THE OUTPUT-COST RELATIONSHIP FOR RETAIL FERTILIZER PLANTS: AN EMPIRICAL APPLICATION OF MULTIPRODUCT FIRM THEORY**

by

Jay T. Akridge and Thomas W. Hertel*

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* Authors are Graduate Research Assistant and Assistant Professor, respectively, at Purdue University, West Lafayette, Indiana.

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Abstract

Retail fertilizer plants produce a number of products and services. To analyze the relationship between cost and output for these multiproduct firms, a short-run, translog cost function is estimated using pooled data. Results indicate plants can lower average cost by increasing output and by diversifying into anhydrous ammonia. Furthermore, preliminary evidence indicates that firms in the sample are over-invested in plant and equipment.

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Volume-cost relationships developed from statistical analysis of accounting data have long provided information for managerial decision making (French). By estimating a cost function to examine volume-cost questions, these studies attempt to provide managers with operating relationships useful in evaluating firm efficiency. Descriptive and statistical analyses of accounting data as well as economic-engineering cost studies have been available to retail fertilizer firms in the past. These previous cost studies have focused on a single product line (Anderson and Miller; Raikes and Heubrock; Williams, et al.), service (O'Rourke) or have simply summarized industry accounting data (Akridge and Downey).

The problems associated with using accounting data to estimate statistical cost functions have been recognized since the technique's inception (French; Johnson; Johnston; Stollsteimer, et al.). Two criticisms in particular stand out: the failure to handle multiproduct firm cost relationships in a satisfactory manner and the dependence of the results on the choice of functional form used in the estimation. Recent cost studies have addressed both issues (Brown, et al.; Christensen and Greene; Cowing and Holtman). Combining advances in multiproduct firm theory with a flexible functional form, statistical cost functions can now be estimated for industries where firms produce more than one output. This richer model captures the cost implications of changing output mix (in addition to the level of output) without imposing arbitrary restrictions on the cost function. The research reported in this paper attempts to determine the relationship between output level/mix and cost for retail fertilizer plants.

Modern retail fertilizer plants market a diverse array of products and services, ranging from dry bag fertilizer, which the retailer simply warehouses until needed by the farmer, to fluid fertilizer which the retailer blends to customer specification. The fluid product may then be custom-applied by the dealer for the farmer. This custom-application service represents yet another product which the firm may or may not provide. Faced with this complex, multiproduct problem, retail fertilizer plant managers have considerable difficulty comparing their firm's performance to that of others. In addition, they lack an adequate framework for sorting out the effects which changes in product mix are likely to have on cost. Such information is important to managers making pricing and promotion decisions.

In this paper, we briefly review the theoretical measures of output-cost relationships for multiproduct firms. We then present the methods and data used to estimate a short-run, translog variable cost function based on time-series, cross-section data from 24 retail fertilizer plants over an eight-year period. The estimated model is then presented along with measures of scope and scale economies. We find that the average plant in the sample exhibits economies of scale and could lower average cost by increasing output. Furthermore, the observed scale economies are primarily the result of cost savings achieved through economies of diversification which exist between product categories. After developing these results in some detail, we discuss the implications of this research for retail fertilizer plant managers.

THEORY

Retail fertilizer firms are assumed to minimize short-run variable cost given vectors of outputs (Y), variable input prices (W), and fixed inputs (F). Solving the firm's short-run cost minimization problem yields the variable cost function:

(1) $C_v(Y,W,F)$

where C_v is the minimal level of cost, conditional on available fixed factors. We examine the short-run problem since retail fertilizer plants experience a high degree of year-to-year variation in operating season length and output level. It is unlikely plants operating under such conditions are in long-run equilibrium. Variable inputs include labor, gasoline, electricity, mechanic services, repair parts, and advertising. Retail fertilizer firms are price-takers in the markets for these types of products, thus input prices are exogenous. Since the investment required to add a new product is substantial, managers have little control over the mix of products marketed in the short-run. Actual quantities of each product are determined by factors affecting the firm's farmer/customers: crop prices, interest rates, government programs, weather, etc. The assumption of exogenous output levels in the short-run appears reasonable.

There are three distinct ways output mix and level can influence cost in the multiproduct firm (Baumol, et al.). Economies of scope (EOS), the first measure of cost-output relationship, are said to exist if it is cheaper for the firm to produce some group of products jointly than it is to produce the same group of products individually. More formally, economies of scope exist if:

(2) $\sum_{i=1}^{K} C_v(Y_{Ti}) > C_v(Y_S)$

where the Y_{Ti} are orthogonal non-negative output vectors, Y_S is an output vector containing all of the Y_{Ti} vectors, and W and F have been supressed for notational convenience.[1] Dividing (2) by $C_v(Y_S)$ provides a scale-free measure of scope economies:

$$(3)\quad EOS = \frac{\sum_{i=1}^{K} C_v(Y_{Ti}) - C_v(Y_S)}{C_v(Y_S)}$$

Economies of scope exist if EOS is greater than zero.

Product-specific economies of scale (PSE) is the second cost-output concept we examine. PSE measures the impact on cost of increased production in a single product line holding all other output levels, input prices, and fixed input quantities constant. The measure is based on the notion of product-specific incremental cost, $IC(y_i)$, which is defined as:

$$(4)\quad IC(y_i) = C_v(\overline{Y}) - C_v(\overline{Y}-y_i)$$

where $C_v(\overline{Y}-y_i)$ is the cost of producing every product at level $\overline{Y}$, except the i^{th} product, y_i, which is not produced. Average incremental cost, $AIC(y_i)$, is then:

$$(5)\quad AIC(y_i) = IC(y_i)/y_i.$$

Finally, product-specific scale economies are given by:

$$(6)\quad PSE = AIC(y_i)/(\partial C_v(\overline{Y})/\partial y_i).$$

Hence, PSE is the average incremental cost of producing the i^{th} output divided by the marginal cost of producing the i^{th} output. If PSE > 1, then product-specific scale economies exist.

The final measure of output-cost relationship corresponds closely to the traditional concept of economies of scale for a single product firm. Multiproduct scale economies (MSE) exist if simultaneously increasing the production of all outputs lowers ray average cost. Ray average cost is the multiproduct equivalent of "average cost" as defined for the single product firm. Formally, ray average cost is expressed as:

$$(7)\quad RAC_v(\overline{Y}) = C_v(\overline{Y}) / \sum_{i=1}^{L} y_i$$

where $RAC_v(\overline{Y})$ is the ray average cost associated with the output vector $\overline{Y}$ and y_i is the quantity of the i^{th} output produced. Multiproduct scale economies exist at $\overline{Y}$ if:

$$(8)\quad dRAC_v(t\overline{Y})/dt < 0$$

where t is evaluated at 1. Writing (8) in elasticity form provides an expression for the elasticity of ray average cost along the output ray defined by $\overline{Y}$. Multiplying this cost elasticity by -1 provides a measure of multiproduct scale economies:

$$(9)\quad MSE = 1 - \sum_{i=1}^{L} \partial \ln C_v(\overline{Y}) / \partial \ln y_i .$$

Hence, multiproduct scale economies exist if MSE is greater than 0 (Cowing and Holtman).

Since MSE measures the cost implications of varying all outputs simultaneously, it is a function of the cost response associated with product-line diversification (EOS) and with changing the production level of individual products (PSE). Thus (9) may be rewritten as:

$$(10)\quad MSE = 1 - \frac{(1 - EOS)}{\alpha_{T1} PSE_{T1} + (1 - \alpha_{T1}) PSE_{T2}}$$

where $\alpha_{T1} = (\Sigma_{j\varepsilon T1} Y_j MC_j)/(\Sigma_{j\varepsilon S} Y_j MC_j)$ and PSE_{T1} and PSE_{T2} are the product-specific scale economies associated with output vectors Y_{T1} and Y_{T2} respectively. This expression links the three output-cost measures (Baumol, et al.). It is clear from (10) that scope economies serve to magnify the effects of scale economies associated with individual products. If EOS is zero, implying there are no gains to diversification, then MSE is simply a function of the weighted average product-specific scale economies associated with Y_{T1} and Y_{T2}. The weights can be roughly interpreted as the proportion of total variable cost expended on production of the respective output vector.

PROCEDURE

We assume a multiproduct translog variable cost function:

$$(11)\quad \ln C_v = \alpha_0 + \sum_r \alpha_r \ln Y_r + \sum_i \beta_i \ln W_i + \sum_k \gamma_k \ln F_k$$
$$+ 1/2\Big[\sum_r \sum_s \alpha_{rs} \ln Y_r \ln Y_s + \sum_i \sum_j \beta_{ij} \ln W_i \ln W_j + \sum_k \sum_l \gamma_{kl} \ln F_k \ln F_l$$
$$+ \sum_r \sum_i \delta_{ri} \ln Y_r \ln W_i + \sum_r \sum_k \Psi_{rk} \ln Y_r \ln F_k + \sum_i \sum_k \Theta_{ik} \ln W_i \ln F_k$$
$$+ \sum_i \sum_r \delta_{ir} \ln W_i \ln Y_r + \sum_k \sum_r \Psi_{kr} \ln F_k \ln Y_r + \sum_k \sum_i \Theta_{ki} \ln F_k \ln W_i\Big]$$

where C_v is (minimal) total variable cost; Y represents a set of six outputs -- dry fertilizer, fluid fertilizer, anhydrous ammonia, chemicals, services, and other farm supplies; W is a set of three input prices for labor, energy, and other variable inputs; and F represents a set of fixed inputs -- management, plant and equipment, and other fixed inputs.[2]

Neoclassical theory suggests the matrix of second-order terms will be symmetric. In addition, the cost function is expected to be homogeneous of degree one in input prices. These restrictions are imposed on the model for estimation (Brown, et al.; Cowing and Holtman). Logarithmic differentiation of the cost function and use of Shephard's lemma yields cost share equations for each variable input:

$$(12)\quad S_i = \partial \ln C_v / \partial \ln W_i = \beta_i + \sum_j \beta_{ij} \ln W_j + \sum_r \delta_{ir} \ln Y_r + \sum_k \Theta_{ik} \ln F_k$$
$$i = 1,2,3$$

where S_i is the proportion of total variable cost expended on the i^{th}

variable input. One of the share equations is dropped for estimation since only two of the three equations are linearly independent (Christensen and Greene).

Imposing homogeneity forces one of the input prices to be defined as a numeraire price. Hence, labor and energy prices are expressed in terms of the other variable input price and the share equation for other variable inputs is dropped. The estimating form of the model consists of equation (11) plus two share equations defined in (12) with symmetry and homogeneity imposed. We assume intercept and slope parameters are invariant across time and plants, and that the disturbance term is contemporaneously correlated between plants within a given year. The 78 parameters in the three equation system were estimated using Zellner's seemingly unrelated regression technique.

DATA

Data for the estimation were collected from 24 Indiana and Illinois retail fertilizer plants over the 1975-1982 period. Annual data for output quantities, expenses, and fixed input levels were obtained from accounting information submitted by the firms to the Purdue Fertilizer Retail Efficiency Data (FRED) Project (Akridge and Downey). Input price proxies were constructed from state and county price data. Summary statistics for the data set are contained in Table 1.

Variable Costs: Total variable costs were defined as those expenditures directly controllable by the firm during the firm's fiscal year. Total variable costs included outlays for labor, repair and maintenance, utilities, fuel and oil, advertising, and miscellaneous operating expenses. Bad debt loss, depreciation, and interest expense were not included in total

Table 1. Descriptive Statistics.

Variable	Sample Mean	Std. Dev.	Minimum Value	Maximum Value	Geometric Mean
VC-Variable Cost ($/yr)	60450	17752	31469	127624	58117
Y1-Dry Fertilizer (Tn/yr)	2435	1082	550	6147	2223
Y2-Fluid Fertilizer (Tn/yr)	1066	514	112	2957	941
Y3-Anhydrous Ammonia (Tn/yr)	464	361	0	1788	127
Y4-Chemicals ($/yr)	140087	81197	24811	570754	122300
Y5-Services (Ac/yr)	15134	8171	3811	47286	13319
Y6-Other Farm Supplies (Ac/yr)	2620	1676	39	8770	2007
W1-Labor Price ($/wk)	98.40	3.65	90.91	102.00	98.33
W2-Energy Price (¢/btu)	.46	.07	.34	.58	.46
F1-Management ($)	16702	8275	7951	53262	15159
F2-Other Fixed Inputs ($)	17942	7564	5077	43783	16422
F3-Plant & Equipment ($)	34108	17268	8556	97002	30301
S1-Labor Share (% of Variable Cost)	63.99	5.82	45.40	77.10	63.72
S2-Energy Share (% of Variable Cost)	11.01	2.31	5.88	16.59	10.75
S3-Other Variable Inputs Share (% of Variable Cost)	25.00	6.42	11.78	46.52	24.19

variable cost. These expenses were either not relevant to the analysis or, due to accrual accounting procedures, were impossible to associate with the appropriate output figures.

Outputs: Output levels for dry fertilizer, fluid fertilizer, and anhydrous ammonia were measured in tons per year. A measure of chemical output was constructed by dividing chemical revenue in dollars per year by a chemical price index. The measure of service quantity (acres per year) was calculated by dividing total service revenue by a weighted average price for custom application.[3] The output measure for the other farm supply category -- typically more than 90 percent hybrid seed corn and seed soybean sales -- was constructed by dividing other farm supply sales by a weighted average seed price. The weights were the proportions of total corn and soybean acreage in corn and soybeans respectively for the county in which the plant was located.

Input Prices: Input price data were not available from the individual plants. The state average weekly wage rate in trade employment was used as a proxy for the price of labor. The weighted average energy price (cents/BTU) was constructed using the state average commercial electricity rate and bulk gasoline price. The weights were the shares of total energy expenditures for utilities and fuel and oil, respectively. Given the range of cost items aggregated into the other variable input category, price movements for the category were assumed to follow the general price level of the economy. The implicit GNP price deflator was used as a proxy for the price of other variable inputs.

Fixed Inputs: A measure of the managerial input was obtained by adding total employee bonuses to the manager's base salary. The bonuses were added to the salary base as an adjustment for management quality differentials.[4]

The book value of plant and equipment as reported on the firms year-end balance sheet was divided by the producer price index for machinery and equipment to calculate the measure of plant and equipment. The residual fixed input category (other fixed inputs) included insurance, local taxes and licenses, professional services, and other fixed overhead expenses. The sum of these items was divided by the implicit GNP price deflator to calculate the level of this input.

Scaling and Zero Observations: We have chosen to scale the data around the geometric mean. This permits us to interpret (11) as an approximation to the true underlying cost function in the neighborhood of this point (Boisvert). However once (11) is estimated, this interpretation is subject to the criticism of White, who argues that traditional regression methods bias the local approximation by giving equal weight to all data points in the sample. Of course, there is also a practical reason for scaling the data. Since the natural logarithm of one is zero, calculation of scope and scale economy measures is greatly facilitated by scaling the data around the point of interest.

Another data transformation was necessitated by the fact that not all firms produced anhydrous ammonia. Since the natural logarithm of zero is not defined, zero observations must be modified in order to estimate the model. We follow Cowing and Holtman by replacing the zero values with a small positive constant (.1) before taking logarithms. Twenty-nine anhydrous ammonia observations were treated in this manner.

RESULTS

Summary: A brief summary of the research findings is in order before developing the results in detail. The remaining 5 subsections will expand on the results presented in this summary. The conclusions we draw are to be interpreted in terms of an average retail fertilizer plant. The average plant produces the geometric mean output vector using the geometric mean levels of fixed inputs while paying the geometric mean price for variable inputs. This is the point of local approximation.

Multiproduct scale economies exist for the average retail fertilizer plant. This plant will find it cost effective to increase production of all outputs simultaneously while holding the mix of outputs constant. From (10), it is clear the existence of MSE requires the presence of either scope economies or product-specific scale economies. Plugging the calculated values for EOS, PSE_{T_1}, PSE_{T_2}, and α_{T_1} into (10) permits MSE to be decomposed into its component parts:

$$(13) \quad MSE = .259 = 1 - \frac{(1 - .848)}{(.076)(1.53) + (.924)(.096)} .$$

Here PSE_{T_1} measures the product-specific scale economies associated with anhydrous ammonia and PSE_{T_2} the scale economies associated with the remaining 5 products.

Equation (13) summarizes the results for the average plant nicely. In the average plant, the cost of producing anhydrous ammonia jointly with the other 5 output categories is 85 percent lower than producing anhydrous ammonia and the remaining 5 products at two separate facilities. This implies the existence of economies of scope between anhydrous and the other outputs. A second implication of (13) is that the average plant will find it cost effective to expand production of anhydrous ammonia due to the presence of product-specific scale economies. At the geometric mean production level for anhydrous (127 tons), the average incremental cost of producing a ton of anhydrous is \$35.89 while the marginal cost at this point is \$23.32.

Since the marginal cost is less than the average incremental cost, pushing production past the geometric mean level will lower the average incremental cost of producing a ton of anhydrous. By contrast, product specific scale economies do not exist for the group of 5 remaining outputs. It is not cost effective to expand production of dry fertilizer, fluids, chemicals, services, and other farm supplies, taken as a group, while holding the output of anhydrous constant.

An additional implication of (13) is that economies of scope are primarily responsible for the existence of multiproduct scale economies. If EOS was zero, implying there were no gains to diversification into anhydrous ammonia, then MSE would equal -3.88. The strong economies of scope overcome the lack of product-specific scale economies for the group of 5 products to generate multiproduct scale economies.

The final results relate to the question of long-run equilibrium in the use of fixed inputs. Our findings indicate the average plant has overinvested in plant and equipment and could lower costs by reducing the investment in this fixed factor.

Statistical Results and Theoretical Consistency: Given the number of parameters estimated, the statistical results presented in Table 2 are quite reasonable. Twenty-three (29.5 percent) of the estimated parameters had t-statistics greater than 1.96 (.05 level) while 27 (34.6 percent) had t-statistics greater than 1.65 (.10 level). This compares to 29.1 and 29.9 percent respectively for the Cowing and Holtman study. The system R^2 was .8626 while the R^2's for the OLS equations were .9519 for the cost function, .2829 for the labor share equation, and .4059 for the energy share equation.[5] The F-test for significance of the regression rejected the hypothesis that all parameter estimates were 0 at the .05 level for each of the OLS equations and the 3-equation system.[6]

Table 2. Parameter Estimates for Cost Function.

Variable	Paramter Estimate	T Ratio
Intercept	-0.097226*	-3.86
Y1	0.310070*	6.52
Y2	0.052349*	2.38
Y3	0.056350*	4.07
Y4	0.173067*	7.21
Y5	0.134228*	3.85
Y6	0.014976	1.06
Y1SQ	1.031577*	3.40
Y2SQ	0.079077	1.08
Y3SQ	0.014840*	3.17
Y4SQ	0.040526	0.33
Y5SQ	0.441698*	2.42
Y6SQ	-0.002361	-0.14
Y1Y2	0.010202	0.09
Y1Y3	-0.013454	-0.82
Y1Y4	0.204283	1.46
Y1Y5	-0.350693**	-1.89
Y1Y6	0.024631	0.36
Y2Y3	0.015622	1.21
Y2Y4	-0.014542	-0.27
Y2Y5	-0.080537	-0.80
Y2Y6	-0.054286	-1.33
Y3Y4	-0.010974	-1.02
Y3Y5	-0.000859	-0.06
Y3Y6	-0.003711	-0.68
Y4Y5	-0.237079*	-2.21
Y4Y6	-0.020735	-0.49
Y5Y6	0.008025	0.17
W1	0.639820*	138.89
W2	0.110155*	66.13
W1SQ	-0.597023*	-3.75
W2SQ	0.032263**	1.71
W1W2	-0.043992	-1.06
F1	-0.142895*	-3.62
F2	-0.045031	-1.58
F3	0.054294*	2.68
F1SQ	-0.214898	-0.84
F2SQ	-0.235596**	-1.83
F3SQ	-0.061882	-0.83
F1F2	-0.150473	-1.09
F1F3	-0.039367	-0.49
F2F3	0.022157	0.30

Table 2. (Continued)

Variable	Paramter Estimate	T Ratio
Y1W1	-0.037799	-1.43
Y1W2	-0.020288*	-2.08
Y2W1	0.004297	0.35
Y2W2	0.000324	0.07
Y3W1	-0.000601	-0.30
Y3W2	-0.001476*	-2.05
Y4W1	-0.015131	-0.99
Y4W2	-0.007110	-1.28
Y5W1	-0.002611	-0.13
Y5W2	0.018417*	2.55
Y6W1	0.003588	0.53
Y6W2	-0.004138**	-1.68
F1W1	0.008875	0.38
F1W2	-0.000932	-0.11
F2W1	0.052624*	3.07
F2W2	0.019155*	3.00
F3W1	-0.038577*	-3.23
F3W2	0.001794	0.42
Y1F1	-0.370255	-1.59
Y1F2	0.094544	0.63
Y1F3	0.011968	0.13
Y2F1	0.247192*	2.65
Y2F2	-0.064207	-0.89
Y2F3	0.010985	0.20
Y3F1	0.050638*	3.03
Y3F2	-0.000044	-0.00
Y3F3	0.007167	0.94
Y4F1	-0.034665	-0.29
Y4F2	0.095312	1.21
Y4F3	-0.035025	-0.54
Y5F1	0.026594	0.16
Y5F2	0.173583	1.54
Y5F3	-0.004125	-0.05
Y6F1	0.018026	0.29
Y6F2	-0.004049	-0.09
Y6F3	-0.010238	-0.32

* Significant at .05 level.

** Significant at .10 level.

Regression diagnostic procedures (Belsley, et al.) were used to examine the OLS equations for influential data points.[7] After correcting data errors located with the diagnostic measures, we found that no single plant consistently generates large residuals or leverage points over the 8 year period.[8] Although some observations were considerably more influential than others, the estimated parameters were reasonably robust to the presence of these influential points.

Taking the logarithmic derivative of the translog cost function with respect to each output yields a set of 6 output elasticities. If all variables are set equal to their geometric mean, this elasticity is simply the parameter on the first-order output term, α_r. A similar procedure using the input prices generates 2 input cost shares, β_i. The output elasticities are all positive implying that an increase in the production of any output will increase total variable cost. With the exception of the residual output category, other farm supplies, all are significant at the .05 level. The labor and energy cost shares are positive and significant, implying the cost function is monotonically increasing in input prices. The cost function was also found to be concave in input prices at the geometric mean.[9] Since homogeneity in input prices and symmetry of the second-order terms were imposed during the estimation, the estimated function satisfies all properties of a theoretically valid cost function (Varian).

Economies of Scope: There is strong evidence supporting the hypothesis of economies of scope between anhydrous ammonia and the other five output categories taken as a group. Calculation of EOS requires determining the variable cost of producing anhydrous as the sole output. Since this sample contains no observations on the stand-alone cost of producing anhydrous, the range of cost over which EOS will exist is presented in Table 3.[10] If the

Table 3. Economies of Scope (EOS) for Anhydrous Ammonia--Average Plant.[a]

	Scope Economies	Variable Cost (1972 Dollars) Anhydrous Only	All Products Except Anhydrous[b]	All Products
Maximum	.91	52732.77	48174.10	52732.77
Minimum	-.09	0.00	48175.10	52732.77
Actual[b]	.85	49264.89	48174.10	52732.77
No EOS	0.0	4558.67	48174.10	52732.77

[a] Cost computed at geometric mean for every variable except the products not produced.
[b] Ten percent of the geometric mean is used as a proxy for zero output.

variable cost of producing anhydrous as a separate product does not exceed the variable cost of producing all 6 outputs (including anhydrous), then the maximum value EOS can take is .91. The minimum value EOS can take is -.09. This lower bound occurs if there are no variable costs associated with the production of ammonia. The critical level of EOS, 0, occurs when the variable cost of producing anhydrous is 8.6 percent of total variable cost. Based on our data, this implies the requirement of one full-time employee for operation of a separate anhydrous facility is sufficient to assure the presence of economies of scope. The evidence of scope economies is further strengthened by observed firm behavior. Although 5 of the plants did not produce anhydrous in 1975, by 1984 every plant was equipped with an anhydrous facility.[11]

The measure of EOS presented above is short-run in nature and does not capture the effects diversification may have on fixed factor requirements. Sufficient conditions for long-run economies of scope involve short-run economies of scope, non-inferior fixed factors, and negative values for the output-fixed input interaction terms (Cowing and Holtman).[12] This sufficient condition is not satisfied for anhydrous ammonia as both the management-anhydrous and plant and equipment-anhydrous interaction terms are positive. Of these two terms, only the management-anhydrous term is significant at the .05 level.[13] Given the stringent assumptions required to obtain this set of sufficient conditions, the evidence regarding existence of long-run economies of scope must be considered inconclusive.

Product-Specific Scale Economies: There also is strong support for the hypothesis of product-specific scale economies in the production of anhydrous ammonia. When anhydrous production is held at the geometric mean level, thereby holding marginal cost constant, the incremental cost associated with anhydrous production will determine the degree of product-specific scale economies which exist. The incremental cost ranges over which PSE will exist for the average firm are presented in Table 4. If there are no

Table 4. Product-Specific Scale Economies (PSE) for Anhydrous Ammonia--Average Plant.[a]

	Product-Specific Economies	Incremental Cost	Geometric Mean Production	Average Incremental Cost	Marginal Cost
		($)	(Tons)	($/Ton)	($)
Maximum	17.75	52732.77	127.4	413.86	23.32
Minimum	0.0	0.00	127.4	0.00	23.32
Actual[b]	1.53	4572.39	127.4	35.89	23.32
No PSE	1.0	2971.48	127.4	23.32	23.32

[a] Cost measured in 1972 dollars. Cost computed at geometric mean for every variable except the products not produced.

[b] Ten percent of the geometric mean is used as a proxy for zero output.

incremental costs involved in the production of anhydrous ammonia, PSE achieves the lower bound of 0. By contrast, if all costs were incremental anhydrous costs, the PSE measure would achieve its maximum value of 17.75. If the incremental cost of producing anhydrous is \$2971.48 (5.6 percent of total variable cost), then the average incremental cost is equal to the marginal cost and PSE equals 1. Thus 5.6 prcent of variable costs is the critical point, above which product-specific scale economies will exist for the average firm.

At the geometric mean output level of 127 tons, the PSE for anhydrous ammonia is 1.53. Thus, the average plant can lower the average incremental cost of producing ammonia by increasing the volume produced.[14] Furthermore, as long as the fixed cost of producing all outputs jointly is greater than the fixed cost of producing every output except anhydrous, the long-run PSE will be greater than the short-run PSE. The measure of PSE reported here is therefore a conservative estimate.

Multiproduct Scale Economies: The estimated measure of multiproduct scale economies (MSE) calculated at the geometric mean for all variables was .26 and significantly different from one. This indicates the presence of short run scale economies. The average firm can therefore simultaneously increase the production of all six outputs holding the mix constant and variable cost will increase less than proportionally.[15] Of course, the presence of the fixed inputs -- management, plant and equipment, and other fixed inputs -- in the short-run suggests that economies of scale will eventually be exhausted as production is pushed past the geometric mean and variable input use is increased. Since MSE is integrally related to marginal cost (footnote 5), the marginal cost equations were examined to explore this hypothesis. When every variable except the output of interest is held constant at its geometric mean, the marginal cost equations take the form:

$$(14)\quad MC_i = \frac{C(Y)}{Y_i} (\alpha_i + \alpha_{ii} \ln Y_i),$$

where $\alpha_i > 0$ if the cost function is monotonic in outputs. Positive values for α_{ii} assure that $\partial MC_i/\partial Y_i$ will be greater than zero at the geometric mean implying that the short-run average cost curves will be U-shaped. Hence, as output levels are expanded past the mean without increasing fixed input use, the multiproduct scale economies will eventually be exhausted and diseconomies of scale will result. The α_{ii} terms are positive for the 5 output categories significantly related to cost. Thus we expect the presence of fixed factors will eventually lead to diseconomies of scale as output is increased.[16]

Long-Run Equilibrium Conditions: Taking the logarithmic derivative of the translog cost function with respect to each fixed input yields a set of 3 fixed input elasticities. The signs on these elasticities are useful in analyzing long-run equilibrium conditions for the firm. Following Cowing and Holtman we note that the first order conditions for long-run cost minimization are satisfied if:

$$(15)\quad \frac{\partial C_v(Y,W,F)}{\partial Fi} = -Wi \qquad i = 1, \ldots, K$$

where Wi is the price per unit of the i^{th} fixed input. Firms substitute fixed inputs for variable inputs until the marginal reduction in variable cost is equal to the price per unit of the fixed input. If retail fertilizer plants are operating in long-run equilibrium, the sign on each fixed input elasticity must be negative. When evaluated at the geometric mean for all variables, the elasticities for management and other fixed inputs are negative while the plant and equipment elasticity is positive. Both the management and the plant and equipment terms are significant while the elasticity for the residual category, other fixed inputs, is not.[17]

Without data on fixed input prices, it is impossible to know if the average firm is using the correct level of management. In addition, since the typical plant employs a single manager, plant volumes could be larger than optimal for one manager, yet still be too low to justify a second manager. However, based on the estimated model, the conclusion for plant and equipment is unambiguous -- the average firm employs too much plant and equipment. This over-investment in plant and equipment may be explained by the length and intensity of the spring operating season (footnote 5). French notes that the optimal level of plant and equipment for a uniform season volume may not be the same as the optimal level when the season volume varies from year-to-year even though the average volume is the same. Weather conditions dictate the number of field-days over which the dealer will be able to service customers. Sufficient equipment to handle a normal season is likely to be inadequate if a wet spring causes the season to be short. Firms without adequate equipment during short seasons may suffer permanent sales losses exceeding the cost of maintaining the extra capacity. Thus it is not surprising that we observe considerable "disequilibrium" in the demand for this fixed factor.[18]

Limitations: Several aspects of the study deserve further comment. The first of these is the question raised by our conclusion that product-specific and multiproduct scale economies exist for the average firm. If firm managers are rational, why have they not increased production to exhaust these scale economies? The answer lies in market constraints or fixed input capacity limitations which prevent firms from capturing all of the gains associated with higher volume. The demand for the additional output may simply not exist. A high proportion of soybean acreage, credit constraints on farmer/customers, heavy competition, and government crop programs are among the reasons firms may not be able to exhaust scale economies in any given year. For the product-specific scale economies associated with anhydrous production, firms may be restricted in the short-run by their investment in the specialized equipment required to sell anhydrous ammonia. Such a constraint prohibits firms from marketing enough anhydrous to achieve all economies of scale.

A second area which merits further discussion is the fact that the EOS and PSE measures employed are "global" in nature (Evans and Heckman). That is, they require information on stand-alone production cost for each product and on cost when individual products are not produced. Data sets containing such global information are rarely available. In addition, the estimated function may be interpreted as a local approximation taken at a single point (the geometric mean for all variables). Hence, calculating EOS and PSE can require extending the estimated function not only far from the point of approximation, but also outside the range of observations over which information is available on cost behavior. Such an extension is questionable at best. By calculating EOS and PSE only for those output categories where observed firms do not produce the output, we avoid the extrapolation problem. In the case at hand, anhydrous ammonia was the only output which was not produced by all firms in every year. For this reason, anhydrous is the only product category for which these measures have been discussed.[19]

A final concern involves the choice of the small positive constant used as a proxy for zero, which is somewhat arbitrary. As indicated earlier, the proxy is required to replace zero output observations in order to estimate the parameters of the model. Sensitivity analysis showed the estimated parameters to be reasonably robust to the choice of the zero proxy. A proxy for zero is also required when using the estimated paramters to calculate the measures of PSE and EOS. Sensitivity to the zero proxy choice was a problem when EOS and PSE were calculated for the 5 output categories for which there were no observations available on the respective incremental production cost. The results for anhydrous ammonia were much more robust to the choice of the zero proxy. This underlines the importance of zero observations for output categories of interest.

CONCLUSION

The use of multiproduct firm theory allows investigation of cost-output relationships which have been impossible to analyze in a single product framework. The theory offers a richer model which can provide managers with more detailed information about how changes in output level and changes in output mix affect variable cost. The average retail fertilizer plant in the sample could become more efficient by increasing output levels to exhaust multiproduct economies of scale. Furthermore, plants not producing anhydrous ammonia should consider the possibility of adding this product line, and many of those already producing anhydrous should consider ways to increase their output. While market constraints may render such changes impossible, these results do serve as useful guides to managers making price and promotion decisions.

The research discussed in this paper has raised several important questions. In particular, what are the costs and benefits to firms of maintaining excess plant and equipment? How does the length and intensity of the operating season influence both short-run costs and long-run plant and equipment investment? Over what range does the estimated function adequately model cost behavior? How does this range affect the usefulness of the tool to managers? Finally, can the results reported in this paper be generalized to other retail fertilizer plants?

Certainly more work remains to be done in this area before the multiproduct cost function becomes a usable tool for managers. Theoretically valid measures of local scale and scope economies need to be developed. The issue of sensitivity to the choice of zero proxy should be pursued in more detail. Finally, a framework to present the results to managers in a readily understandable form must be developed. Although no small order, these issues appear tractable, and we are led to conclude that the multiproduct cost function is a promising tool for use in managerial decision making.

FOOTNOTES

[1] The orthogonality condition implies the Y_{Ti} output vectors have no positive components in common.

[2] The translog functional form places no a priori restrictions on substitution possibilities among the factors of production, nor does it imply the corresponding production structure is homogenous. The more restrictive, but far simpler, Cobb-Douglas functional form imposes unit elasticities of substitution between inputs as well as imposing homogeneity of the production structure. Eliminating all second-order output terms imposes homogeneity of the production structure on the translog form (Christensen and Greene). Unit elasticity of substitution among inputs is imposed on the translog form by eliminating the second-order price terms. When all second-order terms are eliminated from the translog form, the resulting expression is Cobb-Douglas. If the production structure displays any of the above characteristics, a simpler model can be used to capture cost behavior. Since all are special cases of the translog form, statistical tests can be conducted to ascertain the validity of each characteristic.

[3] The primary services marketed by plants were soil testing, fertilizer delivery, equipment rental, and custom application. Of these, custom application of dry bulk and fluid fertilizer provided nearly 90 percent of total service revenue in every year. Therefore, the per-acre prices for dry and fluid custom application were weighted by the respective proportion of total dry bulk and fluid sales to arrive at the weighted average custom application price.

[4] Although bonuses were paid to all employees, industry contacts indicated plant performance was primarily determined by the plant manager. Hence, bonuses are a measure of managerial quality.

[5] Two alternative labor price series were tested in an attempt to improve the fit of the labor share equation. These series were calculated from Bureau of the Census County Business Pattern data. Neither the county-level annual wage data from the Retail Trade series or the Wholesale Trade-Nondurable Goods (SIC 51) series improved the fit of the labor share equation.

An alternative explanation for the poor fit of the labor share equation may be found in the absence of some measure of operating season length and intensity in the model. French points out that adjustments made in the length of operation and rate of output in response to seasonal changes in farm production can influence short-run cost. Annual accounting data tends to average out these changes in operating season length thereby removing an important explanatory variable from the cost function. Typically, about 60 percent of a retail fertilizer plant's business is done in the April-June quarter (Akridge and Downey). Wet weather in this three-month period may cause a short, intense operating season. Labor expenditures will increase due to outlays for overtime compensation. A revised model with a measure of operating season length and intensity included was developed to examine this hypothesis.

Using the number of field days suitable for field work as a measure of retail dealer operating days, various measures of season length and intensity were developed and incorporated into the model. The hypothesis that the number of field days would be inversely related to cost was not supported. In most cases, cost and field days were not significantly related. Furthermore, the parameter estimates from the original model remained virtually unchanged when the season length/intensity measure was included. For these reasons, the variable was not included in the final model.

However, the economic argument for inclusion of a measure of season length/intensity in a short-run cost function is sound. The measure incorporated in this model may simply not accurately describe the dealer's operating season. Alternatively, the effect of variations in season length/intensity may be confounded with changes in output. (More information on the measures used to model season length/intensity is available from the authors on request.)

[6] Homogeneity, unit elasticity, and Cobb-Douglas restrictions were all rejected at the .05 level. The test statistics and critical values are shown in Appendix Table 1. We conclude that the translog functional form is required to adequately model cost behavior for the plants in this sample.

[7] Influential points justify careful examination when using the translog functional form. The nature of the logarithmic transformation and tendency for most data points to show little variation in a cross-section sample of fairly homogeneous firms can lead to parameter estimates entirely dependent on a small number of extreme observations.

[8] A leverage point is an observation which is significantly more influential in determining the estimated parameters than the average observation (Belsley, et al.). Leverage points may or may not be associated with large residuals.

[9] The cost function was also found to be concave in input prices at all 192 observations.

[10] EOS for the average firm is calculated holding every variable except those products not produced at the geometric mean.

[11] Since EOS, like the cost function, can be heavily influenced by extreme data points, a subset of 80 observations was constructed where every

variable for each observation was between the .05 and .95 quantiles. EOS was then calculated for each observation in this subset. When 10 percent of the geometric mean was used as a proxy for zero anhydrous output and 25 percent of the geometric mean was used to represent zero output in the other 5 product categories, all observations exhibited economies of scope with respect to anhydrous, although 21 (26.3 percent) had EOS measures greater than one. The magnitude of the EOS measure proved quite sensitive to the choice of the zero output proxy; however, the sign on EOS did not. EOS was always positive for every observation. These results support findings for the average firm.

[12]The sufficient condition for long-run economies of scope is:

$$\frac{\partial^2 C}{\partial Yn \partial Ym} < 0 \qquad Ym \neq Yn.$$

This cost complementarity condition implies that increasing the quantity of the m^{th} output produced will reduce the marginal cost of producing the n^{th} output. If we evaluate the short-run cost function at the point of tangency with the long-run cost function -- i.e., at long-run equilibrium levels for the fixed factors -- the above derivative can be expressed as:

$$\frac{\partial^2 C}{\partial Y_n \partial Y_m} = \frac{\partial^2 C_v}{\partial Y_n \partial Y_m} + \sum_{i=1}^{K} \frac{\partial^2 C_v}{\partial Y_n \partial Q^*_i} \frac{\partial Q_i^*}{\partial Y_m}.$$

Short-run economies of scope imply the first term is negative -- short-run cost complementarities are present. If fixed inputs are not inferior then $\partial Q_i^*/\partial Y_m$ is greater than zero. Hence $\partial^2 C_v/\partial Y_n \partial Q_i^*$ must be negative for the sufficient condition to be satisfied. Note the requirement of the envelope condition needed to generate this result.

[13]The manner in which the managerial input is measured may explain the positive sign on the interaction term. If economies of scope exist with respect to anhydrous, then plants diversifying into this product category will be more profitable, hence generating a larger bonus for employees. Since this bonus is added to the managers salary as a quality adjustment, then a positive management/anhydrous interaction term would be expected.

[14]PSE was calculated for the 80 observation subset where all variables for each observation were in the .05 to .95 quantile (footnote 11). When 10 percent of the geometric mean was used as a proxy for zero anhydrous output, 79 (98.8 percent) of the observations exhibited product-specific scale economies. Both the magnitude and the sign of the PSE measure proved sensitive to the choice of the zero anhydrous output proxy. When evaluated at levels less than 10 percent of the geometric mean, the average incremental cost was negative in many cases, implying a negative PSE. Since zero is a reasonable lower bound on PSE, the 10 percent figure was used. As for the average firm, given a reasonable proxy for zero output, most plants exhibit product-specific scale economies with respect to anhydrous.

[15]Marginal cost and MSE are integrally related notions. The measure for MSE defined in (9) can be expressed as:

$$MSE = 1 - \sum_{i=1}^{L} MC_i \left(\frac{Y_i}{C_v}\right)$$

where MC_i is the marginal cost of the i^{th} product. For the full set of observations, MSE was positive in 151 cases (78.6 percent). However, out of 1152 marginal cost estimates, 202 (17.5 percent) were negative. These were concentrated in the fluids and other farm supplies categories where 122 of the 202 negative marginal cost values were found. The remaining 86 were primarily for those firms not producing anhydrous. For these firms, the zero output level took the approximation far from the mean. Since negative marginal cost estimates bias the MSE measure upward, marginal cost and MSE were computed for the 80 observation subset where the extreme values had been dropped (footnote 11).

Multiproduct scale economies existed for 63 (78.8 percent) of the observations in the subset. The upward bias on MSE due to negative marginal cost estimates was offset by a downward bias on MSE due to extremely high positive marginal cost estimates. For the 80 observation subset, there were no negative marginal cost estimates for chemicals or anhydrous, while 4 were negative for dry fertilizer and 7 were negative for the service category. However, 17 of the fluid fertilizer marginal cost estimates were negative while 25 were negative for other farm supplies. The relationship between other farm supplies and cost was not significant so perverse results are not surprising in this category. The results for fluids are more puzzling. Since the fluids category encompasses several products including liquid nitrogen, regular fluid fertilizer and suspensions, aggregating these different products may be distorting the link between the output of fluids and cost. Even though the marginal cost results for fluids are somewhat implausible we conclude that most plants can lower ray average cost by expanding output within capacity and market constraints.

[16]A more rigorous approach involves checking the curvature of the cost function in the 5 outputs significantly related to cost at the geometric mean. The cost function is not convex in outputs at the geometric mean. Nor is it convex at any of the 192 observations when all 5 of the outputs significantly related to cost are considered as a group.

[17]The fixed input elasticities were evaluated for each sample observation. The management elasticity was negative in 141 cases (73.4 percent), the other fixed input elasticity was negative in 131 cases (68.2 percent), while the plant and equipment elasticity was positive in 170 cases (88.5 percent). When the extreme values were dropped and the elasticities were calculated for the 80 observation subset (footnote 11), 56 (70 percent) of the management elasticities and 48 (60 percent) of the other fixed input elasticities were negative while 79 (98.8 percent) of the plant and equipment elasticities were positive. These results support the results for the average firm.

[18]The plant and equipment elasticity remained virtually unchanged in the revised model which included the measure of season length/intensity. In addition, the term measuring the interaction between plant and equipment and season length/intensity was not significant. The revised model offered no insight into the relationship between plant and equipment levels and the length and intensity of the operating season.

[19]EOS and PSE were calculated for the remaining products to gain insight into the behavior of the cost function away from the point of approximation and outside the range over which information was available. The EOS and PSE measures, shown in Appendix Table 2, were calculated using 25 percent of the geometric mean as the proxy for zero output levels. (The measures were quite unstable when smaller values were used.) All product categories exhibit economies of scope but both dry fertilizer and services have an EOS measure greater the reasonable upper bound on the measure of 1. Chemicals and other farm supplies, along with anhydrous ammonia, exhibit product-specific economies. However, the PSE measures for dry, fluids, and services were negative, due in all cases to negative average

incremental cost. The above implausible results imply that observations where firms do not produce the full complement of products are critical if EOS and PSE are to be used in analyzing firm performance.

To further examine the sensitivity of the results, the measures for PSE, EOS, and MSE were simulated over a portion of the observed range for anhydrous ammonia. Since the fixed input complement -- along with all other variables -- was held constant at the geometric mean, we examined the behavior of the measures as anhydrous output varied from .1 to 5 times the geometric mean output level. The results of these simulations are presented in Appendix Table 3.

Over this range for anhydrous, marginal cost fall continuously. Average incremental cost is increasing at very low levels of anhydrous output and then declines from about one-half of the geometric mean through 5 times the mean. PSE is therefore increasing over this range. EOS falls slightly over the range while MSE remains unchanged. The information provided by the simulations implies the measures are stable over a reasonably wide range of anhydrous output.

APPENDIX TABLES

Appendix Table 1. Summary of Hypothesis Tests for Cost Function.

Test	Statistic/ Degrees Freedom	Calculated Value	Critical Value[a]
Homogeneity of Production Set	F(51,498)	6.32	1.38
Unit Elasticity of Substitution	F(3,498)	8.72	2.62
Cobb-Douglas Form	F(66,498)	7.56	1.34

[a] All critical values at .05 level.

Appendix Table 2. Economies of Scope (EOS) and Product-Specific Scale Economies (PSE) for All Products -- Average Plant.[a]

Product Category	Economies of Scope	Product-Specific Scale Economies
Dry	1.20	-2.43
Fluids	.78	-.07
Anhydrous Ammonia	.61	1.10
Chemicals	.69	1.05
Services	1.99	-2.01
Other Farm Supplies	.66	1.52

[a] Calculated using 25 percent of the geometric mean as a proxy for zero output. All variables held at geometric mean except the products not produced.

Appendix Table 3. Product-Specific Scale Economies (PSE), Economies of Scope (EOS), and Multiproduct Scale Economies (MSE) for Various Anhydrous Ammonia Output Levels - Average Plant.[a]

Anhydrous Output Level	Proportion of Geometric Mean	Average Incremental Cost[b]	Marginal Cost	Product-Specific Economies	Economies of Scope	Multi-Product Scale Economies
(Tons)		($/Ton)	($/Ton)			
12.70	.10	0.00	83.85	0.0	.87	.26
25.5	.20	25.96	62.54	.58	.87	.26
63.71	.50	42.69	36.80	1.16	.86	.26
127.42	1.00	35.77	23.32	1.53	.85	.26
191.13	1.50	30.57	17.63	1.73	.84	.26
254.84	2.00	26.90	14.38	1.87	.83	.26
382.26	3.00	22.05	11.02	2.05	.82	.26
509.68	4.00	18.96	8.73	2.17	.81	.26
637.10	5.00	16.77	7.41	2.26	.80	.26

[a] Cost measured in 1972 dollars. Cost computed at geometric mean for every variable except the products not produced.

[b] Ten percent of the geometric mean is used as a proxy for zero output.

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