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Even Bergseng, Grethe Delbeck,

Hans Fredrik Hoen (eds.)

Ås

Implications of Extreme and Mean Ratio in Near-Natural Forest Management Planning – Using a Diameter Class Modelling Approach

Peter Tarp
University of Copenhagen, Faculty of Life Sciences, Forest & Landscape

Abstract

Near natural forest management is performed by use of domestic tree species that are able to regenerate naturally. One of the long-term key objectives is to achieve more structural richness in the form of uneven-aged mixed species forests dependent on so-called forest development types that are allocated with support in ecological mapping. A goal of perfection or an ideal structure may be sought by replication of the normal forest structure that takes the form of an inverse J-shape when analysing the stem number ha^{-1} as a function of diameter. Here alternative approaches applying the extreme and mean ratio (1.61803) are developed with the purpose to describe and analyse the ideal or perfect structure of an uneven-aged forest of European beech (*Fagus sylvatica L.*) site index 1. The so-called golden ratio is applied for example in determining midpoints of diameter classes based on the assumption that the relation between the stem number ha^{-1} in adjacent diameter classes is equal to the golden ratio. Solutions are found by use of Markov chains through solution of recursive equations applying linear programming. Alternative approaches where the golden ratio is applied as a structural goal are analysed.

Keywords Diameter class models, extreme and mean ratio, Fibonacci numbers, golden ratio, linear programming, Markov chains, modelling, near-natural forest management planning, recursive equations

Introduction

The purpose of this paper is to explore the implications of the golden ratio (GR) within near-natural forest management planning (NNFMP).

The GR is assumed to express the ideal form or structure within many different spheres of life such as art, nature, architecture, painting, geometry etc. The arrangement of flowers and seeds in golden spirals in certain types of sunflowers making efficient use of the space available is a good example from nature of the GR. Another well-known example is the proportions of the human body. The GR is often found in well-functioning and aesthetically attractive architectural designs e.g. expressed as the relation between the height of houses in relation to the width of the street in cities. Many more examples of the presence of the GR have been presented since its recognition more than 2,000 years ago.

Pythagoras and Euclid in ancient Greece, Leonardo of Pisa, and Johannes Kepler studied the GR intensively (Livio 2002) and Fibonacci (1170-1250) mentioned the numerical series now named after him in his Liber Abaci. The first known calculation of the GR as a decimal number was described by Michael Maestlin 1597 at University of Tübingen to his former student Johannes Kepler (Wikipedia 2008).

The GR may be seen as the consequence of genetic improvement through survival of the fittest, i.e. forms or structures fulfilling the GR are more apt for our environment than alternative forms such as e.g. symmetric ones. The GR is viewed as an appealing attribute of aesthetics, which is often seen in daily life when combining aesthetical and functional qualities. E.g. the proportions of the monitor of most laptop computers fulfil the GR.

The GR may be expressed in numerical terms for various stand factor attributes such as e.g. stem number per ha, standing volume per ha, area, height and diameter as illustrated in Fig. 1 (see mathematical definitions in the Materials and Methods chapter below).

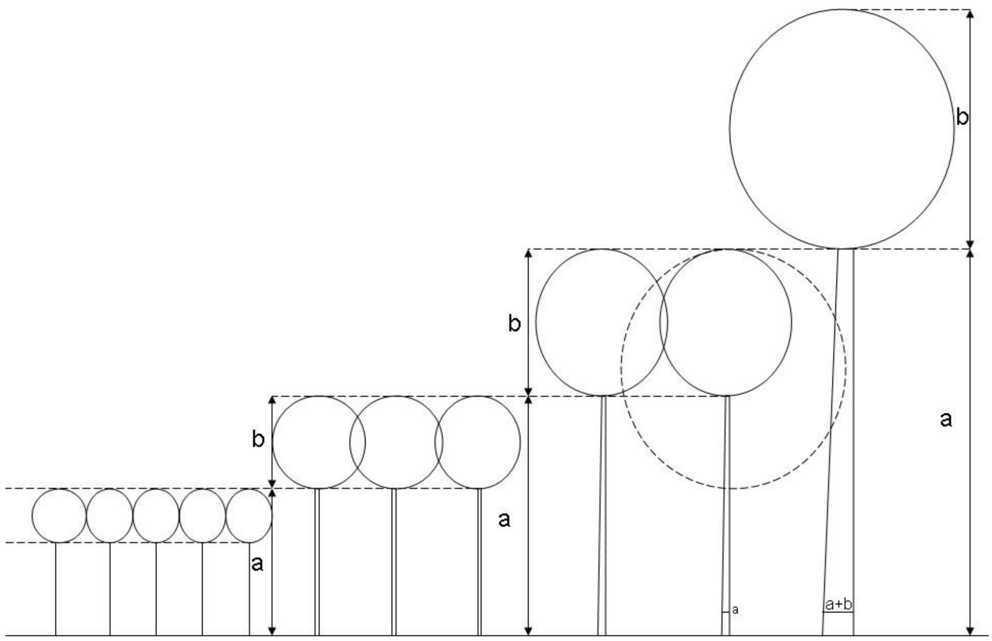

Figure 1. Golden ratio of stem number, height, and diameter.

The approach applied here is based on the GR of stand factor attributes being represented in adjacent diameter classes. E.g., if the stem number per ha of diameter class i is N_i then the stem number per ha of diameter class N_{i+1} is equal to N_i/φ, where φ is the GR. The stem number per ha is assumed to be determined by the stem number per ha in the largest diameter class defined for beech, site index 1, at the age of 100 years. As shown in Table 1 the stem number per ha at 100 years is 131 ($4B/(\pi D^2)$, where B is basal area per ha and D is diameter – at breast height). The stem number per ha in smaller diameter classes is consequently computed recursively.

The smallest diameter class is defined such that this class includes trees with a height equal to breast height. The stem number per ha is assumed to be lower than or equal to 10,000.

The relationships between stand factor values are assumed to be in accordance with the growth and yield table data shown in Table 1. E.g., the height of a diameter class at the age of 100 years is 31.8 m, diameter is 49.3 cm, basal area is 25.0 m^2 per ha, stem number per ha is 131, form factor is 0.531, standing volume is 422 m^3 per ha, thinning harvest is 7.8 m^3 per ha per year, and increment is 10.1 m^3 per ha per year. I.e., it is implicitly assumed that the growth and the thinning regime of the forest fulfil the data shown in Table 1.

The GR may also be expressed in geometric terms as illustrated in Fig. 2 with golden rectangles or circles. The geometric representation may be applied in relation to the shape and size of e.g. regeneration harvests. The shape and size of regeneration openings could alternatively be determined as a function of tree height where the relation between tree height and the length of the side of a rectangle or the radius of a circle fulfils the GR. Interestingly, it is found that when the sides of golden rectangles and the radius of golden circles fulfil the GR then the relative size of the areas of these rectangles and circles fulfils the GR as illustrated in Fig. 2.

Table 1. Growth and yield table for beech, site index 1 (A=Age, H=Height, D=Diameter, B=Basal area, bh=before harvest, ah=after harvest, F=Form factor, V=Volume >5 cm, I=Increment).

A	H	D	B	B	F	V	B	V	I
			bh	ah			H	h	
yrs	M	cm	m^2 ha^{-1}	m^2 ha^{-1}		m^3 ha^{-1}	m^2 ha^{-1}	m^3 ha^{-1}	m^3 ha^{-1} yr^{-1}
4	0.3	-	-	-	-	-	-	-	-
5	0.6	-	-	-	-	-	-	-	-
6	1.0	-	-	-	-	-	-	-	-
7	1.5	0.9	0.3	0.3	-	-	-	-	-
8	1.9	1.2	0.6	0.6	-	-	-	-	-
14	4.9	3.6	5.3	5.3	-	-	-	-	-
18	6.8	5.5	12.3	12.3	0.365	31	-	-	7.7
20	7.8	6.6	16.7	13.9	0.454	49	2.9	8	14.7
22	8.8	7.8	18.1	14.9	0.485	63	3.2	13	13.4
24	9.8	8.9	18.8	15.7	0.491	75	3.1	14	13.0
27	11.2	10.7	21.1	16.7	0.493	92	4.5	24	13.6
30	12.6	12.5	21.5	17.6	0.495	110	4.0	24	14.0
33	14.0	14.2	22.0	18.3	0.497	127	3.7	25	14.2
36	15.3	16.0	22.3	19.0	0.499	145	3.3	25	14.3
39	16.6	17.8	22.6	19.5	0.502	163	3.1	26	14.4
42	17.9	19.5	22.9	20.1	0.504	181	2.8	25	14.3
45	19.1	21.2	23.2	20.5	0.507	199	2.6	25	14.3
48	20.2	22.9	23.4	21.0	0.509	216	2.4	25	13.9
52	21.7	25.0	24.5	21.5	0.511	238	3.0	33	13.8
56	23.0	27.2	24.8	22.0	0.513	259	2.8	33	13.5
60	24.3	29.3	25.0	22.4	0.515	280	2.6	32	13.1
64	25.4	31.3	25.2	22.8	0.517	299	2.5	32	12.7
68	26.4	33.3	25.4	23.1	0.519	317	2.3	31	12.3
72	27.4	35.3	25.6	23.5	0.520	334	2.1	30	11.9
76	28.2	37.3	25.9	23.8	0.522	350	2.1	31	11.5
80	29.0	39.3	26.0	24.0	0.523	365	2.0	30	11.2
85	29.8	41.7	26.8	24.4	0.525	382	2.4	37	10.9
90	30.6	44.2	27.0	24.7	0.527	397	2.4	38	10.6
95	31.2	46.7	27.2	24.8	0.529	410	2.4	39	10.3
100	31.8	49.3	27.3	25.0	0.531	422	2.3	39	10.1
105	32.2	51.9	27.5	25.2	0.533	433	2.3	39	9.9
110	32.7	54.6	27.6	25.4	0.534	443	2.3	39	9.8
115	33.0	57.5	27.8	25.5	0.536	451	2.3	41	9.8
120	33.3	60.5	27.9	25.6	0.538	458	2.3	41	9.8
125	33.6	63.6	28.0	25.7	0.540	465	2.3	42	9.8
130	33.8	66.9	28.1	25.7	0.543	472	2.4	43	9.9
135	34.0	70.3	28.2	25.8	0.544	477	2.4	44	10.0
140	34.1	73.7	28.3	-	0.546	528	-		

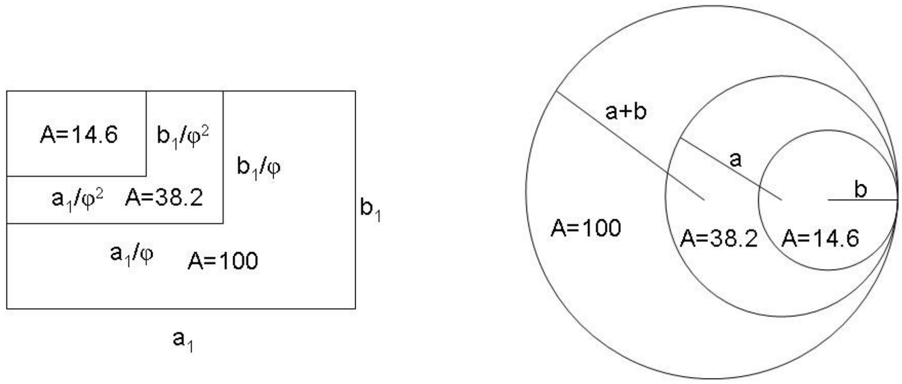

Figure 2. Golden rectangles and circles. (A=area.)

Material and Methods

The GR is defined as the proportion of a to b where a is to b as (a+b) is to a. The numerical value is the solution to the equation

$$\frac{a+b}{a} = \frac{a}{b} = \varphi = \frac{1+\sqrt{5}}{2} = 1.61803398874989 \quad (1)$$

The implications of the GR defining the ideal structure of the forest are analysed assuming that the following relative stand factor values in adjacent diameter classes is equal to φ:

1) Stem number per ha (N).
2) Standing volume per ha (V).
3) Area (A) – decreasing (A-) and increasing (A+).
4) Height (H).
5) Diameter (D).

The growth and yield table of Moeller (1933) that is applied in a slightly modified form in Anon. (2003) is applied for beech, site index 1 as shown in Table 1.

The growth of beech is modelled as a Markov chain using the following equation

$$\mathbf{p_t} = \mathbf{p_{t-1}P}, \text{ for } t=1,\ldots,T \quad (2)$$

where
$\mathbf{p_t}$ is the state vector defined by the diameter distribution in terms of the number of trees per unit area in different size classes at time t
P is the transition probability matrix of trees between states of trees

T is the number of periods in a projection

The target diameter is represented by the midpoint diameter of the largest diameter class i.e. 49.3 cm at the age of 100 years (see Table1).

The relative area distribution to diameter classes is computed recursively by solution of formula 2 where the transition probabilities determine the area distribution to diameter classes assuming steady state. The solution to the corresponding set of n recursive equations (n = number of diameter classes) is obtained by application of linear programming (LP) hereby assuring sustainability of the optimal solution.

The size of the regeneration harvest is computed by use of the transition probability estimated for the largest diameter class.

The Berger-Parker index is viewed as one of the most satisfactory indices of diversity (Berger & Parker 1970). It is defined as:

$$BP_i = \frac{\sum_{i=1}^{N} A_i}{Max(A_i)} \qquad (3)$$

where

A_i proportion of area in diameter class i

N number of diameter classes

The maximum value of the index is found when the distribution to classes is even. In this case the value is equal to N. The minimum value is 1 representing no diversity with the entire area is found in one single class.

Results

The solutions fulfilling the GR with respect to stem number per ha (N), standing volume per ha (V), decreasing area distribution (A-), increasing area distribution, height (H), and diameter, - in adjacent diameter classes, respectively, are shown in Table 2 and illustrated in Figs. 3-8. The characteristics of the normal forest (NF) are given as reference.

Table 2. Diameter class midpoint (DC), stem number per ha (N), area (AR), height (H), standing volume per ha (V), age (A), annual thinning volume per ha (TV), and annual target diameter harvest volume per ha (TD) – by diameter class - for GR structural strategies (S): stem number (N), volume (V), decreasing area (A-), increasing area (A+), height (H), diameter (D), and normal forest (NF). The Berger-Parker index (BP) of each strategy is shown in the first column. (Total=T, mean=M.)

BP	DC	N	AR	H	V	A	TV	TD
	cm	ha^{-1}	ha	m	m^3 ha^{-1}	yrs	m^3 ha^{-1} yr^{-1}	m^3 ha^{-1} yr^{-1}
S	N							
4.7	1.9	555.4	0.056	2.8	0.0	9.6	0.0	
	4.3	275.2	0.045	5.4	0.0	14.3	0.0	
	6.8	170.1	0.045	7.8	2.2	19.9	0.2	
	9.3	117.0	0.050	10.0	4.0	24.5	0.4	
	12.3	94.1	0.065	12.6	7.1	30.0	0.5	
	16.5	74.3	0.083	15.9	12.5	37.2	0.7	
	21.5	62.2	0.112	19.4	22.8	45.9	0.9	
	28.9	49.8	0.145	24.0	40.2	59.0	1.2	
	37.6	39.4	0.186	28.2	64.7	76.0	1.4	
	49.6	28.2	0.215	32.0	91.6	101.6	1.7	4.1
T/M	27.5	1,465.5	1.000	21.1	245.1	58.0	7.0	4.1
S	V							
2.7	2.1	209.7	0.021	1.3	0.0	6.7	0.0	
	2.9	209.7	0.021	2.1	0.1	8.3	0.0	
	3.8	183.4	0.018	2.8	0.1	9.9	0.0	
	4.6	103.7	0.016	3.5	0.1	11.4	0.0	
	5.5	25.6	0.005	5.8	0.1	15.9	0.0	
	5.8	33.7	0.007	6.3	0.2	16.9	0.0	
	6.3	59.5	0.014	7.2	0.5	18.7	0.0	
	7.5	112.9	0.034	8.8	2.1	22.0	0.2	
	10.5	160.1	0.084	11.7	8.4	28.1	0.7	
	17.8	110.7	0.141	16.5	22.8	38.7	1.2	
	27.9	96.6	0.268	23.1	69.8	49.5	2.2	
	50.7	45.9	0.371	31.8	156.7	100.3	2.8	3.6
T/M	30.3	1,351.5	1.000	22.0	260.8	60.2	7.1	3.6
S	A-							
2.6	20.2	248.4	0.390	18.5	73.7	43.5	3.2	
	32.6	66.4	0.241	25.9	74.6	66.0	1.9	
	40.3	28.4	0.149	29.3	54.8	81.6	1.1	
	45.1	14.2	0.092	30.9	36.8	91.0	0.7	

	48.0	7.8	0.057	31.6	23.8	96.3	0.4	
	49.8	4.5	0.035	32.0	15.0	99.2	0.3	
	51.0	2.7	0.022	32.2	9.4	104.4	0.2	
	51.7	1.6	0.013	32.4	5.9	105.8	0.1	2.2
T/M	32.2	373.9	1.000	24.7	294.2	66.1	8.0	2.2
S	A+							
2.6	0.7	134.4	0.013	1.5	0.0	7.1	0.0	
	1.8	217.5	0.022	2.8	0.0	9.8	0.0	
	3.6	283.0	0.035	4.7	0.0	13.7	0.0	
	6.6	231.1	0.057	7.6	2.5	19.5	0.2	
	11.3	154.6	0.092	11.8	9.2	28.2	0.7	
	19.0	104.5	0.149	17.7	26.4	41.6	1.2	
	31.5	70.5	0.241	25.4	72.4	64.3	1.9	
	51.7	46.7	0.390	32.4	170.4	105.8	3.1	2.2
T/M	32.2	1,242.3	1.000	23.1	280.9	67.5	7.2	2.2
S	H							
2.4	0.5	89.8	0.009	1.1	0.0	6.1	0.0	
	1.1	145.3	0.015	1.8	0.0	7.6	0.0	
	2.0	309.9	0.041	2.9	0.0	10.0	0.0	
	4.7	180.1	0.029	4.6	0.0	13.6	0.0	
	6.6	200.0	0.049	7.5	2.1	19.4	0.2	
	11.0	223.2	0.127	12.1	13.3	29.0	1.0	
	23.0	154.7	0.305	19.7	63.2	46.5	2.5	
	50.5	53.0	0.425	31.8	179.5	100.0	3.3	3.5
T/M	30.4	1,356.0	1.000	21.7	258.1	62.3	7.0	3.5
S	D							
3.1	0.6	104.8	0.010	1.3	0.0	6.5	0.0	
	1.0	94.3	0.009	1.7	0.0	7.6	0.0	
	1.7	143.8	0.014	2.5	0.0	9.2	0.0	
	2.7	174.5	0.023	3.7	0.0	11.8	0.0	
	4.4	308.9	0.047	5.5	0.0	15.4	0.0	
	7.2	216.1	0.061	8.1	3.3	20.6	0.3	
	11.6	158.4	0.099	12.0	10.2	28.7	0.8	
	18.8	113.9	0.159	17.6	27.9	41.2	1.3	
	30.5	79.8	0.258	24.8	75.1	62.3	2.1	
	49.3	41.5	0.319	31.9	135.4	101.0	2.5	3.8
T/M	28.5	1,435.9	1.000	21.5	251.9	60.2	7.0	3.8
S	NF							
10.0	4.3	687.0	0.100	5.3	0.0	15.2	0.0	
	9.3	233.6	0.100	10.0	8.0	24.6	0.7	
	14.3	114.5	0.100	14.2	12.9	34.0	0.8	

	19.3	68.5	0.100	17.9	18.0	43.4	0.8	
	24.3	45.9	0.100	21.2	23.2	52.9	0.8	
	29.3	33.1	0.100	24.2	28.1	62.3	0.8	
	34.3	25.3	0.100	26.8	32.3	71.7	0.8	
	39.3	19.9	0.100	28.9	36.1	81.1	0.8	
	44.3	15.9	0.100	30.6	39.5	90.6	0.8	
	49.3	13.1	0.100	31.9	42.5	100.0	0.8	4.5
T/M	26.8	1,256.8	1.000	21.1	240.4	57.6	7.1	4.5

The number of diameter classes for the different distribution strategies varies between 8 for the A strategies and the H strategy to 12 for the V strategy.

The A strategies have the largest mean diameter of 32.2 cm and the NF strategy the smallest of 26.8 cm.

The greatest diameter class width of mature or near-mature trees is observed for the H strategy, which is not surprising because a height increase corresponding to the GR is relatively large when the height is large. The largest distance between diameter midpoints is found for the H strategy. See Fig. 3.

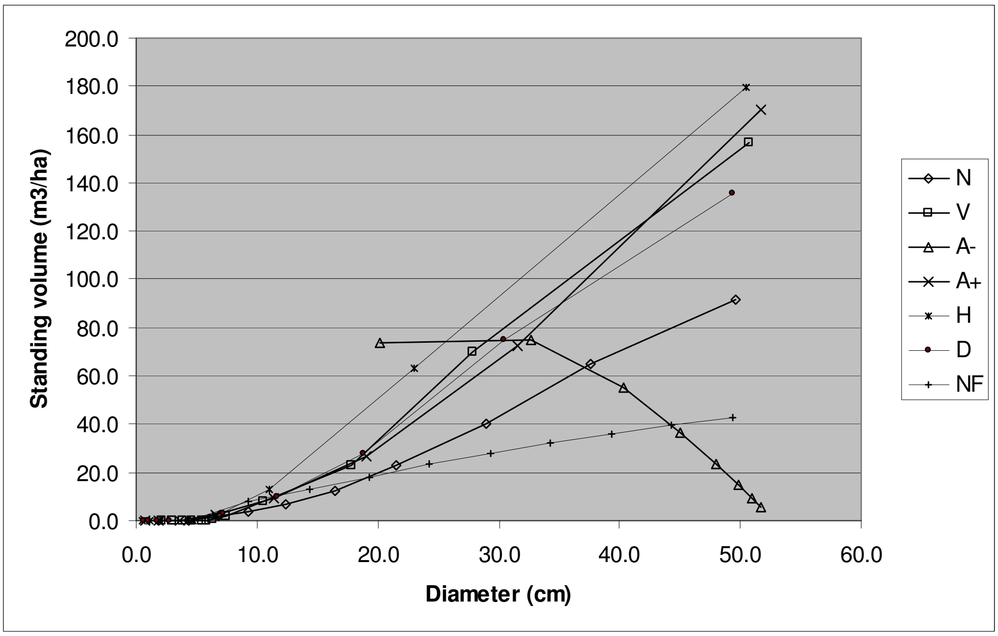

Figure 3. Standing volume per ha by diameter class.

The largest diameter class width of small trees is observed for the A-strategy where the area in the smallest diameter class and the next smallest fulfils the GR.

The stem number per ha fluctuates between a minimum of 374 for the A- strategy to 1,466 for the N strategy. Fig. 4 shows that the N strategy and NF strategy have a high stem number per ha because there are many small diameter classes in the N strategy and the NF strategy comprises relatively large area proportions of small diameter classes.

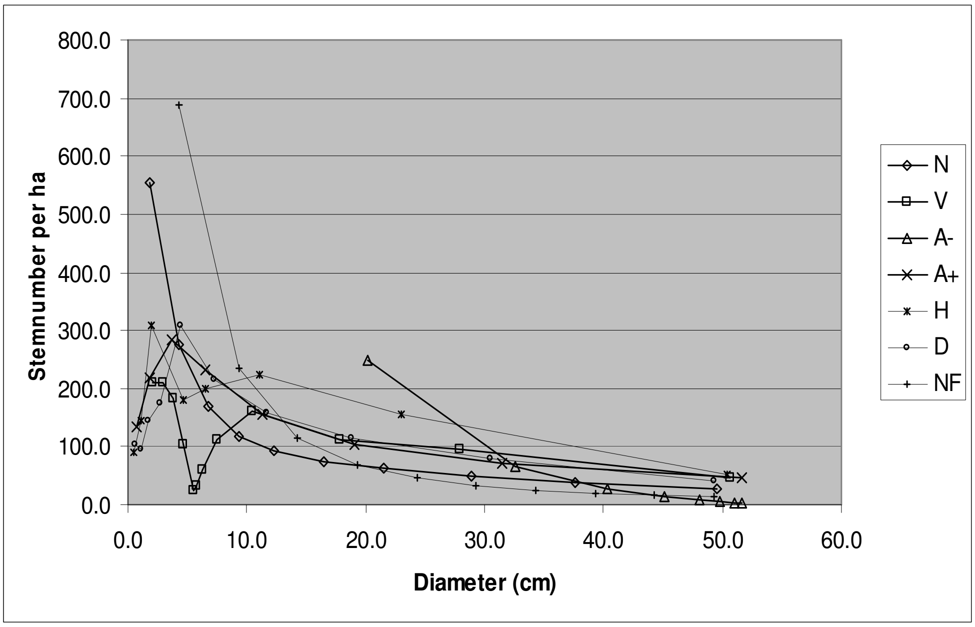

Figure 4. Stemnumber per ha by diameter class.

The area distributions are characterised by an overweight of large diameter classes except for the A- strategy with 1.3 per cent in the largest diameter class. The proportion of the largest diameter class of the remaining strategies varies between a maximum of 43 per cent for the H strategy and a minimum of 22 per cent for the N strategy. See Table 2 and Fig. 5. Fig. 5 also shows that the largest proportion of area in the mature and near-mature diameter classes is found for the H strategy.

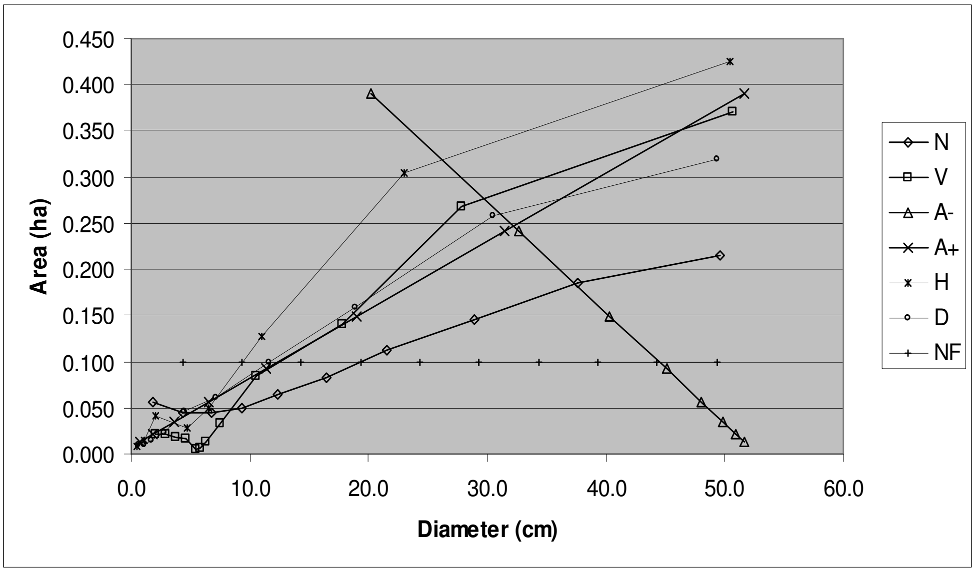

Figure 5. Area distribution by diameter class.

The area-weighted height varies from a minimum of 21.1 m for the N strategy to a maximum of 24.7 m for the A- strategy caused by a large mid-diameter of the smallest diameter class of this strategy.

The A- strategy results in the highest standing volume per ha of 294 m^3 ha^{-1} and the lowest is 245 m^3 ha^{-1} for the N strategy (the reference, NF, has a standing volume of 240 m^3 ha^{-1}). The high standing volume of the A-strategy is attributed to a relatively high standing volume in the small to medium diameter classes. See Figs. 3 and 8.

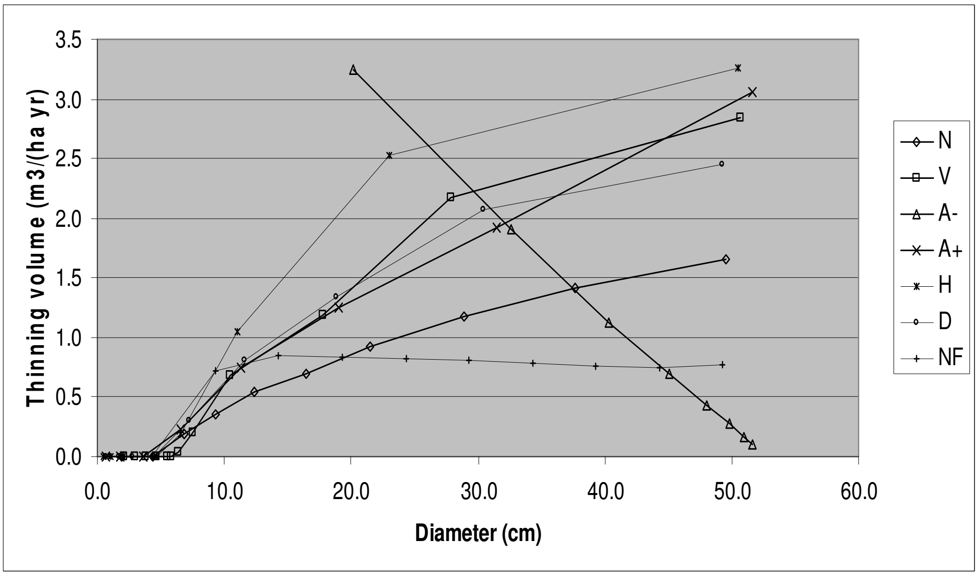

Figure 6. Thinning volume by diameter class.

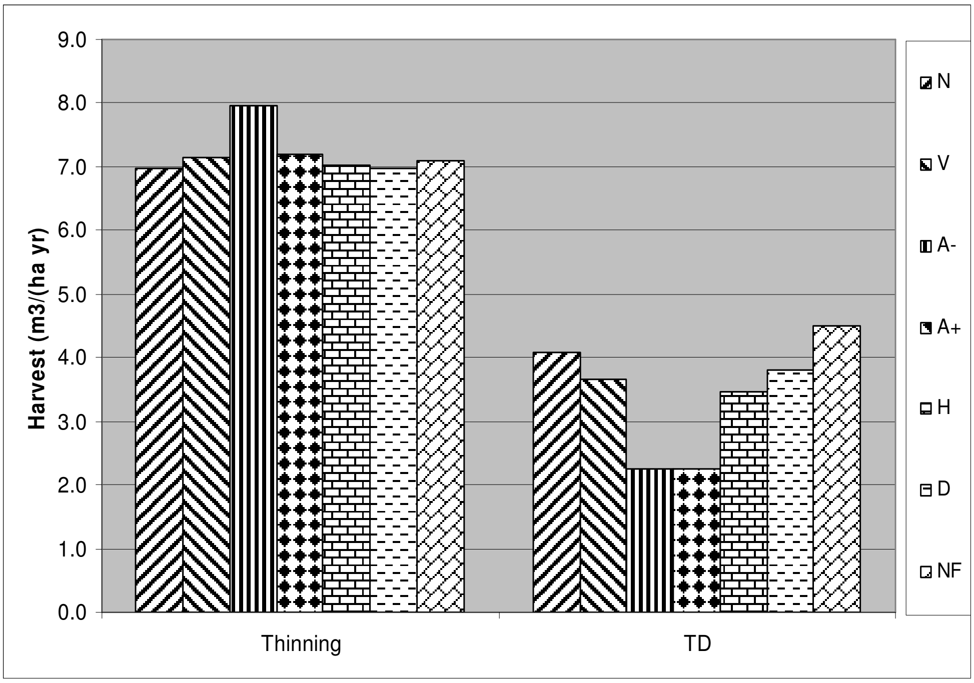

Figure 7. Total thinning and target diameter (TD) harvest per ha per year.

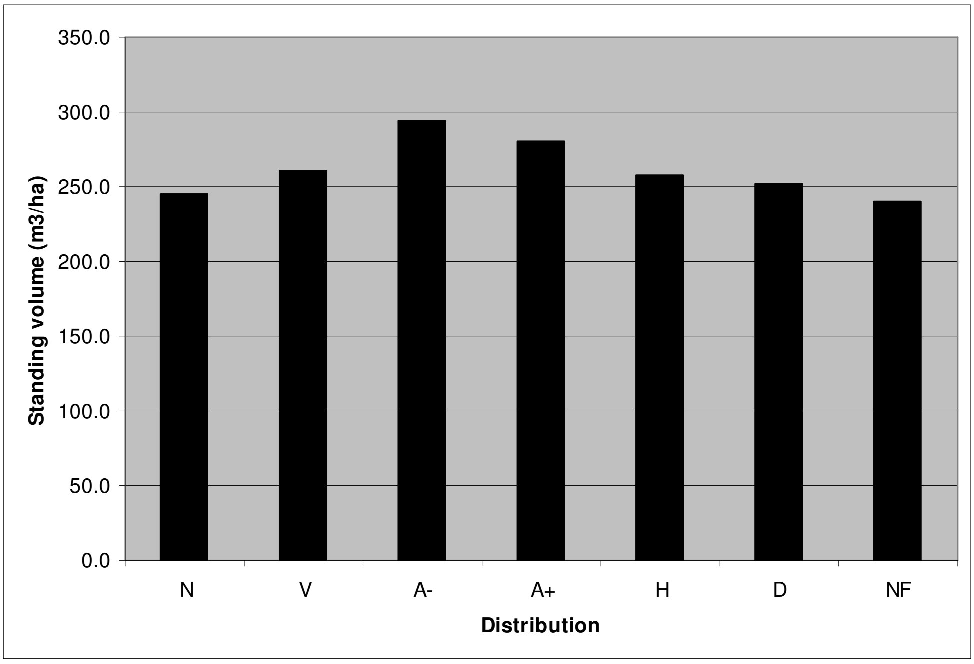

Figure 8. Total standing volume per ha.

The A+ strategy will result in the structure with the highest mean age of 68 years whereas the youngest forest with a mean age of 58 years is obtained when using the N strategy (the mean age of the NF reference strategy is also 58 years). The high mean age of the A+ strategy is attributed to the large and growing proportion of mature and near-mature diameter classes.

The largest thinning volume per ha per year of 8.0 m^3 ha^{-1} is achieved by the A- strategy because the area proportions of medium to small diameter classes dominate and the size of target diameter regeneration harvest volume is relatively small. This result is caused by the small proportion of mature and near-mature diameter classes. The thinning volume per ha per year does not differ much among the remaining strategies with an annual thinning volume per ha of around 7.0 m^3 ha^{-1}. See Fig. 7.

Fig. 7 also shows that the A- and A+ strategies result in the smallest regeneration harvest of 2.2 m^3 ha^{-1} $year^{-1}$ and the N strategy in the largest at 4.1 m^3 ha^{-1} $year^{-1}$ (the regeneration harvest volume of the NF reference strategy is 4.5 m^3 ha^{-1} $year^{-1}$).

Conclusion

The stem number distribution following from the N strategy characterised by the stem number per ha of adjacent diameter classes fulfilling the GR does not comply with the GR itself because the area distribution is uneven – as a consequence of the steady state requirement of the solution. The strategy results in an overweight of large diameter classes parallel to the majority of the remaining strategies. E.g., the relative stem number per ha is 2.02 (555/275) for diameter classes 1 and 2 and 1.40 (39.4/28.2) for diameter classes 9 and 10.

The A- strategy is an outlier in terms of most of the stand factors. It may, therefore, be questioned whether this strategy is realistic or attractive? The most conspicuous attribute is that the proportion of the smallest diameter class is 39 per cent with a mean height of 18.5 metres and a diameter of 20.2 cm. The strategy is realistic because the mean residence time of this extremely wide diameter class covering 39 per cent of the area is 76 years and the transition probability of moving to the next larger diameter class is equal to 0.01316 assuming a period length of 1 year. Similarly, the mean residence time of the largest diameter class, which covers 1.3 per cent of the area, is 2.6 years because the transition probability that a tree in this class moves into the smallest diameter class - interpreted as regeneration harvest – is 0.3819. Due to the fact that the largest proportion of the thinning volume comes from the smallest diameter class it is likely that this strategy performs poorly when applying the profitability criteria of maximizing the land expectation value.

The H strategy is assumed to be the most appealing from a physical or aesthetic point of view because the relative height of adjacent diameter classes fulfils the GR. However, the area distribution is dominated by the large diameter classes parallel to most of the other strategies – 32 per cent is covered by the largest diameter class and only 1 per cent by the smallest diameter class.

The H and A+ strategies are the most capital intensive because the area distribution is dominated by large diameter classes that hold large amounts of valuable volume. The standing volume per ha in the largest diameter class amounts to 180 and 170 m^3 ha^{-1}, respectively, corresponding to 70 and 61 per cent of the total standing volume (see Table 2). The distribution obtained by use of the V strategy is less capital intensive because the standing volume in the largest diameter class is lower than that of the A+ strategy and the regeneration harvest of the V strategy is more than 50 per cent larger.

The relative age of adjacent age classes of the D strategy approaches the GR as the age increases. If the age is adjusted by the age at breast height of 6.1 years then the GR is approximately fulfilled for the relation between diameter and age (see Table 2). E.g., the relation between the diameters of 7.2 cm and 4.4 (1.64) is approximately as the relation between the ages above breast height of 14.5 years (20.6 – 6.1 years) to 9.3 years (15.4 – 6.1 years) (1.56), i.e. approximately the GR. The GR is expressed more closely for the two largest diameter classes. The ratio of diameter mid-point values is 1.6164 and the ratio of the breast height adjusted ages is equal to 1.6886 ((101 – 6.1) years/(62.3 - 6.1 years)). The reason is that the thinning regime implies an approximate linear relationship between age and diameter.

The best performing strategy with respect to diversity is the N strategy with a weighted (by number of diameter classes) BP index of 0.47 (see Table 2) and the lowest diversity is obtained by use of the H strategy with a weighted BP index of 0.24. The NF reference strategy naturally comprises maximum diversity with a weighted BP index of 1.000. It is concluded that the best performing strategy for a forest owner emphasizing ecological objectives is the N strategy.

The proportion of annual regeneration areas for the six strategies is: 0.95 per cent (N), 0.43 per cent (V), 0.51 per cent (A- and A+), 0.82 per cent (H), 0.90 per cent (D), and 1.06 per cent (NF reference), which indicate that the N strategy implies the highest rate of regeneration establishment and consequently the highest regeneration costs per ha per yr.

Discussion

The results described above are based on the assumption of the silvicultural treatment encompassed in Table 1.

An alternative approach consists of seeking a strategy based on the assumption of fulfilling the GR of a given stand factor while adjusting the silvicultural treatment. This approach has been applied for stem number (N). I.e., it is assumed that N in adjacent diameter classes fulfils the GR. Thinning volume is derived from the resulting N reduction per period where the diameter of thinning is assumed to be 0.9 times the diameter of the stand before thinning. A fixed period length of 10 years is assumed. The Eichorn rule is assumed, i.e. the total volume production is a function of height alone. The form factors applied as a function of height as shown in Table 1 are assumed.

Based on these assumptions it is found that the diameter growth is strongly affected by the stem number per ha in the largest diameter class, when this stem number is selected as the final goal of the distribution characterized by the GR. E.g., when the stem number per ha following from the data in Table 1 at the age of 140 years (66) is chosen as the goal for the largest diameter class then the diameter of the largest diameter class is equal to 42.4 cm. This is not realistic in terms of target diameter. A minimum target diameter of 50 cm is assumed to be realistic.

It is found that the target diameter is highly sensitive to the stem number per ha in the smallest diameter class. Therefore, the stem number distribution fulfilling the GR is alternatively constructed on the basis of the initial stem number – in the smallest diameter class, instead of the final stem number – in the largest diameter class. The criteria for the stem number per ha in the smallest diameter class is that the stem number should be in the interval from 5,000-10,000 per ha. The regeneration stem number based on planting according to Anon. (2003) is 5,600.

Natural regeneration establishment is assumed to be initiated when the basal area becomes lower than 19 m^2 ha^{-1}. Based on the approach above, it is found that the age of natural regeneration is independent on the initial stem number per ha, when the stem number in the lowest diameter class is in the interval from 5-10,000 per ha. Natural regeneration starts at the age of 64 years (with basal area = 19 m^2 ha^{-1}). However, the target diameter is obtained earlier (at a lower age) when the stem number per ha in the smallest diameter class is small. Assuming an initial stem number of 10,000 per ha results in a target diameter of approximately 50 cm obtained at the age of 115 years. An initial stem number of 5,000 per ha results in a target diameter of 50 cm at the age of around 99 years. The minimum basal area over the age of 64 years is in this case 15 m^2 ha^{-1}, which makes the assumption of the Eichorn rule questionable because the growth and yield of beech is reduced considerably when the basal area is lower than approximately 22.5 m^2 ha^{-1} (Tarp et al. 2005). However, the relationship between basal area and growth is not linear. E.g., a basal area reduction from 22.5 m^2 ha^{-1} to 20.3 m^2 ha^{-1} (10 per cent) results in a growth reduction of less than 10 per cent.

The GR may be represented vertically by comparison of tree height with stem height. Obviously, relative stem height is dependent on thinning intensity. The stronger the thinning intensity the lower the relative stem height. Henriksen (1988) presents a summary of Danish thinning experiment data where the relative stem height is approximately 40 per cent when the applied thinning intensity corresponds to 50 per cent of the maximum basal area (50 m^2 ha^{-1}). The site index is approximately 1 and may therefore be compared with the data in Table 1. It is seen that the thinning intensity applied by Anon. (2003) corresponds to a relative basal area of approximately 50 per cent – from 21.5 to 25.8 m^2 ha^{-1} in the age interval from 50 to 140 years. The relative stem height reflected by the thinning regime is in accordance with the GR in the sense that the ratio of tree height to crown height is approximately the GR (100/60 = 1.67). The stem height/crown height data of Henriksen (1988) seem to be independent of age for the thinning regime described above, with a ratio of height to crown height at 1.63 at 38 years, 1.59 at 68 years, and 1.61 at 87 years. If the thinning regime depicted in Table 1 is normal and common in practice, then it is concluded that the GR is reflected in the goal of practical forest management. The vertical tree form obtained by application of the regime represents the GR as illustrated in Fig. 1 on tree number two from the right with the crown height marked with a dashed line. However, as illustrated by e.g. Larsen & Nielsen (2007) the crown form of a managed beech forest is normally longitudinal, i.e. the crown height is longer than the crown width, rather than circular.

Zelic (2006) found that the diameter growth of beech in Croatia as a function of age follows the golden section. This result is confirmed by the analyses of the D strategy above for beech in Denmark.

Future research with alternative approaches concerning the implications of the GR with respect to the different stand factors could potentially contribute to improved forest management through formulation of new thinning/harvesting regimes and guidelines. E.g., the scenario described above with a rotation age of around 99 years may be realistic but an economic evaluation would be desirable. Investigation of the economic implications is, however, beyond the scope of this paper. The approach may be constructive in analysing the structural characteristics of forest development types as outlined by Larsen & Nielsen (2007).

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