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THE OPTIMAL CONTROL OF A STOCK POLLUTANT WITH SUNK CAPITAL AND ENDOGENOUS RISK OF CATASTROPHIC DAMAGES

by

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Abstract

In this paper we characterize the optimal rate of emission of a pollutant whose stock poses a threat of damages of an unknown magnitude in the future, when abatement capital is sunk, the stock of pollutant is non-degradable, and there is an endogenous risk of catastrophic damages. The economic agent wants to avoid two situations: (i) building up a large stock of abatement capital when damages turn out to be negligible; (ii) allowing too large a build-up in the stock of a non-degradable pollutant when damages are revealed to be catastrophic. Unfortunately, the stock of pollutant cannot be reduced unless the agent invests in abatement capital. Given this trade-off, and the added feature that the probability of a catastrophe occurring may be endogenous, i.e., dependent on the pollutant stock which is in turn dependent on past investment decisions, our paper asks what should be the optimal rate of emission of the pollutant. Loosely speaking, we find a stronger effect on the optimal rate of emission associated with the accumulation of the pollutant than with investment in abatement capital.

1. Introduction

In this paper we characterize the optimal rate of emission of a pollutant whose stock poses a threat of damages of an unknown magnitude in the future, when abatement capital is sunk, the stock of pollutant is non-degradable, and there is an endogenous risk of catastrophic damages. These are features of the decision problem with regard to a number of different environmental threats. Policymakers in charge of nuclear waste management must decide whether to sink resources into building underground nuclear waste repositories or to leave the radioactive waste at the plant site where people will be exposed to low levels of radiation. This decision must take into account the fact that radioactive waste decays slowly, that there is uncertainty about whether or not exposure to low levels of radiation over a long period of time can result in radiation poisoning, and that the risk of radiation poisoning is avoidable (Makhijani 2000). Policymakers concerned with the as-yet-uncertain threat posed by global climate change need to decide whether to invest more heavily in abatement capital or to allow a further build-up of a stock of greenhouse gases. This decision must allow for the possibility that climate change events may lead to catastrophic damages. Finally, since anthropogenic sources are leading to the accumulation of the stock of greenhouse gases in the atmosphere, the risk posed by global climate change is endogenous. This fact too must enter the decision calculus. Similar issues arise with the management of toxic chemicals and with the control of emissions of pollutants that deplete stratospheric ozone.

Abatement capital is considered to be sunk if resources, once invested, cannot be converted to consumption or other forms of capital. The stock of pollutant is said to be non-degradable if it cannot be reduced through abatement and if it does not decay naturally. Policymakers want to avoid two situations: (i) building up a large stock of abatement capital when damages turn out to be negligible; (ii) allowing too large a build-up in the stock of a non-degradable pollutant when damages are revealed to be catastrophic. Unfortunately, the stock of pollutant cannot be reduced unless policymakers invest in abatement capital. Sunk capital and non-degradable pollutant stocks thus confront policymakers with a tradeoff. A third important concern is the extent to which the risk of potentially catastrophic damages is endogenous. If the probability of damages occurring depends on the behavior of economic agents, then the risk should be considered to be endogenous. The effect of endogenous risk and the potential for catastrophic impacts on the optimal rate of emission are a major focus of our paper.

In order to characterize the optimal rate of emission, we develop a multi-period stochastic model that incorporates the three features of the decision making environment that we have drawn attention to—sunk capital, non-degradable stocks of pollutant, and endogenous risk of catastrophic damages. The main differences from earlier studies of the optimal control of stock pollutants are our focus on rigidities, or irreversibilities, in both investment and pollution, and the joining of these rigidities with endogenous risk of catastrophic impact. Conrad (1992) studies the effect of non-degradable pollutant stocks on the optimal rate of emissions, but his model does not include capital, the potential for catastrophic damages, or endogenous risk. Clarke and Reed (1994) consider what we have called endogenous risk, in a model of an accumulating pollutant that can trigger an irreversible environmental catastrophe, but both capital and the stock of the pollutant are assumed to be fully reversible. A similar model is developed by Aronsson, Johansson and Lofgren (1997), with catastrophic damages, endogenous risk and two types of capital—trees and production capital. However, their model does not include abatement capital or the sorts of irreversibilities we emphasize here.

Our model does not allow agents to act on new information, but nonetheless generates changes in the optimal steady state rate of emission. Although we do find an 'irreversibility effect' (defined as a change in the optimal rate of emissions caused by capital and pollutant stock rigidities) even in the absence of learning, we recognize that a model with learning would perhaps be more realistic. Adding learning to our model, which already has two state variables, capital and the stock of pollutant, would however make the analytics intractable. In order to incorporate learning we would have to either restrict ourselves to a two-period model or to numerical simulations, as in recent contributions to the literature on control of greenhouse gases (Kolstad (1996a, 1996b); Ulph and Ulph (1997)). Although we mentioned this application earlier, our view is that the primary issue with respect to control of greenhouse gases is essentially how soon, and how stringently, to control emissions. The steady state, which may be well in the future, is probably of less concern. Elsewhere we develop a two-period analytical model, and a multi-period numerical simulation model, a revised and extended version of Nordhaus' (1994) DICE model, to address the question of whether and how stringently to control greenhouse gases today and in the near future. Learning about future damages is a feature of both models.

Briefly, in the present paper we find that with abatement capital and non-degradable pollutant stocks, an increase in the degree of capital 'sunkness' does not affect the optimal steady state rate of investment while an increase in the degree of stock non-degradability calls for an increase in the optimal steady state rate of investment. These results continue to hold, under certain sufficient conditions, when the risk of a catastrophe is endogenous, but not under alternative definitions for sunk abatement capital and non-degradable pollutant stocks that have been applied in the literature.

The rest of the paper is organized as follows. The next section contains a description of the theoretical model. Section 3 considers the effect of sunk capital, and section 4 the effect of a non-degradable pollutant stock on the optimal rate of investment. Section 5 repeats the analysis in sections 3 and 4 with the added feature that the risk of a catastrophe is endogenous. Conclusions are presented in section 6.

2. Theoretical Model

Agents derive utility from consumption and disutility from the stock of pollutant so long as a catastrophe does not occur. If and when a catastrophe occurs utility is driven to zero forever.[1] Catastrophic damages are therefore irreversible; once catastrophic damages have occurred they exist forever, and at the same magnitude as at the time of their occurrence. The only source of uncertainty in the model is whether or not a catastrophe will occur. The extent of catastrophic damages, should a catastrophe occur, are known in advance.

The agent receives a fixed endowment of resources in every period which she allocates between consumption and investment in capital used to abate the flow of the pollutant. Abatement capital is either convertible or sunk. Only the former can be converted back into consumption, though at a cost. If not abated, emissions, a by-product of consumption, add to the stock of pollutant which is either degradable or non-degradable. Only degradable stocks decay naturally over time and neither type of stock can be abated.[2] In addition to causing disutility in every period up until a catastrophe occurs, the stock of pollutant also affects the probability that a catastrophe will occur when the risk of a catastrophe is endogenous.

2.1. **Primitives.** Up until a catastrophe occurs, a representative agent derives utility from consumption, C, and disutility from the stock of pollutant, M. Let the momentary utility function $U = U(C, M)$ satisfy the conditions

$$\begin{aligned} &U_1(C,M) > 0, \qquad U_{11}(C,M) < 0, \qquad U_{12}(C,M) < 0 \\ &U_2(C,M) < 0, \qquad U_{22}(C,M) < 0 \end{aligned}$$

where subscripts denote differentiation. As long as there is no catastrophe, catastrophic damages are zero and the utility function is unaffected. After a catastrophe however, utility goes to zero forever.

[1]The assumption that the catastrophe drives utility to zero does not significantly affect our results. Any constant, nonzero level of utility could be substituted instead with no effect on results. We can show that a weaker, and perhaps more plausible, assumption, that utility after the catastrophe is a concave function of the levels of the stock of abatement capital and the stock of pollutant at the time of the catastrophe, leads to a similar (though a bit messier) result. For simplicity of exposition we use the assumption of zero utility.

[2]We do not allow the stock of pollutant to be abated in order to simplify our analysis. This assumption allows us to move from a model where the stock of pollutant is degradable to one where the stock is non-degradable by simply changing the rate of decay of the stock of pollutant. In the absence of this assumption we would have to introduce a different equation of motion for degradable stock of pollutant and thus a new set of optimality conditions. Our results are not affected by this simplification.

A fixed amount of output, R, is available each period for either consumption or investment, I, in abatement capital. The budget constraint is thus $C + I = R$. Abatement capital, K, changes from one period to the next as a result of investment and depreciation according to

$$\dot{K} = I - \delta_K K \tag{1}$$

where δ_K is the rate of depreciation of capital. For the base model we assume that capital is reversible. This means that at any time consumption can be greater than the fixed amount of resource R or that investment can be negative. Specifically,

$$C \leq R + \Phi K \tag{2}$$

where Φ is a parameter that governs the cost of converting capital into consumption. When $\Phi = 0$ it is infinitely costly to convert capital into consumption and when $\Phi \geq 0$ capital can be converted into consumption, though at a cost.[3]

Emissions are a by-product of consumption. Let $g(C)$ be the emissions function where $g_1(C) > 0$ and $g_{11}(C) = 0$. If unabated, emissions increase the stock of pollutant. Let $H(K)$ be the abatement function where $H_1(K) > 0$ and $H_{11}(K) \leq 0$. Capital abates only the flow and not the stock of pollutant, implying that the amount of pollutant abated in a period cannot exceed the amount emitted in that period. The abatement function $H(K)$ thus lies between zero and one. The stock also decays naturally. Consequently, the law of motion for the stock of pollutant is given by[4]

$$\dot{M} = g(C)\big(1 - H(K)\big) - \delta_M M \tag{3}$$

where $g(C)(1 - H(K))$ are net emissions and δ_M is the natural decay rate of the pollutant. If the rate of decay is close to zero, then the stock of pollutant is considered to be non-degradable.

[3]Our representation of sunk capital assumes symmetric but increasing adjustment costs. Irrespective of whether investment is positive or negative a unit of investment can be exchanged for a unit of consumption. This implies that adjustment costs are symmetric. However, depending on the magnitude of Φ adjustment costs are low when a small amount of capital is converted into consumption and infinitely large as the amount converted increases. An alternative formulation would be to have asymmetric but constant adjustment costs. Asymmetric adjustment costs imply that when investment is positive a unit of consumption can be exchanged for a unit of investment and when investment is negative a unit of investment gives less than a unit of consumption. However, irrespective of the quantity of capital converted into consumption, the adjustment costs remain constant. We feel both formulations are valid representations of sunk capital. We have chosen to use the former for its mathematical tractability.

[4]By restricting the decay rate to be a linear function of the stock of pollutant we are in fact assuming that there is a unique steady state for the stock of pollutant. See Tahvonen (1995) for a discussion of multiple steady states with non-convex decay functions.

Finally, there always exists the possibility of a catastrophe occurring. The occurrence of a catastrophe is captured by a jump process as

$$dD = \begin{cases} 1 & \text{with probability } pdt, \\ 0 & \text{with probability } (1 - pdt). \end{cases} \tag{4}$$

where D is an indicator of the catastrophe (initially $D = 0$) and p is the conditional probability for an immediate occurrence of a catastrophe. If the risk of a catastrophe is exogenous then p is a constant. With endogenous catastrophic risk the conditional probability for an immediate occurrence of a catastrophe is an increasing and convex function of the stock of pollutant. That is, $p = p(M)$ with $p_1(M) > 0$ and $p_{11}(M) > 0$.[5]

2.2. **Objective.** The agent chooses a stream of consumption to maximize expected intertemporal utility subject to equations (1)–(4). The only source of uncertainty is whether or not a catastrophe will occur. The Bellman-Hamilton-Jacobi equation for this problem is

$$(r + p)V(K, M) = \max_{C \leq R + \Phi K} \left[U(C, M) + \mu_1 \dot{K} + \mu_2 \dot{M}\right] \tag{5}$$

where $V(K, M)$ is the value function, r is the discount rate, μ_1 is the co-state variable associated with the stock of abatement capital and μ_2 is the co-state variable associated with the stock of pollutant.[6]

2.3. **Optimality Conditions.** In this subsection we establish optimality conditions for consumption, capital and the stock of pollutant. For now we assume that risk is exogenous and so p is a constant. We begin by differentiating equation (5) with respect to the choice variable—consumption. This gives the following first order condition

$$U_1(C, M) - \lambda - \mu_1 + \mu_2 g_1(C)\big(1 - H(K)\big) = 0 \tag{6}$$

where λ is the Lagrange multiplier on the consumption constraint given by equation (2). The co-state equations of motion, obtained by differentiating equation (5) with respect to the state variables, K and M, are

[5] Tsur and Zemel (1996) consider the effect of a different type of catastrophic risk on the optimal rate of emissions. In their model uncertainty stems from not knowing what is the exact level of stock needed to trigger a catastrophe. They call this risk endogenous and contrast it with the stochastic process we consider which they refer to as an exogenous risk. We consider a situation where there is a lag between stock build-up and the time when the effects of that level of stock are felt. Consequently, we believe that modeling the risk as a stochastic process is appropriate.

[6] μ_1 is in fact equal to $V_1(K, M)$ while $\mu_2 = V_2(K, M)$. The derivation of the Bellman-Hamilton-Jacobi equation is given in the appendix.

$$\dot{\mu}_1 = \mu_1(r + \delta_K + p) - \lambda\Phi + \mu_2 g(C) H_1(K) \tag{7}$$

$$\dot{\mu}_2 = \mu_2(r + \delta_M + p) - U_2(C, M) \tag{8}$$

We are now in a position to characterize the steady state.

2.4. **Steady State.** Equation (6), and equations (7) and (8) at the steady state, combine to give the Euler equation, which in turn with the steady state laws of motion gives a system of equations which determines the steady state level of consumption, capital and stock of pollutant (the arguments are suppressed for compactness)

$$U_1 = -\frac{U_2}{(r + \delta_M + p)}\left(g_1(1 - H) + \frac{gH_1}{r + \delta_K + p}\right) + \lambda\left(\frac{\Phi}{(r + \delta_K + p)} + 1\right) \tag{9}$$

$$K^* = \frac{R - C^*}{\delta_K} \tag{10}$$

$$M^* = \frac{g(1 - H)}{\delta_M} \tag{11}$$

where stars denote steady state levels. When the constraint on consumption is not binding, $\lambda = 0$ and the Euler equation states that along the steady state consumption trajectory there is nothing to gain by increasing consumption. Equation (10) states that at the steady state, investment is equal to capital depreciation while Equation (11) states that net emissions are equal to the decay in the steady state stock of pollutant.

When the constraint on consumption is binding, $\lambda > 0$ and steady state consumption is equal to $C^* = R + \Phi K^*$. Since negative investment period after period drives the steady state capital stock to zero, steady state consumption is in fact equal to R.

3. Sunk Capital

In this section we explore the implications of capital being sunk. We find that these are weaker under our suggested definition than they are under an alternative definition employed in a related literature.

3.1. **Defining Sunk Capital.** One possible definition of sunk capital is that capital is sunk if it is durable, that is, if it has a low rate of depreciation (Kolstad (1996a, 1996b)). In our judgment this definition fails to capture the essential problem faced by policymakers, who we assume wish to avoid a situation where resources invested in abatement capital cannot be converted back into some productive use in the future, or into consumption, if damages turn out to be negligible. What matters is the cost of conversion, not depreciation. Durable capital may still have a low conversion cost. We therefore prefer to define sunk capital as capital that is prohibitively costly to convert into consumption.[7] Or, capital is considered to be sunk if investment is constrained to be positive in every period.

3.2. **Positive Investment.** We now show that, under our definition, a decrease in capital convertibility has no effect on the optimal steady state rate of investment. We capture convertibility through the parameter Φ, with an increase in Φ implying greater convertibility. Capital is perfectly sunk when $\Phi = 0$.

The parameter Φ enters the steady state system of equations only as part of the coefficient on the consumption multiplier, λ. This implies that if the constraint on consumption is not binding then Φ does not affect the steady state level of consumption or investment. The constraint on consumption is less likely to bind the greater is the disutility associated with the stock of pollutant because agents will want to undertake investment to reduce the disutility from the stock of pollutant.

In fact, this result holds even when the constraint on consumption is binding at the steady state. Recall that when the constraint binds the steady state level of consumption is in fact equal to R and is thus independent of Φ. Out of steady state and when the consumption constraint binds, Φ may have an effect on the rate of consumption. So long as $\Phi > 0$, agents will drive the inherited capital stock to zero very quickly. The level of investment, while the system is out of steady state, remains independent of the level of Φ at zero. Out of steady state, the level of consumption (and thus emissions) however now depends on Φ. This is because the level of Φ determines how much consumption can be had from a given amount of capital. When the consumption constraint binds and $\Phi = 0$, agents have no option but to wait for capital depreciation to drive the stock of capital to zero. Once again, there is no effect on the level of investment out of steady state. Even the level of consumption out of steady state is unaffected. Capital cannot add to consumption since it is perfectly sunk.

Simulations based on a simple parameterization of the model presented in section 2 help illustrate these results.[8] The first subplot in Figure 1 shows that when the initial stock of abatement capital is low, the ratio of consumption under convertible capital to consumption under sunk capital is one during the transition to and at the steady state. However, if the initial stock of capital is high then consumption is at first higher and then lower under convertible capital as compared to consumption under sunk capital (see subplot 2 of Figure 1). This is because consumption is constrained to be less than or equal to R (for the simulations $R = 20$) when capital is perfectly sunk. The steady state level of consumption is the same for both types of capital. That said, the first subplot is usually

[7] Pindyck (1991) defines capital to be sunk if it cannot be used productively by a different industry. To avoid having to add another state variable we define non-convertibility in terms of the ability to switch between capital and consumption. Otherwise our definition matches that of Pindyck.

[8] The details of the simulation model and the Matlab code used for the simulation are available from the authors.

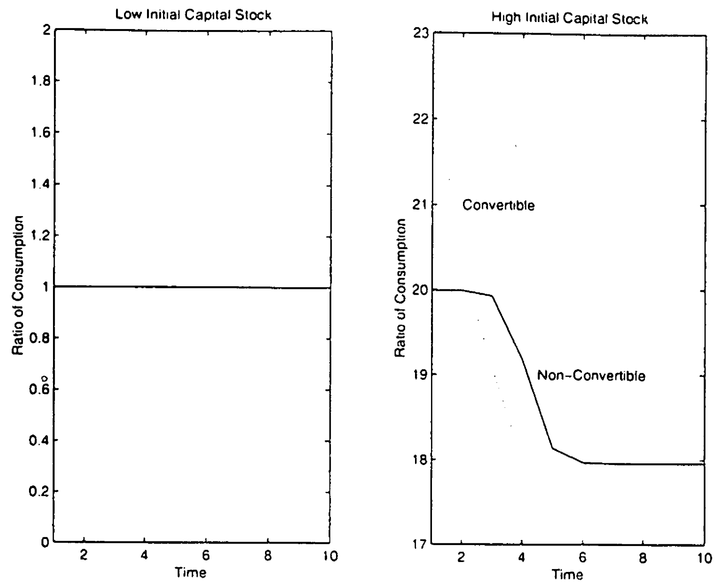

FIGURE 1. Consumption Paths for Convertible and Non-Convertible Stocks of Capital with Exogenous Risk

more reflective of the policy environment facing the policy maker. Rarely does the policymaker have to worry about there being too much abatement capital.

3.3. **Durable Capital.** We now explore the impact of durable capital on the optimal rate of investment. We begin by writing the Euler equation, equation (9), as a function of steady state consumption and system parameters (steady state capital and stock of pollutant gases are both functions of steady state consumption and system parameters from equations (10) and (11), respectively).[9] With this simplification the effect of a change in the degree of durability of capital on consumption is given by differentiating the Euler equation with respect to the rate of depreciation. The result is formalized in the following proposition.[10]

Proposition 1. *If* $-H_{11}(K^*)\frac{K^*}{H_1(K^*)} \geq \frac{\delta_K}{(r+\delta_k+p)}$ *then* $\frac{dC^*}{d\delta_K} < 0$.

In words the proposition states that steady state consumption will increase (and steady state investment will decrease) as capital becomes more durable if the gain from the increase in capital, caused by the decline in the rate of depreciation, is greater than the loss caused by the decline in the marginal product of capital. This result is fairly straightforward since the presence of durable capital reduces the need for new investment.

[9]We consider only an interior solution.

[10]Proofs for all the propositions are in the appendix.

The effect of capital becoming more durable on steady state stocks of capital and the stock of pollutant is ambiguous. Consequently, the effect on the optimal rate of net emissions, $g(C^*)(1 - H(K^*))$, of a change in the durability of capital is unknown.

4. Non-degradable Pollutant Stock

We next explore whether or not there is a change in the optimal rate of investment associated with a change in the degree of degradability of the pollutant stock. We find that the effect is stronger under our preferred definition of non-degradablility than under definitions employed in the literature.

4.1. **Defining Non-degradable Stock.** We require a near-zero decay rate and capital to abate only the flow of emissions for the stock of gases to be considered non-degradable. If the stock decays or if emissions are permitted to be negative, then the stock will dissipate over time and cannot be considered non-degradable. Conrad (1992) defines a non-degradable pollutant stock as one that has a near-zero decay rate but since his model does not allow for abatement he does not impose the additional requirement that capital abate only the flow of emissions. Kolstad (1996b) and Ulph and Ulph (1997) define a non-degradable stock of greenhouse gases as one where emissions are restricted to be non-negative and place no restriction on the rate of decay. Since Clarke and Reed (1994) and Aronsson et al. (1997) do not consider stock irreversibilities, they do not define non-degradable pollutant stocks.

4.2. **Rate of Decay.** We capture non-degradability of the stock of pollutant through the parameter δ_m with a decline in δ_m implying an increase in stock non-degradability. The effect, then, of a change in the degree of stock non-degradability can be studied by differentiating the Euler equation (expressed in terms of consumption and system parameters) with respect to the rate of decay. This yields the following proposition.

Proposition 2. *Steady state consumption is an increasing function of the rate of decay of the stock pollutant.*

As the stock of pollutant becomes non-degradable, consumption decreases while investment in abatement capital increases. A lower rate of decay implies that any emissions that are released into the atmosphere remain for a longer period of time. This in turn implies that agents have to incur the disutility of these emissions for a longer period. Agents thus choose to reduce consumption, the source of these emissions, and increase investment to reduce the stock of pollutant.

Figure 2 illustrates this result. The rate of decay is higher for the solid line and lower for the dotted line, though for both curves the rate of decay is positive. For our simulation model, under

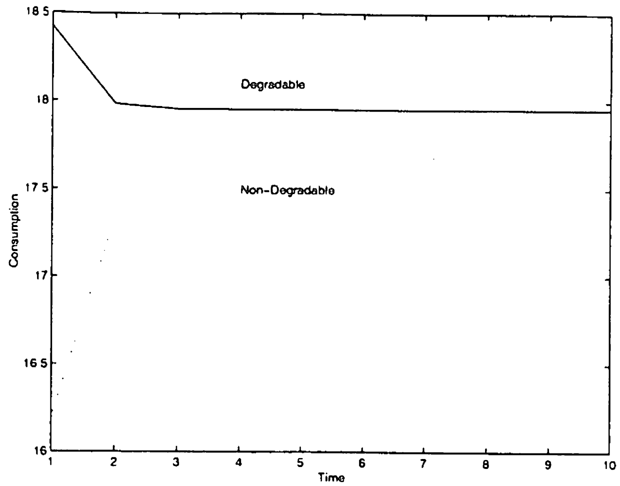

FIGURE 2. Consumption Paths for Degradable and Non-degradable Pollutant Stocks with Exogenous Risk

non-degradable pollutant stocks, not only is the steady state level of consumption lower, but so is consumption along the approach path, as compared to that under degradable pollutant stocks.

Along with investment the stock of capital increases and the level of net emissions decreases. However, the effect of an increase in the non-degradability on the stock of pollutant itself is ambiguous.

4.3. **Non-Negative Emissions.** If instead stock non-degradability is defined in terms of non-negative emissions with no restriction on the rate of decay, then an increase in non-degradability has no effect on the steady state level of consumption or investment. The constraint on emissions simply does not bind at the steady state. This can be seen from the following equation for the steady state stock of pollutant (assuming an interior solution).

$$M^* = \frac{g(C^*)(1 - H(K^*))}{\delta_M} \geq 0 \tag{12}$$

The steady stock of pollutant is restricted to be non-negative which in turn implies that steady state emissions will be non-negative (positive if the stock is positive and zero if the stock is zero). The constraint on emissions simply does not bind. A change in the degree of degradability does not affect steady state consumption or investment.

If agents choose to reduce the stock of pollutant to zero, and want to do so quickly, then in the transition to steady state the non-negativity constraint will bind. This situation will also arise when agents inherit a large stock of pollutant and want to reduce the stock quickly. In both these situations agents will prefer to emit negative amounts of the stock to reduce the stock as fast as possible. This will not be possible if the stock is non-degradable. Consequently, stock non-degradability may have an effect away from the steady state. However, for this effect to hold it must be true that a drastic reduction of the stock is optimal or that agents begin with a large endowment of the stock of pollutant.

5. Endogenous Risk

If it is true that the probability of catastrophe is increased with an increase in the stock of pollutant, then the threat can be mitigated by reducing the stock, that is, by economic agents changing their behavior. In other words, the risk of a catastrophe is endogenous. We now explore the implications of adding endogenous risk to the model.

5.1. **Optimality Conditions.** As with exogenous risk, we begin by differentiating equation (5) with respect to the choice variable—consumption. This gives the first order condition

$$U_1(C,M) - \lambda - \mu_1 + \mu_2 g_1(C)(1-H(K)) = 0 \tag{13}$$

The equations of motion for the co-state equations, obtained by differentiating equation (5) with respect to the state variables, K and M, are

$$\dot{\mu}_1 = \mu_1\big(r + \delta_K + p(M)\big) - \lambda\Phi + \mu_2 g(C) H_1(K) \tag{14}$$

$$\dot{\mu}_2 = \mu_2\big(r + \delta_M + p(M)\big) - U_2(C,M) - V(K,M)p_1(M) \tag{15}$$

To allow for endogenous risk, the conditional probability for an immediate occurrence of the catastrophe, p, is now a function of the stock of pollutant.

5.2. **Steady State.** Since equation (15) contains $V(K,M)$, to obtain the Euler equation we need an additional equation that relates the value function to the primitives of the economy. This additional equation is obtained by evaluating the Bellman-Hamilton-Jacobi equation, equation (5), at the steady state.

$$(r + p(M))V(K,M) = U(C,M) + \lambda(R + \Phi K - C) \tag{16}$$

Now equations (13) and (16), and equations (14) and (15), evaluated at the steady state, combine to give the Euler equation, which in turn with the steady state laws of motion gives a system of equations which determines the steady state level of consumption, capital and stock of pollutant.

(17)

$$U_1 = \frac{(U + \lambda(R + \Phi K - C))p_1 - (r+p)U_2}{(r+p)(r+\delta_M+p)}\left(g_1(1-H) + \frac{gH_1}{(r+\delta_K+p)}\right) + \lambda\left(\frac{\Phi}{(r+\delta_K+p)} + 1\right)$$

(18)

$$K^* = \frac{R - C^*}{\delta_K}$$

(19)

$$M^* = \frac{g(1-H)}{\delta_M}$$

When $\lambda = 0$ (the constraint on consumption is not binding) the Euler equation states that, along the optimal consumption path, net utility from an increase in consumption is zero. Equations (18) and (19) give arbitrage conditions for optimal stocks of capital and the pollutant. When the consumption constraint is binding ($\lambda > 0$), steady state consumption is once again equal to R.

5.3. **Positive Investment.** With or without a binding constraint on consumption, Φ, the parameter governing the cost of converting capital to consumption, does not affect steady state consumption, capital or stock of pollutant. Away from the steady state too, Φ does not affect investment decisions. If agents choose to consume all their endowment and devote nothing to investment then they will run down the capital stock at a rate dictated by Φ (quickly if Φ is high and slowly if Φ is low). However, Φ does not affect an agent's decision to invest nothing and consume all. As with exogenous risk, the degree of convertibility of abatement capital does not affect decisions to consume or emit the pollutant.

Figure 3 illustrates this result. The ratio of consumption under a convertible stock of abatement capital to consumption under a sunk stock is one. For our numerical model, at the steady state and away from it, a change in the degree of convertibility of capital does not affect the level of consumption or investment.

5.4. **Durable Capital.** Now let us consider the case where risk is endogenous and the rate of depreciation is used to capture capital convertibility. Once again steady state capital can be expressed as a linearly decreasing function of steady state consumption, equation (18), and the steady state stock of the pollutant as an increasing function, equation (19). This in turn implies that we can write the steady state Euler equation as a function of steady state consumption and some parameters.

To analyze the effect on the optimal rate of consumption of a change in the degree of durability of capital, we differentiate the Euler equation, equation (17), with respect to the rate of depreciation. The differentiation yields a complicated expression with an ambiguous sign. Adding endogenous

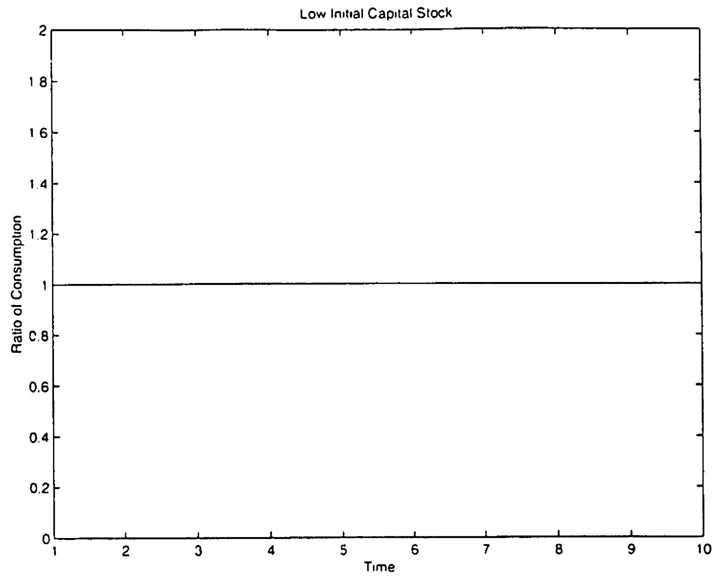

FIGURE 3. Consumption Paths for Convertible and Sunk Stocks of Capital with Endogenous Risk

risk thus dilutes the result that durable capital leads to a decrease in investment. Individuals may choose to increase investment in order to reduce the risk of the catastrophe. This counters the need to decrease investment as capital becomes more durable. Models that do not account for endogenous risk will find a stronger effect of durable capital on the optimal rate of emissions.

5.5. **Decay Rate.** Now let us consider how consumption at the steady state changes with a change in the degree of degradability of the stock of pollutant. This result is obtained by differentiating the Euler equation with respect to the rate of decay of the pollutant stock.

Proposition 3. *If* $\frac{-\partial p}{\partial M}\frac{\partial M}{\partial \delta_M}\left(\frac{(r+\delta_M+p)}{(r+\delta_K+p)}+\frac{(2r+2p+\delta_M)}{(r+p)}\right)<1$ *then* $\frac{dC^*}{d\delta_M}>0$

In words, as the stock of pollutant become non-degradable, consumption decreases while investment increases if a reduction in the rate of decay leads to a relatively small increase in the conditional probability of a catastrophe. Consequently, an increase in the degree of degradability of the stock of pollutant lowers consumption when the risk is endogenous so long as the increase in non-degradability does not cause a large increase in the conditional probability of catastrophe. If risk does increase rapidly, then it may be optimal to increase consumption today rather than wait for a tomorrow that may never come.

Figure 4 illustrates that a change in the degree of stock degradability can affect the level of consumption at, as well as away from, the steady state. The solid line corresponds to a higher

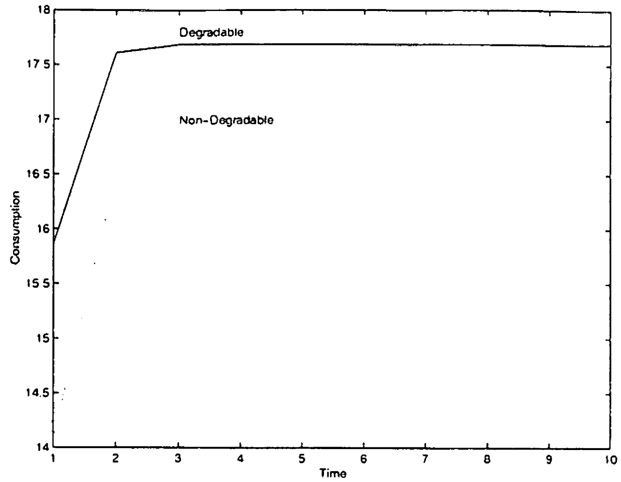

FIGURE 4. Consumption Paths for Degradable and Non-degradable Stocks of Pollutant with Endogenous Risk

rate of decay while the dotted line to a lower rate of decay. An increase in the degree of non-degradability decreases the amount of resources devoted to consumption and thereby increases the amount devoted to investment.

5.6. **Non-negative Emissions.** Once again the non-negativity constraint does not bind at the steady state or during the transition to the steady state unless the agents begin with a large stock of pollutant that they want to reduce drastically. Consequently, defining non-degradability of the stock of pollutant in terms of non-negative emissions weakens the effect of the stock of pollutant on the optimal rate of emissions.

6. Conclusions

We have shown that under our definitions of sunk abatement capital and a non-degradable stock of pollutant, a decrease in the degree of capital convertibility has no impact on the optimal steady state rate of investment while an increase in the degree of pollutant stock non-degradability leads to an increase in the optimal steady state rate of investment. These results hold even when the conditional probability for an immediate occurrence of a catastrophe is endogenous, though under sufficient conditions. The upshot of these results is that for an environmental problem characterized by sunk abatement capital, non-degradable pollutant stocks and endogenous risk of catastrophic damages, the optimal steady state rate of investment in abatement capital is increased.

With different definitions of sunk abatement capital and non-degradable stocks of pollutant, as we show, the policymaker would come to a different conclusion. When sunk capital is equivalent to durable capital, an increase in durability leads to a decrease in the optimal steady state level of investment. On the other hand, when pollutant stock non-degradability is captured by a non-negativity restriction on emissions then an increase in the degree of stock non-degradability has no effect on the optimal steady state rate of investment. A policymaker using these alternative definitions and faced with sunk capital, non-degradable pollutant stocks and potentially catastrophic damages would decrease steady state investment. This result is opposite to the one we reach with the definitions that we have argued better reflect both physical realities and the decision-making environment. Adding endogenous risk corrects for some of the bias introduced by the alternative definitions, but these can still give misleading results.

Appendix A. Derivation of the Bellman-Hamilton-Jacobi Equation

An agent chooses a stream of consumption to maximize expected intertemporal utility subject to equations (1)–(4).

$$\max_C E_t \int_t^\infty S(C, M, D, \tau) d\tau$$

where $S(C, M, D, t)$ is the momentary utility function. Let $J(K, M, D, t)$ denote the corresponding value function. To derive the appropriate Bellman-Hamilton-Jacobi equation we begin by splitting the dynamic program into two parts[11]

$$J(K, M, D, t) = \max_C E_t \left[\int_t^{t+dt} S(C, M, D, \tau) d\tau + \int_{t+dt}^\infty S(C, M, D, \tau) d\tau \right] \tag{20}$$

Since

$$E_{t+dt} \int_{t+dt}^\infty S(C, M, D, \tau) d\tau = E_{t+dt} J(K + dK, M + dM, D + dD, t + dt)$$

equation (20) simplifies to

$$\begin{aligned} J(K, M, D, t) = \max_C \Big[& S(C, M, D, t)dt + J\big(K + dK, M + dM, D + a, t + dt\big)pdt \\ & + J\big(K + dK, M + dM, D, t + dt\big)\big(1 - pdt\big) \Big] \end{aligned} \tag{21}$$

Next we take a first order Taylor series expansion of the last two terms on the right hand side of equation (21) around $dt = 0$. This gives the following expression

$$\begin{aligned} J(K, M, D, t) = \max_C \Big[& S(C, M, D, t)dt + J(K, M, D + a, t)pdt + J(K, M, D, t) \\ & + J_1(K, M, D, t)(R - C - \delta_K K)dt \\ & + J_2(K, M, D, t)\big(g(C)(1 - H(K)) - \delta_M M\big)dt \\ & + J_4(K, M, D, t)dt - J(K, M, D, t)pdt + h.o.t. \Big] \end{aligned}$$

where $J_1(K, M, D, t)$ is the derivative of the value function with respect to its first argument. $J_2(K, M, D, t)$ and $J_4(K, M, D, t)$ are similarly defined and h.o.t. denotes higher order terms in the Taylor expansion.[12] Subtracting $J(K, M, D, t)$ from both sides, dividing through by dt and letting

[11] This derivation draws heavily on Mangel (1985) and Karp (1997).

[12] Note that because damages take on integer values we do not differentiate the value function with respect to damages.

dt approach zero with the added assumption that $lim_{dt\to 0}\frac{h.o.t.}{dt} = 0$ gives

$$
\begin{aligned}
0 = \max_C\Big[& S(C,M,D,t) + \big(J(K,M,D+a,t) - J(K,M,D,t)\big)p \\
& + J_1(K,M,D,t)(R - C - \delta_K K) \\
& + J_2(K,M,D,t)\big(g(C)(1-H(K)) - \delta_M M\big) + J_4(K,M,D,t)\Big]
\end{aligned}
\tag{22}
$$

For the autonomous problem the value function $J(K,M,D,t)$ can be written as $e^{-rt}W(K,M,D)$. Making this substitution into equation (22) and multiplying through by e^{rt} gives the following version of the Bellman-Hamilton-Jacobi equation

$$
\begin{aligned}
rW(K,M,D) = \max_C\Big[& S(C,M,D) + \Big(W(K,M,D+a) - W(K,M,D)\Big)p \\
& + W_1(K,M,D)\big(R - C - \delta_K K\big) \\
& + W_2(K,M,D)\big(g(C)(1-H(K)) - \delta_M M\big)\Big]
\end{aligned}
$$

Up until the time when the catastrophe occurs $D = 0$ and once the catastrophe has occurred utility goes to zero forever, or that, $W(K,M,a) = 0$. With these and the final simplification that $W(K,M,0) = V(K,M)$ and $S(C,M,D) = U(C,M)$ the Bellman-Hamilton-Jacobi equation can be written as

$$
\begin{aligned}
rV(K,M) = \max_C\Big[& U(C,M) + V_1(K,M)\big(R - C - \delta_K K\big) \\
& + V_2(K,M)\big(g(C)(1-H(K)) - \delta_M M\big) - V(K,M)p\Big]
\end{aligned}
$$

Appendix B. Proofs for Propositions and Corollaries

B.1. **Proof for Proposition 1.** Differentiating the Euler equation with respect to δ_K yields the condition that

$$
\frac{dC^*}{d\delta_K} < 0 \quad \text{if} \quad -H_{11}(K^*)\frac{K^*}{H_1(K^*)} \geq \frac{\delta_K}{(r + \delta_K + p)}
$$

B.2. **Proof for Proposition 2.** Differentiate equation (9) with respect to δ_M. This gives the result that $\frac{dC^*}{d\delta_M} > 0$.

B.3. **Proof for Proposition 3.** Differentiate equation (17) with respect to δ_M. This expression is omitted here because of its complexity. Its denominator is negative while its numerator is negative if

$$\frac{-\partial p}{\partial M}\frac{\partial M}{\partial \delta_M}\left(\frac{(r+\delta_M+p)}{(r+\delta_K+p)}+\frac{(2r+2p+\delta_M)}{(r+p)}\right)<1$$

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