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SPORT AND COMMERCIAL FISHING INTERACTIONS; AN OPTIMAL CONTROL MODEL WITH IMPLICATIONS FOR POLICY AND RESEARCH

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SPORT AND COMMERCIAL FISHING INTERACTIONS: AN OPTIMAL CONTROL MODEL WITH IMPLICATIONS FOR POLICY AND RESEARCH

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Abstract:

A recreational sector is added to a standard commercial fishing model to identify public decision variables which are important in dealing with sport-commercial conflicts. Shortcomings of current ecomomic inputs to policy making are identified and future research topics to remedy this are derived from the model.

Because fish caught recreationally are more valuable than fish caught commercially, sports fishing should be favored whenever the two groups clash. So goes the argument heard by policy makers who are being asked with increasing frequency to allocate scarce fishery resources between sport and commercial interests.[1] And, although this argument is distinctly economic in character, economists themselves have been mostly silent on the topic.[2] This void in the literature is curious given the

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[1] An interesting alternative interpretation is that sport fishing is "just" recreation and produces nothing economic, whereas commercial fishing produces food, profits and employment. This argument, however, appears to be completely devoid of economic content.

[2] Two unpublished papers have recently appeared. See McConnell and Sutinen for an optimal control approach to this question that is somewhat similar to the present effort. A very interesting static model has been developed by Anderson. Other relevant works include those by Copes and Knetsch; Brown and Hussen; Mathews and Wendler; Southey; and Rothschild, et al.

large amount of research currently being devoted to commercial fishing and to the estimation of demand functions for recreation of all kinds. It is also unfortunate, since--as the analysis in this paper will show--the allocation of fishery resources involves economic relationships that are not adequately captured by comparing dockside commercial values of fish with expenditures or recreation benefits per pound.

The present paper develops a bioeconomic model of recreational-commercial fishing interactions. The results of the model should be of interest to both decision-makers and econimists. The model explicitly describes some of the variables that ought to be considered when comparing the economic contributions of sport and commercial fishing. As such, it shows the decision-maker what is important from an economic point of view and why arguments like that given above are economic oversimplifications. The economist will immediately recognize the possibilities for important additional research on both the theoretical and empirical levels.

The Clark-Munro Linear Model

The model presented here adds a recreational sector to the linear optimal control model developed by Clark and Munro and later discussed at length by Clark. Let $F(x)$ = the natural rate of growth of the fish population as presented in the Schaefer model of population dynamics, where x is the size of the fish population in question. Let p = the price per pound of fish caught commercially, where demand is assumed to be perfectly elastic. Let $c(x)$ = the cost of catching a pound of of fish as a function of the population size, $c'(x) < 0$. Let h = the fish harvest and h_{max} = the maximum possible rate of harvest, so that $0 \leq h \leq h_{max}$. To simplify the notation, let $p - c(x) = C(x)$, the rent per pound of fish caught commercially. Thus, $C'(x) > 0$. Let t denote specific points in time and δ the social rate of discount. Except as otherwise stated below, all functions are assumed to be continuous and differentiable.

The objective functional is given by

$$\max J(h) = \int_0^\infty e^{-\delta t} C(x)\, h\, dt$$

subject to

$$\frac{dx}{dt} = \dot{x} = F(x) - h$$

$$0 \leq h \leq h_{max}$$

$$x(0) = x_0$$

The final constraint is an initial condition, that at $t = 0$, x stands at some given value x_0. The problem is to choose an optimal control $h(t)$ for all t such that J is maximized without exceeding the natural productivity of the fish population or the productive capacity of the fishing fleet.

The relevant Hamiltonian function is

$$H = e^{-\delta t} C(x)\, h + \lambda(t)[F(x) - h],$$

where $\lambda(t)$ is the adjoint variable. Clark and Munro apply the basic theorems of optimal control theory to derive a set of three necessary conditions for the optimum, assuming that one exists with positive x over all time and positive h over at least part of the time. These necessary conditions are:

I. that h is chosen over all $0 \leq t \leq \infty$ such that H is maximized.

II. $\frac{\partial H}{\partial x} = -\dot{\lambda}(t)$

III. $\dot{x} = F(x) - h$

Clark and Munro ahow that these conditions imply a "singular solution" for x, symbolized by x^*, which is defined implicitly by

$$F'(x^*) + \frac{C'(x^*)F(x^*)}{C(x^*)} = \delta$$

and that the optimal time path for commercial harvest is given by the piecewise continuous function

$$h(t) = \begin{cases} h_{max} & \text{for all } t \text{ such that } x(t) > x^* \\ F(x^*) & \text{for all } t \text{ such that } x(t) = x^* \\ 0 & \text{for all } t \text{ such that } x(t) < x^* \end{cases}$$

This is the optimal control for the case where there is only a commercial fishery. Now let us add a recreational sector.

The Model with Recreational Fishing Added

Let g = the rate of catch in the recreational fishery. Let g_{max} be the maximum rate of harvest by recreational fishers, so that $0 \leq g \leq g_{max}$. Let $R(x)$ = the rent per pound of fish caught recreationally, with $R'(x) > 0$. As in commercial fishing, presumably the cost of catching a pound of fish would be expected to decline, the larger the biomass. Furthermore, it is intuitively appealing to assume that recreational demand for fish is positively related to the rate of angler success, as measured, say, by fish caught per fishing day. Empirical studies by Stevens and Talhelm support this view. One would expect that normally angler success and biomass would be positively related as well. In the present model we in effect assume that the recreational demand for fish is perfectly elastic with respect to quantity caught (g) but shifts with biomass (x).

One other assumption must be made before we state the revised model formally: it is assumed that society's objective is to maximize the present value of the rents from the fish population in question. Over-emphasis on the efficiency goal can lead to problems as discussed by Bromley and Bishop. Nevertheless, optimization provides useful benchmarks for later analysis. Furthermore, remarks in the introduction demonstrate that efficiency is already playing a role in the policy debate over allocation of fishery resources between commercial and recreational interests. Hence, we proceed on that basis.

The revised problem can now be stated.

$$\max J(g,h) = \int_0^\infty e^{-\delta t}[R(x)g + C(x)h]dt$$

subject to

$$(1) \qquad x = F(x) - g - h$$

$$0 \leq g \leq g_{max}$$

$$0 \leq h \leq h_{max}$$

$$x(0) = x_0$$

The new Hamiltonian function is

$$H = e^{-\delta t}[R(x)g + C(x)h] + \psi(t)[F(x) - g - h]$$

where $\psi(t)$ is the adjoint variable in the new problem. At this point, however, let us rearrange H as follows:

$$H = \sigma_1 g + \sigma_2 h + \psi(t)F(x)$$

where σ_1 and σ_2 are "switching functions",

$$\sigma_1 = e^{-\delta t}R(x) - \psi(t)$$

$$\sigma_2 = e^{-\delta t}C(x) - \psi(t)$$

In order to describe the characteristics of a solution, the first necessary condition must be modified to allow for two control variables, g and h.

I'. The values of g and h at all points in time $0 \leq t \leq \infty$ must be such that H is maximized.

Necessary condition II remains the same and III is modified to include recreational harvest:

III'. $\dot{x} = F(x) - g - h$

Some Characteristics of the Solution

Assuming that a solution to our problem exists which involves positive x at all future times and positive g and/or h for at least part of the time, we can begin to explore the characteristics of that solution by examining I'. One approach to maximizing H would be

$$(2) \qquad \frac{\partial H}{\partial g} = e^{-\delta t}R(x) - \psi(t) = 0$$

$$(3) \qquad \frac{\partial H}{\partial h} = e^{-\delta t}C(x) - \psi(t) = 0$$

These equations help define the singular solution to the problem and yield our first important result. The adjoint variable $\psi(t)$ is interpreted as the value at the margin of increasing the fish population by one pound along the optimal time path (Clark; Dorfman). It is hard to imagine a realistic case where it would not be a single-valued function of time. Hence, (2) and (3) can hold simultaneously only in the rather uninteresting special case where $R(x^*) = C(x^*)$ where x^* is the optimal biomass.

Here society is indifferent between sport and commercial fishing once x^* is attained. Otherwise, and this is a more interesting result, once the singular time path is reached, either h or g must go to zero, except under special circumstances discussed below. Stated differently, once the steady state is achieved, either recreational or commercial fishing must cease. This is a direct outgrowth of the linearity of the model.

What characterizes the singular time path? If we take equation (3) and necessary conditions II and III' and follow the same steps as Clark and Munro, the result is the familiar

$$F'(x_2^*) + \frac{C'(x_2^*)F(x_2^*)}{C(x_2^*)} = \delta$$

where now x_2^* is the optimum level of biomass if commerical fishing is optimal along the singular time path. Likewise, let x_1^* = the optimal biomass if recreational fishing is socially preferred. Following the same procedures for (2), II and III' yields

$$(4) \qquad F'(x_1^*) + \frac{R'(x_1^*)F(x_1^*)}{R(x_1^*)} = \delta$$

Society's problem is to choose between x_1^* and x_2^* as the long term goal for biomass and to choose an optimal plan to approach the preferred steady state. There are two sets of critical factors here. The first relates to the sensitivity of the net values of sport and commercial catches to biomass. The second set relates to the capacities of the fleets. The impacts of these factors will be illustrated with two cases.

<u>Case 1</u>. If we assume that $R(x) > C(x)$ for all x, then a strong case exists for devoting the fish resource completely to recreation so long as the fishery is in the steady state x_1^* as defined in Equation (4) and the recreation sector has the capacity to harvest the catch, i.e. $g_{max} \geq F(x_1^*)$. Even here, however, commercial fishing may be economically efficient under certain conditions.

First, consider the path to the steady state. Suppose, for example, that $x(0) = K$, the carrying capacity of the environment in the absence of exploitation. The model would signal this by showing $\sigma_1 > 0$, i.e. rent per pound exceeds the value of ψ, the adjoint variable. So long as this holds, condition I' means that g must be set at g_{max}. This is the so-called "bang-bang" approach to equilibrium. In addition, the appropriate value for the other control variable h must be considered. There are three possibilities. The first is that $e^{-\delta t}C(x) < \psi(t)$ for all $x \geq x_1^*$ in which case commercial fishing is never economical. On the other hand, it is conceivable that $e^{-\delta t}C(x) > \psi(t)$ for some values of x larger than x_1^*. This would make $\sigma_2 > 0$ and I' would require that $h = h_{max}$. A possible scenario would then be to have both sport and commerical fishing at the maximum rate for an initial period of time, a second phase of development where commercial fishing is eliminated but sport fishing continues at full capacity, and a third, equilibrium phase where $h = 0$ and $g = F(x_1^*)$ in perpetuity.

There is a third possibility and a rather interesting one. This would be the case if the recreational fishery has insufficient capacity to move to the steady state, prescribed by Equation (4), i.e. $g_{max} < F(x_1^*)$. This would mean the $\psi = e^{-\delta t}C(x)$ in the steady state. Then conditions II and III' plus the necessity that $g = g_{max}$ must hold ($\sigma_1 > 0$ continues to be true in the long-run) can be manipulated algebraically to define a new optimal biomass $\tilde{x}$ defined by

$$F(\tilde{x}) + \frac{C'(\tilde{x})[F(\tilde{x}) - g_{max}]}{C(\tilde{x})} + \frac{R'(\tilde{x})g_{max}}{C(\tilde{x})} = \delta$$

Commercial fishing continues in the steady state at a level $h = F(\tilde{x}) - g_{max}$.

It should be noted that cases where $C(x) > R(x)$ for all x are also conceivable. Here the analysis would be symmetrical except that commercial fisheries would tend to dominate. It is also possible that at some values of x, $R(x) > C(x)$, while at others the reverse holds. This brings us to the second case.

<u>Case 2</u>. Let us suppose that for larger levels of biomass, say $\hat{x} \leq x \leq K$, $R(x) > C(x)$ but that for $0 \leq x \leq \hat{x}$, $C(x) > R(x)$. While the existence of the case in reality is an empirical question, it has some intuitive justification. Fishing with rod and reel is not a very effective method for catching most species. A fairly large biomass may be required to maintain success rates at sufficiently high

levels and to keep costs down. Also, the average size of the fish may be critical in recreational fisheries and size usually decreases as biomass declines. Thus, while $R(x)$ may be quite large when x is relatively high, it may decline rapidly with decreases in biomass. Commercial rent per pound, on the other hand, may be much less sensitive to biomass. Thus this appears to be a case of some practical interest.

We will assume that there are no capacity problems such as those discussed toward the end of Case 1, above. Also, recall that x_1^* and x_2^* are the optimal levels of biomass for recreational and commercial fishing as defined above. We must consider three possibilities.[3]

(a) It may hold that x_1^* and x_2^* are both less than $\hat{x}$. This would mean that once x falls below $\hat{x}$, $C(x) > R(x)$ and the results would follow Case 1. That is, x_2^* is optimal and recreational fishing will be economically (i.e. $\sigma_1 \geq 0$) if at all, only during the phase before the steady state is achieved. Commercial fishing continues at h_{max} until x_2^* is reached and at $h = F(x_2^*)$ thereafter.

(b) Suppose now that both x_1^* and x_2^* exceed $\hat{x}$. Then, since $R(x) > C(x)$ for all relevant biomass levels, the situation is

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[3] A fourth possibility, that $x_1^* < \hat{x} < x_2^*$, is mathematically inconsistent with the assumptions. The definition of x_1^* implies $R(x_1^*) = \psi(t)$ if it is the optimal steady state. However, by definition $R(x_1^*) < C(x_1^*)$ when $x_1^* < \hat{x}$. Thus $x_1^* < \hat{x}$ implies $C(x_1^*) > \psi(t)$ which means that x_1^* could not be the optimal solution for the overall problem. Having $x_2^* > \hat{x}$ would likewise imply that $R(x_2^*) > C(x_2^*) = \psi(t)$ so that neither point could be optimal.

again analagous to Case 1 but now recreational fishing is the goal in the steady state and commercial fishing is economical only in disequilibrium if at all.

(c) If $x_2^* < x < x_1^*$, the problem becomes more complex and the solution cannot be determined on a priori grounds.

A conclusion which follows from discussion of the two cases above is that the relative values of $R(x)$ and $C(x)$ for any given biomass x do not alone provide a clear indication of whether recreational or commercial fishing is most efficient in the steady state. Also necessary to consider are the values of $R(x)$ and $C(x)$ at the optimal biomass levels x_1^* and x_2^*, as well as whether there exists sufficient harvesting capcity in each fishery. But this raises another question. In the absence of a binding capacity constraint is it necessarily true that a fishery must be devoted exclusively to either sport or commercial fishing if efficiency is to be achieved? As noted previously, this outcome is a direct result of the linearity of our model. Once nonlinearities are introduced, efficiency may well call for a multiple-use fishery.

Multiple Use in the Steady State

To construct the more realistic non-linear model it is necessary to change the definitions of our benefit functions. Let $R(x,g)$ equal the marginal net benefits of recreational fishing and $C(x,h)$ equal the marginal net benefits of commercial fishing. We assume that over relevant values of x, g, and h, that

$$\frac{\partial R(x,g)}{\partial x} > 0, \quad \frac{\partial R(x,g)}{\partial g} < 0.$$

$$\frac{\partial C(x,h)}{\partial x} > 0, \quad \frac{\partial C(x,h)}{\partial h} < 0.$$

Total net benefits given some level of x are functions of the catch rates such that

$$\hat{R} = \int_0^g R(x,g)dg$$

$$\hat{C} = \int_0^h C(x,h)dh.$$

The problem is then set up as:

$$\max J(g,h) = \int_0^\infty e^{-\delta t}[\hat{R} + \hat{C}]dt$$

subject to $\dot{x} = F(x) - g - h$

$$x(0) = x_0$$

The Hamiltonian expression for this problem is then

$$H = e^{-\delta t}[\hat{R} + \hat{C}] + \psi(t)[F(x) - g - h]$$

where $\psi(t)$ is again the adjoint variable.

For our purposes it is only necessary to explore some of the characteristics of the steady state solution to this problem. Assuming that such a solution exists, conditions I', II, and III' above can be used to derive necessary conditions which can be rearranged to show that

(5) $$R(x^*,g^*) = C(x^*,h^*)$$

(6) $$F'(x^*) + \frac{\partial\hat{R}/\partial x + \partial\hat{C}/\partial x}{C(x^*)} = \delta$$

(7) $$F(x^*) = g^* + h^*$$

Expression (6) implicitly defines the optimal biomass, x^*, for the population of fish in question. The harvestable surplus $F(x^*)$ is then divided between sport fishing at a rate of g^* and commercial fishing at a rate of h^* such that marginal net benefits from the two fisheries are equal (Equation 5).

Thus, once benefits come to depend on the catch levels, it is quite conceivable that efficient exploitation of fish population could involve joint use by sport and commercial fishers after the steady state has been reached.

Conclusions

The paper began by pointing out that there is a tendency in policy debates to compare sport and commercial fishing on the basis of recreation benefits per pound caught versus the dockside values of commercially caught species. We are now ready to draw some conclusions about the potential pitfalls of this type of argument and to suggest some lines of research that will facilitate more meaningful comparisons.

The first conclusion--and we really did not need an optimal control model to tell us this--is that costs are important. Dockside values of commercially caught fish do not reflect costs. Recreation benefits when calculated in conventional ways do exclude direct costs to the recreationists, but may neglect management costs born by the taxpayer. Meaningful economic comparisons of sport and commercial fishing must be based on net benefits.

Secondly, and this is particularly apparent from the non-linear model, it is net benefits at the margin that must be compared. To use average recreation benefits per pound is a flagrant violation of this principle and probably exaggerates recreation values. Going back to our non-linear model, if the assumptions are correct,

$$\frac{\hat{R}}{g} > R(x,g).$$

Thus, meaningful comparisons must be made on the basis of marginal net values.

Thirdly, before economic contributions can be adequately compared, recreational fishing benefits have to be related empirically to recreational catch. This is critical because most fishing demand studies relate price to recreation days not fish caught. Fish are only one input into the production of recreation days. Simply dividing total recreation benefits by catch to get average benefits per pound is like dividing gross farm income by pound of fertilizer to get the ecomomic contribution of fertilizers to farm income. Meaningful comparisons must be based on the <u>derived demand</u> for fish.

Furthermore, the economic contributions of sport and commercial fisheries depend on x, g, and h. For one vector of values of these variables, sport fishing may look advantageous, while for another vector, commercial fishing may appear more economically attractive. Meaningful comparisons must take account of the <u>sensitivity</u> of marginal and total net benefits to changes in biomass, sport catch, and commercial catch. The importance of the relationships between biomass and total net benefits is illustrated by the various equations for determining x^* (e.g. Equations (4) and (6)). The models show that conditions outside the steady state and the fishing capacity of two interest groups may also be important.

Comparison of dockside values and recreation benefits per pound caught has occurred, of course, to fill a void left by the scarcity of more adequate economic analyses. This leads to topics for future research. A top priority is the quantification of the derived demand for fish in recreational uses. Some important work relating to this issue are currently underway at Michigan State, Wisconsin, and elsewhere, but much more work is needed. Also, better demand curves for commercially caught species and further investigations of the cost of both types of fishing are needed. In addition, we presently know little about the distributional implications of alternative policies.

Many theoretical problems also remain that must be solved if we are to have solid theoretical principles to guide empirical research. What if there are two species of fish instead of one and the recreational species is predator and the commercial species is prey? What if the two species compete? What if one or both interest groups continue to operate under open access? What if commercial fishing has recreational benefits through seafood restaurants and the contribution the fleets make to the coastal atmosphere?[4]

Clearly economists have much work to do before they can adequately assist public decision-makers confronting conflicts between sport and commercial fishers.

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[4]McConnell and Sutinen consider the parallel case where recreationists can market their catches.

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