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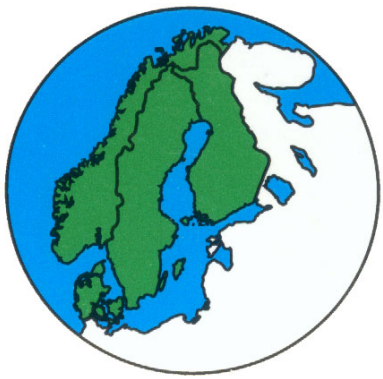

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Copenhagen

Threat, discount premiums, net present value and rotation: a real-world example

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Abstract
Similarly to financial risk in markets, physical risk (threat or hazard) has sometimes been treated by adding a premium to the discount rate used for the NPV calculations applied to forestry options. As it happens, a discount rate premium reflecting the rate of threat gives the correct rotation and, by simple adjustment, the correct land expectation value, but only if the threat occurs at a constant rate throughout the rotation and if destruction – if it happens at all – is complete. If some value can be salvaged following the destructive event, neither optimal rotation nor land expectation value is correctly determined by using a threat premium. The same is true when the threat level changes during the rotation, even if the threat rate used would correctly predict the probability of crop survival to the age of optimal rotation. These findings may be illustrated with calculations based on wind-susceptible crops in the UK.

Keywords: Risk, threat, discount premium, optimal rotation

1. Introduction

In financial markets, risk interpreted as variability of annual return from investment has often been treated by adding a premium to the discount rate. While this stratagem has the desired general effect of making risky investments seem less attractive, this is not a suitable way to treat risk when returns from investment are long delayed, as in forestry. Mean expected utility may be compromised by risk of this kind. But the path of mean expected utility is not well tracked by the exponential process that discount premiums reflect (Price, 1993, chapter 11).

Rather to the contrary, it has often been shown that, in the absence of risk aversion, the fluctuation of timber prices (according to various models) increases both NPV and rotation, essentially because of the greater possibilities of selecting a favourable time at which to terminate the rotation (Lohmander, 1987). These results are the opposite of the effects of a discount premium.

This paper does not treat such a form of risk, but rather deals with what has sometimes been called *threat* or *hazard*, to distinguish it from risk in the sense described above. Numerous papers have dealt with the climatic, biological, anthropogenic and political factors which may suddenly reduce or eliminate the value of a forest crop. Unlike the risk due to variability, the effect of threat is monotonic decline in mean expected value, compared with value in a threat-free condition. Some papers have assumed that the threat causes loss of all value, as by catastrophic fire (Routledge, 1980; Martell, 1980; Reed, 1984) or by human transgression (Price and Trivedi and Price, 1994). These and others have also examined the effects of relaxing assumptions about total loss of value under threat, and about rate of threat throughout the crop rotation. The effect of spatial interaction between stands has also been examined (Lohmander and Helles, 1987; Meilby et al., 2001), as has that of management interventions which may mitigate the threat (Thorsen and Helles, 1998), and the implications of threat for non-market values (Englin et al., 2000).

The paper does not attempt to add to this body of often sophisticated modelling work. Instead it returns to any earlier era, when it was sometimes proposed that the rate of physical threat could simply be added to the discount rate to give a risk-adjusted discount rate in NPV calculations, again, particularly for optimal rotation determination (Reed, 1984). It derives that result for those limited circumstances (catastrophic loss plus threat which is independent of crop age) in which it is appropriate, by somewhat more straightforward algebra than has been presented previously. It then demonstrates that the deficiencies of the threat premium approach increase when more realistic and complex representations of crop history are adopted. The cases are illustrated numerically by the threat from storms for a conifer crop in the UK.

2. The approach

Forest economists, when addressing the optimal rotation problem, often formulate it in marginal or incremental terms (e.g. Reed, 1984, Olschewski and Benitez, 2010): at the optimal rotation, the increase in (discounted) value of the present crop (the first derivative of the discounted revenue function) should equal the cost of delaying crop replacement (generally, the discounted net value of a perpetual series of successor crops, multiplied by the discount rate). Where threat to the crop exists, the marginal condition for optimality is:

$$V'(t) = \frac{(\lambda + \rho)(V(t) - C)}{1 - e^{-(\lambda+\rho)t}} \tag{1}$$

where $V(t)$ is an expression for crop value at age t, λ is the continuous annual rate of threat, ρ is the continuous rate of discount and C is the regeneration cost.

And so page after page in academic journals is filled with elegant and (often) general equations, sometimes represented graphically, which in practice there is no hope of solving by analytical means. I played around with these formulations as an undergraduate, but never reached a useful conclusion – and not, I suspect, merely because of limited mathematical competence. The fact of the matter is, that in order to fulfil the marginal condition for optimal rotation, one first needs to calculate and insert the discounted value of a perpetual series of successor crops *on the [yet-to-be-derived] optimal rotation*. This approach becomes particularly tiresome for stochastic circumstances. So, why not go directly to the tedious but straightforward direct calculation of the optimal rotation, by inelegant but serviceable numerical methods?

The base result, about the effect of threat on rotation, can in fact be derived without recourse to the marginal conditions, or to differential calculus, or to optimal control theory. However, the algebra is bulky (rather than complicated). Which of the available approaches is least likely to terrify forestry students, I am not able to judge.

In what follows, a numerical approach is adopted from the outset, based on non-parameterised models of windthrow threat in the UK. The optimal rotation, according to the NPV of a perpetual series of rotations subject to stochastic threat of termination and replacement, is identified by inspection of values computed for each possible rotation. This is then compared with the optimal rotation given by supplementing the "normal" discount rate by a rate related to the level of threat.

3. The case of catastrophic loss

The focus example is the crop most extensively planted in the UK, Sitka spruce (*Picea sitchensis*), which is taken to have maximum productivity of 12 m^3 ha^{-1} $year^{-1}$ and to be grown according to a non-thin regime on a site moderately susceptible to windthrow. This is typical of the conditions of commercial forestry. In all cases a schedule of "reasonable" prices for different dimensions of timber, a low planting cost and a base discount rate of 3% are used. Profitability is represented as net present value (NPV) in £ per ha. Annual management costs, which are invariant between options, are not included.

Catastrophic windthrow is assumed: if it happens, there is no salvage value. Discounted mean expected value of final revenue, V_T, at age T, less planting cost, C, is given as

$$\mathrm{MEV} = -C + V_T \times e^{-\lambda T} \times e^{-\rho T} \qquad (2)$$

where λ is the annual probability of destruction. Figure 1 shows NPV, calculated by adding a risk premium of λ to the discount rate, and the mean expected value (MEV) based on the probability of survival until T.

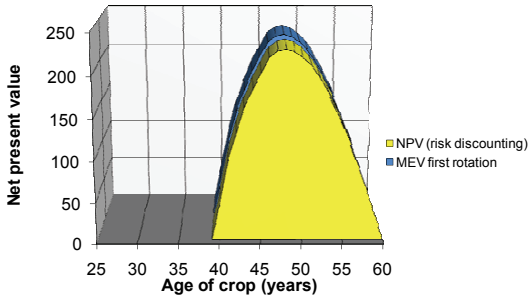

Figure 1: Risk premium and explicit threat

Unsurprisingly, the explicitly calculated MEV (£227) is identical to the NPV that uses the rate of threat as a discount premium (which simply aggregates the two exponential terms in (2)), and the optimum is 48 years in both cases.

4. Planned and unplanned replacement

A threat discount, applied universally, not only affects relative values *within* a crop rotation, but relative values *between* one rotation and the next. Figure 2 shows the effect of treating risk by adding a premium to the discount rate. By contrast, "explicit threat probability" recognises that, in the second rotation, the crop experiences the same cycle of threat as in the first rotation: discounting between second and first rotation is due only to time preference, or whatever is deemed to underlie the normal discount rate.

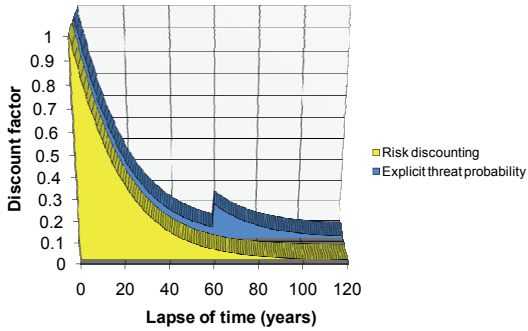

Figure 2: Discount factors using a threat premium and using explicitly calculated threat

It is clear that adding a premium to the discount rate does not capture the real nature of value variation.

Consider now an indefinite sequence of rotations, in which a replacement crop is planted at a planned time, T, irrespective of whether the crop survives to that planned rotation, or whether it is destroyed earlier. This provision might result from a very rigid management regime, where the paramount need is seen as being to maintain the age class structure of each working circle. In calculating the mean expected value, MEV_T, the multiplier, $1/(1-e^{-\rho T})$ for subsequent rotations uses a 3% discount for the lapse of time between successive rotations: the threat discount applies only over the duration of each rotation. This yields equation (3).

$$\mathrm{MEV}_T = \frac{-C + V_T \times e^{-(\lambda+\rho)T}}{1 - e^{\rho T}} \tag{3}$$

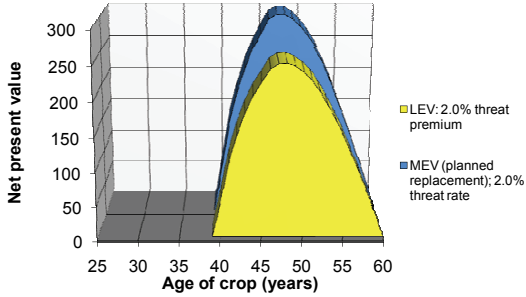

Figure 3: LEV with threat premium, and explicit MEV with planned replacement

The optimal rotation for planned replacement is now slightly shorter (47 years) than under the threat premium (48 years). This is because planned replacement entails "waste of growing space" in the event that windthrow occurs before planned replacement, and shortening the rotation reduces the mean expected waste. The difference in MEV is significant, £298.3 for explicit probability, and £250 for threat premium. Even if both are calculated for a 48-year rotation, "explicit probability" still gives £298.1: the difference from the threat premium value is due to the much reduced discounting of successor rotations' value.

However, if catastrophe destroys the crop prematurely, and if management plans are adaptive, a replacement crop may be planted *earlier than expected*, and this will enhance the regime's profitability.

In figure 4, MEV_T is calculated by explicitly considering the probabilities of the crop's surviving to various ages, and of its being replaced immediately if it does *not* survive, following the protocol of Madsen (1984, 1985), as also described in Price (1989).

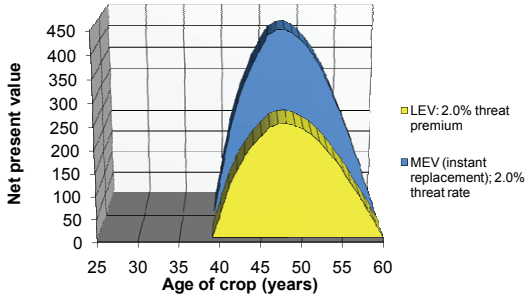

Figure 4: LEV with risk premium and explicit MEV with instant replacement

MEV is now further increased, to £417 (while LEV remains at £250), because of the value of immediately initiating successors for destroyed crops. However, the optimal rotations are now identical, at 48 years.

As well as by the numerical method used above, the identity of risk premium and explicit probability rotations has been noted by Reed (1984), working via marginal conditions – see his equations (2) and (15). What Reed does not explicitly note is the large and readily calculable margin by which MEV exceeds LEV. The following is an algebraic *demonstration* of the identity of rotation, and *derivation* of the margin of difference in value.

Land expectation value (L), for rotation length T, is given as:

$$L = \frac{-C + V_T \times e^{-\rho T}}{1 - e^{-\rho T}} \tag{4}$$

Treating the annual probability of catastrophic destruction (λ) as a discount premium gives

$$L = \frac{-C + V_T \times e^{-(\lambda+\rho)T}}{1 - e^{-(\lambda+\rho)T}} \tag{5}$$

The mean expected value, M, of an indefinite sequence of rotations is the mean expected value of the first rotation (2), plus the mean expected value of all possible replacement sequences, initiated either following catastrophe, or at the planned end of the rotation if there is no catastrophe.

The mean expected value of replacement at the end of the planned rotation is the value of an indefinite sequence of successor rotations, M, multiplied by the probability of surviving to the planned rotation end, discounted over the planned rotation period:

$$M \times e^{-\lambda T} \times e^{-\rho T} = M \times e^{-(\lambda+\rho)T} \tag{6}$$

The probability density of catastrophic destruction at age t is

$$\lambda \times e^{-\lambda t} \tag{7}$$

The value summed across the rotation period (t = 0 to T), of mean expected values of an indefinite sequence of replacement rotations following catastrophe, multiplied by the probability of catastrophe at every t, discounted for t years (i.e. to the beginning of the first rotation) is:

$$M \times \int_{t=0}^{t=T} \lambda \times e^{-\lambda t} \times e^{-\rho t} dt = M \times \pi \times \frac{e^{-(\lambda+\rho)T} - 1}{-(\lambda+\rho)} \tag{8}$$

Combining (2), (6) and (8),

$$M = -C + V_T \times e^{-(\lambda+\rho)T} + M \times e^{-(\lambda+\rho)T} + M \times \pi \times \frac{e^{-(\lambda+\rho)T} - 1}{-(\lambda+\rho)} \tag{9}$$

Collecting terms in M,

$$M \times \left(1 - e^{-(\lambda+\rho)T} - \frac{\lambda}{(\lambda+\rho)} \times \left(1 - e^{-(\lambda+\rho)T}\right)\right) = -C + V_T \times e^{-(\lambda+\rho)T}$$

Multiplying out the bracketed term in the LHS,

$$M \times \left(1 - \frac{\lambda}{(\lambda+\rho)} + \frac{\lambda}{(\lambda+\rho)} \times e^{-(\lambda+\rho)T} - e^{-(\lambda+\rho)T}\right) = -C + V_T \times e^{-(\lambda+\rho)T}$$

factorising,

$$M \times \left(1 - \frac{\lambda}{(\lambda+\rho)}\right) \times (1 - e^{-(\lambda+\rho)T}) = -C + V_T \times e^{-(\lambda+\rho)T}$$

and simplifying,

$$M \times \frac{\rho}{(\lambda+\rho)} \times (1 - e^{-(\lambda+\rho)T}) = -C + V_T \times e^{-(\lambda+\rho)T}$$

whence

$$M = \frac{-C + V_T \times e^{-(\lambda+\rho)T}}{\frac{\rho}{(\lambda+\rho)} \times (1 - e^{-(\lambda+\rho)T})} \tag{10}$$

Dividing (4) by (10), $\frac{L}{M} = \frac{\frac{-C + V_T \times e^{-(\lambda+\rho)T}}{1 - e^{-(\lambda+\rho)T}}}{\frac{-C + V_T \times e^{-(\lambda+\rho)T}}{\frac{\rho}{(\lambda+\rho)} \times (1 - e^{-(\lambda+\rho)T})}} = \frac{\rho}{\lambda+\rho}$

Since for each rotation the multiplier from LEV to MEV is the same constant, $(\lambda + \rho) / \rho$, the rotation maximising LEV also maximises MEV, even though LEV and MEV are themselves very different. As a short-cut to optimising, one could identify the rotation maximising LEV, then apply the conversion factor $\times (\lambda + \rho) / \rho$ to derive maximum MEV.

This same result has been derived using optimal control methods by Rubin (unpublished material).

5. Threat with salvage value

Windthrow is not usually catastrophic in the sense of destroying the entire value of the crop. Some stems are snapped in their most valuable length; there are additional costs in harvesting and replanting; markets may be depressed if windthrow happens on a regional scale. But in reality the consequent loss is less extreme than is implied by raising the discount rate by the windthrow probability. At the opposite extreme, take an event which, although it brings growth of the crop to an end, it does not affect the harvest value available at that time. The resulting MEV is shown in figure 5.

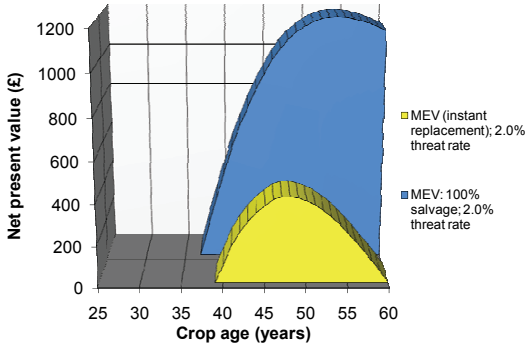

Figure 5: Mean expected value with and without salvage

Not only is NPV greatly enhanced, from £417 to £1184, by allowing salvage value: the optimal rotation is also increased, from 48 to 54 years, because now extending the rotation by a year does not risk any loss of value at all.

Next, in figure 6, riskless LEV (without any discount premium) is compared with explicit MEV when 100% salvage is possible. Although explicit MEV is much less than riskless LEV (£3014), they appear to show a similar optimal rotation. In fact, numerical inspection shows that explicit MEV reaches a maximum slightly *later* (54 years) than riskless LEV (52.5 years). This adjustment is due to the reduced opportunity cost of successors: even with 100% salvage, MEV of successors is reduced by the possibility that the rotation will be curtailed prematurely. The effect becomes somewhat more pronounced with a lower discount rate.

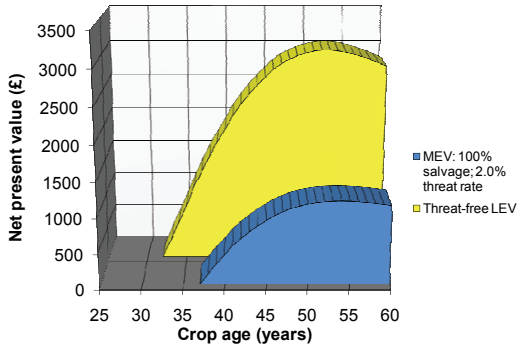

Figure 6: Mean expected value with salvage, and land expectation value without threat

The outcome with partial salvage value can now be readily predicted. For figure 7, a windthrown crop is deemed to lose a "reasonable" 25% of value, compared with its value if felled conventionally at that same age. The effect is to shorten the rotation, to 52 years, as well as to reduce NPV to £968, compared with values with 100% salvage.

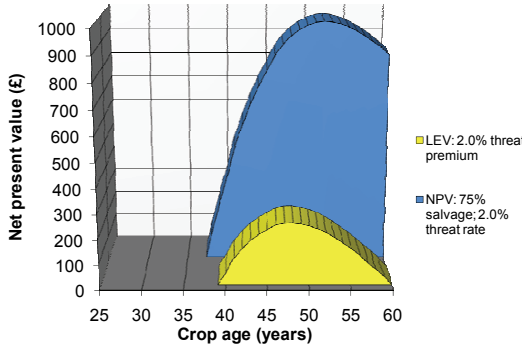

Figure 7: MEV with 75% salvage, compared with LEV with threat premium

6. Age-dependent threat

For most threats, the level of threat varies through the crop's life (Martell, 1980). For example, susceptibility to lethal insect defoliation depends on stage in crop cycle. Deer browsing damage may cause crop failure at the beginning of the economic rotation; but bark stripping, with subsequent fungal invasion and stem breakage, occurs late in the rotation.

Windthrow rarely happens early in the rotation; and then only on very susceptible sites, such as waterlogged peat soils in Scotland. More usually it is a problem in the crop's later life, when timber value would otherwise be increasing rapidly (Price, 1981). The UK Forestry Commission (Gardiner et al., 2004) has developed a predictive physical model, ForestGALES, on which the following evaluations are based.

Below are shown empirical probabilities from ForestGALES of the crop's surviving windthrow to various ages, compared with survival probabilities implied by a 2% threat premium. Clearly discounting for threat at a constant rate underestimates survival probability in the short term, but overestimates it in the long term: importantly, the changeover occurs close to the guideline economic rotation.

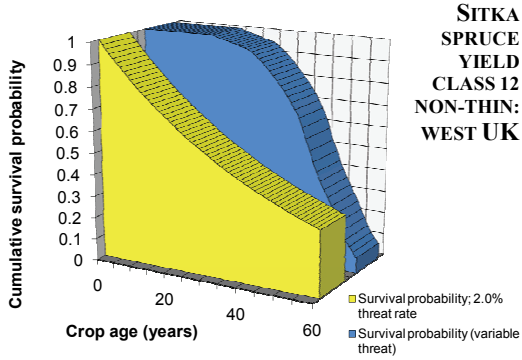

Figure 8: Survival probabilities under constant and variable threat

Calculating MEV now becomes more time consuming, because the well-behaved function with a constant rate of threat is replaced by a stepped function showing periodically increasing annual threat.

Formula (2) for the MEV of the first rotation

$$-C + V_T \times e^{-\lambda T} \times e^{-\rho T}$$

is replaced by the discontinuous formula (2a)

$$-C + V_T \times \left(\prod_{\theta=1}^{\theta=T} (1-\lambda_\theta) \right) \times e^{-\rho T} \tag{2a}$$

where the time indices $\theta = 1$ to T refer to the 1st to T^{th} years of the rotation and λ_θ to the probabilities of windthrow during these periods.

Formula (8) for the discounted value of post-destruction successor crops

$$M \times \pi \times \frac{e^{-(\lambda+\rho)t} - 1}{-(\lambda+\rho)}$$

is replaced by (8a)

$$M \times \sum_{t=1}^{t=T} \left\{ \left(\prod_{\theta=1}^{\theta=t-1} (1-\lambda_\theta) \right) \times \lambda_t \times e^{-\rho t} \right\} \tag{8a}$$

Formula (7) for the discounted value of successor crops following survival to planned rotation end

$$M \times e^{-(\lambda+\rho)T} \tag{6}$$

is replaced by (6a)

$$M \times \left(\prod_{\theta=1}^{\theta=T} (1-\lambda_\theta) \right) \times e^{-\rho T} \tag{6a}$$

Combining (2a), (6a) and (8a), we get

$$M = -C + V_T \times \left(\prod_{\theta=1}^{\theta=T} (1-\lambda_\theta) \right) \times e^{-\rho T} + M \times \sum_{t=1}^{t=T} \left(\prod_{\theta=1}^{\theta=t-1} (1-\lambda_\theta) \right) \times \lambda_t \times e^{-\rho t} + M \times \left(\prod_{\theta=1}^{\theta=T} (1-\lambda_\theta) \right) \times e^{-\rho T}$$

The derived equation (10) for M is now replaced by

$$M = \frac{-C + V_T \times \left(\prod_{\theta=1}^{\theta=T}(1-\lambda_\theta)\right) \times e^{-\rho T}}{1 - \left(\sum_{t=1}^{t=T}\left(\prod_{\theta=1}^{\theta=t-1}(1-\lambda_\theta)\right) \times \lambda_t \times e^{-\rho t} + \left(\prod_{\theta=1}^{\theta=T}(1-\lambda_\theta)\right) \times e^{-\rho T}\right)} \qquad (10a)$$

This is not sufficiently similar to equation (4) to allow general results for relative values for MEV and LEV to be derived: numerical evaluation is required, which is accomplished reasonably easily by spreadsheet.

For the sake of precision, in the spreadsheet, equation (10a) is modified further, so that crop replacement may occur any time within each individual year; and, where appropriate, salvage values are also distributed across the year. In reality, windthrow probability varies through the year, and replanting would only be done at a suitable season, so the discontinuous form, (10a) may be just as realistic. An alternative approximation is to discount values resulting from windthrow from halfway through the period, i.e. from t–0.5, and to use salvage values based on mean crop value between beginning and ending of period: see Price (1981) for a discussion.

Now that a varying empirical threat rate is being used, it is not appropriate to set a threat rate on the basis of any *a priori* conception of the value it should take. Instead, a representative threat rate is derived, that gives the same probability of survival to T as the cumulative empirical survival probability:

$$\overline{\lambda_t} = \frac{-\ln\left(\prod_{\theta=1}^{\theta=T}(1-\lambda_\theta)\right)}{T} \text{ for } t = 1 \text{ to } T \qquad (11)$$

Because $\overline{\lambda_t}$ is both a *function* of optimal rotation, and a *factor determining* it, its value is calculated iteratively. Near the optimal economic rotation, the threat premium rate so calculated is less than the *current windthrow threat*.

Figure 9 shows the result with no salvage value. Maximum MEV of £1312 occurs at 43 years, curtailed by the rapidly rising threat. Maximum LEV of £465 occurs at 48 years, by which age MEV has plummeted to little more than 50% of its maximum: using the threat premium approach could evidently have serious financial consequences. On the other hand, the relatively high empirical survival probability *before* this age makes the MEV of short rotations much greater than that indicated by the threat premium.

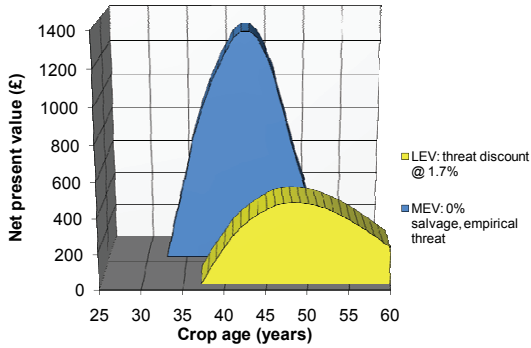

Figure 9: MEV with no salvage value compared with LEV using mean threat rate

7. Combining all factors

Previous sections have considered two separate reasons why in practice the discount premium approach may be an unsatisfactory way to deal with threats facing forest crops. In practice, these factors operate together. The case illustrated below uses empirical survival probabilities generated by ForestGALES, and a 75% value for salvage fellings.

Now, the optimal rotation is a little longer, 46 years, than that with no salvage value, but the major difference is that there are smaller penalties in extending the rotation by a few years.

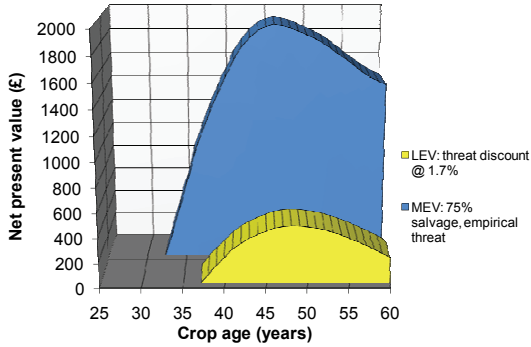

Figure 10: MEV with 75% salvage compared with LEV using mean threat rate

The resulting optimal rotation remains shorter than under the threat premium approach (48 years): the high empirical risk in the late rotation, which tends to shorten the explicit MEV rotation, outweighs the salvage value factor, which tends to prolong it. However, the value of the site for forestry, £1919, is greatly underestimated by the risk premium approach (£465), because salvage values are ignored, because successor rotations are inappropriately threat-discounted, and because no early-replacement successors are encompassed.

For the sake of completeness, the result with empirical risk and 100% salvage is shown in figure 11. The MEV is £2315, and the optimal rotation is 52 years, longer than that under LEV with a threat discount, and only slightly shorter than that under LEV without any threat discount at all.

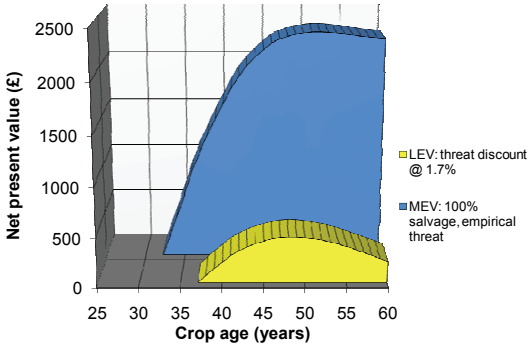

Figure 11: MEV with 100% salvage compared with LEV using mean threat rate

8. Conclusion

For forest economists, there is beguiling attraction in simple and easily manipulated formulas as ways of representing the world. They offer great possibilities of tidy solutions to classical problems of forest management. But the world of real forests is rather messy, not inhabited by well-behaved mathematical functions. This is not to say that the traditional tools of algebraic analysis have no place – far from it, they are very productive of insights that may, among other things, point their users towards the real world data on which important decisions sensitively depend. But to elegance should be added relevance, and this inevitably requires acquisition of data

which may not fall along regular functions. In the case of forest threats, the data are likely to be more-than-usually irregular.

Adding a threat premium to the discount rate only yields the correct optimal rotation in the restricted case of catastrophic loss and invariant rate of threat. Even for this case, the expedient does not produce the correct MEV, but at least the MEV can be obtained by a simple adjustment of LEV calculated with a threat premium.

When some value can be salvaged, or when risk is age-variant – which seems to be so for most European threats – the effect of threat on rotation and MEV is best modelled explicitly. Manipulation of the discount rate does not give anything like a good approximation to the maximum MEV: for threats that do not decline through time, calculated MEV is invariably higher under explicit treatment than under a threat premium that reflects mean threat up to the adopted rotation. However, according to circumstances, the MEV rotation may be either longer or shorter than the threat premium rotation.

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