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Insuring Heat Related Risks in Agriculture with Degree-Day Weather Derivatives

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Abstract

This paper presents a model and framework for pricing degree-day weather derivatives when the weather variable is a non-traded asset. Using daily weather data from 1840-1996 it is shown that a degree-day weather index exhibits stable volatility and satisfies the random walk hypothesis. The paper compares the options prices from the recommended model and compares it to a typical insurance-type model. The results show that the insurance model overprices the option value at-the-money and this may explain why the bid-ask spreads in the weather derivatives market is sometimes very large.

Keywords: weather derivatives, degree-day options, weather risk.

Insuring Heat Related Risks in Agriculture with Degree-Day Weather Derivatives

The role of weather as a source of business risk has resulted in an emerging market for weather based insurance and derivative products. Applications are wide spread among the natural gas, oil, and electricity sectors, and more and more such products are being used for agricultural and other weather sensitive industries such as ski resorts and snow mobile manufacturing. The main attraction of weather derivatives is that it insures volume rather than price. Too cool or too hot, too dry or too wet affects energy demand in utilities, production of crops and processing inventory in agriculture.

With weather being one of the most significant risks facing agricultural producers, marketers, and processor, there is an increased interest in examining ways in which specific weather events can be insured (Turvey 2001). A general form of rainfall insurance applied to agricultural risks has been presented in Turvey (2001) and Martin et al (2001) have examined rainfall insurance in terms of actuarial pricing methods.The types of contracts used to insure weather events are varied and include both swaps and options. In terms of heat-based options there are two different types. First, there are multiple event contracts. A utility company may want to insure against a specific event such as daily high temperatures being below 5°F for 3 days straight, and the contract might stipulate that up to 4 events would be insured over a 90 day period, or an agribusiness firm may want to insure against multiple events of the daily high temperature exceeding 90°F for 4 days straight in order to compensate for yield and/or quality loss.

Second, are straight forward derivative products based upon such notions as cooling degree-days above 65°F (an indication of electricity demand for air conditioning), heating degree-days below 65°F (an indication of electricity, oil, and gas demand required for heating), and growing degree-days or crop heat units above 50 degrees Fahrenheit (an indication of maximum crop yield potential in agriculture).

One of the problems facing the weather derivatives markets, and ultimately how they are priced to agricultural firms, is how these derivatives should be priced in the market. In the absence of a tradeable contract in weather an equilibrium price cannot be established using conventional means (Dischel 1998). At one end of the pricing spectrum, Cao and Wei (2000) develop a pricing model based on expected utility maximization with an equilibrium developed from Lucas’s (1978) model. Davis (2001) also concludes that a Black-Scholes type framework is not appropriate for pricing weather derivatives as a matter of course, but under the assumptions of Brownian motion, expected utility maximization, a drift rate that includes the natural growth rate of the degree day measure, the natural growth rate in the spot price of a commodity (e.g. fuel price) and the natural growth rate in firm profits, then degree day options can be priced by a Black-Scholes analogue. Considine (undated) provides some simpler formulas based on the historical probability distribution of weather outcomes as well as a gaussian (normality) model that he claims can be sufficient at times. Turvey (2001) presents a number of flexible rainfall and heat related option contracts based upon historical probabilities. There has been little published on how to price weather insurance, because there is virtually no agreement on how the derivatives should be priced in the first place.

There are empirical issues related to weather derivatives and a large part of this paper is dedicated toward resolving these issues in general, and the pricing of degree-day options in particular. First, until the CME started trading weather futures there was no forward market for weather. Individuals speculate on what a heat index might be 90 days hence, but unlike stock market indexes there is no mechanism for transparent price discovery on which to base such a prediction, and nature is under no obligation to comply with subjective market assessments. Second, rain or heat or any other insurable condition does not have a tangible form that is easily described (in contrast with common stock or a commodity futures contract). Third, for cities in the U.S.A. and elsewhere that are not listed on the CME, there does not exist a forward market weather index that would allow brokers, traders, and insurers to price such derivatives on an ongoing and transparent basis. This can impact liquidity in the OTC and insurance markets and can also have an impact on the appropriate market price of risk with which to price the contract. Fourth, the mechanics of brokering weather contracts depends specifically on the nature of the contract. A common approach is to use historical data and from this use traditional insurance ‘burn-rate’ methods to determine actuarial probabilities of outcomes. This convention limits trade. For the most part counterparties must agree on a price prior to the opening contract date and are in general restricted by lack of data to efficiently price and trade the contract during the period in which it is active.

For pricing put and call options on cumulative weather outcomes a limiting factor is in the transparency of a forward weather index. A forward weather index such as the HDD and

CDD futures at the CME would operate like any other index and would be used to provide a current estimate of what the final weather index settlement would be. In so doing it would provide a mechanism for counterparties to trade on a continuous basis, and would also provide a mechanism for the continual pricing of the options' intrinsic values.

This paper develops an option pricing model based on such an index even if it is not traded. The model is based on the notion of equilibrium pricing and under the assumption of Brownian motion a formula similar to a classical Black model is used. However, unless the index is formally traded, deriving option values from it will require consideration of the natural diffusion rate and the market price of risk as per lemma 4 in Cox, Ingersoll and Ross (1988). This paper discusses the properties of such an index, shows the evolution of the index in a dynamic context, and develops an options pricing model. The theoretical model is then applied to the pricing of degree-day derivatives for Toronto, using daily mean temperatures from 1840 to 1996. The equilibrium pricing model is compared to the Guassian insurance-type model. However, as stated above, the mere presentation of a Black-Scholes type equilibrium model does not imply that it represents the status quo in pricing weather derivatives.

The 'Burn Rate' Method for Pricing Weather Derivatives

In the absence of a forward weather index the pricing of weather derivatives is relatively straightforward. Using historical data cumulative degree-days (heating days, cooling days or crop heat units) are calculated for the time period in question and the options are priced as

$$(1) \qquad V_p = e^{-pT} E\{Max[Z - W_T^*, 0]\}$$

for a put option, and

$$(2) \qquad V_c = e^{-pT} E\{Max[W_T^* - Z, 0]\}$$

for a call option where p is the appropriate risk adjusted discounted rate, T is time or duration of contract in years, Z is the strike level in degree-days, and W^*_T is the value of the index at expiration also measured in degree-days. Since V measures the expected value of in-the-money degree-days, the actual price of the option is calculated by multiplying V by a dollar value with units \$/degree-day. In equations (1) and (2) it is assumed that the payoff is \$1/ degree-day. The probabilities that establish V are assumed to be stationary priors drawn from historical weather patterns and can be defined as either discrete or continuous.

Weather Indices, Futures Hedging, and Options Pricing

The burn-rate models will typically be purchased prior to the insured period, and will be traded infrequently, if at all. The reason that such contracts will not be traded results from the fact that there is no transparent mechanism to update or revise the probabilities during the insured period and hence no opportunity to arbitrage risk. The opportunity to arbitrage requires liquidity and liquidity requires observable volatility in an expected weather index W^*_T. IF W_T is the value of a degree-day weather index at expiration then for any $t<T$ there must exist an expectation about W_T, that is $W^*_T = E[W_T|t]$, conditional on weather information up to and including time t. Observable volatility in W^*_t requires first the existence of a forward weather index, and secondly that it be defined by an inter-temporal stochastic process.

The continuous time stochastic differential equation for the weather index can be described by Brownian motion and the Ito process

$$(3) \qquad dW^*_t = \mu W^*_t dt + \sigma W^*_t dZ_t$$

The stochastic process described by (3) describes a random walk and is fundamental to the design of new derivative products for entities that follow a Markov process. As shown by Merton (1993), Black and Scholes (1973), Black (1976) and others, if the underlying assumptions in (3) hold then it can be used to price options. In Equation (3) μ is the mean change in cumulative degree-days and σ is the variance of the daily change in degree-days. The assumptions, which are empirically tested in Turvey (2002), are that the diffusion rate μ is constant over time and σ^2 increases linearly in time.

Equilibrium Pricing Formulas for Degree-Day Derivatives

With the introduction of the CME degree-day future contracts there will be, at least for specific locations, a spanning asset for which a classical options pricing formula can be derived. However, there are more jurisdictions without contracts than with, and this implies that not all risks can be spanned and risk-neutral valuation techniques cannot readily be used without modification. Under such circumstance it is necessary to apply a different set of rules to price options on non traded assets. In particular, an options pricing model when the underlying asset cannot be spanned by traded assets requires including the market price of risk. This has lead some practitioners to declare that modern options theory in the form of Black (1976) or Black-Scholes (1973) will not work (Nelken 1999, Dischel 1998) for pricing weather derivatives.

To capture the market price of risk, equation (3) is represented by

(4) $dW^*_t = (\mu - \lambda\sigma)\, W^*_t\, dt + \sigma W^*_t\, dZ$

where λ represents the market value of risk, and $\lambda\sigma$ the risk premium. The call option value of F(W, X, t) that solves this equation for a strike price $X=W_z$ is

(5) $C(W,t) = F(W,t) = \theta\, [e^{-pt}\, N(d_1)\, X - e^{-(P- (\mu-\lambda\sigma))t}\, N(d_2)\, W]$

where t is time remaining until option enquiry, θ is the value per tick, X is the strike price in degree-days, p is the discount rate, N( ) is the value of the standard normal cumulative distribution function evaluated at d_1 or d_2,

$d_1 = [\ln (W/X) + (\mu - \lambda\sigma + .5\sigma^2)t]/\sigma\sqrt{t}$

and

$d_2 = d_1 - \sigma\sqrt{t}$

Since the market price of risk is explicitly included in the solution, the appropriate discount rate 'p' for a risk-neutral valuation is the risk free rate, r. However this still leaves unresolved the problem of determining the market price of risk λ. In a more general framework the diffusion $\mu - \lambda\sigma = r$ is called the risk neutral growth rate (Cox and Ross, 1976) and is a necessary condition for equilibrium pricing. In contrast μ is viewed as the natural growth rate in the value of the underlying. The value $\lambda = (\mu - r)/\sigma$ is then the market price of risk.

If the market price of risk so defined is applied to freely traded assets then $p = r = \mu-\lambda\sigma$ can be substituted into equation (5) and the resulting formula is identical to Black-Scholes. A more general argument is required for assets that are not-traded. For this we appeal to the security market line of the capital asset pricing model. Then we can define the market price of risk λ as

(6) $\lambda = \beta\, [r_m - r]/\sigma$.

As indicated above, equation (5) is a general solution to pricing all assets in equilibrium. For the particular case of weather derivatives its form becomes simplified. If the underlying is a futures contract then

(7) $d_1 = [\ln (W/X) + (r + .5\sigma^2)t]/\sigma\sqrt{t}$

and (5) becomes the standard Black-Scholes pricing model with p=r. Using $W(t) = e^{-rt}\, W(T)$ and substituting this into (5) gives Black's model for pricing options on futures.

When the weather index is not a traded variable we rely on the direct relationship between the non-traded weather index and the market portfolio. Since the impact of weather events in localized regions will not be correlated with the market portfolio, then it to will have a beta of zero. This is consistent with the empirical findings in Cao and Wei (2000). The result and conclusion does not imply that the conditional underlying risks of economic outputs are zero, but that in equilibrium the source of the risk can be diversified away. However, unlike a futures contract the non-traded weather variable will not grow at the risk-free rate. In fact the spot value at time t will simply equal the expected value at time T, that is $W^*_t = E[W^*_T]$. This implies a natural tendency towards mean reversion so $E[\mu]=0$. By substituting $\beta=0$ and $\mu=0$ into equation (5) and setting p=r to account for risk neutral valuations, the pricing model for call option on a non-traded weather index is given by

(8) $C(W,t) = \theta e^{-rt} [N(d1)X-N(d2)W]$

where

$d1 = [\ln (W/X) + .5\sigma^2 t]/\sigma\sqrt{t}$

and

$d_2 = d_1 - \sigma\sqrt{t}$

As a reminder the parameter θ is the tick value measured in $/degree and the bracketed term is measured in degrees. The equivalent put option value is

(9) $P(W,t) = \theta e^{-rt} [N(-d_2)W - N(-d_1)X]$

The solution value of the option pricing models rests on three assumptions that are evaluated in the empirical section. Assumption 1 is that the natural dynamics of dW originates from a random walk and hence unanticipated changes in W are not serially correlated. If strong and predictable autocorrelation is present then asymmetric information between buyers and sellers of the option will allow for risk free arbitrage opportunities. In Turvey (2002) I provide strong evidence that W* evolves as a random walk that is consistent with geometric Brownian motion. The second assumption is that volatility is non-stochastic. In Turvey (2002) I show that volatility is stable within and between years. This assumption is consistent with the assumption of time dependence in Merton (1993) and Wilmott (1998). The third assumption is that $E[\mu]=0$ This assumption simply states that $W^*_O = E[W_T]$ and investors in weather options will use the mean of the historical sampling distribution as an unbiased estimate of the initial condition for dW. This is exactly how the opening prices of the CME exchange traded degree-day future prices are set. A less naïve condition is that $W^*_O = E[W_T|\Omega]$ where the expectation is now based on the conditional mean based on the information set Ω at time t=0. This is relevant when counterparties believe that degree-days will be higher or lower than the historical average. This may or may not come about as a variance preserving shift. However, in Turvey (2002) I provide evidence that the volatility of the degree-day index, at least for Toronto, is remarkably stable.

Defining a Weather Index

In the previous section the existence of a forward weather index was presumed. While possibility rather than existence is sufficient to support the development of an option pricing model, it is obviously a limitation to implementation and practice. The CME futures contracts will satisfy the spanning requirement of a correlated underlying derivative security, but CME contracts do not exist for many regions or cities. Hence the foregoing is a generalized solution that can be used to price options even if a formal futures contract does not exist. In this section a general approach to constructing a weather index using historical data is presented. In the next section the index model will be applied to a case study of degree-day contracts for Toronto.

The challenge for any broker or exchange to accurately price weather options is in the construction of an appropriate weather index which can be observed on a daily basis, and provide representative measures of volatility. To construct such an index it is useful to draw on the unique characteristic that the weather index cannot be influenced by human speculation. In this context the index is observable, objective, and representatively transparent. For example, settlement of the CME contracts is based exclusively on the data collected by Earth Satellite Corporation. Furthermore, a consistent characteristic of weather is that it is seasonal and systematic; summer, for example, always starts of with low temperatures that rise to a peak, and then decreases towards autumn. A naïve hedger planning a hedge in early spring would naturally assume that the summer weather pattern would follow the average pattern as dictated by history. Critical to this is the additional assumption that temperature is mean reverting: In the absence of any contrary information it is not unreasonable to assume that if the average temperature on June 30th is 70 degrees Fahrenheit, then in the current year the best unbiased estimate is that it will also be 70 degrees. The notion of mean reversion is also a natural phenomenon; the tendencies for temperature to fall to within a normal range following a heat wave, or to rise to normal temperatures following a cold snap is clearly the norm rather than the exception.

The absence of predictability and the assumption of mean reversion suggest that the best initial (t=0) unbiased estimate of the forward index is the historical average of the index over the specified contract time horizon. Indeed the opening trade on the CME futures contracts will most likely be the long-run average cumulative degree-day with some adjustment for long-term forecasts or revised expectations. The initial index value is given by equation (10):

$$(10)\qquad W_0^* = E[W_T] = \sum_{t=0}^{T} E[W_t]$$

where W represents the weather index (e.g. cooling degree-days, heating degree-days, growing degree-days or cumulative rainfall). After 1 day the observed weather condition at t=0 is recorded and the index value is appropriately adjusted to include the actual outcome plus the projected outcome;

$$(11)\qquad W^*{}_1 = W_0 + \sum_{t=1}^{T} E[W_t]\ .$$

and for any time increment k in the sequence

$$(12)\qquad W^*{}_k = \Sigma^k{}_{t=0}\, W_t + \sum_{t=k+1}^{T} E[W_t].$$

As the index evolves with time the instantaneous percentage change in the weather index can be calculated as

$$(13)\qquad E[\mu] = E[(W^*{}_k - W^*{}_{k-1})/\, W^*{}_{k-1}]$$

and daily volatility is

$$(14)\qquad \sigma^2 = E[\mu - E[\mu]]^2.$$

Finally, the path described by $\sum_{t=0}^{T} E[W_t]$ needs to be estimated. This can be done by using historical data directly but since this has to be recalculated for each day in the contracts life it is computationally intense. In the alternative, $\sum_{t=0}^{T} E[W_t]$ can be estimated from a simple regression equation to get the same result. In this study the estimated equation describing the evolution of temperatures during the summer months was quadratic .

The Pricing of Cooling Degree-Day Options

In this section option premia are calculated for Toronto Ontario using Environment Canada daily mean temperatures from 1840 to 1996. The contracts examine summer cooling degree-day call (put) spreads. With this option the buyer agrees to pay a fixed premium in exchange for payment from the seller if the defined Weather Index settles above (below) the Index Strike for the Contract Term. The payment equals the number of Weather Units the Weather Index falls above (below) the Index Strike times the Unit Price. There may be a payout limit but this is not considered in this study.

First the temperature history from June 1 through August 31 is described from a historical perspective. As history will always be the source of weather patterns it is important to understand how more recent trends compare to past trends. Second, using a cooling degree-day measure of heat above 65 degrees Fahrenheit, degree-days are calculated for each day and cumulative degree-days are calculated for each year. Third, a quadratic regression equation is estimated with mean daily degree-days as the dependent variable and time and time squared (within the contract term) as the independent variables. Fourth, using mean cumulative cooling degree-days as the initial index value, observed daily degree-days, and the regression equation, the forward index value for each day, in each year was calculated. Fifth, using the daily forward index values, the empirical volatility of the index is calculated from the variance of the daily percentage change in index values. This is done for each year. And, sixth, assuming a discount rate of 6.5%, the historical mean volatility, 92 days to expiration, and a strike price (which is varied), call and put option premiums are calculated. As a point of comparison premiums using the ‘burn rate’ approach are also calculated.

Toronto’s Weather History

This section describes the weather history from June 1 to August 31 for the years 1840-1996 in Toronto. The data used were obtained from Environment Canada and represents one of the longest available weather data series in Canada. Figure 1 plots the data. The plot shows an overall increase in mean daily temperature over this time period, with temperatures increasing at an increasing rate until approximately 1930 and then increasing at a decreasing rate. Since Approximately 1950 there does not appear to be a significant rise in mean daily temperatures.

Figure 2 shows the cumulative cooling degree-days in Toronto between 1840 and 1996. The cooling degree-days increase with the mean temperature as would be expected, but the graph also illustrates the variability and unpredictability of the measure. The graph shows that cooling degree-days increased at an increasing rate throughout most of the 19th century but appear to be quite stable or decreasing in terms of mean value towards the end of the 20th century. Table 1 summarizes the key statistics for the entire 1840-1996 period and the sub period from 1930 to 1996. From 1840 the average cooling degree-days ranged from 107 to 787 with a mean of 379 and a standard deviation of 147. The period since 1930 has cooling degree-days ranging from 186 to 787 with a higher mean of 489 and a standard deviation of 114.

Figure 3 illustrates the mean actual and predicted daily degree-days within the 92-day period from June 1 to August 31. The pattern is parabolic and the statistical fit (using a quadratic equation) of predicted to average was approximately 93% (R-squared)[1]. Figure 4 illustrates the cumulative degree-day effect throughout the time period. The degree-day value used in options pricing is the total sum recorded on the 92nd day.

Calculating the Cooling Degree-Day Weather Index

This section describes how the CDD weather index was calculated. The index was calculated for each year in order to assess the range of CDDs and to measure volatility. The cooling degree-day weather index was generated from a combination of observed daily data in each year, the seasonal regression equation, and the average cumulative degree-day value across all years. The initial index value at t=0 is assumed equal to the average cumulative degree-day value. This is identical to the sum of the marginal degree-days illustrated in figure 3. The smooth parabola in figure 3 illustrates how the regression equation smoothes the variability in daily degree-day measures and acts as an unbiased predictor of the most likely temperature path based on the assumption that weather patterns are mean reverting. To calculate the index the degree-day above 65f is calculated from the first observation (day 1). Then the sum of the predicted daily degree-days is calculated along the parabola from day 2 through to day 92. Assuming that the day one degree-day measure is small this will provide a day 1 index value very close to the long run average. On day 2, the actual degree-day measure is taken and is added to the day 1 value. The sum of the predicted is then taken from day 3 to day 92 and added to the actual day 1 plus day 2 values. The procedure is repeated for each of the 92 days, and is repeated for each year in the sample.

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[1] With daily temperatures about 65°F as the dependent variable the equation is $Temp = -.38 + .21T - .002T^2$ where T is day number (e.g. 1-92). Only the intercept is not statistically significantly different from zero.

Figure 5 illustrates the results for three recent years in Toronto; 1986 was an average year with cooling degree-days of 386. The summer started of quite cool and this caused the index to fall below the average until about day 55 where a warming trend caused a slight increase in the value of the index; 1988 was a hot year and the index was above average throughout the season. A short cooling spell from day 31 to about day 40 caused the index to decrease but beyond that cooling degree-days were significantly higher than average. The 1988 index peaked at approximately 750 on day 80, but a cooling trend caused the index to fall to 725 by day 92; In contrast to 1986 and 1988, 1992 was unusually cool with cumulative degree-days of 186 by day 92. The index was average for the first 3 weeks of June, but after that a long cooling trend caused the index to fall to a low of about 180 before ending at 186.

Calculating Volatility

Volatility is measured relative to the percentage change in the value of the index on a daily basis and then converted to an annualized (365 day) basis for convenience. Table 2 and Figure 6 show the estimated average volatility for Toronto cooling degree-days from 1840 to 1996 and from 1930 to 1996.

The results indicate that the weather has actually been less variable since 1930 than in the previous 90 years. From 1840 to 1996 annualized volatility was .2063 or 20% per year, but this decreased to .1739 or 17% per year in the mid to latter part of the 20th century. For the entire period the minimum volatility was found to be 16.62% with a maximum of 29.61%, while the latter part of the century the range was as low as 14.14% but only went as high as 23.5%. Combined with the information in Table 1, weather averages in Toronto saw an increase in mean summer temperatures and degree-days, but this increase did not come with increased variability. In fact, the standard deviation of cumulative degree-days (Table 1) is lower for the 1930-1996 period than the 1840-1996 period. Importantly, these observations signify that when options on weather are being priced it is important to match recent weather trends on index values and volatility. In the next section, which calculates option premia, an approach, which mitigates this problem, is discussed.

Volatility Stability

Use of the options pricing model requires stability in the index's volatility within a given year and across years. The first item is important because if daily volatility is a function of time or is characterized by discernable jumps the proposed pricing model will be mis-specified. The second is important because stability in volatility across years means that the sample volatility can be used as an unbiased estimate of volatility.

Volatility stability was measured by calculating the percentage daily change in the weather index in each year (91 days), i.e. In $[W_T/W_{t-1}]$. To determine the stability of volatility rolling 30-day standard deviations of the percentage change were calculated and annualized to a 365 day year. Thus for 91 days used in this study there were 61 volatility estimates for each year. Table 4 shows the results from this evaluation over the 1840-1996 period and two sub periods 1840-1935 and 1936-1996.

The annualized volatilities have been stable across years, with the average 30 day volatility being about 20%. This compares to the average volatility over the whole 91 days of .2063 as shown in Table 2. The results also show that the standard deviations are low relative to the mean. For example a standard deviation of .023 for 1840-1996 indicates that the average 30 day volatilities ranged from .178 to .223 approximately 67% of the time. The within year coefficient of variation (mean/standard deviation) reveals that the means are 6.42, 5.98 and 7.13 times the within-year 30-day standard deviations for each of the periods. These numbers imply that not only is volatility stable across years but they are quite stable within each year as well.

Estimates of Cooling Degree-Day Option Premia

This section reports actual option premiums calculated for Toronto, Ontario. The contracts considered are 92-day put and call options with contract terms from June 1 with an expiry on August 31. Each tick in-the-money (θ) was valued at \$5,000 per degree-day. Several empirical considerations are illustrated in the results. First, premium estimates are calculated using the both the inter-year ‘burn-rate’ method used in the insurance industry (equations 8 and 9) and the intra-year Black’s option pricing model (equations 22 and 23). Second, in order to illustrate the importance of ‘relevant time horizon’, estimates are provided for the 1840-1996 data period and the 1930 to 1996 sub-period. Third, the options pricing model is sensitive to the initial index value, W^*_0, and using a simple average in all cases would not be prudent. For the options pricing model only, a range of initial values of W^*_0 are examined. This type of sensitivity analysis is important because weather agencies such as Environment Canada and the U.S. Weather Service cannot generally predict forward temperatures with reasonable accuracy. However, they can and do provide three or four-month forecasts that state whether conditions are going to be normal, below normal, or above normal. If the prediction is above normal, for example, the buyer of a call may want to increase the initial expectation of W^*_T to match the forecast and reduce the premium.

Tables 4 for 1840-1996 and 6 for 1930-1996 present results for base case at-the-money option pricing calculations as well as a range of strike prices above and below this value. The at-the-money strike is defined as the average cooling degree-days across the years sampled. This is 379.39 for 1840-1996 and 489.50 for 1930-1996. The option premiums differ between the options model and the burn rate model as well as across the two time periods. When the sampling period was represented from 1840 the at-the-money put and call price was $77,073 for the 379-CDD strike option model and approximately $297,030 for the burn rate model (Table 5). The maximum payoff for the put option under either case would have been $1,361,450 for the put option and $2,038,150 for the call option. As the strike price was increased put options would be issued in-the-money and the put option premiums would rise as the call premiums fell. For a strike of 600 CDD the option model put premium was $1,085,126 while the burn-rate model was $1,136,421. The maximum put payoff increased to $2,464,500. The corresponding call option for the option model was $0 and for the burn-rate model it was $33,405. The maximum payoff that would have possibly occurred with this strike over this period was $935,100. A lower than average strike implies that put options are issued out-of-the-money, while call options are issued in-the-money. At a strike of 250 CDD the put options price is negligible, while the call option price is $636,438. Using the burn-rate model the corresponding put and call prices were $63,947 and $710,420 with maximum payoffs being $714,500 for the puts and $2,685,100 for the calls.

A similar pattern was observed for the 1930-1996 period (Table 5). The at-the-money option price (489.5 CDD) for the put and call was $83,835 and using the burn-rate model the put-call price was approximately $220,358. The maximum put and call payoffs would have been $1,516,900 and $1,487,600 respectively. For in-the-money calls with a strike of 250 CDD the call option was $1,178,041 and the corresponding put was $0. The burn-rate put and call prices were $4,767 and $1,202,279 respectively, with maximum payoffs of $319,400 and $2,685,100. For in-the-money puts at 600 CDD the put option price was $544,298 and the call price was only $776. The burn-rate premiums were $624,900 and $72,412 for the put and call respectively.

These results illustrate some important and critical details regarding the pricing of degree-day derivatives and the selection of a time period over which to analyze heat. The difference between options pricing and burn-rate models is striking, especially when priced at-the-money. Using the 1840-1996 period the burn-rate model prices the insurance at 3.85 times the option pricing model whereas the 1930-1996 period the pricing multiple is 2.63. The ratio converges to 1 for policies that are in-the-money and infinite for options out-of-the-money. The results illustrate why different approaches to pricing weather options can result in large bid-ask spreads.

The explanation for these differences lies in how risk is measured and what risks are actually being traded. The burn rate model assumes that history will repeat itself and the variability and probability distribution of the past will be replicated in the future. It rests upon an actuarial structure, which is seemingly predictable, but one, which also carries with it some significant variability. In contrast the options pricing model is not backward looking in the sense of a memorized historical probability distribution. It assumes an infinite of random weather patterns, which can occur in any season. The role of history is vague only in its use to establish seasonal norms and a range of volatility measures, but once these are established history's role is done. Another key difference is the assumption of a starting point. The options pricing model assumes a numerical starting position from which variability in a weather index is measured, and the price of the option is sensitive to this initial position. In Turvey (2002) I provide tables that illustrate how differences in initial expectations can affect option values.

Conclusions

This paper addressed the pricing issue of degree-day weather derivatives. The market for weather insurance products has increased dramatically in past years for several reasons. First weather derivatives are directed at hedging production or volume versus price risk. In the natural gas and energy sectors, utilities will often fix prices to the consumer or face regulated prices to consumers. Electrical utilities must of ten pay peak-load prices when energy demand exceeds contracted supplies, and natural gas and oil companies must pay higher spot prices when extreme cold causes excess demand in those markets. Agriculture is also an industry that faces weather related production risk. A crop insurer might have to pay increased indemnities if weather is either too hot or too cool, and might use weather derivatives as a reinsurance product, or a food processor might require a hedge against undeliverable forward contracts resulting from weather conditions.

The approach used in this paper differs markedly from an insurance approach to pricing weather derivatives. The 'burn-rate' approach, prices premiums based upon what would have occurred over a recent time period. It was pointed out that the key difference between the burn-rate model and the options pricing model is in how risk is defined. Under the burn-rate model it is assumed that history will repeat itself with the same likelihood, but not necessarily the same order, as the time horizon selected for pricing. In other words, the approach assumes that the relevant measure of risk is the inter-year variability in weather. The options pricing model developed in this study makes no such assumption and is in fact based on intra-year risks. As with conventional options pricing, volatility and the initial value of the weather index are the key drivers of risk. History is used only to measure volatility and determine a range of index values, but once a measure of volatility is selected and the initial condition determined, history has no further role to play in the pricing process. For example the 1840 to 1996 period had mean cooling degree-days (above 65f) of 379 CDD and an annualized volatility of 20.63% for the period June 1 to August 31. Using the 1930-1996 period the average cooling degree-days was 489 CDD with a volatility of 17.39%. Under no year was volatility found to exceed 29.6%, yet the implied volatility that would equate the options pricing model to the burn rate model was 80% for the 1840-1996 period and 45.8% for the 1933 to 1996 period.

It was shown that there is a significant and often large difference between the burn-rate model and the options pricing model, particularly for products priced at or near-the-money. It was shown that the burn rate model prices options as much as 2 to 3 times higher than the options pricing model. The two approaches converge only for options that are priced in the money or out of the money. It is consistent with the various theories of pricing non-traded assets in equilibrium, and in a risk-neutral economy. Statistical analyses confirmed that the underlying assumptions required for pricing degree-day weather options are empirically valid.

The options pricing model presented in this paper is new. On one hand it is an improvement over the traditional burn-rate approach in that it places much more emphasis on risk and for a derivatives market which is essentially designed to manage the buying and selling of risk there can be efficiency and liquidity gains if the model is implemented in practice. On the other hand the traditional approach is easy to implement and even easier to comprehend. However, if a formal derivatives market for weather insurance is going to emerge it is very likely that the approach developed in this study will provide foundation for pricing weather derivative products.

Table 1: Historical Summary of Toronto Cooling Degree-Days

	Mean	Standard Deviation	Minimum	Maximum
1840-1996	379.39	146.67	107.10	787.02
1930-1996	489.50	114.69	186.12	787.02

Table 2: Historical Summary of Toronto Cooling Degree-Days' Volatility

	Mean	Standard Deviation	Minimum	Maximum
1840-1996	.2063	.0012	.1662	.2961
1930-1996	.1739	.0009	.1414	.235

Table 3: Seasonality and Stability in Volatility

	1840-1996	1840-1936	1936-1996
Mean (365 days)	.201	.207	.193
Standard Deviation	.023	.022	.021
Coefficient of Variation Mean	6.42	5.98	7.13
Coefficient of Variation Standard Deviation	3.19	3.03	3.32

Table 4: European Options and Burn Rate Premiums: 1840-1996, Tick = $5,000

	Option Value		Burn Rate		Maximum Payoff	
	Put	Call	Put	Call	Put	Call
200	0	882,374	18,215	915,190	464,500	2,935,100
250	0	636,438	63,647	710,420	714,500	2,685,100
300	692	391,264	135,264	533,239	964,500	2,435,100
350	23,156	167,718	229,910	376,885	1,214,500	2,185,100
379.39	77,073	77,073	297,054	1,361,450	2,038,150	2,038,150
400	139,950	38,574	352,121	249,096	1,464,500	1,935,100
450	351,674	4,361	508,943	155,918	1,714,500	1,685,100
489.50	542,100	497	657,13	106,788	1,912,000	1,487,600
500	539,513	263	698,560	95,534	1,964,500	1,435,100
550	839,198	10	908,806	55,781	2,214,500	1,185,100
600	1,085,126	0	1,136,421	33,405	2,464,500	935,100
650	1,331,062	0	1,370,114	17,089	2,714,500	685,100

Table 5: European Options and Burn Rate Premiums: 1930-1996, Tick = $5,000

	Option		Burn Rate		Maximum Payoff	
	Put	Call	Put	Call	Put	Call
200	0	1,423,978	1,035	1,448,548	69,400	2,935,100
250	0	1,178,041	4,767	1,202,279	319,400	2,685,100
300	0	932,103	8,498	956,010	569,400	2,435,100
350	2.52	686,168	20,119	717,631	819,400	2,185,100
379.39	94	541,697	841,749	39,261	2,319,400	685,100
400	670	440,897	50,519	498,031	1,069,400	1,935,100
450	17,974	212,266	121,639	319,150	1,319,400	1,685,100
489.5	83,835	83,835	220,358	220,370	1,516,900	1,487,600
500	113,047	61,400	249,622	197,134	1,569,400	1,435,100
550	306,996	9,411	417,813	115,325	1,819,400	1,185,100
600	544,298	776	624,900	72,412	2,069,400	935,100
650	789,497	37	841,749	39,261	2,319,400	685,100

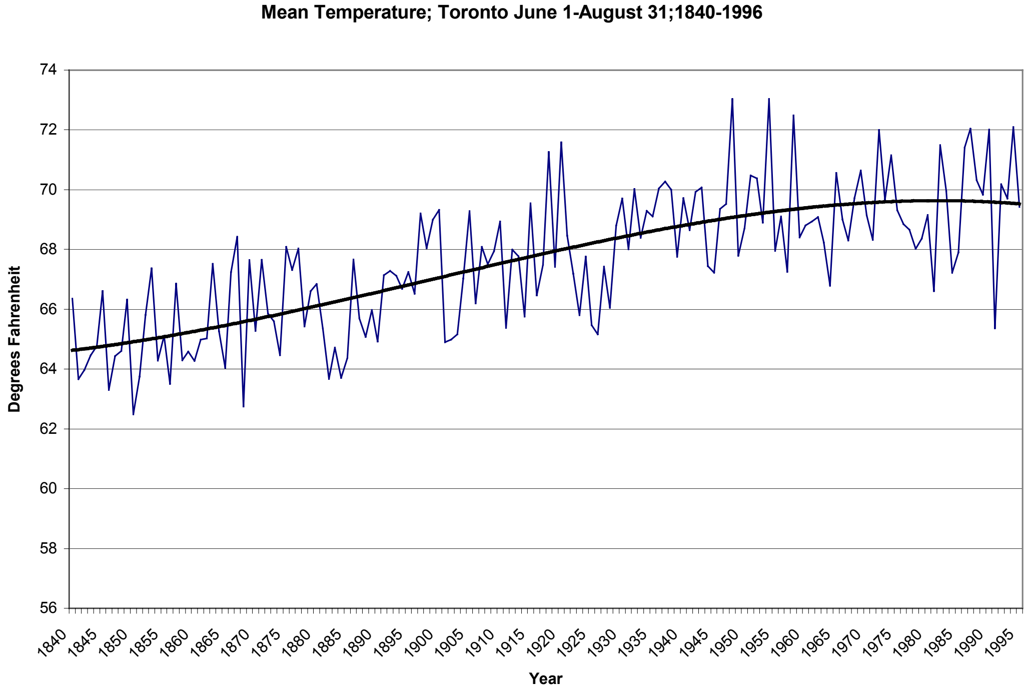

Figure 1:Mean Seasonal Temperature, Toronto, June 1 to August 31

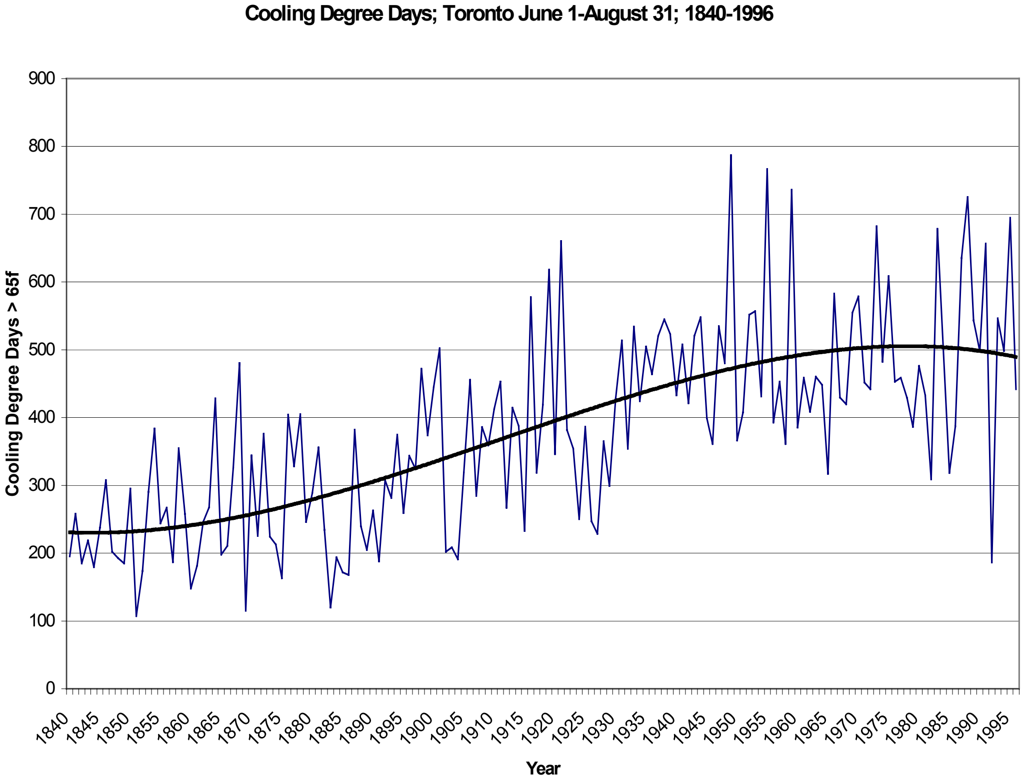

Figure 2:Mean Actual and Predicted Daily Degree-Days, Toronto, June 1 to August 31

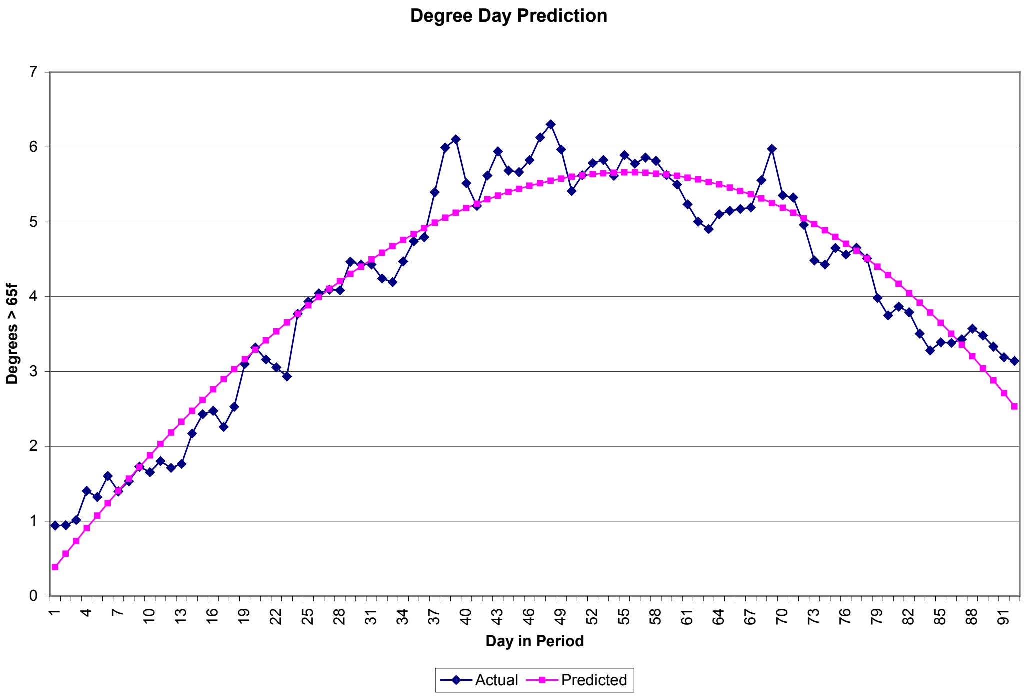

Figure 3:Mean Daily Cooling Degree-days, actual and predicted, Toronto, June 1 to August 31

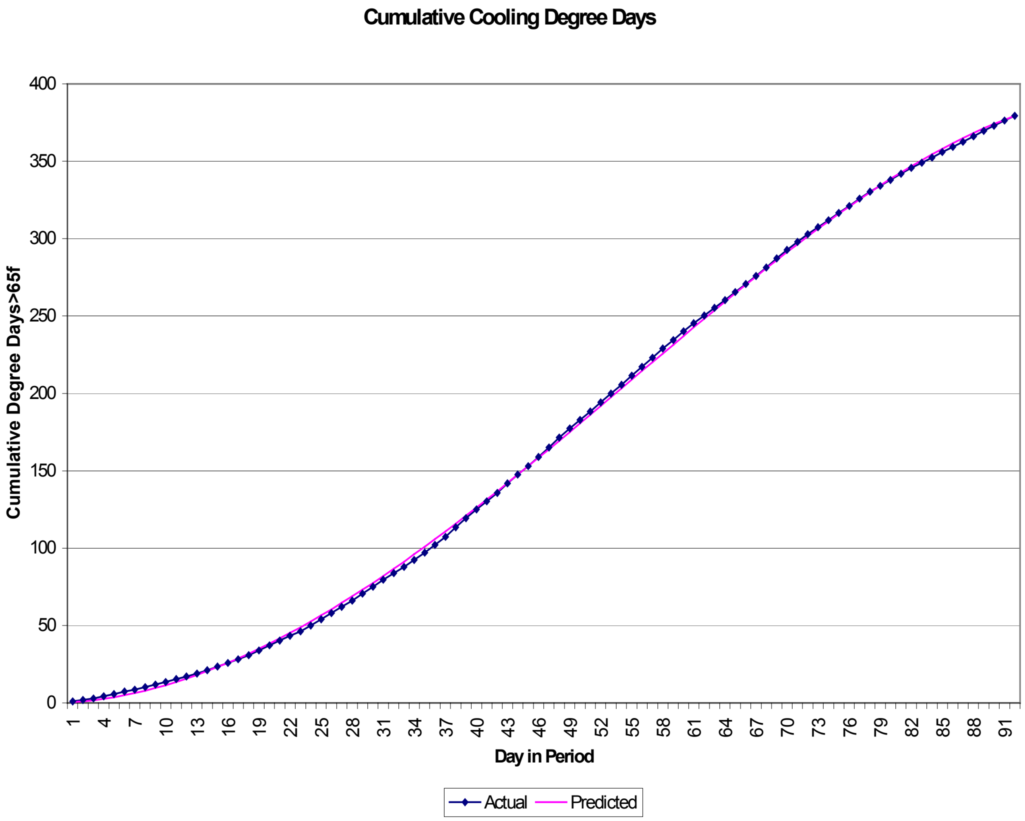

Figure 4:Cumulative Cooling Degree-days, Actual and Predicted, Toronto, June 1 to August 31

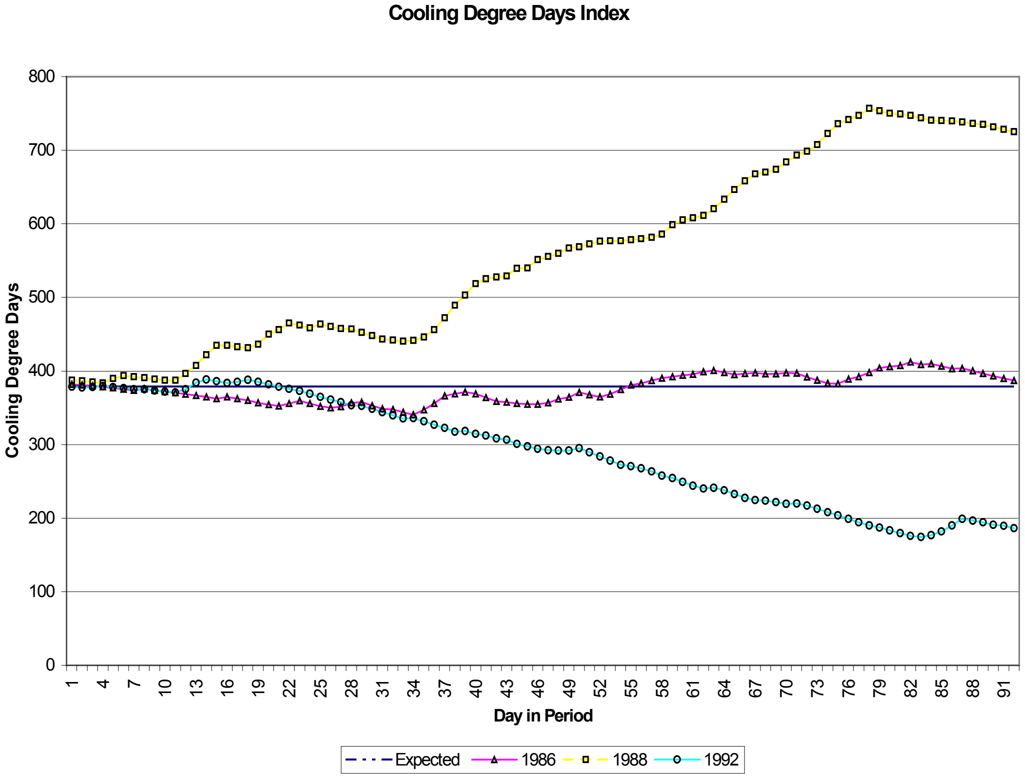

Figure 5: Cooling Degree-Day Weather Indexes for 1986 (average), 1988 (above average) and 1992 (below average), Toronto, June 1 to August 31

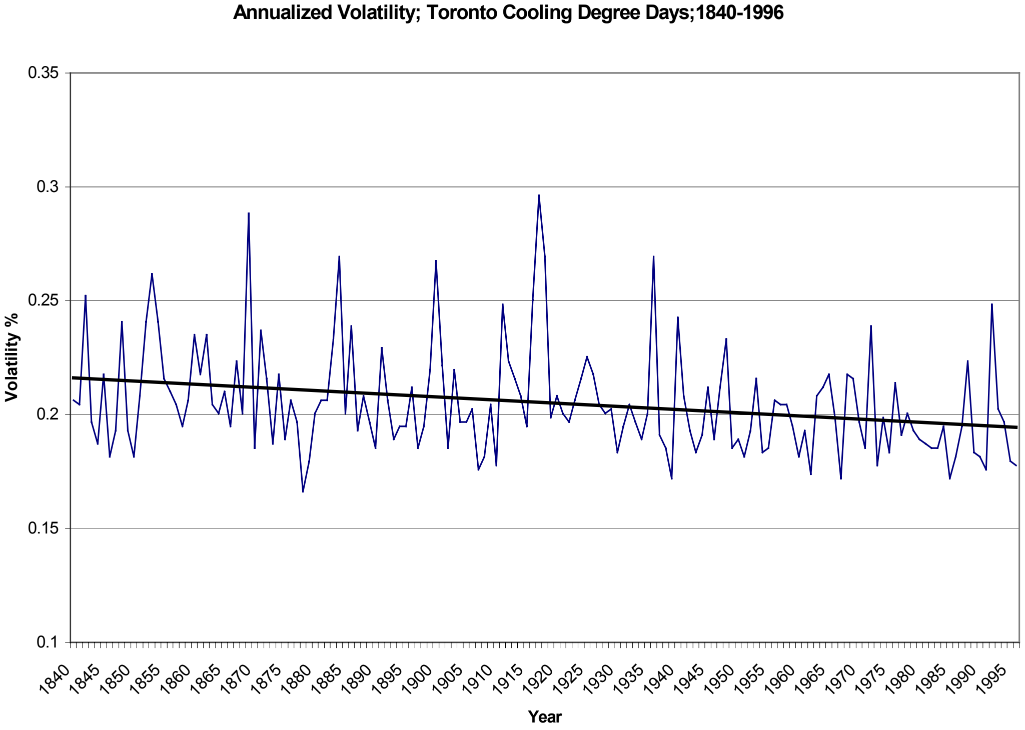

Figure 6:Mean Annualized (365 day) Volatility, Toronto, June 1 to August 31. 1840-1996

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