). Under such conditions, the optimal solutions to problems (2) and (3) are unique and the following result is obtained from lemma 1 and lemma 2:

Proposition 1: Consider a differentiable, strictly increasing and strictly quasi-concave direct utility function $u(x)$ such that

$$x_t \text{ solves } \max \{u(x) : v_t'x \leq 1\}, \quad t=1, \dots, T.$$

Also, consider the dual differentiable, strictly decreasing and strictly quasi-convex indirect utility function $g(v) = \max \{u(x) : v'x \leq 1\}$ such that

$$v_t \text{ solves } \min \{g(v) : v'x_t \leq 1\}, \quad t=1, \dots, T.$$

Then, there exists V_t, λ_t such that

$$\lambda_t > 0 \quad (5a)$$

$$V_t \leq V_s + \min(\lambda_s(v_s'x_t - 1), \lambda_t(v_s'x_t - 1)), \quad s, t=1, \dots, T \quad (5b)$$

Proof: Equations (5) are obtained from (1) and (4), given the duality relationships $V_t = \bar{V}_t$ and $\lambda_t = \bar{\lambda}_t$. Q.E.D.

In contrast to previous nonparametric demand analysis (based on the primal problem (2) and equations (1), as in Varian), equations (5) reflect the empirical implications of both the primal problem (2) and the dual problem (3) given a finite number of data points. As we will see below, equations (5) are more restrictive than equations (1) in the primal problem or equations (4) in the dual problem. Thus, using (5) (instead of (1) or (4)) should provide more precise results for the nonparametric analysis of consumer behavior. This nonparametric primal-dual representation of consumer theory under differentiability assumptions typically made in parametric analysis will be called the "differentiable duality" formulation in the rest of the paper. Empirical applications to demand analysis are discussed next.

III - Empirical Implementation:

Equations (1), (4), and (5) all consist in a set of linear inequality restrictions. Finding a solution to these inequality restrictions is equivalent to testing the consistency of consumption data with a corresponding formulation of consumer theory. Although these tests are not statistical tests (i.e. tests attaching a probability to rejection of a null hypothesis), they can provide useful insights in the analysis of consumer behavior without requiring an ad hoc specification of functional form for demand or underlying preference functions. In this section, we discuss briefly the empirical implementation of these nonparametric tests.

First, note that equations (1b), (4b) or (5b) can be written as $A'y \geq c$, where y is a vector of the V 's and λ 's, while A and c are an

appropriately defined matrix and vector. The existence of a solution to (1), (4) or (5) can then be checked by solving the following linear programming problem (see Diewert, 1983):

$$\begin{array}{l} \text{Min } (b'y: A'y \geq c, y \in C) \\ y \end{array} \quad (6)$$

where the cone domain $C = \{y | (\lambda_1, \dots, \lambda_T) \geq \alpha > 0; (v_1, \dots, v_T) \geq 0\}$ and the vector b is arbitrarily defined such that problem (6) is bounded. Checking equations (1), (4) or (5) is therefore equivalent to finding whether or not the linear programming (6) has a feasible solution. It has a feasible solution if and only if the data $(v_t, x_t; t=1, \dots, T)$ are consistent with the corresponding formulation of consumer theory.

Although the linear programming problem (6) can be solved by standard computer algorithms, note that for $T > 1$ the number of constraints in (6) will exceed the number of activities. In this case, it will be computationally convenient to consider the linear programming problem dual to (6):

$$\begin{array}{l} \text{Max } (c'z: b-Az \in C^*; z \geq 0). \\ z \end{array} \quad (7)$$

where C^* is the polar cone of C .

It is well known that (7) has an optimal solution if and only if (6) has an optimal solution (e.g. Luenberger; Sposito). Alternatively, if (6) is infeasible, then (7) is either unbounded or infeasible (Sposito, p. 95). The number of constraints in the dual problem being only equal to $2T$, it will be easier to solve (7) (instead of (6)) by the simplex method. In this context, the existence of an optimal solution

to (7) would be interpreted as the consistency of the consumption data $(v_t, x_t; t=1, \dots, T)$ with the corresponding formulation of utility theory.

It should also be noted that the dimension of the linear programming problem (i.e., number of activities and constraints) is determined by the number of observations, not the number of commodities. Thus, in contrast to the parametric approach, nonparametric analysis is not constrained by the number of commodities. This lack of a "dimensionality curse" facilitates nonparametric analysis of very disaggregate commodity specifications.

IV - Interpretation of the Nonparametric Test:

The nonparametric tests presented above consist in checking the existence of a solution to a set of linear inequalities (i.e. (1), (4), or (5)). If a set of consumption data $(x_t, v_t; t=1, \dots, T)$ passes the test, it implies the existence of a preference function that rationalizes the data in the sense discussed in section II. Alternatively, if a data set does not pass the test, it implies that the data are not consistent with the corresponding formulation of consumer theory (Landsburg). What can we expect to learn for these nonparametric tests?

First, equations (1), (4) and (5) reflect alternative implications of consumer theory under alternative assumptions about the nature of the objective function of the consumer. Hence, empirical testing of these implications can provide evidence on the appropriateness of alternative formulations of consumer theory for the analysis of particular

consumption data. Previous nonparametric analysis has been limited to the primal implications of equations (1) (see Afriat; Diewert, 1973; Varian, 1982, 1983; Diewert and Parkan). If we are willing to take the existence of duality relationships under differentiability as a maintained hypothesis, then the empirical testing of (5) may reveal some information on consumer behavior not provided by (1) or (4).

Second, how can we interpret the results of the nonparametric tests? Useful insights on the structure of consumer preferences (as reflected by various weak separability hypotheses)^{3/} as well as the stability of preferences can be obtained by considering tests associated with different consumption bundles (Varian, 1983; Diewert and Parkan; Swofford and Whitney). To see this, consider a household choosing the consumption vector $x = (x_1, x_2)$ given prices $p = (p_1, p_2)$ and the direct utility function $u_2(x_1, x_2, \beta)$ where β denotes other variables (beside x_1 and x_2) that affect household preferences. It is well-known that, under stable preferences, weak separability of $u(x)$ in N_1 implies the existence of a sub-utility function $u_1(x_1)$ satisfying $u(x) = u_2(u_1(x_1), x_2)$, which is equivalent to the second stage allocation for x_1 in a two-stage budgeting process (Deaton and Muellbauer; Blackorby et al.). Consider that two nonparametric tests are performed: test "a" using the consumption vector x_1 with corresponding normalized prices $v_1 = p_1/(p'_1 x_1)$, p_1 being the price vector for x_1 ; and test "b" using the consumption vector $x = (x_1, x_2)$ with corresponding normalized prices $v = p/(p'x)$. A given consumption data can either pass or fail each test. The results can be interpreted in the light of the results discussed in section II. It will be convenient to limit our discussion here to the

implications of the analysis for the direct utility function.^{4/} Four cases are possible.

Case 1: The data pass both tests "a" and "b". In the context of the primal problem (2), this implies that existence of a direct utility function $u_2(u_1(x_1), x_2)$ that rationalizes the data. This utility function is characterized by the separability of x_1 from (x_2, β) where the existence of the sub-utility function $u_1(x_1)$ follows from the result of test "a", and the result from test "b" implies the separability of $x = (x_1, x_2)$ from β . Equivalently, this case can be interpreted as evidence of preference stability with respect to x_1 as well as x_2 across data points.

Case 2: The data fail test "a" but pass test "b". In the context of the primal problem (2), this implies the existence of a direct utility function of the form $u_2(u_1(x_1, x_2), x_2)$ where x_1 is separable from β but not separable from x_2 . In addition, x_2 is separable from β . This can be interpreted as a situation where preferences are stable with respect to $x = (x_1, x_2)$, but where x_2 influences the preferences for x_1 .

Case 3: The data pass test "a" but fail test "b". In the context of the primal problem (2), this shows the existence of a direct utility function $u_2(u_1(x_1), x_2, \beta)$ that rationalizes the data. This utility function implies the separability of x_1 from (x_2, β) , while x_2 is not separable from β . Equivalently, this case can also be interpreted as

evidence of preference stability with respect to x_1 , but not with respect to x_2 .

Case 4: The data fail both tests "a" and "b". In the context of the primal problem (2), this implies the existence of a direct utility function of the general form $u_2(x_1, x_2, \beta)$ where x_1 is not separable from (x_2, β) and $x = (x_1, x_2)$ is not separable from β . Equivalently, this case can also be interpreted as evidence that preferences for x_1 as well as x_2 are changing across data points.

Thus cases 1 and 2 imply the existence of a stable utility function that can rationalize the data in terms of price and income effects alone. In contrast, cases 3 and 4 provide evidence that preferences are changing, i.e. that there exist shifters of the utility function not accounted for in the analysis. For cross-section data, this would mean that preferences vary from one household to another. For time series data, this would mean structural change in the sense that preferences are changing over time. In this context, finding data inconsistency with consumer theory would be interpreted as evidence of the existence of other factors (i.e., other than the prices and total expenditures accounted for in the analysis) influencing consumer behavior. In other words, the results of the nonparametric tests can provide indirect evidence on the effect of these other factors as they contribute to shifts in consumption patterns. Nonparametric demand analysis can therefore provide heuristic insights on the structure and stability of

consumer preferences. This is further illustrated in the following sections with an application to U.S. food demand.

V - Data:

The above nonparametric methods are used to analyse the stability and structure of U.S. food consumption. Given the considerable parametric based research concerning structural change in U.S. meat demand (e.g., Braschler; Chalfant and Alston; Chavas; Dahlgran; Eales and Unnevehr; Nyankori and Miller; Moschini and Meilke; Thurman; Wohlgenant), we focus our nonparametric structure and stability analysis around the meat group. To establish comparability with the recent primal nonparametric work of Chalfant and Alston, we analyse the same 1947-83 time series on beef, pork, poultry and fish from Wohlgenant (1985).^{5/}

Since much parametric research on U.S. food demand (e.g. George and King; Eastwood and Craven; Huang and Haidacher) has maintained various weak (or strong) separability hypotheses without much scrutiny, we also include several non-meat food commodities and a non-food aggregate to nonparametrically evaluate alternative "complete" demand specifications. These alternatives are summarized in Table 1. PROTEINS adds eggs and dairy products (including butter) to the MEATS group: it consists of the commodities that are expected to constitute a major share of the protein intake of consumers. The remaining non-protein food commodities are comprised of fruits and vegetables (including potatoes and excluding melons), cereals and bakery products, sugars and sweeteners, fats and oils (excluding butter), and non-alcoholic beverages. Last, we add a

non-food implicit quantity aggregate defined as personal consumption expenditures (PCE) on all items less food deflated by the associated BLS price index for all items less food.

Price and quantity data for the non-meat food items are from U.S.D.A.'s Food Consumption, Prices, and Expenditures: 1964-1984. All quantities are measured in (retail weight equivalent) pounds per capita with the exception of non-alcoholic beverages (gallons per capita) and non-food (implicit quantity index). Corresponding commodity prices are derived using the at-home consumer price indexes for the above food groups.^{6/} Hence, in the absence of quantity and price information which distinguishes at-home and away-from-home consumption for each food commodity, we assume that the appropriate prices for the corresponding aggregate disappearance data follow at-home retail price movements.

VI - Results:

The proposed nonparametric procedures allow considerable flexibility in evaluating data consistency with the various demand theory formulations (i.e., equations 1, 4, or 5) and the structure and stability of consumer preferences over time or across alternative commodity groupings (see section IV). We proceed to evaluate data consistency of a particular commodity grouping with equations 1, 4, and 5 over the 1964-83 time period. Note that data consistency over a time series implies data consistency over its component time periods. If the nonparametric test of data consistency fails (i.e., if corresponding dual LP formulation is unbounded or infeasible (see section III)), then we sequentially test component time periods for data consistency. In

particular, if data consistency is not found, then we sequentially evaluate the 1964-79 and 1972-83, and then the 1964-71, 1972-79, and 1980-83 periods. One motivation for the choice of these sub-periods is to isolate the 1972-79 period of relatively high inflation from the more stable earlier period and the early 1980's economic context. This sequential testing procedure can also be viewed as a nonparametric heuristic tool to identify data consistent time periods for parametric estimation.

Table 2 summarizes the nonparametric test results for the time periods and commodity groupings analysed. Given that the direct (equations 1) and indirect (equations 4) utility formulations are the least restrictive, they are evaluated first. Next, the differentiable duality formulations (from equations 5) are evaluated. Recall that lemma 1 (equations 1) concerns data consistency with the existence of a continuous, strictly increasing direct utility function and the associated quantity dependent Marshallian demand correspondences as solutions to the primal problem (equation 2). Similarly, lemma 2 (equations 4) concerns data consistency with the existence of a continuous, strictly decreasing indirect utility function and the associated price dependent Marshallian demand correspondences as solutions to the dual problem (equation 3). Neither of these conditions guarantees the existence of differentiable demand functions usually encountered in parameteric demand analysis (i.e., they are necessary but not sufficient conditions). Assuming differentiable, strictly increasing (decreasing), and strictly quasi-concave direct (quasi-convex indirect) utility functions, Proposition 1 (equations 5) concerns data

consistency with the existence of duality based, differentiable demand functions. Again, data consistency with both the direct and indirect utility formulations are necessary but not sufficient for data consistency with the nonparametric differentiable duality formulation.

We begin our discussion of Table 2 with the Afriat/Varian direct utility (equations 1) and the dual indirect utility (equations 4) formulations. The nonparametric results from equations (1) or (4) are identical for all commodities and time periods analysed. These results indicate that the data are consistent with either hypotheses for all commodity groups and time periods analyzed except PROTEINS over the 1964-83 and 1964-79 period. Thus, these results provide nonparametric evidence of the existence of separable and stable preferences and the associated second stage demand correspondence for MEATS over the 1964-83 period analysed.^{2/} The PROTEIN and ALL FOOD results, however, provide nonparametric evidence that a stable preference function for PROTEINS existed but was not separable from all other foods over the 1964-79 period (see Case 2, section IV). Since the PROTEINS results for each sub-period (i.e., 1964-71 and 1972-79) indicate a stable and separable demand relationship, we infer that the separability structure of the PROTEINS demand correspondence was different in these two periods. In contrast, the ALL FOOD results provide nonparametric evidence of stable and separable preferences (e.g., from non-foods and preference shifters) for the ALL FOOD commodities over the 1964-83 period.

Thus, the nonparametric results from equations (1) and (4) imply that these data are consistent with the existence of a stable second stage demand correspondence for MEATS, ALL FOODS, and FOODS & NONFOOD

over the 1964-83 period. The second stage demand correspondence for PROTEINS, while stable, did not exhibit separability from the non-protein foods during the 1964-79 period. Although these results provide nonparametric evidence that much of the U.S. food consumption behavior analysed follows the theory, they do not give much insight on how to proceed in empirical demand analysis. This in part reflects the low power of the Afriat-Varian nonparametric test of consumer behavior (Bronars; Thurman). Stronger restrictions are required to generate demand functions as opposed to the demand correspondences implied by the direct or indirect utility formulations (see lemma 1 and lemma 2).

The differentiable duality results for the years 1964-83 or 1964-79 in table 2 can be interpreted as nonparametric evidence that empirical demand analysis based on a stable, differentiable, strictly increasing (decreasing), and strictly quasi-concave direct (quasi-convex indirect) utility specification would not be consistent with these data for all commodity groupings evaluated. This indicates that the nonparametric test proposed in proposition 1 has higher power compared to the Afriat-Varian nonparametric test. Further analysis of the 1964-83 sub-periods reveals that the lack of data consistency in the 1964-71 PROTEINS, ALL FOODS, and FOOD & NONFOOD specifications are partly responsible for these findings. These rejections of data consistency imply that preferences were not stable and/or were not separable in the manner specified.^{8/} In contrast, the nonparameteric differentiable duality results for 1972-83 indicate that these data are consistent with the existence of second stage demand functions for PROTEINS and FOODS & NONFOOD over this period.

The MEATS differentiable duality results indicate a lack of data consistency with the existence of second stage demand functions over the 1972-83 period.^{9/} Comparison of the MEATS and PROTEINS results for 1972-83 reflects our Case 2 (Section IV) discussion. That is, since the PROTEINS data are found to be separable from non-protein commodities and to be consistent with the existence of second-stage demand functions over the 1972-83 period, we infer that the associated preference function for MEATS was stable but not separable from non-meat proteins (i.e., eggs and dairy products) during this period. Given the MEATS data consistency over the 1964-71, 1972-79 and 1980-83 periods (implying the existence of stable and separable (from non-meats) preferences over these separate sub-periods), we find non-parametric evidence that the separability structure of MEATS demand was different in each of these periods. Indeed, if this separability structure was the same (particularly for 1972-79 and 1980-83, given the 1972-83 MEATS and PROTEINS results), then we would not expect to find the lack of MEATS data consistency in 1972-83 or 1964-79. Hence, these results suggest that the relative price and income effects characterizing 1964-71, 1972-79 and 1980-83 MEATS demand reflect distinct separability structures on preferences. Given the economic adjustments of the highly inflationary 1972-79 period, these results are not unreasonable.

From these MEATS and PROTEINS nonparametric tests of differentiable duality, we infer that previous evidence of structural change in meat demand during the 1970's (e.g. Nyankori and Miller; Chavas) is likely due to model misspecification. While this could reflect an inappropriate parametrization of the demand functional forms, our

nonparametric results suggest that additional attention to the commonly maintained hypothesis of MEATS demand separability is warranted. Our nonparametric differentiable duality results suggest that the 1972-83 data do support parametric estimation of MEATS demand in the context of a second stage PROTEINS demand system.

The 1972-83 differentiable duality results for PROTEINS, ALL FOODS, and FOODS & NONFOOD reflect our Section IV discussion of Case 3 (PROTEINS and ALL FOODS) and Case 2 (ALL FOODS and FOODS & NONFOOD). That is, all three commodity specifications are found to be consistent with separability and the existence of a stable, differentiable utility function over the 1972-83 period with the exception that ALL FOODS are found to be stable but not separable with respect to the non-food aggregate. Thus, our results suggest that parametric estimation of PROTEINS or FOODS & NONFOOD demand systems using the 1972-83 period are supported by these data.

We also analysed a specification with two commodities: aggregate food and aggregate non-food. This specification is motivated by other nonparametric work concerning the data consistency and separability of fairly aggregated commodities (e.g., Swofford and Whitney; Barnhart and Whitney).^{10/} Our nonparametric tests indicate that these aggregate data are consistent with equations (1), (4) and (5) hence, with the existence of differentiable and stable, dual preferences over the 1964-83 period. Given our more negative results concerning "disaggregate" FOODS & NONFOOD (see table 2), this finding raises questions about aggregate demand analysis. Although separability assumptions are often considered "less offensive" when applied to broad aggregates (Deaton and

Muellbauer), our results suggest that working with broad aggregates may induce significant aggregation errors. This would be consistent with Eales and Unnevehr's findings that inappropriate maintained separability hypotheses can distort parametric measures of structural change. Our results illustrate that nonparametric procedures can provide heuristic insight for the analysis of these issues.

VII - Concluding Remarks:

This paper illustrates the usefulness of a nonparametric approach to demand analysis based on a finite number of consumption data points. This approach allows an evaluation of consumer theory and its dual formulation by generating empirical tests that do not depend on the choice of a functional form for utility or demand functions. The duality formulation proposed here is of interest since it relies on differentiability assumptions typically made in parametric demand analysis (Chiappori and Rochet).

The results indicate that this extension of nonparametric demand analysis is clearly more restrictive than the direct (or indirect) utility formulation, as expected. By increasing the power of nonparametric methods, the proposed duality approach can provide more precise results and is more likely to find evidence against the standard theory. Given the low power of previous nonparametric tests, this appears to be a desirable characteristic (Bronars). If one is interested in the investigation of the stability and structure of preferences under restrictions commonly imposed in applied, parameteric

demand analysis, then the proposed nonparametric, differentiable duality approach may prove very useful.

The approach is easy to implement empirically using a standard linear programming algorithm. The size of the resultant LP problem depends on the number of observations, not on the number of commodities analyzed. Thus, in contrast to the parametric approach, the investigation of a large number of very disaggregate commodities is quite feasible in nonparametric demand analysis.

Our analysis suggests that the proposed nonparametric approach can be a powerful heuristic tool when used prior to the parametric specification and estimation of demand functions. For example, by providing evidence on the validity of various weak separability hypotheses, the methodology can help choose the appropriate commodity aggregates and/or periods for parametric demand analysis. The methodology can also provide useful information in the analysis of structural change in consumer preferences as demonstrated in the application to U.S. meats demand.

At this stage of its development, however, nonparametric analysis does not allow estimation of price nor income elasticities of demand. Neither does it provide much insight on how factors other than prices and income influence consumption behavior. Hence, the nonparametric approach is clearly not a substitute for the parametric approach. Rather, when appropriately used, the two approaches should prove complementary. We hope that the nonparametric duality framework and the empirical application presented in this paper will stimulate further use of this potentially powerful heuristic tool in applied demand analysis.

References

- Afriat, S.N. "The Construction of Utility Functions from Expenditure Data." International Economic Review 8(1967):67-77.
- Anderson, R.W. "Some Theory of Inverse Demand for Applied Demand Analysis." European Economic Review 14(1980):281-290.
- Barnhart, Scott W. and Gerald A. Whitney. "Nonparametric Analysis in Parametric Estimation: An Application to Translog Demand Systems." Review of Economics and Statistics 70(1988):149-153.
- Blackorby, C., D. Primont and R.R. Russell. Duality, Separability and Functional Structure: Theory and Economic Applications. North Holland Publ. Co., Amsterdam, 1978.
- Braschler, C. "The Changing Structure for Pork and Beef in the 1970's: Implications for the 1980's." Southern Journal of Agricultural Economics 15(1983):105-10.
- Bronars, S.G. "The Power of Nonparametric Tests of Preference Maximization." Econometrica 55(1987):693-698.
- Chalfant, James and Julien Alston. "Accounting for Changes in Taste." Journal of Political Economy 96(1988):391-410.
- Chavas, J.P. "Structural Change in the Demand for Meat." American Journal of Agricultural Economics 65(1983):148-153.
- Chiappori, Pierre-Andre and Jean-Charles Rochet. "Notes and Comments: Revealed Preferences and Differentiable Demand." Econometrica 55(1987):687-91.
- Christensen, L.R., D.W. Jorgenson and L.J. Lau. "Transcendental Logarithmic Utility Functions." American Economic Review 65(1975):367-383.

- Dahlgran, Roger A. "Complete Flexibility Systems and the Stationarity of U.S. Meat Demands." Western Journal of Agricultural Economics 12(1987):152-63.
- Deaton, A. and J. Muellbauer. Economics and Consumer Behavior. Cambridge University Press, Cambridge, 1980.
- Diewert, W.E. "Afriat and Revealed Preference Theory". Review of Economic Studies 40(1973):419-426.
- Diewert, W.E. "Applications of Duality Theory" in Frontiers of Quantitative Analysis, Vol. III. M.D. Intriligator and D.A. Kendrick, eds., North Holland Publ. Co., Amsterdam, 1974, 106-171.
- Diewert, W.E. "Duality Approaches to Microeconomic Theory" in Handbook of Mathematical Economics, Vol. II, K.J. Arrow and M.D. Intriligator, eds., North Holland Publ. Co., Amsterdam, 1982, 535-599.
- Diewert, W.E. and C. Parkan. "Tests for the Consistency of Consumer Data." Journal of Econometrics 30(1985):127-147.
- Eales, Janes S. and Laurian J. Unnevehr. "Demand for Beef and Chicken Products: Separability and Structural Change." American Journal of Agricultural Economics 70(1988):521-532.
- Eastwood, D.B. and J.A. Craven. "Food Demand and Savings in a Complete Extended LES." American Journal of Agricultural Economics 63(1981):544-49.
- George, P.S. and G.A. King. Consumer Demand for Food Commodities in the United States with Projections for 1980. Gianni Foundation Monograph No. 26, Berkeley: University of California Press, 1971.

- Huang, K.S. and R.C. Haidacher. "Estimation of a Composite Food Demand System for the United State." Journal of Business and Economic Statistics 1(1983):285-91.
- Landsburg, S.E. "Taste Change in the United Kingdom, 1900-1955." Journal of Political Economy 89(1981):92-104.
- Lau, L.J. "Duality and the Structure of Utility Functions." Journal of Economic Theory 1(1970):373-396.
- Luenberger, D.G. Linear and Non-Linear Programming, 2nd edition, Addison-Wesley Publ. Co., Reading, Massachusetts, 1984.
- McMillan, Melville, L. and Joe Amoako-Tuffour. "An Examination of Preferences for Local Public Sector Outputs." Review of Economics and Statistics 70(1988):45-54.
- Moschini, G. and K.D. Meilke. "Parameter Stability and the U.S. Demand for Beef." Western Journal of Agricultural Economics 9(1984):271-282.
- Nyankori, Y.C.O. and G.H. Miller. "Some Evidence and Implications of Structural Change in Retail Demand for Meats." Southern Journal of Agricultural Economics 14(1982):65-70.
- Samuelson, P.A. "Using Full Duality to Show that Simultaneously Additive Direct and Indirect Utilities Implies Unitary Price Elasticity of Demand." Econometrica 33(1965):781-786.
- Sposito, V.A. Linear and Non-Linear Programming, Iowa State University Press, Ames, Iowa, 1975.
- Swofford, J.L. and G.A. Whitney. "Nonparametric Tests of Utility Maximization and Weak Separability for Consumption, Leisure and Money." Review of Economics and Statistics 69(1987):458-464.

- Thurman, Walter N. "The Poultry Market: Demand Stability and Industry Structure." American Journal of Agricultural Economics 69(1987):30-37.
- Varian, H. "Nonparametric Analysis of Optimizing Behavior with Measurement Error." Journal of Econometrics 30(1985):445-458.
- Varian, H. "Nonparametric Tests of Consumer Behavior." Review of Economic Studies 50(1983):99-110.
- Varian, H. "The Nonparametric Approach to Demand Analysis." Econometrica 50(1982):945-973.
- Weymark, J.A. "Duality Results in Demand Theory." European Economic Review 11(1980):377-395.
- Wohlgenant, M.K. "Estimating Cross Elasticities of Demand for Beef." Western Journal of Agricultural Economics 10(1985):322-329.
- U.S.D.A. Food Consumption, Prices and Expenditures: 1964-84. ERS Statistical Bulletin 736, Dec. 1985.

Table 1: Composition of Commodity Groups Analyzed.

MEATS: Wohlgenant (1985) Meat Data

- Beef&Veal Pork Poultry Fish

PROTEINS: Proteins (with Wohlgenant's Meat Data)

- Beef&Veal Pork Poultry Fish
- Eggs Dairy (including butter)

ALL FOODS: All Foods (Disaggregated)

- Beef&Veal Pork Poultry Fish Eggs Dairy
- Fruit&Vegetables Cereals&Bakery Sugar&Sweets
- Fats&Oil Non-AlcoholicBeverages

FOOD/NON-FOOD: ALL Foods (Disaggregated) and Non-Food (Aggregated)

- Beef&Veal Pork Poultry Fish Eggs Dairy
 - Fruit&Vegetables Cereals&Bakery Sugar&Sweets
 - Fats&Oil Non-AlcoholicBeverages
 - Aggregate Non-Food
-

Table 2: Nonparametric Test Results: U.S. Food Consumption Per Capita, 1964-83.¹

<u>Direct Utility or Indirect Utility:</u> ²				
<u>YEARS</u>	<u>MEATS</u>	<u>PROTEINS</u>	<u>ALL FOODS</u>	<u>FOODS & NONFOOD</u>
1964-83	OK	Unbd	OK	OK
1964-79	*OK	Unbd	*OK	*OK
1972-83	*OK	OK	*OK	*OK
1964-71	*OK	OK	*OK	*OK
1972-79	*OK	*OK	*OK	*OK
1980-83	*OK	*OK	*OK	*OK

<u>Differentiable Duality:</u>				
<u>YEARS</u>	<u>MEATS</u>	<u>PROTEINS</u>	<u>ALL FOODS</u>	<u>FOODS & NONFOOD</u>
1964-83	Unbd	Unbd	Unbd	Unbd
1964-79	Unbd	Unbd	Unbd	Unbd
1972-83	Unbd	OK	Unbd	OK
1964-71	OK	Unbd	Unbd	Unbd
1972-79	OK	*OK	Unbd	*OK
1980-83	OK	*OK	OK	*OK

¹ "Unbd" indicates that the dual LP was unbounded, nonparametric evidence that the data are not consistent with the hypotheses being tested over the period. Conversely, "OK" indicates that the dual LP converged, thus providing nonparametric evidence that the data over this period are consistent with the hypotheses being tested. "*OK" indicates that data consistency is implied by a prior result.

² The nonparametric results for the direct (equations 1) and indirect (equations 4) utility formulations are identical for these data over the commodities and periods analysed.

Footnotes

- 1/ See Chiappori and Rochet for similar arguments concerning the primal non-parametric tests.

- 2/ Then the indirect utility function $g(v)$ is differentiable, strictly decreasing and strictly quasi-convex.

- 3/ A direct utility function is weakly separable if the marginal rate of substitution between any two commodities within a group is independent of the commodities outside that group. Equivalently, the utility function $u(x_1, x_2)$ is weakly separable in x_1 if it can be written as $u_2(u_1(x_1), x_2)$ where $u_1(x_1)$ is a sub-utility function for the group x_1 (e.g. see Deaton and Muellbauer; Blackorby et al.).

- 4/ By duality, the existence of a direct utility function $u(x)$ (or $u_1(x_1)$) implies the existence of an indirect utility function $g(v/v')$ (or $g_1(v_1/v'_1, x_1)$). As a result, the discussion below could be presented alternatively in terms of indirect utility functions. Also, it should be noted that our analysis does not impose homothetic or homothetically separable preferences as discussed in Varian (1983).

- 5/ Since we analyse time-series data, we assume that aggregate consumption decisions are made by an optimizing representative consumer in a way consistent with his/her preferences. The influence of population size is taken into consideration by defining quantities and income on a per capita basis. The associated implicit prices are derived as the ratio of per capita expenditures to per capita quantity disappearance.

- 6/ All price indexes are first adjusted to 1964 = 100. Commodity expenditure proportions for 1964 are derived from George and King (p. 36) as the average of December 1963 and December 1965. These expenditure proportions are then used to allocate 1964 PCE per capita across foods. Expenditures per capita for 1964 are then divided by the corresponding 1964 quantities per capita to yield implicit prices which satisfy Fisher's weak factor reversal test for 1964. These prices are then projected forward through 1983 using the corresponding at-home consumer price indexes with base 1964 = 100.

- 7/ We also nonparametrically analyse the Wohlgenant (1985) meat data over the 1947-83 period. We find data consistency with the indirect approach to the demand correspondence as well as with the Afriat/Varian direct utility formulation reported in Chalfant and Alston using the same data.

We do not report these 1947-83 and 1947-63 MEATS results in table 2 to avoid clutter.

- 8/ This separability inference could reflect various maintained hypotheses in the current specifications. One possibility is the lack of a labor/leisure/consumption commodity specification (i.e., the implied separability of consumption from labor/leisure decisions). Another possibility is that the non-food aggregate may be inappropriate (e.g., further disaggregation may be needed following Barnhart and Whitney, or Swofford and Whitney).
- 9/ As noted above, we also evaluated MEATS over the 1947-83 and 1947-63 periods, as did Chalfant and Alston. While the MEATS data were found to be consistent with the existence of a direct or indirect utility function and the associated second stage demand correspondence over the 1947-83 period (as found by Chalfant and Alston), our differentiable duality results (not reported in Table 2) indicate a lack of data consistency with the existence of second stage MEATS demand functions for this period. This implies that parametric demand analysis using stable second stage demand functions is not supported by the data. However, our nonparametric procedures do indicate that the MEATS data are consistent with a stable duality specification over the 1947-63 period. Given these results, Table 2 and the ensuing discussion focuses on the stability and structure of MEATS demand over the 1964-83 period and sub-periods.
- 10/ Aggregate food per capita is measured as an implicit quantity index where per capita PCE on food is deflated by the BLS price index for all food.