Valuing Multiple-Exercise Option Contracts: Methodology and Application to Water Markets

Michele T. Villinski
DePauw University
mvillinski@depauw.edu

Selected paper prepared for presentation at the
American Agricultural Economics Association Annual Meeting
Denver, Colorado, August 1-4, 2004

Abstract: Option contracts for water are emerging in some U.S. states as institutional and legal modifications allow water users to devise new mechanisms to increase reliability of water supply in dry years. Option contracts for water, though, are structurally distinct from financial derivatives and often entail a lengthy lifespan and the opportunity for multiple exercise. In this paper I present the framework and results of a finite-horizon, discrete-time, stochastic dynamic programming methodology for valuing multiple-exercise option contracts. I use data from short-term water markets in the Texas Lower Rio Grande to estimate parameters for two different price processes: mean reversion and geometric Brownian motion. Key findings of the analysis include non-zero contract values for both price processes and higher contract values under geometric Brownian motion than under mean reversion.

1. Option Contracts for Allocating Water

Option contracts for water are emerging in some states, California for instance, as institutional and legal modifications allow water users to devise innovative mechanisms to increase reliability of water supply in dry years. These option contracts for water are an exciting addition to the range of market-based mechanisms for transferring water. Clearly they are related to similar derivative instruments in financial markets. However, option contracts for water are structurally distinct from financial options and therefore require an innovative pricing approach.

Evidence from the western United States shows that option contracts for water can be even more exotic than many exotic options considered in finance. Watters (1995), for example, uses a draft of an option contract between MWD and Areias Dairy Farms (in the Central California Irrigation District) in her application of the Black-Scholes framework to water transfers. This MWD agreement differs substantially from the Dudley Ridge example: it spans fifteen years and allows for exercise in any seven of those years. The maximum amount transferred per year is 5,000 acre-feet; MWD must give notice of intent to exercise by March 15 of each year. This decision timing allows Areias to adjust agricultural production plans accordingly. The payment structure of the contract is moderately complex, with installments paid by MWD at various stages of negotiation and government approval. In effect, though, the payments amount to an option premium of $87.50 per acre-foot and a strike price of $87.50 per acre-foot. MWD is also obligated to pay conveyance costs and environmental fees. This is a highly non-standard option; there is no simple pricing formula to calculate its value. Standard European call options involve a binary decision of whether or not to exercise on the expiration date. In contrast, there are more than 6,400 ways to exercise the water district's call option seven or fewer times in fifteen years. Clearly this complicated payoff structure increases the difficulty of putting a value on the option contract.

Recent formulations of option valuation in water markets naturally mimic the Nobel Prize winning conceptual and mathematical framework developed by Black, Scholes, and Merton (see Howitt (1998) and Watters (1995)). Watters's express purpose is to use financial models to analyze water transfer contracts in Southern California. She focuses on binomial tree estimation for discrete time analysis and the Black-Scholes formulation for continuous time analysis of the Palo Verde, Dudley Ridge, and Areias Farms contracts with the MWD. She translates the detailed contract documents into relevant option pricing parameters and uses price elasticity of demand and hydrologic data to construct a volatility estimate for water returns.

Howitt (1998) focuses on the role of uncertainty as he builds on Watters's research. He pinpoints two types of risk that hamper development of robust water markets: supply uncertainty and price uncertainty. Supply uncertainty refers to the stochastic nature of the quantity of water that will be available from a certain water right at a specific future time. Price uncertainty is the unknown future value of water, given a supply quantity. Option contracts for water, Howitt asserts, can "spread the supply and price risk between the current water user and the potential buyer and thus stimulate a fuller and more efficient market" (p. 127).

The option pricing models that Waters and Howitt heavily rely on is the Black-Scholes formulation. However the assumptions of Black-Scholes option pricing formulas may not be fully justifiable when valuing options in nascent or thinly traded water markets. Furthermore, the original Black-Scholes model cannot be applied to non-standard options with complex payoff structures.

2. Multiple-Exercise Option General Framework

The fundamental innovations of my work, setting it apart from both the financial economics literature and the water option contract research, are the procedure's ability to accommodate multiple-exercise options and my examination of both geometric brownian motion and mean reversion in the price process. The multi-year, multiple-use structure that many option contracts for water exhibit demands such an extension of pricing methodology. Moreover, my multiple-exercise pricing program is not dependent on the context of water options in any way and can be used in other financial engineering applications.

This model generalizes the "seven times in fifteen years" option contract to a stylized "multiple-exercise" option. As its name implies this option can be used more than one time during the life of the option contract, generally no more than $\bar{R}$ times out of m periods, with $m \geq \bar{R} > 0$. The multiple-option contract must specify in advance the strike price for each period. To make the problem non-trivial, m must be greater than zero. The option holder knows that he may exercise the call option once per period, up to $\bar{R}$ times in m periods. The option contract also specifies the strike price, K. The holder knows the current price of the asset, S_t, but must estimate next period's price based on assumptions about the future price path. Exercising the option today yields a payoff equal to the difference between S_t and K while the payoff from exercising in later periods is uncertain. In contrast, foregoing use of the option today earns the option holder zero monetary reward in the current period but preserves the exercise right for possible use in the future. After the contract expires at the end of period m, however, any exercises of the option that remain unused are worth nothing.

In each period during the life of the contract, then, the option holder must decide whether or not to strike. Though he knows the immediate reward from striking in the current period, $(S_t - K)$, his future payoffs depend on the realized price of the asset in each future period and on his option use history. Since he cannot exceed the maximum number of exercises for his contract, striking today has an irreversible and limiting effect on his ability to secure higher payoffs later. Uncertainty, irreversibility, and the importance of timing characterize this derivative holder's decision, making it an excellent candidate for a dynamic programming solution algorithm.

2.1. Standard Dynamic Programming Formulation

The decision of whether and when to exercise a multiple-exercise option is a variation on the common problem of dynamic programming: optimizing the sum of current and expected future rewards. Translating this multiple-exercise option-pricing problem into a standard dynamic programming formulation requires one control variable and two state variables. The control variable x_t represents the decision of whether to strike ($x_t = 1$) or hold ($x_t = 0$) in the current time period. The two states are P_t, the price of water in the current period, and R_t, the number of option exercises left in the contract.

The state variables both change from period to period but their dynamics are quite distinct. The number of exercises remaining decreases whenever the option buyer strikes, evolving according to:

$$R_{t+1} = R_t - x_t. \tag{2.1}$$

The state equation that describes the price path for water is more complicated, for example, it may be an Ornstein-Uhlenbeck mean-reverting process:

$$dP = \eta\left(\bar{P} - P\right)dt + \sigma dP, \tag{2.2}$$

with η and $\bar{P}$ denoting the speed of reversion and mean-reverting price level.[1] These parameters are estimated from price data..

The objective is to maximize, over the life of the contract, the present value of the sum of expected returns from the option by making the optimal decision each period regarding whether or not to exercise,

$$\max_{x\in\{0,1\}} \sum_{t=0}^{T} \left(\frac{1}{1+r}\right)^{t} x_t\left(P_t - K\right)$$

subject to the state equations above, non-negativity of P, K, and R, and the following constraint:

$$\sum_{t=0}^{m} x_t \leq \bar{R}.$$

In any given period the reward function is a function of the two state variables, asset price, and number of uses remaining:

$$f(x_t, p_t) = x_t\left(P_t - K\right). \tag{2.3}$$

For this problem the value function $V_t(P_t, R_t)$ is the value of the option at time t if the price of water is P_t and there are R_t uses of the call remaining. The value function must

[1] The conceptual model is not limited to a mean-reverting price process. Geometric brownian motion or other stochastic processes can be incorporated.

satisfy Bellman's equation, balancing the immediate payoff from exercising the option against the present value of expected payoffs from using the option in future periods:

$$V_t(P_t, R_t) = \max_{x\in\{0,1\}} \left\{ x_t \left(P_t - K\right) + \frac{1}{1+r} E\left[V_{t+1}\left(P_{t+1}, R_{t+1}\right)\right] \right\}. \tag{2.4}$$

The boundary condition needed to solve the dynamic programming problem arises from the terms of the option contract since the value function equals zero with certainty for $t > m$:

$$V_t\left(P_t, R_t\right) = 0 \text{ for } t > m.$$

3. Parameter Estimation for The Texas Rio Grande Spot Market for Water

The dataset for my research is from the office of the Rio Grande Watermaster and represents spot market[1] exchanges of water from January 1, 2001 through June 30, 2002. The Rio Grande Watermaster Office (RGW), an arm of the Texas Natural Resource Conservation Commission (TNRCC), monitors water use and availability, coordinates water transfers, and manages reservoirs in the Rio Grande basin. The watermaster regulates water distribution, approves and implements short-term transfers, monitors water usage to ensure that users do not exceed their allocations, allocates water during shortages, and can penalize users who violate the terms of their rights.

The RGW provided a text file containing a log of all approved spot market transfers from December 22, 2000 through July 23, 2002, with information on effective date and termination date of the trade, acre-feet transferred, price per acre-foot, and type of use intended for the water. In all, 565 transactions were approved: five for mining purposes, 34 for municipal use, one for industrial processes, one unclassified, and the rest for irrigation. After discarding transactions with missing data and the handful of mining transactions, the final dataset included 312 trades.

To facilitate later analysis of model parameters, I computed a single transaction price for each of the 78 weeks in my dataset. The weekly price is equal to the weighted average of the prices for all trades that occurred Sunday through Saturday. Each transaction's price for a given week is weighted by the number of acre-feet exchanged during that trade. This resulted in 55 weeks with associated prices; 23 weeks have no price because no trade occurred in that seven-day period. The Texas dataset serves as the raw material for calculating four parameters of the multiple-exercise option valuation program: discount rate and three parameters that describe the stochastic water price process.

[1] Carlos Rubinstein confirms that his office makes no record-keeping distinction between spot and lease transfers and that prices and uses for these two types of transactions are the same (Rubinstein, 2003).

3.1. Analysis of Stochastic Water Price Process

Financial derivative pricing models based on the Black-Scholes framework typically make restrictive assumptions about how the price of the underlying asset evolves over time. My multiple-exercise option valuation methodology also requires information about the stochastic price process. However, rather than assume from the start that water prices follow a random walk, I conducted a series of tests to see whether or not water prices from Texas show this type of unpredictability. I then built the appropriate price process into my computerized implementation of the valuation algorithm. Assessing the characteristics of the stochastic process driving the price of water supplies necessary parameters to the valuation algorithm.

I used Matlab to implement Campbell *et al.'s* (1997) formulation of the variance ratio test to examine the random walk hypothesis on the Texas dataset. To make the data usable, however, I had to address nonsynchronous trading in the dataset since non-trading effects "increase the variance of observed individual security returns that have non-zero means" (Campbell *et al.*, 1997; p. 129). See Villinski (2003) for details of my approach to using Monte Carlo methods to simulate the 23 missing observations in the dataset. This overwhelmingly rejected RW1; the standard assumption of geometric brownian motion in the underlying asset price does not fit weekly spot market water data from Texas.

4. Valuation Results: Mean Reversion

Using the Texas dataset to compute the price process parameters for mean reversion and to determine the appropriate discount rate completes the set of parameter inputs for the multiple-exercise option valuation program. The base case parameters are listed in Table 4.1.

Parameter	**Description**	**Value**
$\bar{P}$	Mean-reverting price	25.969
η	Speed of reversion	1.941
r	Annual interest rate	0.0543
μ	Mean of price shock distribution	0
σ_ϵ	Standard deviation of shocks	8.5
T	Lifespan of option (years)	15
$Rmax$	Maximum number of strikes	7
k	Strike price	25
n	Discretization points for price	500
a	Minimum starting price	7
b	Maximum starting price	115

Table 4.1: Parameter Values for Base Case Analysis of Texas Data Under Mean Reversion

Parameters a and b are taken from the Texas data; T and $Rmax$ come from the MWD-Areias Farms contract. Based on these parameters and a mean-reverting price process, the option valuation program reports the contract values in Table 4.2.

Starting Price for Water	**Value in Initial Period**
\$6.94 per acre-foot	\$35.54
\$25	\$36.69
\$49.89	\$61.39
\$99.92	\$116.90

Table 4.2: Option Contract Values for Base Case Texas Data, Under Mean Reversion

As Table 4.2 shows, the value of a 7/15 option contract at its initiation depends on the price of water at that time. The mean-reverting price process causes price level to fall toward $26.00 when the starting price is high, and causes the price level to rise toward $26.00 when the starting water price is low. For a low initial price, roughly $7.00 per acre-foot for instance, the buyer expects water prices to rise toward $26.00 then level off. He will not exercise the option until mean reversion has pulled water price above $25.00. The value of the 7/15 option contract for a starting price of $6.94 is $35.54.

When the starting price of water is above the mean-reverting level, the option buyer expects the mean-reverting process to force prices to fall and to eventually hover around the mean-reverting price. In this case the buyer will strike quickly to take advantage of temporarily elevated prices and large payoff to exercising the option. When the initial price of water is near $100.00, for example, the 7/15 option contract's value is $116.90.

Using the base case parameters of Table 4.1, I compared the value of the 7/15 option with the value of option contracts with fewer than seven strikes in fifteen years. In Figure 4.1 the seven lines correspond to the value of the option contract, as assessed in the initial time period, when the option has one to seven strikes. The lowest line on the graph represents one strike available in fifteen years; the highest line shows contract values for seven uses. As expected, additional strikes add value to the option contract; notice that contract value rises as the number of exercises increases for any given initial price for water. The marginal value of additional strikes decreases rapidly then becomes almost constant. Again this is due to the mean-reverting process: in time periods after the price approaches the mean-reverting level, the undiscounted expected payoff from exercising the option is small.

The Texas data reveal that most trades with a transfer price different from $25.00 per acre-foot have prices higher than the mean-reverting level of $26.00. This suggests that options written in the Texas Rio Grande water market would most often entail initial water prices above the mean-reverting price. Thus the option contract values would likely be similar to the base case results for a starting water price of about $50.00 per acre-foot, about $61.39 for a 7/15 option.

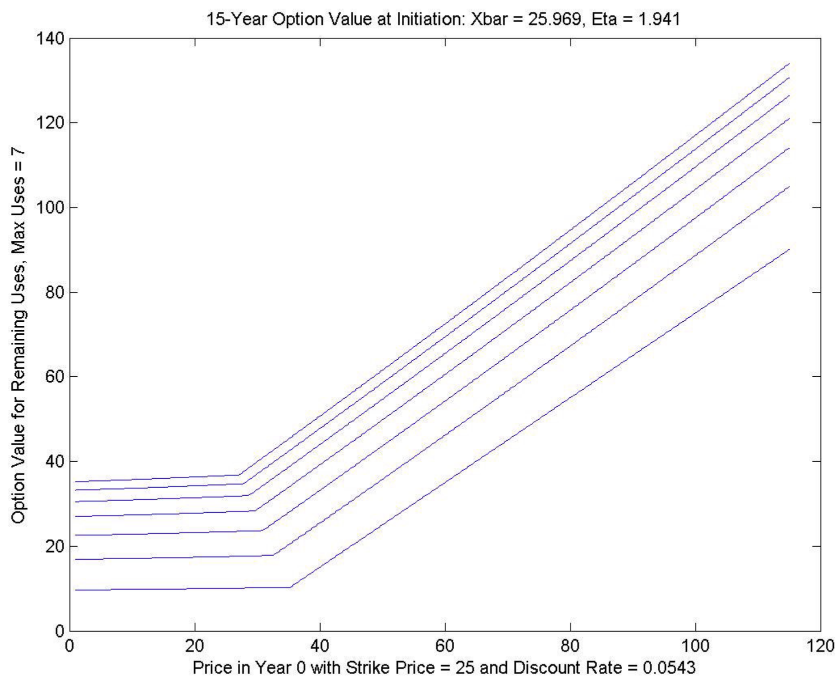

Figure 4.1: Baseline Option Contract Values Under Mean Reversion

5. Valuation Results: Geometric Brownian Motion

How important to the empirical results of this model is the assertion of mean reversion? Since most financial models and the Black-Scholes framework assume that prices evolve according to a random walk, I tested my model using the Texas data but based on an alternative price process: geometric brownian motion.

Since expected rate of return, μ, and volatility, σ, are standard parameters in financial economics, I used Hull's procedures to estimate price volatility and rate of return for the Texas time series data (Hull, 1997; p. 233). I found that the Texas spot market data exhibit annual expected rate of return of -0.01419 and annual volatility equal to 0.4418, on average.[1]

The base case I computed using geometric brownian motion and the multiple-exercise option valuation program began with the parameters in Table 5.1.

Parameter	**Description**	**Value**
σ	Annual volatility	0.4418
μ	Annual expected rate of return	-0.01419
r	Annual interest rate	0.0543
T	Lifespan of option	15
$Rmax$	Maximum number of strikes	7
k	Strike price	25
n	Discretization points for price	501
a	Minimum starting price	7
b	Maximum starting price	100

Table 5.1: Parameter Values for Base Case Analysis of Texas Data Under Geometric Brownian Motion

Note that the geometric brownian motion base case parameters are identical to the mean reversion base case parameters in Table 4.1, except for the price process

[1] I computed volatility for 27 sets of 52 consecutive weeks in my sample. These volatilities ranged from 0.5125 to 0.3571. I chose to use the average result.

parameters. Varying the smallest possible number of inputs facilitates comparisons between the results of the valuation algorithm under the two price processes.

Table 5.2 displays the value of a 7/15 multiple-exercise option contract under the base case parameters and using geometric brownian motion to drive the price of water over time. Figure **??** presents the value in period zero of option contracts with from one

Starting Price for Water	**Value in Initial Period**
\$6.94 per acre-foot	\$24.94
\$25	\$111.40
\$50.14	\$221.42
\$100	\$425.51

Table 5.2: Option Contract Values for Base Case Texas Data, Under Mean Reversion

to seven strikes over fifteen years, for each possible starting price for water. The highest curve in the graph applies to a 7/15 option; the lowest curve values a 1/15 option.

Comparing the base case results for the two price processes reveals some notable differences. For the selected water prices in Tables 4.2 and 5.2, option contracts are worth more under geometric brownian motion than under mean reversion. The exception to this is the lowest starting water price, roughly \$7.00 per acre-foot. In general, geometric brownian motion allows for the possibility of higher water prices over time than mean reversion does. This translates into higher expected payoffs to exercising the option since the price of water is unlikely to hover around \$26.00 per acre-foot under geometric brownian motion as it does under the mean-reversion base case. For very low initial water prices, expected future price rises quickly under mean reversion. Under geometric brownian motion, however, expected future prices from the lowest starting price (\$7.04 per acre-foot) do not often reach high enough levels to make striking worthwhile.

The range of contract values is much larger under geometric brownian motion than under mean reversion. A 7/15 option contract's value ranges from a minimum of \$35.54 to a maximum of \$116.90 under mean reversion, but falls between \$24.94 and \$425.51 under geometric brownian motion, for the displayed prices. Again, this is due to the difference in variability of prices inherent in the two stochastic processes. Mean reversion is more predictable than geometric brownian. Thus, we expect option prices to be lower under mean reversion to reflect the relatively low level of additional flexibility the buyer gains.

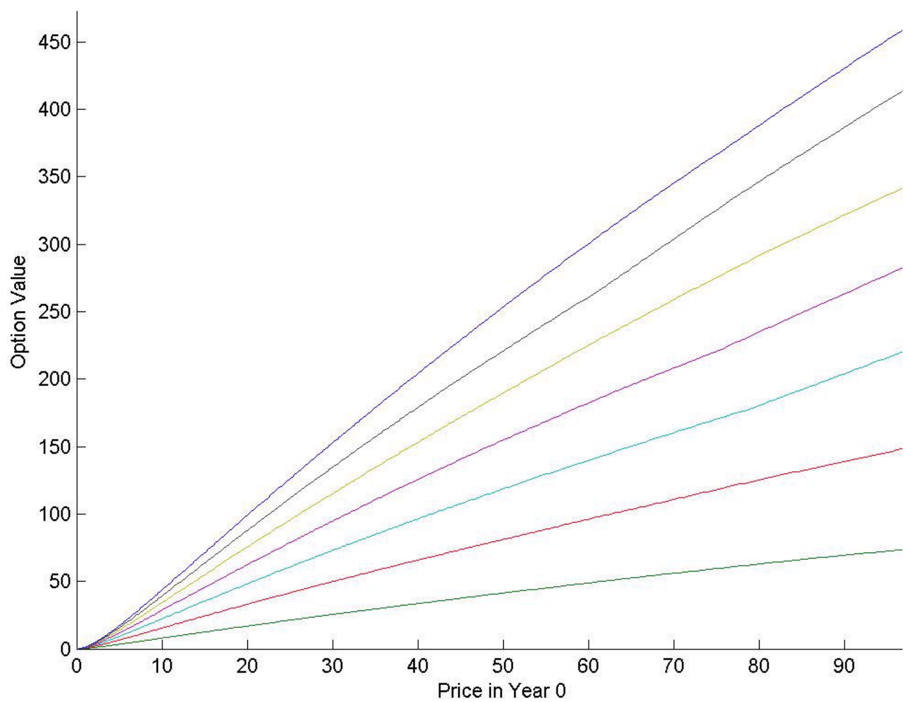

Figure 5.1: Baseline Option Contract Values Under Geometric Brownian Motion

6. Key Findings

These research results clearly show that the distinction between mean reverting water prices and water prices that exhibit geometric brownian motion has important implications for the value of option contracts in the Texas Rio Grande. The two price processes yield notably different dollar values for multiple-exercise option contracts. Sensitivity of contract values to changes in key parameters also varies depending on the underlying price process. Correctly assessing the stochastic nature of the price path for water is therefore a critical consideration for applying this valuation model to actual option transactions.

Empirical analysis of the Texas dataset casts doubt on the assertion that short-term water prices in this region follow geometric brownian motion. The variance ratio test in particular suggests that the prices in the dataset do not follow a random walk. However, limitations of the timespan of the data make it difficult to diagnose mean reversion with confidence. Despite the concern of a relatively short time series, mean reversion is indeed the more relevant model for the underlying stochastic process in the price data from the Texas Rio Grande. This assertion rests on two pieces of evidence: 1) strong and consistent rejection of RW1 through the variance ratio test, and 2) the observations of the Watermaster of the Rio Grande. Carlos Rubinstein's first-hand experience with water allocation in southwest Texas and his formal role in administering water transfers have led him to conclude that water prices tend to follow a somewhat predictable pattern, generally centering near \$25.00 acre-foot (Rubinstein, 2003).

The fact that option contracts for water have not yet emerged in the Texas Lower Rio Grande may also confirm the conclusion of mean reversion in the water price process. Recall from Table 4.2 that a 7/15 call option is worth less than \$53.00 when the price of water is \$50.00 per acre-foot or less. The low value of the contract may thus be overwhelmed by the transaction costs of individually negotiating and formalizing the contracts. The higher values for option contracts that result under geometric brownian motion would make it more likely that such contracts would be appearing as a transfer mechanism in southwest Texas.

Bibliography

[1] Campbell, J.Y., A.W. Lo, and A.C. MacKinlay. *The Econometrics of Financial Markets.* Princeton NJ: Princeton University Press, 1997.

[2] Howitt, R.E. "Spot Prices, Option Prices and Water Markets: An Analysis of Emerging Markets in California." *Markets for Water: Potential and Performance.* K.William Easter, Mark W. Rosegrant, and Ariel Dinar, eds., pp.119-40. Massachusetts: Kluwer Academic Publishers, 1998.

[3] Miranda, M.J., and P.F. Fackler. *Applied Computational Economics and Finance.* Cambridge MA: The MIT Press, 2002.

[4] Rubinstein, C. Personal e-mail correspondence, 17 March 2003.

[5] Villinski, M.T. "A Framework for Pricing Multiple-Exercise Option Contracts for Water." PhD Dissertation: University of Minnesota, 2003.

[6] Watters, P.A. "Efficient Pricing of Water Transfer Options: Non-Structural Solutions for Reliable Water Supplies." PhD Dissertation: University of California-Riverside, 1995.