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A Non-Parametric Analysis of Productivity:

The Case of U.S. Agriculture

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University of Wisconsin-Madison, Department of Agricultural Economics Staff Paper #281, January 1988.

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I. Introduction:

Empirical analysis of productivity and technical change has generally proceeded in two directions. On the one hand "accounting data" analysis has been used to create input and output quantity indices which, in turn, define total factor productivity (TFP) measures. On the other hand, direct specification and estimation of production technology via production, cost, or profit function approaches have been used to obtain parametric measures of total factor productivity and technical change. Both of these procedures are known to impose implicit structure on aggregate production technology (Capalbo and Vo).

Diewert (1976) has argued in favor of superlative indices, defined to be exact and derived from a "flexible" second order approximation to the underlying aggregator function. For example, Diewert has shown that the discrete Divisia TFP proposed by Christensen and Jorgenson can be interpreted as a superlative quantity index derived from an homogenous translog transformation function that is separable in outputs and inputs, and exhibits Hicks neutral technical change. Caves, Christensen and Diewert have argued that separability and Hicks neutrality are not required to justify the Christensen and Jorgenson TPP index. However, this commonly used TFP index still requires that technology can be appropriately represented by a homogeneous translog transformation function.

Parametric measures of TFP imply similar _a priori_ structure for aggregate production technology. Translog or Generalized Leontief specifications of production, cost or profit functions have been commonly estimated in the investigation of technical change (e.g. Binswanger; Berndt and Christensen; Stevenson; Lopez; Antle; Norsworthy and Malmquist). Again, the implication for the analysis of productivity is that results are conditional on the parametric functional form chosen. In this context, it appears desirable to develop TFP measures from a methodology that does not depend, as much as it is possible, on the parametric specification of the underlying technology.

This paper proposes an alternative measure of TFP based on extensions of the non-parametric work of Hanoch and Rothschild, Diewert and Parkan, and Varian. In particular, we use a "generalized augmentation" hypothesis to extend non-parametric production analysis so that it incorporates both Hicks neutral and biased technical change. These results are stated (and proved) as a proposition concerning the implications of profit maximization under technical change without making a priori assumption about the parametric form of the underlying technology. For the empirical application to aggregate U.S. agricultural data, we focus on input and/or output translating (additive augmentation) as an empirically convenient specification of the "generalized augmentation" hypothesis.

Non-parametric analysis of productivity and technical change under profit maximization and additive augmentation hypotheses yields a set of implied linear inequalities that must be met if the data are consistent with the specification of technology. These linear inequalities are easy to evaluate using standard linear programming solutions. If an optimal solution is found, these procedures yield non-parametric estimates of the (additive) augmentations to inputs and/or outputs for each time period (e.g., year) of the data. These non-parametric estimates can be interpreted in terms of rates of technical change (for each input and/or output, for each year) which are consistent with the data. In particular, these non-parametric estimates can generate TFP measures which are derived under considerably less restrictive assumptions than extant TFP measures.

Section II presents the extensions to the Hanoch and Rothschild, and Varian non-parametric results to incorporate technical change under the generalized augmentation hypothesis. Section III discusses the empirical implementation and interpretation of the non-parametric results using linear programming techniques. Section IV discusses the data used for the empirical analysis of U.S. agricultural productivity and the results. Conclusions are presented in Section V.

II. Non-Parametric Production Analysis under Technical Change:

Consider a competitive firm producing an output y sold at a market price p, and using a set of n inputs $x = (x_1,...,x_n)'$ with corresponding prices $r = (r_1,...,r_n)'$. The firm faces a production technology represented by the production frontier

$$Y = g(X) \tag{1}$$

where $Y = Y(y,A)$ denotes "effective output", $X = X(x,B) = \{X_i(x_i,B_i), i=1,\ldots,n\}$, X_i denoting the $i^{\underline{th}}$ "effective input", and A and $B = (B_1,\ldots,B_n)$ are technology indices. We assume that Y is a strictly increasing function of y and that X_i is a strictly increasing function of x_i, $i=1,\ldots,n$. This formulation of technology corresponds to the augmentation hypothesis where technical change (as reflected by changes in A and B) influences the transformation of actual inputs (or output) into effective inputs (or output). In this context, technical progress can be characterized by increasing the effectiveness of inputs in the production of output.

Note that the representation (1) is fairly general. Although it implies that the marginal rate of substitution between any x_i and B_i is independent of the values of all (x_j, B_j), $j \neq i$, it imposes no a priori restriction on the functional form g(X). Also, changing A while holding B constant corresponds to the hypothesis of Hicks neutral technical change where the marginal rate of substitution between any two inputs is independent of the technology index A. Alternatively, changing values of B imply a bias in technical change as the marginal rate of substitution between inputs is affected by the technology indices B.

Consider that the firm maximizes profit

$$V(p,r,A,B) = \max_{x,y} \{py - r'x : Y(y,A) \leq g(X(x,B)),\ y \geq 0,\ x \geq 0\} \qquad (2)$$

where $x^*(p,r,A,B)$ and $y^*(p,r,A,B)$ are the profit maximizing input demand and output supply functions and $V(p,r,A,B) = py^* - r'x^*$ is the indirect profit function.

Let $y(Y,A)$ and $x_i(X_i,B_i)$ be the inverse functions of $Y(y,A)$ and $X_i(x_i,B_i)$, $i=1,\ldots,n$. Then, expression (2) can be alternatively written as

$$V(p,r,A,B) = \underset{X,Y}{\text{Max}} \{py(Y,A) - r'x(X,B) : Y \le g(X),$$

$$y(Y,A) \ge 0,\ x(X,B) \ge 0\} \tag{3}$$

Assume that the firm is observed choosing (x,y) T times, each observation (x_t,y_t) being associated with a situation t characterized by input prices r_t, output price p_t and technology (A_t,B_t), $t=1,\ldots,T$. It is of interest here to investigate under what conditions the decision set $\Omega = (x_1,y_1;\ldots;\ x_T,y_T)$ is consistent with profit maximization as stated in (2) or (3). Under the profit maximization hypothesis, checking the consistency of actual decisions Ω with (2) or (3) can be done in the context of non-parametric tests as proposed by Hanoch and Rotschild or Varian.

A basis for a non-parametric test is presented in the following proposition (see the proof in the Appendix).

Proposition 1: Given a set of decisions $\Omega = \{x_t,y_t;\ t=1,\ldots,T\}$ each (x_t,y_t) corresponding to a situation $\{p_t,r_t,A_t,B_t\}$, $t=1,\ldots,T$, then:

a/ if (x_t,y_t) solves $\max \{p_t y - r_t'x\colon Y(y,A_t) \le g(X(x,B_t));\ x \ge 0,\ y \ge 0\}$, then

$$p_t[y(Y_t,A_t) - y(Y_s,A_t)] - r_t'[x(X_t,B_t) - x(X_s,B_t)] \ge 0 \tag{4}$$

$s,t = 1,\ldots,T.$

b/ if (4) is satisfied for a particular production data set, then there exists a function G(X) that rationalizes the data in the sense that (x_t, y_t) solves

$$\max \{p_t y - r_t' x: Y(y, A_t) \leq G(X(x, B_t)),\ x \geq 0,\ y \geq 0\}.$$

Equation (4) gives a set of necessary and sufficient conditions for the decisions $\Omega = \{x_1, y_1; \ldots; x_T, y_T\}$ to be consistent with profit maximization as defined in (2) or (3) for some production technology. Testing for consistency then involves checking whether the inequalities in (4) are satisfied. Expression (4) provides a non-parametric test of production decisions in the sense that a priori specification of the functional form g(X) in the characterization of production technology is not required. Although this non-parametric test is not a statistical test (with associated probability statements), it can provide useful information on technology. In particular, by allowing for technical change, the above results extend the non-parametric analysis of production decisions proposed by Hanoch and Rothschild or Varian (see below).

Checking whether the inequalities in (4) are satisfied requires prior information on the functions Y(y,A) and $X_i(x_i, B_i)$, i=1,...,n. In this paper, we focus on the translating hypothesis where Y(.) and $X_i(.)$ are specified in linear form: $Y = y - A$ and $X_i = x_i + B_i$, i=1,...,n. This implies that expression (4) takes the form

$$p_t[y_t - A_t - y_s + A_s] - r_t'[x_t + B_t - x_s - B_s] \geq 0. \qquad (5)$$

Under the translating hypothesis, expression (5) is linear in A and B, which greatly facilitates its empirical application.[1/]

In the absence of technical change, i.e., where $A_s = A_t$, $B_s = B_t$, $\forall s \neq t$, expression (5) reduces to the axiom of profit maximization proposed by Hanoch and Rothschild and Varian. In other words, for a given production data set, if the inequalities in (5) are satisfied with $A_s = A_t$, $B_s = B_t$, $\forall s \neq t$, then proposition 1 implies the existence of a stable production function that would rationalize the data according to (2) in the absence of technical change. Alternatively, violations of the inequalities in (5) in the absence of technical change, would imply that there does not exist a stable production function that rationalizes the data. Assuming profit maximization and additive augumentations as maintained hypotheses, this non-parametric test would then provide evidence that the production function is not stable, i.e. that technical change has taken place.

As noted above, technical change can take place in several ways. For example, finding $A_s \neq A_t$ for some $s \neq t$ but $B_s = B_t$, $\forall s \neq t$, such that expression (5) is satisfied for some production data would imply the existence of a production function exhibiting Hicks neutral technical change that rationalizes the data. Alternatively, if such a set of A's does not exist, this would imply that a production function $y = A + g(x)$ which rationalizes the data does not exist. Assuming profit maximizing behavior and the translating hypothesis as given, this non-parametric test would then provide evidence that technical change is not Hicks neutral, i.e. that technical change is biased as the marginal rate of substitution between any two inputs is affected by the change.

Finally, assume that some A's and B's are found which satisfy expression (5) given a particular data set. What interpretation can be given to these values? Since y = Y + A, it is clear that higher values of A are associated with higher productivity. More specifically, if $B_s = B_t$, $\forall s \neq t$, then $(\frac{A_s - A_t}{y_t})$ can be interpreted as the rate of change in output between situation s and situation t due to technical change alone. More generally, given $y(A,X) = A + g(X) = A + Y$, note that $(\frac{A_s - A_t}{y_t}) = \frac{A_s + g(X_t)}{y_t} - 1 = \frac{y(A_s, X_t)}{y(A_t, X_t)} - 1$. It follows that $\frac{y(A_s, X_t)}{y(A_t, X_t)} = \frac{A_s - A_t}{y_t} + 1$.

Thus, given the effective inputs X_t, the expression $(\frac{A_s - A_t}{y_t} + 1)$ can be interpreted as a productivity index for situation s measuring the impact of technical change on production, using t as a base (reference) situation. An empirical evaluation of this productivity index will be presented in section III.

Similarly, since $X_i = x_i + B_i$, it follows that an increase in B_i increases the effectiveness of the actual input x_i in the production process. Thus, given a higher B_i, the firm could produce the same output ceteris paribus by reducing the actual input x_i. In this sense, increasing B_i can be interpreted as a bias in technical change that is "factor saving" for the $i^{\underline{th}}$ input. Alternatively, a decreasing value of B_i would imply that, ceteris paribus, the firm would have to increase the actual use of x_i in order to produce the same output. Thus, a decrease in B_i can be interpreted as a bias in technical change that is "factor-using" for the $i^{\underline{th}}$ input. Thus the sign and magnitude of the changes in B allow investigation of the nature of the bias in technical change.

These examples illustrate the potential usefulness of the non-parametric tests just discussed in the analysis of productivity and technical change. An empirical implementation of these tests is presented next.

III. Empirical Implementation:

From proposition 1, the inequalities in (4) (or (5) under the translating hypothesis) are necessary and sufficient for the existence of a production function that would rationalize a particular set of production data under technical change. Non-parametric testing thus involves checking the existence of a solution to these inequalities.

In the absence of technical change where $A_s = A_t$, $B_s = B_t$, $\forall s \neq t$, the empirical implementation of (4) or (5) is straightforward as the inequalities in (4) or (5) involve only observable variables p,r,x,y. In this case, it is a simple matter to check whether the inequalities in (4) or (5) are satisfied for all observations.

However, in the presence of technical change where at least some of the A's and B's change across observations, the A's and B's are typically not directly observable. In this case, the non-parametric test consists in finding whether there exists a set of values for the A's and B's which would satisfy the inequalities in (4). Note that, under the translating hypothesis, (4) becomes expression (5) which is linear in the unobserved variables (i.e. the A's and B's). This linearity is particularly convenient and is the main reason for focusing our paper on the translating hypothesis (see footnote 1). Given the linearity in (5), checking the existence of a solution to the inequalities (5) for the A's and B's can be conveniently formulated as a linear programming problem.

Let $q = (A_1,\ldots,A_T;\ B_1^+,\ldots,B_T^+;\ B_1^-;\ldots,B_T^-)$ be the vector of unobserved variables in (5) where $B = B^+ - B^-$, $B^+ \geq 0$, $B^- \geq 0$. Allowing for positive or negative B can support factor-saving as well as factor-using bias in technical change. Expression (5) can be written as $D'q \geq c$, given appropriate definitions of the matrix D and the vector c. Then, consider the linear programming problem

$$\underset{q}{\text{Min}}\ \{b'q : D'q \geq c,\ q \geq 0\} \tag{6}$$

where $b \geq 0$, such that problem (6) is necessarily bounded. It follows that either problem (6) has a solution, or if it does not, it must be infeasible. In other words, the inequalities $D'q \geq c$ have a solution for q if and only if problem (6) has a feasible solution. In this context, checking the existence of a solution to the non-parametric inequalities is performed by evaluating the existence of a feasible solution to the linear programming problem (6) (e.g. using the simplex method). Choosing appropriate values for the b's (the coefficients of the objective function in (6)), can yield useful information concerning the nature and magnitude of technical change (see below).

Note that, even for a moderate number of observations T, the number of constraints in the linear programming (6) will typically exceed the number of activities. In this case, it will be computational convenient to consider the linear programming problem dual to (6)

$$\underset{\bar{q}}{\mathrm{Max}} \{c'\bar{q} : D\bar{q} \leq b, \ \bar{q} \geq 0\} \tag{7}$$

It is well known that (7) has an optimal solution if and only if (6) has an optimal solution (e.g. Luenberger; Sposito). Alternatively, if problem (6) is infeasible, then (7) is either unbounded or infeasible.

Here, we propose to solve the dual problem (7) by the simplex method. If (7) has an optimal solution for a production data set, then the data are consistent with the existence of a production function exhibiting a particular type of technical change depending on the solution values for the A's and B's. The usefulness of this approach in the analysis of productivity and technical change is illustrated next in the context of U.S. agriculture.

IV. Application to U.S. Agricultural Data:

Aggregate time series data for the U.S. agricultural sector for the years 1950-1983 are taken from Capalbo and Vo. The data analyzed include quantity indices (1977=1.00) and associated implicit price indices for U.S. agricultural output and 9 inputs: family labor, hired labor, land, structures, other capital, materials, energy, fertilizers, pesticides, and miscellaneous (see Capalbo and Vo for a description of the data).

First, the full 1950-83 period is analyzed with each non-parametric hypothesis. If these data are not consistent with a particular hypothesis, then sub-periods of the data are evaluated. In particular, the 1950-71 and 1960-83 sub-periods are analyzed first, and then the 1950-59, 1960-71, and 1972-83 sub-periods are evaluated in order to isolate the energy price shocks, high inflation, and surging export demand of the 1970's from the considerably more stable earlier time periods. Note that data consistency over a set of years implies that all component sets of years are also consistent with the non-parametric hypothesis.

The analysis begins with Varian's axiom of profit maximization which implies the existence of a stable production function in the absence of technical change. The non-parametric form of this hypothesis implies that $A_s = A_t$ and $B_s = B_t$, $\forall$ $s \neq t$ in (5). As indicated in table 1, these data are found to be inconsistent for all time periods analyzed. Thus, assuming profit maximizing behavior as a maintained hypothesis, this non-parametric result provides strong support for the existence of technical change in U.S. agriculture.

Next, we evaluate the existence of Hicks neutral technical change using the additive output augmentation hypothesis, where the A's are unrestricted but $B_s = B_t$, $\forall$ $s \neq t$ in (5). As indicated in table 1, these data are not found to be consistent with this specification of Hicks neutrality for any time period analyzed. Thus, given profit maximization and additive output augmentation as maintained hypotheses, these non-parametric results indicate that U.S. agriculture was not characterized by Hicks neutral technical change over the time periods analyzed. We interpret this as evidence of biased technical change in U.S. agriculture during the 1950-1983 period.

Given that these data are inconsistent with Hicks neutrality specified as additive output augmentation, we next evaluate the existence of a production function exhibiting additive augmentations in both outputs and inputs under the assumption of profit maximization. This yields the general specification of equation (5) where the A's and B's are unrestricted. We also allow the additive input augmentations to reflect both factor saving (positive B_t's) or factor using (negative B_t's) biased technical change. As indicated in table 1, the data are found to be consistent with this specification over the full 1950-83 time period. We interpret this result as non-parametric evidence that U.S. agricultural technology over the 1950-83 period can be characterized by biased technical change of a translating nature.

Annual estimates of input and output augmentations that are consistent with these data can be generated by the solution to the linear programming problems (6) or (7). This is done here by choosing the elements of the vector b in (6) to be equal to k if they are coefficients of A, and equal to k^2 if they are coefficients of B, where k is a large positive scalar.2/ In this context, the linear programming solutions for the B's can be interpreted as the "smallest bias" in technical change that is consistent with the data, while the solutions for the A's can be interpreted as the "smallest output augmentations" (given the B's) that rationalize the data. These estimates of the A's and B's are the primal activity levels in problem (6), or equivalently, the "shadow prices" on the constraints in the dual formulation (7). Table 2 summarizes these estimates, the associated total factor productivity (TFP) index proposed in Section II and computated as

$(\frac{A_s - A_{1977}}{y_{1977}} + 1)$, the discrete Divisia TFP created by Capalbo and Vo using these same data, and the TFP index reported by USDA (USDA, ERS p.75).

Three inputs -- family labor, land and other capital -- are found to exhibit biased technical change with the proposed non-parametric procedures (see table 2). Note the flexibility of the approach: contrary to most parametric analysis, the non-parametric procedure yields a different estimate of factor bias in technical change for each input and each year.

The estimated biases in family labor and other capital cease by the early 1960's. Other capital is estimated to be generally of an input using nature (i.e., negative augmentations) while family labor exhibits input saving (i.e., positive augmentations) technical change in 1956 and 1960. The estimated biases in land inputs are found to be negative (input using) over the 1974-82 period. These estimates increase considerably (in absolute value) between 1977-78 (second oil price shock) and start declining in 1982 before turning positive in 1983. Thus, these non-parametric estimates of the factor bias in land utilization suggest that the export driven agriculture of the mid to late 1970's was characterized by land using technical change.

The estimated non-parametric additive output augmentations are conveniently summarized in table 2 using the non-parametric TFP index proposed in Section II.3/ Figures 1 and 2 compare this non-parametric TFP estimate with other commonly used TFP measures. In Figure 1, the non-parametric TFP index is contrasted to the USDA TFP index. Note that the USDA index indicates relatively more variation in productivity change for most of the period analyzed, in particular from 1957 through

1975. Also note that the USDA TFP index lies above the non-parametric index for most of the same period (except for 1974). The difference between these two TFP measures may reflect different quality adjustments in the measurement of several inputs in the Capalbo and Vo data compared to the USDA data. For example, the USDA index of labor input does not reflect quality changes while the Capalbo and Vo does (see Capalbo and Vo). Besides these data differences, the fixed weight Laspeyres index used by USDA is also a likely source of the observed differences. Ball and others have argued the Laspeyres index may not be appropriate because of the a priori restrictions it imposes on the structure of production.

Figure 2, in contrast, compares the non-parametric TFP measure to the Christensen and Jorgenson discrete Divisia TFP index generated by the same (Capalbo and Vo) data. In this case, the Christensen and Jorgenson TFP index tends to increase more slowly than our non-parametric TFP. Part of the differences between the non-parametric and Christensen and Jorgenson TFP indexes in Figure 2 may reflect the input biases indicated in table 2: allowing for the bias in technical change may affect the measurement of productivity growth.[4/] Alternatively, the difference between the two measures presented in figure 2 could reflect the fact that a translog specification does not provide an appropriate global representation of agricultural technology during the period considered. By not requiring a priori specification of the production technology, the proposed non-parametric approach thus appears to provide a useful and flexible tool for the analysis of productivity.

V. Summary and Conclusions:

This paper proposes a non-parametric procedure for calculating total factor productivity (TFP) measures and investigating technical change. Commonly used TFP measures such as the discrete Divisia TFP (Christensen and Jorgenson), although superlative, still require possibly stringent assumptions on the form of the underlying technology. In contrast, the proposed non-parametric procedure only assumes profit maximizing behavior and very general hypotheses concerning the nature of technical change.

The proposed procedures extend the non-parametric results of Hanoch and Rothschild, Diewert and Parkan, and Varian to include explicit hypotheses about the nature of technical change in production through a generalized augmentation hypothesis. We explore an empirically convenient form of this augmentation hypothesis that generates non-parametric tests of technical change as a system of linear inequality constraints amenable to solution using standard linear programming techniques.

Application of the proposed methodology to aggregate U.S. agricultural data for 1950-83 indicates that the hypotheses of the absence of technical change or Hicks neutral technical change are not consistent with these data. Hence, these non-parametric results indicate that U.S. agriculture was characterized with biased technical change over the periods analyzed. The non-parametric test of this hypothesis under additive input <u>and</u> output augmentation is consistent with the data over the full 1950-83 time period analyzed.

The proposed methodology can yield non-parametric estimates of annual rates of technical change in outputs and/or inputs that are consistent with the data. Annual factor saving <u>and</u> factor using bias in technical change are identified for family labor, land and miscellaneous capital inputs. The magnitudes of the estimated factor using biases for land in the middle 1970's to the early 1980's appear quite reasonable given the export driven agriculture of that period.

Non-parametric estimates of rates of additive output augmentation yield TFP measures that imply considerably less restrictive assumptions than commonly used measures. In particular, the non-parametric TFP index explicitly allows for biased technical change without a priori specification of the underlying technology. Comparison of this non-parametric TFP measure for aggregate U.S. agricultural data (1950-83) with USDA TFP and the discrete Divisia TFP of Capalbo and Vo indicates that the alternative indexes are not identical. While the proposed non-parametric procedures do not yield statistical hypotheses tests in the usual sense, similar criticisms hold for traditional TFP measures such as the discrete Divisia. Given the reasonableness of the results compared to the alternative TFP indexes considered, the less restrictive "non-parametric" nature of the approach, and the potential to identify rates of input and output augmentations that are data consistent, the proposed non-parameteric methodology appears worthy of consideration in the analysis of productivity and technical change.

Appendix

Proof of Proposition 1:

Note that expression (4) is simply stating that, given input and output prices in situation t, profit is at least as high choosing (x_t, y_t) compared to any other choice (x_s, y_s). This proves a/.

Define

$$G(X) = \underset{t}{\text{Min}}\ Y(y_t + \frac{r_t'}{p_t}[x(X,B_t) - x_t],\ A_t).$$

Treating G(X) as a production frontier where $Y \leq G(X)$, it follows that

$$p_t y(Y,A_t) - r_t' x(X,B_t) \leq p_t y(Y(y_t + \frac{r_t'}{p_t}[x(X,B_t) - x_t],\ A_t),\ A_t) - r_t' x(X,B_t).$$

But y(Y,.) and Y(y,.) are inverse functions, implying that

$$p_t y(Y,A_t) - r_t' x(X,B_t) \leq p_t y_t - r_t' x_t,$$

which proves b/.

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Table 1: Results of Non-Parametric Tests for Data Consistency With Several Specifications of Profit Maximization Under Additive Augmentation (Technical Change) Hypotheses: Aggregate U.S. Agriculture 1950-83.

TECHNICAL CHANGE HYPOTHESES:	TIME PERIOD: RESULT[1]
No Technical Change	1950-83: Reject
	1950-71: Reject
-- $A_s = A_t$, $B_s = B_t$, $\forall\ s \neq t$	1960-83: Reject
	1950-59: Reject
	1960-71: Reject
	1972-83: Reject
Hicks Neutral Technical Change (Output	1950-83: Reject
Translating):	1950-71: Reject
-- $B_s = B_t$, $\forall\ s \neq t$	1960-83: Reject
-- the A's are unrestricted	1950-59: Reject
	1960-71: Reject
	1971-83: Reject
Biased Technical Change (Output and Input Translating):	1950-83: Accept
-- the A's and the B's are unrestricted.	

1) Reject (accept) implies unbounded (optimal) solution to the dual linear programming formulation of equation (5) via problem (7). Thus, the data are found to be inconsistent (consistent) with the non-parametric test of the technical change hypothesis given profit maximization and additive augmentations as maintained hypotheses.

Source: Computations by the authors using data from Capalbo and Vo.

Table 2: Non-Parametric Estimates of Rates of Technical Change in U.S. Agriculture 1950-83, Assuming Profit Maximization and Additive Input and Output Augmentation.[1]

	B_t: BIASED INPUT AUGMENTATIONS[2]:			A_t: ADDITIVE OUTPUT AUGMENTATION			CAPALBO AND VO	USDA ERS
YEAR	FAMILY LABOR	LAND	OTHER CAPITAL	OUTPUT AUGMENTS	CHG IN AUGMENTS	NON-PARM TFP INDEX[3]	(DIVISIA) TFP INDEX[4]	(FIXED WT) TFP INDEX[5]
1950	0	0	0.030	0.000	0	0.564	0.670	0.580
1951	0	0	0	0.032	0.032	0.596	0.689	0.600
1952	0	0	-0.024	0.066	0.033	0.630	0.719	0.620
1953	-0.069	0.023	0	0.090	0.024	0.654	0.743	0.640
1954	-0.125	0	0	0.090	0.001	0.655	0.742	0.650
1955	0	0	0	0.086	-0.004	0.650	0.735	0.660
1956	0.035	0.044	0	0.138	0.052	0.703	0.791	0.670
1957	0	0	0.055	0.112	-0.026	0.676	0.772	0.680
1958	0	0.014	0	0.153	0.041	0.717	0.786	0.740
1959	0	0	-0.042	0.141	-0.012	0.705	0.758	0.730
1960	0.062	-0.014	-0.046	0.162	0.021	0.727	0.780	0.760
1961	0	-0.015	-0.014	0.174	0.012	0.738	0.786	0.780
1962	0	0	0	0.182	0.008	0.746	0.802	0.780
1963	0	0	0	0.197	0.016	0.761	0.815	0.820
1964	0	0.005	0	0.214	0.017	0.778	0.834	0.810
1965	0	0.003	0	0.220	0.006	0.784	0.826	0.840
1966	0	0.002	0	0.232	0.012	0.796	0.835	0.830
1967	0	0	0	0.254	0.022	0.818	0.854	0.850
1968	0	0	0	0.264	0.010	0.828	0.856	0.870
1969	0	0	0	0.283	0.019	0.847	0.866	0.880
1970	0	-0.001	0	0.257	-0.026	0.821	0.844	0.870
1971	0	0	0	0.316	0.059	0.880	0.887	0.950
1972	0	0	0	0.330	0.014	0.894	0.897	0.940
1973	0	0.003	0	0.350	0.021	0.915	0.912	0.950
1974	0	-0.017	0	0.353	0.003	0.917	0.944	0.900
1975	0	-0.010	0	0.391	0.038	0.955	0.963	0.990
1976	0	-0.018	0	0.388	-0.003	0.953	0.963	0.980
1977	0	-0.010	0	0.436	0.047	1.000	1.000	1.000
1978	0	-0.031	0	0.433	-0.002	0.998	0.973	1.010
1979	0	-0.038	0	0.507	0.074	1.071	1.025	1.050
1980	0	-0.041	0	0.495	-0.012	1.060	1.023	1.010
1981	0	-0.041	0	0.590	0.094	1.154	1.096	1.160
1982	0	-0.027	0	0.606	0.017	1.171	1.107	1.160
1983	0	0.008	0	0.452	-0.154	1.017	1.001	0.980

1) These estimates of the B's and A's are the dual "shadow prices"of problem (7).

2) These estimates reflect either factor using (B-) or factor saving (B+) biases in technical change.

Table 2 (Continued):

3) This non-parametric TFP index computed $(1 + (A_s - A_{1977})/y_{1977})$ reflects the base 1977=1.00.

4) Source: Capalbo and Vo, p. 3-37.

5) Source: U.S.D.A. ERS. Economic Indicators of the Farm Sector. (ECIFS 5-5) APRIL, 1987, p. 75.

Source: Computations by the authors using SAS Proc LP and data from Capalbo and Vo.

Figure 1: Comparison of Alternative Total Factor Productivity Indexes (TFP) For Aggregate U.S. Agriculture, 1950–83.

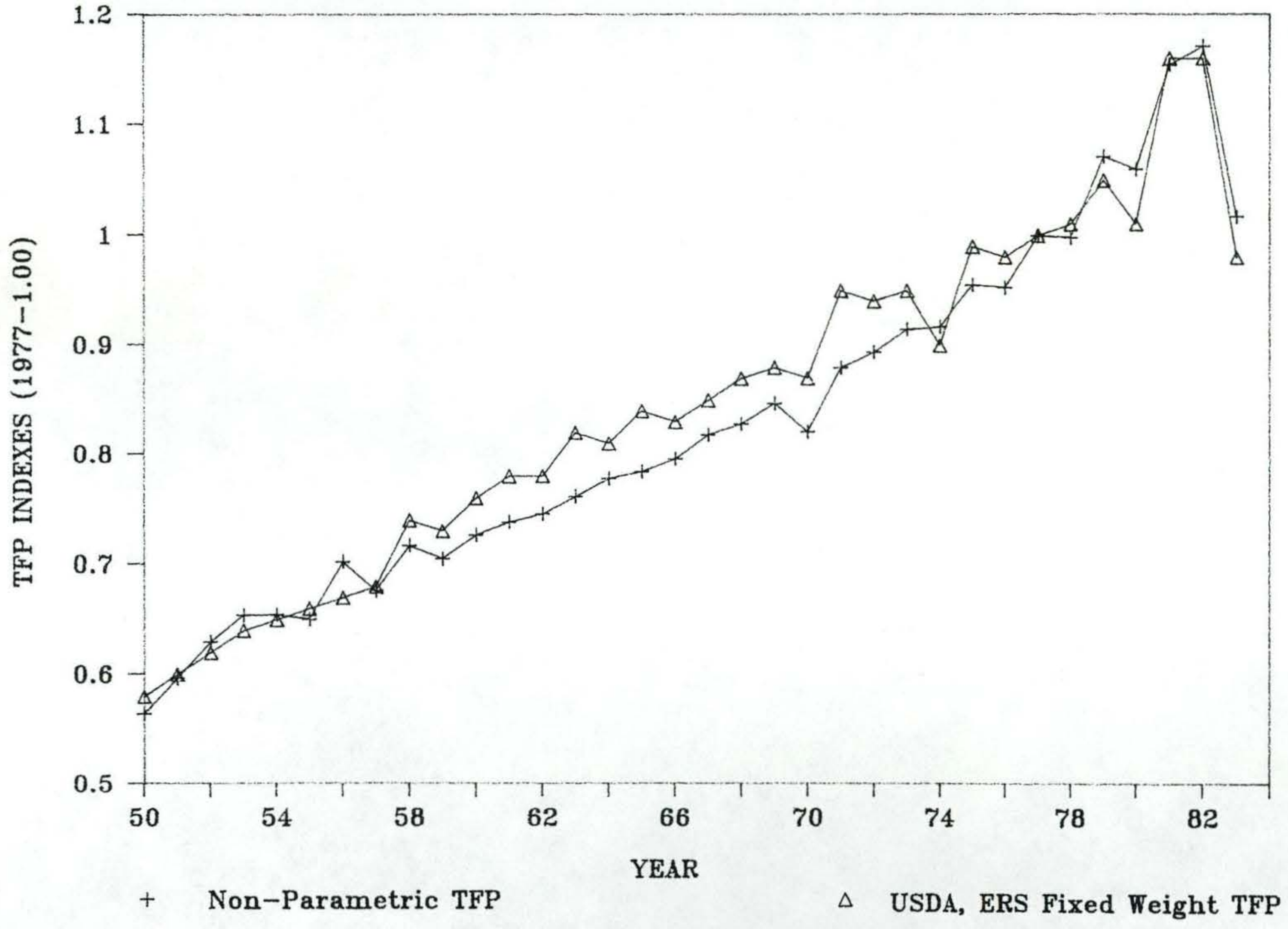
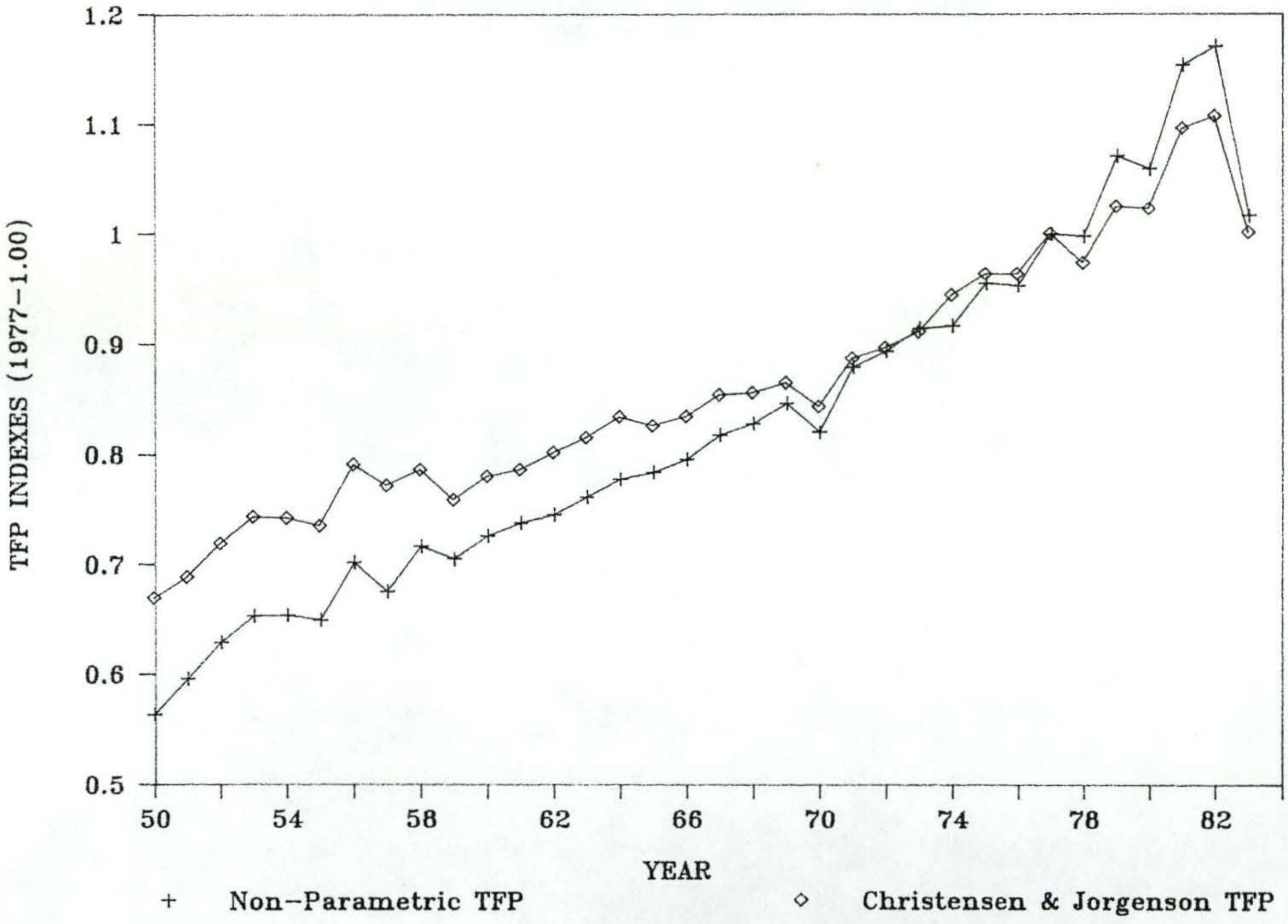


Figure 2: Comparison of Alternative Total Factor Productivity Indexes (TFP) For Aggregate U.S. Agriculture, 1950–83.



Footnotes

1/ An alternative hypothesis to the specification of $Y(y,A)$ and $X_i(x_i,B_i)$ is the scaling hypothesis where $Y = y/A$ and $X_i = B_i.x_i$, $i=1,\ldots,n$. Under this specification, expression (4) becomes

$$p_t[y_t - \frac{A_t}{A_s}.y_s] - \sum_{i=1}^{n} r_{it}[x_{it} - \frac{B_{is}}{B_{it}}.x_{is}] \geq 0$$

Compared to (5), this expression is non-linear in A and B and is therefore considerably more complex to use empirically.

2/ The results presented below correspond to $k = 1000$. Choosing larger values of k did not affect the results.

3/ We should also note that very similar A_t results (and associated TFP indexes) were generated when allowing only input using or only input saving technical change. This indicates that the A_t estimates are fairly robust to the B_t specifications evaluated.

4/ Caves et al. have shown that the Christensen and Jorgenson TFP index can be derived from a translog transformation function allowing for bias in technical change through the influence of technology on the first order translog parameters. However, if the bias in technological change affects also the second order translog parameters, then the Christensen and Jorgenson index is no longer an exact TFP index associated with a translog transformation function (see Caves et al., p. 77, footnote 2).