Treating irregularities in carbon price and discount schedule: resolving a nightmare for forest economics?

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Abstract

Classical forest economics posits an optimal sequence of constant rotations. Projected variation of discount rate changes optimal rotation through time, as does projected relative price change for multiple products. These factors greatly increase the difficulty of calculating NPV of multi-rotation forestry projects, for traditional timber production or for multiple purposes. If, however, a given schedule of time-dated carbon prices is used, combined with set discount rates, a spreadsheet solution is practical and feasible. A given sequence of rotation lengths can be evaluated and compared with alternative sequences. Beyond when both discount rate and carbon prices are projected to stabilise, classical formulas can be used for perpetual series of constant rotations. Among the consequences of using government-mandated values are: very high value for most commercial forest crops; very long optimal rotation; favour for no-thinning regimes. Bizarre consequences include a negative carbon account for regimes which are carbon-neutral; a positive or negative value of a regime, depending on start date; a crop's being more valuable if infected with a serious disease.

Keywords: irregular cash flows, carbon price, discounting, tree disease

Introduction

Classical forest economics is based on regular sequences of cost and benefit, with prices normally constant. The necessary calculations are aided by compact formulas for cash flows repeating over a period – and, in the limit, perpetually. For example:

$$\begin{bmatrix}\text{Discounted value of}\\ \text{perpetual series of}\\ \text{rotations of } T \text{ years}\end{bmatrix} = \frac{\begin{bmatrix}\text{Discounted value}\\ \text{of first rotation}\end{bmatrix}}{1-e^{-\rho T}} \quad \text{where } \rho = \text{discount rate}$$

Under these conditions, the optimal sequence of activities comprises repeated rotations of constant length, often called the Faustmann rotation.

How readily we forest economists take such convenient formulas and such conventional assumptions for granted! How they pervade the pages of our journals! But how easily they are invalidated by governmental edicts about appraisal procedures! This paper outlines a spreadsheet approach to evaluating a sequence of rotations when the key values of discount rate and price of carbon transactions (fluxes) change irregularly through time. It shows how the changes can be encompassed, for forest crops grown over as many rotations as may reasonably be desired. It notes some consequences for forestry of the UK government's approach; then examines some bizarre results that arise from using such irregular values, particularly the possibility that the carbon account of forestry may be negative, even though the tree crop returns no more carbon to the atmosphere than it fixes initially. Finally, some applications to appraisal of a recently-serious disease of pines are given.

Declining discount rates: sophistication or frustration?

The UK government and that of France have recently advised that discounting in public project appraisal should use a schedule of declining rates (Treasury, undated; Lebègue et al., 2005). The UK's prescribed annual rate declines from 3.5% for the first 30 years, 3% for 30-75 years, 2.5% for 75-120 years, 2% for 120-200 years, 1.5% for 200-300 years, and 1% thereafter.

For classical forest economics, application of these rates is highly disruptive.

- It invalidates equations into which a single discount rate can be substituted over perpetual time.
- As the rate varies discretely and irregularly, equations in continuous time are differentiable and integrable only across limited periods.
- This prevents one-step analytical approaches to long-period optimisation.
- Numerical time series summations cannot be applied if the series crosses a step between discount rates: tedious, year-by-year calculations are indicated instead.
- The summation formula for a perpetual series of rotations is inapplicable.

Is there a forest economist on earth whose past work would not be compromised by these restrictions?

The most-addressed problem in theoretical forest economics, that of the unique optimal rotation (see Newman, 2002), no longer has a solution. Instead, as discount rate declines, so optimal rotation lengthens. Nor is the optimal schedule of lengthening rotations susceptible to either algebraic or simple numerical solution. Instead, a cumbersome forward-recursive simulation solution is required, and it is uncertain whether this solution will be consistently maintained as time moves forwards (Price 2011). A similar result arises when the relative prices of products change (Price, 2012a).

Moreover, even a stipulated silvicultural regime cannot be evaluated by short-cut formulas for recurring cash flows – particularly if that specified regime is to be repeated in perpetuity. Instead, onerous year-by-year calculations are required even for a single rotation, and the value of each subsequent rotation can only be found by repeating these calculations.

Climate change: a new analytical nightmare

The irregular discount profile creates a nightmare for valuing continuous flows of cost or benefit. Particularly this is so for the social cost of climate change, as would be imposed on the world economy by flux of CO_2 into the atmosphere (Clarkson and Deyes, 2002; Stern, 2006). In deriving this cost, several exponential processes interact (Price, 2012b). Despite these processes' complexity, the continuously integrable form of the underlying equations allows compact results: for example, the capitalised value of an individual carbon flux could be represented as an equation embodying several exponents, variously combined.

$$P_0 = C_0 \times \left(\frac{1}{\rho} - \frac{1}{\theta + \rho} \right) \times \left(1 - \sum_{b=1}^{b=5} p_b \times \left(\frac{\mu_b}{\mu_b + \rho} \right) \right)$$

where

P_0 = carbon price at time 0

C_0 = economic damage under present conditions caused by one additional tonne of carbon in the atmosphere

ρ = carbon discount rate (normal rate, minus rate of carbon price rise)

θ = oceanic thermal adjustment coefficient

p_b = proportion of atmospheric CO_2 "allocated" to carbon sink b
μ_b = uptake coefficient from the atmosphere into sink b

Although more elaborate than the naïve exponentials of decline promulgated by Nordhaus (1992) and criticised in Price (1995), this is still a compact formulation for a base carbon price. A price so derived was often projected to increase exponentially through time: particularly, with size of the world economy, which was implicitly deemed to be affected proportionately by temperature (Cline, 1992; Fankhauser, 1995). This rate of increase can be embedded in the carbon discount rate. Single carbon-relevant events, such as timber harvest, can be valued by one equation, in which base carbon price, the carbon discount rate and timber decay rates are arguments.

With an irregular discount profile, however, deriving a price for a carbon flux in any year is daunting: each ensuing cost to the world economy must be assessed and discounted year-wise and summed, onwards to the time (300 years in future for the UK) when the discount rate stabilises. Price (2012b) details the problems of encompassing all the chains of consequence, for each carbon flux. For each year's flux, the lagged consequences for the world economy, via uptake of CO_2 into sinks and thermal inertia, must be projected for each individual future year, and discounted with the mix of discount rates appropriate to that year. The irregular discount steps prevent these consequences being combined into a single integral.

Irregular carbon price schedule: bane or blessing?

The UK government has recently added a new dimension to the problem. The social cost of carbon approach has been supplanted by one where a price schedule, varying irregularly, is set by the UK Department of Energy and Climate Change (DECC) (2009). The prices are shadow prices, which if used pervasively would meet year-wise targets for reduced CO_2 emissions (DECC, 2013). Prices so derived relate to politically negotiated targets as well as to characteristics of climate systems and economies. They may rise, fall, or stabilise through time. Separate price series are defined for two sectors, "traded" and "non-traded". The components of each sector seem arbitrarily derived, with forests themselves in the non-traded sector, but certain wood-using industries, particularly biomass burning, in the traded sector (Valatin and Price, 2014). And yet, from a global perspective, the flux of a tonne of CO_2 into or out of the atmosphere affects climate equally, whichever sector the transaction takes place with. By 2030, the series converge, but they continue to rise irregularly thereafter.

One may have a dissenting view on the validity of declining discount rates (Price, 2004, 2005); of distinguishing traded and non-traded prices; and on whether the previously-used social cost of carbon was actually a more rational basis for carbon price. However, appraisals made under contract to the UK government must now use the prescribed values, for both discount and carbon price schedules, even if the results are unwanted ones.

This irregularity of schedules seems further to obstruct any analytical or straightforward numerical appraisal of forestry options. Such is the multiplicity of numerical calculations needed, that only computer approaches are feasible. The spreadsheet approach described below builds on understandings developed during an earlier numerical solution of carbon flux problems in forestry (Price and Willis, 1993).

The structural solution

The initial need is to align time scales for the diverse data. The Treasury discount schedule starts from "the present". DECC carbon prices are dated in historical time, from AD2008 onwards. A forest crop's formation may start at any future time, or (importantly for analysis of crop disease) may already have occurred, so the baseline time is the current crop's age. All baselines must be made "contemporary": crop age, carbon price and discount rate all must be those in force at the project start. Then, from each baseline, data from relevant files are rolled forwards annually.

Cash flows are discounted by factors compiled from Treasury discount rates for the appropriate period: for example for the first 80 years the factor is

$$\frac{1}{(1+3.5\%)^{30}}\times\frac{1}{(1+3\%)^{45}}\times\frac{1}{(1+2.5\%)^{5}}=0.083271$$

Discrete-period discounting format is used, to distinguish this declining discount rate process from the continuous form common in theoretical forest economics: it accords with practice promulgated by UK Treasury (undated).

Irregular discounting and carbon prices preclude the earlier model's compact equations. Instead, annual values for forest carbon increment are credited. At each harvest (thinning or final felling) material is allocated by formula, according to size of tree, among three further carbon pools: large roundwood (LRW), small roundwood (SRW) and non-timber biomass (branches, stump, roots). (A greater variety of products could readily be encompassed in the same structure.) A facility exists for a specified proportion of each product to displace high-embodied-carbon materials, and thus permanently to reduce atmospheric CO_2. (See Price and Willis (2011) for more details of this.)

Carbon is accumulated into the three product pools at each harvest, and carbon volatilises (decays or burns) from each pool at the pool's assigned rate, irrespective of the time when the original addition was made to the pool. This is mathematically equivalent to applying the same decay rate individually to each time-signed addition to the pool. For greater accuracy, carbon decay should be allocated to the mid-point of the period from $t-1$ to t, or continuous discounted carbon accretion and decay could be integrated across carbon stock during the period. (This is possible, because the same carbon price and discount rate prevail *within* the year.) Integration is thus done first across *all sources* of a pool for *a single time period's flux*, rather than across *all time periods* for *a single source*. This yields a net change of sequestered carbon, summed across pools, for each year, allowing compact application of the appropriate carbon price and discount factor for the year.

Variation of discount rates also precludes a multiplier from the NPV of one rotation to that of a perpetual series. Therefore the defined rotation, with its cash flows and carbon fluxes, is repeated in successive sequences of spreadsheet rows, for as many rotations as might be of significance. The usual result obtains, that any effects after 500 years are completely trivial, even with the very low (1%) discount rate prevailing after 300 years.

The result is shown in figure 1: the accumulated discounted value of carbon fluxes fluctuates at first, with growth then decay of crops, but stabilises within four or five rotations, long before the 500-year time horizon adopted arbitrarily for the spreadsheet.

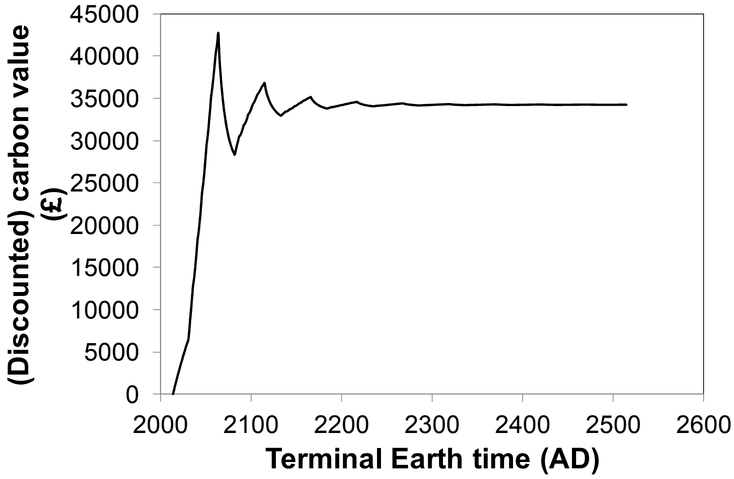

Figure 1. Accumulated value of discounted carbon fluxes, thinned Corsican pine yield class (YC) 14, 50-year rotation

Prolonging the sequence of rotations in perpetuity is technically feasible. Beyond the time when both discount rate and carbon price are projected to stabilise (or no reason is adduced for them to change further), the classical formula can convert the value of the first subsequent rotation to that of a perpetual series of rotations of constant length. That summary value is then discounted by the appropriate factor to the present.

The procedure could use, instead of DECC values, a schedule of prices based on social cost of carbon. These prices would be compiled via a separate line-by-line calculation for each future year, with the appropriate discount profile for the economic effect, in that year, of a particular temperature change. This embodies two-stage discounting:

- discounting, at period-appropriate rates, the far-future social costs of atmospheric CO_2 flux, back to specified nearer-future dates, so deriving a time-dated carbon price, then
- discounting the resulting price back further to the appraisal's time zero.

The procedure does not *optimise* (though its structure facilitates manual iteration, or use of the spreadsheet's iteration ability). Earlier simulation models, which have no carbon element, could be adapted to yield a recursive solution. But the procedure described is intended to value a *given* silvicultural regime, as often required by public, corporate and private forestry agents.

Resulting values and indications

DECC prices are very high, compared with those previously recommended, rising to equivalence of around £1000 per tonne of carbon (*not* CO_2) by 2100. Predictably, the carbon account for most crops modelled shows remarkably high values: the rest of this paper shows typical results. For high productivity crops values may reach £100,000 per ha. Even low productivity crops on poor sites (e.g. lodgepole pine of YC4 (4 m^3 increment per ha per year)) achieve social profitability. Carbon values overwhelm those of timber production, and heavily outweigh crop formation costs.

There are major consequences for rotation. The values plotted in figure 2 show no optimum, though a maximum NPV finally exists at about 200 years. However, if timber prices increase by a factor of three (as might be expected, in absence of felling of commercial crops), an optimum occurs at about 105 years. Also, if 25% of harvested carbon displaces high-embodied-carbon products, earlier harvesting again becomes carbon-advantageous: the optimum is about 115 years. By contrast with all these, the optimal rotation without carbon values (but using Treasury discount rates) is 55 years, with NPV only £2500/ha. However, this great prolongation of rotation is partly a function of *high* carbon prices and low discount rates, not just of *irregular* ones (see Price and Willis, 2011). For example, a carbon price of £100 and a constant 3% discount give an optimal rotation of 125 years, with the usual high NPV.

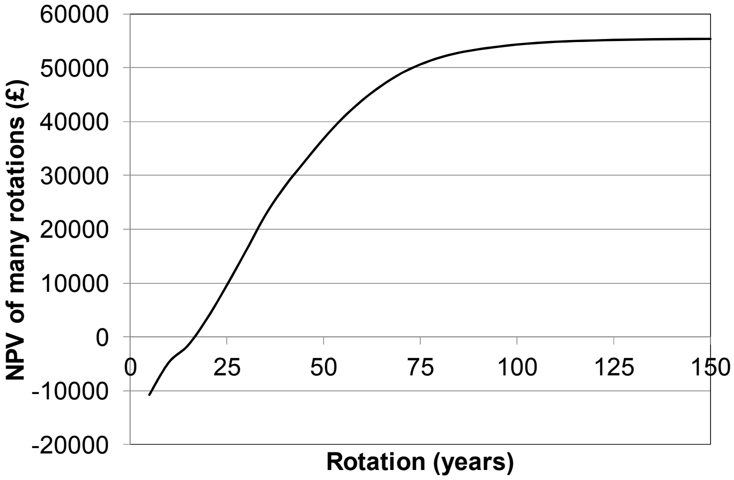

Figure 2. NPV of thinned Corsican pine YC14 on various rotations

High carbon prices have other silvicultural results. Table 1 compares values for thinned and unthinned Douglas fir YC20 on repeated 50-year rotations, showing that high-priced carbon reverses the superiority of crown thinning.

Table 1. Influence of DECC carbon prices on thinning of Douglas fir

Regime	**DCF (no carbon)**	**Discounted carbon value**	**NPV (including carbon)**
Crown thinning	£6,245	£51,355	£57,599
No thinning	£5,427	£58,586	£64,013

Bizarre outcomes from irregular changes

Some unexpected results, however, do arise simply from irregularity of carbon prices. A single thinned crop of Sitka spruce, YC24, starting from 2010, on a 30-year wind-constrained rotation has a discounted carbon account of –£1,357 (undiscounted carbon account of –£109,320). However, if replacement crops are included, the carbon account becomes positive.

More bizarrely, a single short (15-year) rotation of black poplar has a negative carbon account if planted in 2014 (£–431), or 2019 (–£1016), but a positive one if planted in 2009 (£546), or 2024 (£38), or 2029 (£2200). However, if a crop planted in 2014 is replaced at the end of succeeding rotations, the carbon account is positive (£17,472). From this it may be deduced that afforestation should start with the "second rotation", in 2029. Contrary to received wisdom, rising carbon price makes replacement crops more important than the first.

Similarly, the sign of carbon value is susceptible to the discount schedule: a single 50-year rotation of Corsican pine YC14 has a negative carbon account (£–23,346) at a constant 1% rate … but a positive one (£17,056) using the Treasury discount schedule. This result may seem counter-intuitive – very low discount rates surely favour environmentally friendly carbon sequestration? It is nonetheless logical, because the 1% discount rate emphasises the higher DECC prices prevailing during the period of decay.

Because of the higher price for carbon fixing which prevails later, the second rotation has a positive carbon account (£76,985) with a 1% discount rate. Once again, it seems optimal to omit the first rotation. The second and all subsequent rotations have an aggregate positive carbon account (£154,016).

These bizarre, often unstable, results and many others, all arise from the irregular profile of discounted carbon price.

Bizarre effects of disease

Some practical results are now summarised in the context of a government-funded project to examine the economic consequences of a disease, *Dothistroma pini*, which presently causes serious damage to pines in the UK.

Take a 25-year-old unthinned crop of lodgepole pine, YC8. Wind constrains its rotation to 50 years. Because of low productivity, at the rotation end subsequent "rotations" are of "bare land" for conservation, a not-uncommon prescription in the UK. However, the disease may kill the crop before planned felling. Figure 3 shows NPV of future cash flows and carbon fluxes, discounted to the present age, of various rotation lengths. Negative NPVs of short rotations are due to low discounted carbon prices for fixing, followed by rising discounted carbon prices for decay. Because of the still-relatively low price of decay, and the small existing volume *available to* decay, infection and felling at 38 is better than the disease-free, wind-constrained norm of 50. If the rotation *could* exceed 50 years, fixing then occurs at a much higher discounted price than the subsequent decay, and the NPV improves.

The result is more "normal" for a rotation only begun in 2014: more fixing in the early rotation is included, and decay occurs in periods of lower discounted price. NPV improves from 23 years onwards, and is positive on the wind-constrained rotation of 50 years. What has changed importantly is *not* the initial age of the crop in itself, but the profile of carbon prices during its remaining transactions with the atmosphere.

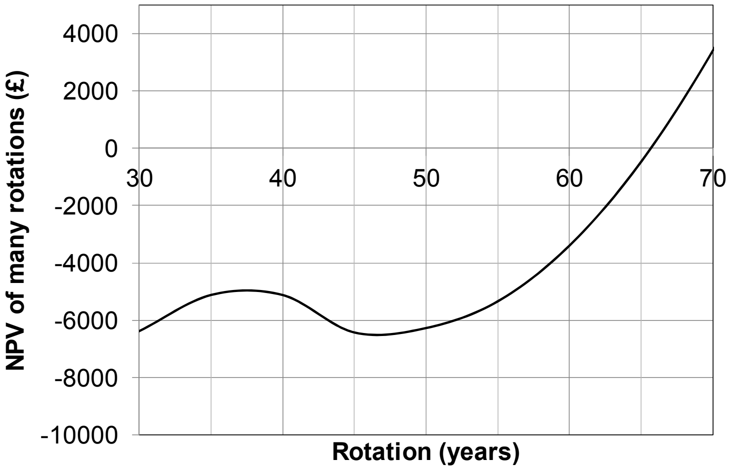

Figure 3. Lodgepole pine YC 8, age 25, DECC prices; various rotations

The bizarre fact here is that optimal feasible rotation changes according to crop age. Seen from age 25, 38 is optimal: seen from age 0, NPV is increasing rapidly at age 38. Shift of base year conventionally shifts absolute NPVs of different rotations, but not their rank order. To repeat, this difference is caused by the irregularity of carbon prices: particularly, of *discounted* carbon prices. By contrast, with a constant £100 carbon price, a 50-year rotation is superior to any shorter one, whether the starting age is 25 or 0.

For Corsican pine the disease usually slows growth, rather than killing trees. The following cases use a thinned crop of YC14, infected at age 30 with the effect of slowing growth to 40% of the previous rate. Normal rotation age is 55 years. The managerial response to disease is to delay subsequent thinning and felling, such that sizes of trees and volumes at each harvest are the same as for an uninfected crop: but the time scale of crop growth is extended.

In the first case (figure 4) the land is abandoned for silviculture after felling. The solid curve is the NPV of a "normal" crop; the dashed curve is that for an "infected" crop. The infected crop performs better, whatever its current age, because the discounted cost of carbon decay at the end of the rotation is postponed and thus reduced by the slower growth.

If planting is delayed for 50 years, however, there is a different balance of discounted carbon prices between fixing and decay, and for most of the rotation it is better if the crop is not infected (figure 5).

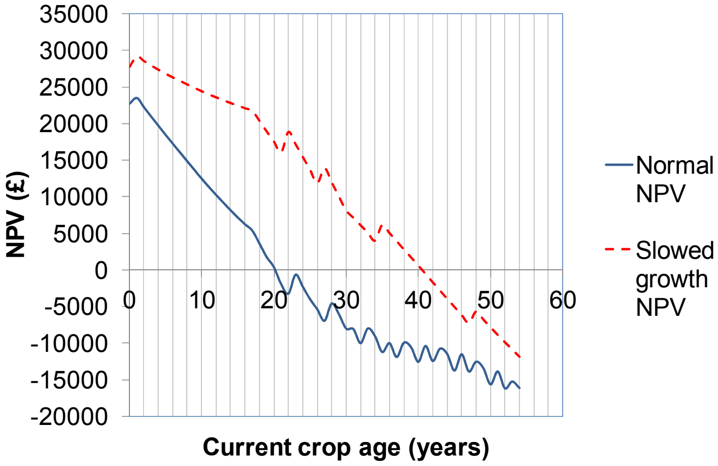

Figure 4. Corsican pine YC14, rotation 55 years, no successor crop, DECC prices; various current ages

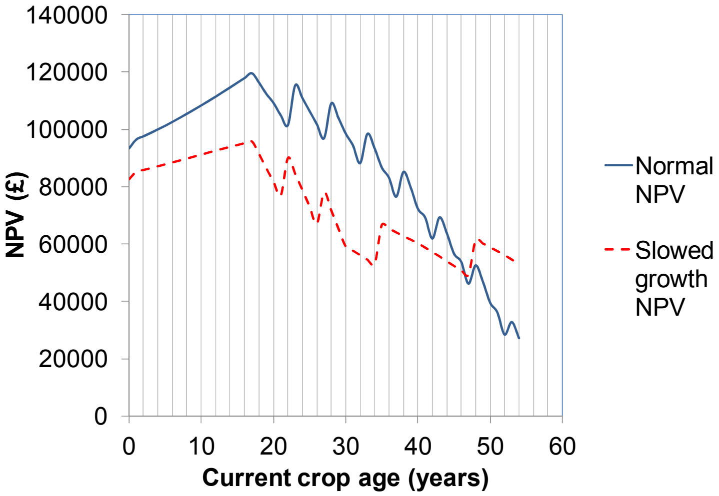

Figure 5. Corsican pine YC14, rotation 55 years, no successor crop, DECC prices, planting delayed 50 years; various current ages

Next, we revert to planting in 2014, but using Douglas fir, also YC14, for all replacement crops (this option accords with current replanting strategy). Now the delay of volatilisation is more-than-balanced by delay of fixing in the following rotations, and slowed growth *reduces* NPV (figure 6).

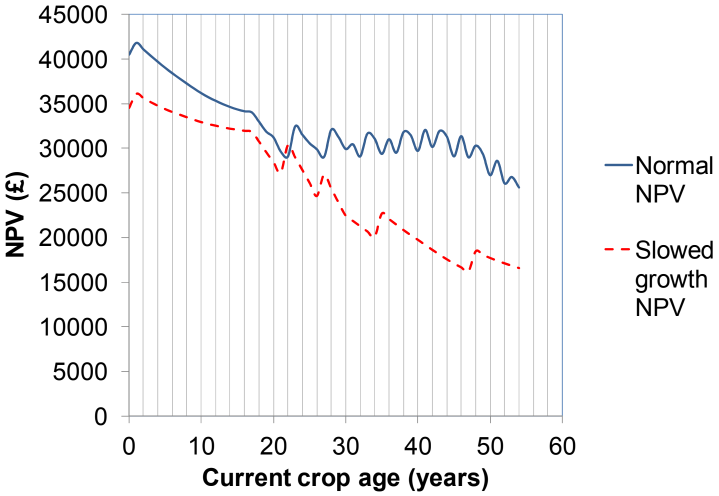

Figure 6. Corsican pine YC 14, rotation 55 years, Douglas fir successor crop, DECC prices; various current ages

That the high performance of slowed growth is due to irregular carbon prices is shown by using a uniform £100 carbon price. With silviculture discontinued after the first crop, only late in the rotation is slowed growth superior, as it still has fixing to achieve, and its decay is delayed (no figure is presented).

With Douglas fir following, slowed growth is never close to competing with normal growth (figure 7).

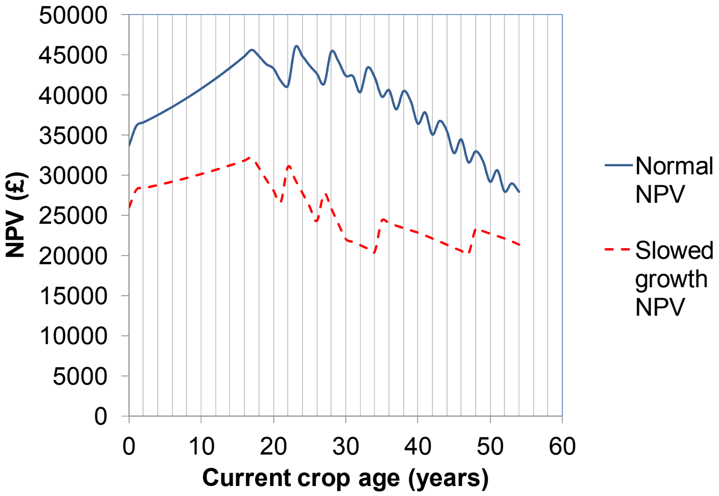

Figure 7. Corsican pine YC14, rotation 55 years, Douglas fir successor crop, £100 price; various current ages

Conclusions

Declining discount rate and irregularly changing carbon price seem to pose near-insuperable problems to economic analysis of carbon transactions, if carbon pricing is based on social cost of raised CO_2 concentrations. However, it can be done in a two-stage way, using prices compiled for each date.

The DECC carbon price schedule may lead physically carbon-neutral forest cycles to have economic carbon deficits. They may cause destructive diseases to seem beneficial. In response to these bizarre cases, the question is: does variation in carbon prices which causes the results reflect the real impact of carbon fluxes on the world economy? If so, they should be accepted, as a date-dependent phenomenon. But if they reflect only the political expediency of meeting arbitrary targets, the further question has to be asked: are there good reasons why a tonne of CO_2 in the atmosphere should have a different value, irrespective of the time when (or the economic activity by which) it is fixed or volatilised? An answer of “yes” might be based on growth of the world economy and population. This indeed could justify apparently bizarre and paradoxical results (Price, 2012a). As for the proposed variation in discount rates, is that also soundly based? Or, as I have argued (Price, 2004, 2005) is it a matter of administrative convenience and political expediency?

If these irregularities do reflect real-world changes, we have to take them seriously, accepting the consequent difficulties of calculation, and the needed remaking of theoretical forest economics. If they are questionably based, we should ask: can we ignore government-mandated procedures, and continue in our cosy world of familiar axioms and helpful formulas? or do we use the instability of results in a challenge to the validity of the mandatory processes?

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References

Clarkson, R. and Deyes, K. 2002. Estimating the Social Cost of Carbon Emissions. Government Economic Service Working Paper 140. HM Treasury/Defra, London.

Cline, W.R. 1992. The Economics of Global Warming. Institute for International Economics, Washington.

DECC 2009. The UK Low Carbon Transition Plan: National Strategy for Climate and Energy. The Stationery Office, London.

Department of Energy and Climate Change 2013. Updated Short-term Traded Carbon Values for Policy Appraisal. The Stationery Office, London.

Fankhauser, S. 1995. Valuing Climate Change. Earthscan, London.

Lebègue, D., Baumstark, L. and Hirtzman, P. 2005. Révision du Taux d'Actualisation des Investissements Publics [Revision of the Rate of Discount for Public Investments]. Commissariat Général du Plan, Paris.

Newman, D.H. 2002. Forestry's golden rule and the development of the optimal forest rotation literature. Journal of Forest Economics 8: 5-27.

Nordhaus, W.D. 1992. An optimal transition path for controlling greenhouse gases. Science 258: 1315-19.

Price, C. 1995. Emissions, concentrations and disappearing CO_2. Resource and Energy Economics 17: 87-97.

Price, C. 2004. Hyperbole, hypocrisy and discounting that slowly fades away. Scandinavian Forest Economics 40: 343-59.

Price, C. 2005. How sustainable is discounting? In Kant, S. and Berry, R.A. (eds). Economics, Natural Resources, and Sustainability: Economics of Sustainable Forest Management. Kluwer, Dordrecht, pp.106-35.

Price, C. 2011. Optimal rotation with declining discount rate. Journal of Forest Economics 17, 307-18.

Price, C. 2012a. Optimal rotation with differently-discounted benefit streams. Paper presented at the biennial meeting of the Scandinavian Society of Forest Economics, Hyytiälä.

Price, C. 2012b. A nightmare waiting in the wings: the social cost of carbon, forest economics and declining discount rates. Paper presented at the 4th Faustmann Conference, Saariselkä, available from the author.

Price, C. and Willis, R. 1993. Time, discounting and the valuation of forestry's carbon fluxes. Commonwealth Forestry Review 72: 265-71.

Price, C. and Willis, R. 2011. The multiple effects of carbon values on optimal rotation. Journal of Forest Economics 17: 298-306.

Stern, N. 2006 The Economics of Climate Change, Cambridge University Press. Also http://www.hm-treasury.gov.uk/stern_review_report.htm.

UK Treasury undated. The Green Book: Appraisal and Evaluation in Central Government. The Stationery Office, London.

Valatin, G. and Price, C. 2014. How cost-effective is forestry for climate change mitigation? In: Fenning, T. (ed.). Challenges and Opportunities for the World's Forests in the 21st Century. Springer, Dordrecht. pp.297-339.