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WORKING PAPER NO. 843

**IRREVERSIBILITY, UNCERTAINTY, AND CATASTROPHIC
GLOBAL WARMING**

by

Urvashi Narain and Anthony C. Fisher

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Irreversibility, Uncertainty, and Catastrophic Global Warming

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Abstract

In this paper we characterize the optimal rate of emission of greenhouse gases when investment in abatement capital is irreversible, some part of the stock of greenhouse gases is non-degradable and there is a possibility of catastrophic damages in the future. The option of investing in abatement capital tomorrow—when learning resolves uncertainty about the extent and possibility of catastrophic damages due to greenhouse gases—is weighed against the option of reducing today the stock of greenhouse gases. The agent wants to avoid two situations: (i) investing in irreversible abatement capital today when the damages tomorrow turn out to be negligible; (ii) not reducing the stock of (irreversible) greenhouse gases today when damages are revealed to be catastrophic tomorrow. Unfortunately, the stock of greenhouse gases cannot be reduced unless the agent invests in abatement capital. Given this trade-off, our paper asks what should be the optimal rate of emission of greenhouse gases. Previous studies have either relied on numerical simulations or failed to capture features of the environment we think important to global warming. Our paper fills these gaps in the literature by developing a stochastic dynamic programming model that allows for irreversible capital, irreversible stock and avoidable catastrophic damages, and yields analytical results. Simulations based on the analytical model introduce the prospect of learning about future damages into the current-period decision, and yield results directly comparable to those of the earlier studies. Loosely speaking, we find a stronger irreversibility effect associated with accumulation of greenhouse gases than with investment in abatement, contrary to the earlier results.

1. INTRODUCTION

Climatologists report that, at current greenhouse gas emission levels, the stock of greenhouse gases in the atmosphere will double the preindustrial level in the next few decades. This will lead to an increase in global mean temperature by a best-guess estimate of 3.5°F (IPCC 1995b).¹ This is a large and sudden increase in mean temperature considering that the world is only 5–9°F warmer now than in the last ice age. The increase in global mean temperature is expected to lead to disruptions in the world's climate.² Whether these disruptions will cause economic damages and whether these damages will be catastrophic in nature is as yet uncertain.³ There are those who believe that global warming will lead to sudden and catastrophic economic damages. Others believe that damages will occur slowly and continuously as the stock of greenhouse gases increases. Still others assert that damages due to global warming will be negligible.⁴

Given a threat of damages of an unknown magnitude, the question facing policymakers is whether they should change the rate at which greenhouse gases are being emitted today.⁵ Four features of the economic environment bear on this decision and make the answer less than obvious: irreversible abatement capital; irreversible stocks of greenhouse gases; avoidable, and potentially catastrophic, damages; and future learning about the nature of damages. Abatement capital is said to be irreversible if resources once invested cannot be re-used for consumption or re-invested in other forms of capital. An obvious concern is whether the presence of irreversible capital alters optimal emission control decisions today. Given the uncertainty, should less be invested in abatement capital if that capital is irreversible?

A second important complicating factor is the irreversibility of the stock of greenhouse gases. The stock of greenhouse gases is said to be irreversible if it cannot be reduced through abatement and if it does not decay naturally. Climatologists claim that some part of the stock of greenhouse gases may be irreversible. The atmospheric concentration of carbon is not expected to return to

¹The range of predicted temperature increase is 2–6°F.

²Predictions about increases in global mean temperature and how disruptive this will be are based on simulations from computer models of the world's climate. While these models are improving over time they still remain rather crude and are plagued by incomplete knowledge of the atmosphere's functioning. One of the biggest limitations is that the net effect of clouds on the planet's temperature is still unknown. By and large the models are able to predict changes at the global level but perform poorly on a regional scale (Stevens 1997).

³IPCC (1995b) identifies possible catastrophic climate change events. A few among these are destabilization of methane clathrates, shutdown of major greenhouse gas sinks, disintegration of the west Antarctic ice sheet and collapse of the North Atlantic thermohaline circulation.

⁴For a discussion of the assessment of socio-economic impacts of climate change see IPCC (1995a).

⁵When policymakers change the rate of emission of greenhouse gases they in fact change the magnitude of damages that may be caused by global warming. This constitutes a policy of prevention. In addition or alternatively, policymakers could wait until after the damages have occurred and then institute policies that mitigate the effects of these damages. Building levies to control flooding due to an increase in sea level is one such policy in the general class of policies of cure. Chichilnisky and Heal (1993) discuss the applicability of insurance markets to mitigate damages after global warming has occurred.

its original (pre-industrial) level but instead is expected to reach a new equilibrium where about 13–18% of total carbon dioxide emitted will remain in the atmosphere for several thousand years (Maier-Raimer and Hasselmann 1987).⁶ Should policymakers reduce greenhouse gas emissions if, once emitted, gases remain in the atmosphere for several thousands of years?

A third important concern for policy makers is the extent to which the risk of future damages is avoidable and whether or not damages will be catastrophic. If the probability of damages depends on the behavior of economic agents, then the risk may be considered avoidable. In the context of global warming, since the probability of damages depends on the stock of greenhouse gases, damages are potentially avoidable.⁷ Recent findings suggest that the probability of catastrophic damage, in particular related to a disintegration of the West Antarctic ice sheet, is greater than economists (and others) have realized (Kerr 1998). This suggestion is strengthened by the prospect that concentrations of greenhouse gases could, over the next couple of centuries, rise well beyond the conventionally assumed doubling of pre-industrial levels (Cline 1992). The implications of the potential for catastrophic impact are a major focus of our study.

A final issue that complicates policy decisions on global warming is how uncertainty is resolved over time. If uncertainty about the extent of damages due to global warming is resolved over time, then policymakers must decide whether they should wait to act until there is better information about the nature of damages. When time resolves uncertainty, Arrow and Fisher (1974) have shown that there is a premium or option value on policies that maintain flexibility.⁸ Irreversibility of capital and the stock of greenhouse gases are two potential sources of inflexibility. Investment in irreversible capital today locks the economy into a particular use of resources which may turn out to be wasteful if tomorrow reveals that damages due to global warming are small. Kolstad (1996b, 1996a) has stressed this possibility. One then expects that investment in irreversible capital will be less than the investment that would be made if capital was reversible. With irreversible accumulation of greenhouse gases, on the other hand, emissions today lock the economy into a level of damages which may be revealed as catastrophic. To maintain the option of not having to bear large damages policymakers might increase investment in abatement today. Both Chichilnisky and Heal (1993) and Fisher and Hanemann (1993) have suggested that this approach may be appropriate.

A rigorous theoretical analysis by Ulph and Ulph (1997) finds however that, counter to these suggestions, irreversible stocks of greenhouse gases may have a zero or positive effect on today's

⁶Farzin and Tahvonen (1996) were the first to incorporate this specification into the economic analysis of optimal carbon taxes.

⁷There has been a great deal of debate about the contribution of human activities to climate change. Two years ago the IPCC declared that human activities influence global climate through the increase in greenhouse gas concentrations.

⁸For other works on option value in the environment see Henry (1974) and Fisher and Hanemann (1990). For a more general treatment see Dixit and Pindyck (1994).

emissions. For a two period theoretical model they provide a sufficient condition for emissions to be lower with irreversible stocks of greenhouse gases, but find that this sufficient condition no longer holds when the model is extended to many periods using numerical techniques. In the multi-period simulations, in fact, when the stock of greenhouse gases is irreversible and there is learning, emissions are higher than when no learning takes place. Similarly, with a numerical model that allows for irreversible stocks of greenhouse gases, irreversible abatement capital and learning about the nature of damages, Kolstad (1996b) finds that investment in abatement capital is lower when capital is irreversible but an irreversible stock of greenhouse gases does not lead to an increase in investment.

The theoretical models are restricted to two periods and do not incorporate features of the decision-making environment which we have suggested may be important. Kolstad (1996a) assumes that the risk of a catastrophe is unavoidable in the sense that individuals cannot alter the probability of occurrence. This seems less realistic than the alternative assumption that decisions on investment in controlling emissions can affect the probability of a catastrophic jump in damages. In Ulph and Ulph (1997), risk is unavoidable and there is no abatement capital. Conrad (1992) does develop a multi-period model and looks at the effect of "non-degradable" or irreversible stocks of greenhouse gases on the optimal rate of emissions. However, his model does not include capital, catastrophic damages, or avoidable risk.

Our paper explores the consequences of changing the assumptions about the nature of irreversibilities and risk in ways that may better reflect reality and economic intuition. Using a multi-period stochastic model we show that previous results are in part driven by definitions used for irreversible capital and stocks of greenhouse gases. We suggest different definitions which imply a stronger irreversibility effect for the stock of greenhouse gases and a weaker effect for capital. Adding avoidable risk to the model further strengthens our results.

Our theoretical analysis is based on a model that does not allow agents to act on new information, but nonetheless generates an irreversibility effect. Previous models that have focused on learning have not considered the possibility that uncertainty and irreversibility alone are sufficient to generate an irreversibility effect, though an early application of dynamic optimization to the problem of environmental preservation did find such an effect (Fisher, Krutilla, and Cicchetti 1972). There, as in our model, learning is not necessary. We then develop a numerical model, a parameterization of the theoretical model, that allows agents to act on new information about the nature of damages. With unavoidable risk, irreversible capital leads to a decrease in investment while irreversible stocks of greenhouse gases lead to an increase. Avoidable risk counters the effect of irreversible capital and strengthens the effect of irreversible stocks of greenhouse gases.

The next section contains a description of the theoretical model. Section 3 considers the effect of irreversible capital under alternative definitions for irreversibility and section 4 considers the effect of an irreversible stock of greenhouse gases. Section 5 adds avoidable risk to the model. Section 6 contains the numerical model, section 7 results of the simulations, and section 8 sums up the conclusions.

2. THEORETICAL MODEL

In this section we develop a stochastic dynamic model where an agent chooses how much greenhouse gas to emit in the presence of uncertainty about whether or not global warming will lead to catastrophic damages. The agent receives a fixed endowment of resources in every period which is allocated between consumption and investment in capital used to abate the stock of greenhouse gases. Abatement capital is either reversible or irreversible. Only reversible capital can be converted back into consumption. If not abated, emissions add to the stock of greenhouse gases which may or may not be reversible. Irreversible stocks of greenhouse gases cannot be abated nor do they decay naturally over time. If either one of these assumptions is relaxed, then the stock of greenhouse gases is considered to be reversible.

The stock of greenhouse gases generates two possible effects. First, it reduces utility in every period. Second, the stock of greenhouse gases can also trigger a catastrophe.⁹ This happens if the likelihood of a catastrophe occurring is a function of the stock of greenhouse gases. With unavoidable risks the stock of greenhouse gases causes only current period damages. We assume that catastrophic damages are independent of the stock of greenhouse gases¹⁰ and that utility is driven to zero forever after the catastrophe has occurred. Catastrophic damages are thus irreversible in our model.¹¹

By assuming that the world comes to an end after the catastrophe has occurred, we do not allow agents to adjust their emission levels after they have learned about the catastrophe. Thus even

⁹Cropper (1976) was the first to draw attention to the effect of catastrophic risks on optimal rate of emissions, though not in the context of global warming.

¹⁰As an aside lets consider one of the implications of the assumption that catastrophic damages are independent of the stock of greenhouse gases. With an unavoidable risk, independence of damages implies that an increase in the probability of a catastrophe will lead to an increase in emissions. It is optimal for agents to increase emissions and not wait for a tomorrow that may never come. This is one of the results shown by Clarke and Reed (1994). However, if damages due to a catastrophe depend on the stock of greenhouse gases then as the probability of an unavoidable risk increases agents may decrease emissions. Even though agents cannot affect the probability of occurrence of a catastrophe their actions do affect the extent of damages due to the catastrophe.

¹¹Clarke and Reed (1994) use a similar model, though with optimal control, to analyze the effects of changes in avoidable and unavoidable risks on the optimal rate of emissions of a pollutant. However, in their model capital and the stock of the pollutant are restricted to be fully reversible. Neither of these restrictions hold for our model. Aronsson, Johansson, and Lofgren (1997) develop a model similar to Clarke and Reed (1994) but with two types of capital neither of which are restricted to be irreversible. Like Clarke and Reed (1994) they do not look for the irreversibility effect.

though uncertainty about damages is being resolved over time, there is no reason to wait for new information as there is no option value of delaying an irreversible decision. However, irreversible capital and stocks of greenhouse gases can still lead to an irreversibility effect which we define as a change in the desired level of emissions.

2.1. **Primitives.** A representative agent derives utility from consumption, C , and disutility from the stock of greenhouse gases, M . Catastrophic damages, D , should a catastrophe occur, cause the utility function to shift. Let the momentary utility function $U = U(C, M, D)$ satisfy the conditions

$$\begin{aligned} U_1(C, M, D) &> 0, & U_{11}(C, M, D) &< 0, & U_{12}(C, M, D) &< 0 \\ U_2(C, M, D) &< 0, & U_{22}(C, M, D) &< 0 \end{aligned}$$

where subscripts denote differentiation. As long as there is no catastrophe, damages are zero and the utility function is unaffected. After a catastrophe however, utility goes to zero forever so that $U(C, M, D > 0) = 0$. To simplify notation we drop damages from the utility function and re-write the utility function as $U = U(C, M)$.

We assume that a fixed amount of output, R , is available each period for either consumption or investment, I , in abatement capital. Abatement capital, K , changes from one period to the next as a result of investment and depreciation according to

$$(1) \quad dK = (I - \delta_K K) dt$$

where δ_K is the rate of depreciation of capital. For the base model we assume that capital is reversible. This means that at any time consumption can be greater than the fixed amount of resource R . Specifically,

$$(2) \quad C \leq R + \Phi K$$

where Φ is a parameter that governs the cost of converting capital into consumption. When $\Phi = 0$ it is infinitely costly to convert capital into consumption and when $\Phi = \infty$ conversion is costless.¹²

Greenhouse gas emissions, E , are a by-product of consumption. Let $g(C)$ be the emissions function where $g_1(C) > 0$ and $g_{11}(C) = 0$. If unabated, emissions increase the stock of greenhouse gases. Let $H(K)$ be the abatement function where $H_1(K) > 0$ and $H_{11}(K) \leq 0$. For the base model we assume that capital abates both the flow and the stock of greenhouse gases implying that the amount of greenhouse gas abated in a period can exceed the amount emitted in that period. The stock also decays naturally. Consequently, the law of motion for the stock of greenhouse gases

¹²Costless conversion is represented by $\Phi = \infty$ because we are considering a continuous time model. For a discrete time model, like the quantitative simulations presented in section 6 below, $\Phi = 1$ denotes costless conversion.

is given by¹³

$$(3) \quad dM = (g(C) - H(K) - \delta_M M) dt$$

where $g(C) - H(K)$ are net emissions and δ_M is the natural decay rate of greenhouse gases. If net emissions are restricted to be non-negative and the rate of decay is close to zero, then the stock of greenhouse gases is considered to be irreversible.

Finally, there always exists the possibility of a catastrophe occurring. The possibility of a catastrophe is captured by a damage function that follows a jump process. The law of motion for damages is given by

$$(4) \quad dD = d\pi$$

with,

$$d\pi = \begin{cases} a & \text{with probability } p dt, \\ 0 & \text{with probability } 1 - p dt. \end{cases}$$

where p is the probability of catastrophic damages occurring and a is the magnitude of the catastrophic jump. If the catastrophe is unavoidable then p is a constant. With avoidable catastrophic risk the probability of a catastrophe occurring is an increasing and convex function of the stock of greenhouse gases. That is, $p = p(M)$ with $p_1(M) > 0$ and $p_{11}(M) > 0$.¹⁴

2.2. Objective. The agent chooses a stream of consumption to maximize expected intertemporal utility subject to equations (1)–(4). The only source of uncertainty is whether or not a catastrophe will occur.

$$(5) \quad \max_C E_t \int_t^\infty U(C, M, \tau) d\tau$$

The Bellman-Hamilton-Jacobi equation¹⁵ for this problem is

$$(6) \quad \begin{aligned} rV(K, M) = \max_{C \leq R + \phi K} & \left[U(C, M) + V_1(K, M)(R - C - \delta_K K) \right. \\ & \left. + V_2(K, M)(g(C) - H(K) - \delta_M M) - V(K, M)p \right] \end{aligned}$$

¹³By restricting the decay rate to be a linear function of the stock of greenhouse gases we are in fact assuming that there is a unique steady state for the stock of greenhouse gases. See Tahvonon (1995) for a discussion of multiple steady states with non-convex decay functions.

¹⁴Tsur and Zemel (1996) consider the effect of a different type of catastrophic risk on the optimal rate of emissions. In their model uncertainty stems from not knowing what is the exact level of stock needed to trigger a catastrophe. They call this risk endogenous and contrast it with the stochastic process we consider which they refer to as an exogenous risk. Since in the context of global warming there exist a lag between stock build up and the time when the effects of that level of stock are felt we believe that modeling the risk as a stochastic process is appropriate.

¹⁵The Bellman-Hamilton-Jacobi equation is derived in the appendix.

where $V(K, M)$ is the value function and r is the discount rate. The left hand side equation of (6) represents the present discounted value of the stocks of capital and greenhouse gases. The right hand side is equal to the sum of momentary utility, the shadow value of capital times the change in capital stock and the shadow value of the stock of greenhouse gases times the change in the stock minus the expected loss from a catastrophe.

2.3. Optimality Conditions. In this subsection we establish optimality conditions for consumption, capital and the stock of greenhouse gases. For now we assume that risk is unavoidable and so p is a constant. We begin by differentiating equation (6) with respect to the choice variable—consumption. This gives the following first order condition

$$(7) \quad U_1(C, M) - \lambda - V_1(K, M) + V_2(K, M)g_1(C) = 0$$

where λ is the Lagrange multiplier on the constraint given by equation (2). The steady state envelope conditions, obtained by differentiating equation (6) with respect to the state variables, K and M , and imposing steady state, are

$$(8) \quad (r + \delta_K + p)V_1(K, M) = \lambda\Phi - V_2(K, M)H_1(K)$$

$$(9) \quad (r + \delta_M + p)V_2(K, M) = U_2(C, M)$$

Equations (7), (8) and (9) combine to give the Euler equation, which in turn with the steady state laws of motion gives a system of equations which determines the steady state level of consumption, capital and stock of greenhouse gases (the arguments are suppressed for compactness)

$$(10) \quad U_1 = \frac{-U_2}{(r + \delta_M + p)} \left(g_1 + \frac{H_1}{(r + \delta_K + p)} \right) + \lambda \left(\frac{\Phi}{(r + \delta_K + p)} + 1 \right)$$

$$(11) \quad K^* = \frac{R - C^*}{\delta_K}$$

$$(12) \quad M^* = \frac{g - H}{\delta_M}$$

where stars denote steady state levels. Equation (10) is the Euler equation. When the constraint on consumption is not binding, $\lambda = 0$ and the Euler equation states that along the steady state consumption trajectory there is nothing to gain by increasing consumption. Utility from the increase in consumption is equal to the disutility from the concomitant increase in greenhouse gases and therefore net utility is zero.¹⁶ Equation (11) states that at the steady state, investment is equal to capital depreciation. Equation (12) states that net emissions are equal to the decay in the steady state stock of greenhouse gases.

¹⁶Specifically, an increase in consumption has three effects: (i) it increases utility; (ii) it increases emissions; and (iii) it decreases investment and consequently capital. The increase in emissions increases the stock of greenhouse gases which in turn decreases utility for every period in the infinite future. The sum of the infinite series of disutility from

When the constraint is binding, $\lambda > 0$ and steady state consumption is equal to

$$(13) \quad C^* = R + \Phi K^*$$

Since negative investment period after period drives the steady state capital stock to zero, steady state consumption is in fact equal to R . The Euler equation then states that there is a gain to be had from increasing consumption. However, it is no longer possible to do so, because consumption is at its maximum. When the constraint is binding, steady state capital is equal to zero and gross emissions are equal to the decay in the stock of greenhouse gases.

Before proceeding to analyze the consequences of irreversible investment in emission control, and irreversible accumulation of greenhouse gases, we need to say a word about our focus here on the steady state. For the generic pollutant that accumulates over time, it is perfectly plausible to define a steady state, as for example Clarke and Reed (1994) do. For greenhouse gases, or at least for the currently most important greenhouse gas, carbon dioxide produced by combustion of fossil fuels, this is not strictly true. Beyond some point, several hundred years in the future, fossil fuels may be largely exhausted, and the accumulated atmospheric concentration of carbon dioxide will presumably decrease. What we study in this part of the paper is then behavior at a steady state that may persist for a very long time—indeed, far beyond the “long run” of most economic models—but not forever. In the simulation model we loosely verify that the time paths of the key consumption and investment variables do in fact approach a steady state.

3. IRREVERSIBLE CAPITAL

In this section we explore the implications of capital being irreversible. We also provide some economic intuition for the somewhat counter-intuitive result given in Kolstad (1996b), namely that, while irreversible abatement capital warrants a decrease in investment, irreversible stocks of greenhouse gases do not warrant an increase in investment.

3.1. Defining Irreversible Capital. This result partly stems from Kolstad’s definition of irreversible abatement capital. He equates irreversibility or “sunkness” of capital with durability, arguing that capital is sunk if it has a low rate of depreciation. We think this definition fails to capture the essential problem faced by policymakers, who we assume wish to avoid a situation

an increase in the stock of greenhouse gases is equal to $\frac{U_2(C,M)}{r+\delta_M+p}$ from the following identity,

$$U_2(C, M) + (1 - (\tau + \delta_M + p))U_2(C, M) + (1 - (\tau + \delta_M + p))^2 U_2(C, M) + \dots \equiv \frac{U_2(C, M)}{r + \delta_M + p}$$

where disutility in the future is discounted to account for time preference, depreciation of the stock of greenhouse gases and the probability of a catastrophic damage. The total disutility is equal to $\frac{U_2(C,M)g_1(C)}{r+\delta_M+p}$. Finally, the decrease in capital decreases abatement for every period in the infinite future which in turn increases the stock of greenhouse gases and thus decreases utility. The total loss of utility from a decrease in the stock of capital is equal to $\frac{U_2(C,M)H_1(K)}{(r+\delta_M+p)(r+\delta_K+p)}$.

where valuable resources invested today in abatement capital cannot be converted back into some productive use in the future if damages turn out to be negligible. What matters is the adjustment cost of conversion, not depreciation. Durable capital may still have a low conversion cost. We therefore prefer to define irreversible capital as capital that is prohibitively costly to convert into consumption.¹⁷ Or, capital is considered to be irreversible if investment is constrained to be positive in every period. We now show that Kolstad's definition implies a stronger irreversibility effect for capital than does our definition.

3.2. Durable Capital. We begin by writing the Euler equation for unavoidable risk (equation (10)) as a function of steady state consumption and some parameters (steady state capital and stock of greenhouse gases are both functions of steady state consumption and some parameters from equations (11) and (12) respectively).¹⁸ With this simplification the effect of a change in the degree of irreversibility of capital on consumption is given by differentiating the Euler equation with respect to the rate of depreciation. This is formalized in the following proposition.¹⁹

Proposition 1. *If $-H_{11}(K^*) \frac{K^*}{H_1(K^*)} \geq \frac{\delta_K}{(\tau + \delta_k + p)}$ then $\frac{dC^*}{d\delta_K} < 0$.*

In words the proposition states that steady state consumption will increase (and steady state investment will decrease) as capital becomes more irreversible if the gain from the increase in capital caused by the decline in the rate of depreciation is greater than the loss caused by the decline in the marginal product of capital. Consequently, there is an irreversibility effect associated with durable capital if this sufficient condition is met. This result is fairly straightforward since the presence of durable capital reduces the need for new investment.

For more intuition, consider a simple concave abatement function— $H(K) = K^\gamma$, where γ is a constant that lies between zero and one. The sufficient condition can be written as $\frac{dC^*}{d\delta_K} < 0$ if $\gamma \leq \frac{(\tau + p)}{(\tau + \delta_k + p)}$. For $\delta_K = 0$ (or close to zero), which implies that capital is infinitely durable, the condition is trivially true and thus $\frac{dC^*}{d\delta_K} < 0$. As δ_K increases, the sufficient condition has more bite and will only be true for small values of γ . This means that for consumption to decrease with an increase in the rate of depreciation it must be true that the increase in the rate of depreciation does not increase the marginal product of capital significantly. A large increase in the marginal product of capital will reduce the need for new investment.

The effect of capital becoming more irreversible on steady state capital and stock of greenhouse gases is ambiguous. Consequently, the effect on the optimal rate of net emissions ($g(C^*) - H(K^*)$)

¹⁷Pindyck (1991) defines capital to be irreversible if it cannot be used productively by a different industry. To avoid having to add another state variable we define irreversibility in terms of the ability to switch between capital and consumption. Otherwise our definition matches that of Pindyck (1991).

¹⁸We consider only an interior solution.

¹⁹Proofs for all the propositions are in the appendix.

of a change in the irreversibility of capital is unknown (see corollary 1 in the appendix for a formal statement).

3.3. Irreversible Capital. We now show that with our definition for irreversible capital there is no irreversibility effect. We consider capital to be irreversible if it cannot be converted into consumption. This translates into a new constraint on consumption, namely that, $C \leq R$, instead of $C \leq R + \Phi K$ (equation (2)).

Recall that Φ does not affect steady state consumption whether the constraint is binding or not. At the steady state individuals will either choose to consume all the resources ($C^* = R$) or consume some and invest the rest ($C^* < R$). Φ does not affect this decision. Suppose it is optimal to consume all the resources and devote nothing to investment in abatement capital. If $\Phi \geq 0$, individuals will convert capital into consumption and drive the capital stock to zero immediately. If $\Phi = 0$, individuals will simply wait for the capital to depreciate away. Investment will be zero irrespective of the value of Φ . In other words, a change in the degree of irreversibility of capital has no effect on steady state consumption or steady state net emissions. Consequently, there is no irreversibility effect associated with irreversible capital.

If it's optimal for agents to consume the entire endowment and devote nothing to investment, then, during the transition to steady state, investment will be non-positive. This once again is not a consequence of Φ and in no way reflects the degree of irreversibility of the capital stock.

To sum up, we have shown that if the rate of depreciation is used to characterize capital irreversibility then there is an irreversibility effect associated with capital. This effect no longer holds if we define irreversible capital by constraining investment to be non-negative, a perhaps more intuitive definition.

4. IRREVERSIBLE STOCK OF GREENHOUSE GASES

We next explore whether or not there is an irreversibility effect associated with the stock of greenhouse gases. Here too we show that the result in Kolstad (1996b)—that there is no irreversibility effect associated with the stock of greenhouse gases—is in part driven by his definition of irreversibility.

4.1. Defining Irreversible Stock. Kolstad (1996b) and Ulph and Ulph (1997) define the stock of greenhouse gases to be irreversible if emissions in a given period are restricted to be non-negative. No restriction is placed on the rate of decay of the stock of greenhouse gases. In contrast, we additionally require a near-zero decay rate for the stock of gases to be considered irreversible. If the stock decays or if emissions are permitted to be negative, then the stock will dissipate over time and cannot be considered irreversible.

We now show that our definition of stock irreversibility results in a stronger irreversibility effect for the stock of greenhouse gases as compared to the definition that relies on non-negative emissions alone.

4.2. **Decay Rate.** With our definition for an irreversible stock of greenhouse gases the equation of motion that governs the evolution of the stock changes to

$$(14) \quad dM = \left(g(C)(1 - \hat{H}(K)) - \delta_M M \right) dt$$

where $\delta_M = 0$ implies that the stock of greenhouse gases is irreversible. $\hat{H}(K)$ is the new abatement function that lies between zero and one. This implies that only the flow of greenhouse gases can be abated and not the stock. We also assume that $\hat{H}(K)$ is an increasing but concave function of capital.

The Bellman-Hamilton-Jacobi equation for an irreversible stock of greenhouse gases also changes to

$$(15) \quad rV(K, M) = \max_C \left[U(C, M) - V(K, M)p + V_1(K, M)(R - C - \delta_K K) \right. \\ \left. + V_2(K, M) \left(g(C)(1 - \hat{H}(K)) - \delta_M M \right) \right]$$

To analyze how an increase in stock irreversibility affects steady state consumption and emissions we first derive optimality conditions for consumption, capital and the stock of greenhouse gases.

4.2.1. *Optimality Conditions.* Using the same procedure as outlined in subsection 2.3 we get the following system of equations that determines the steady state level of consumption, capital and stock of greenhouse gases (the arguments are suppressed for compactness and we consider only an interior solution)

$$(16) \quad U_1 = -\frac{U_2}{(\tau + \delta_M + p)} \left(g_1(1 - \hat{H}) + \frac{g\hat{H}_1}{\tau + \delta_K + p} \right)$$

$$(17) \quad K^* = \frac{R - C^*}{\delta_K}$$

$$(18) \quad M^* = \frac{g(1 - \hat{H})}{\delta_M}$$

The Euler equation, (equation (16)) implies that there is no utility to be gained from increasing consumption once at the steady state. Increased consumption while increasing utility also increases the stock of greenhouse gases which generates disutility. The stock of greenhouse gases increases because: (i) emissions, holding abatement capital constant, increase; and (ii) abatement, holding

emissions constant, decreases because the stock of capital decreases. At the steady state, investment in abatement capital is equal to capital depreciation (equation (17)) and emissions, net of abatement, are equal to the decay of the stock of greenhouse gases (equation (18)).

From equation (17) we can write the steady state stock of capital as a function of steady state consumption and the rate of depreciation, $K^* = k(C^*, \delta_K)$. Similarly, the stock of greenhouse gases can be expressed as a function of steady state consumption and some parameters, $M^* = m(C^*, \delta_M, \delta_K)$ (from equation (18)). The existing assumptions imply that $k_1 < 0$, $m_1 > 0$ and $m_2 < 0$. The Euler equation, once again, is a function of steady state consumption and some parameters.

4.2.2. *Results.* We now consider how consumption at the steady state changes with a change in the degree of irreversibility of the stock of greenhouse gases.

Proposition 2. *Steady state consumption is an increasing function of the rate of decay of greenhouse gases.*

In words, as the stock of greenhouse gases becomes irreversible, consumption decreases while investment in abatement capital increases. Consequently, irreversibility leads to a change in the optimal policy—the irreversibility effect.

Along with investment the stock of capital increases and the level of net emissions decreases. However, the effect of an increase in irreversibility on the stock of greenhouse gases itself is ambiguous (this is formalized in corollary 2 in the appendix).

4.3. **Non-Negative Emissions.** If stock irreversibility is defined in terms of non-negative emissions only, then there is no irreversibility effect associated with the stock of greenhouse gases at the steady state. Let's consider the optimality conditions for the stock of greenhouse gases at steady state. If the constraint on consumption is not binding then the steady state stock of greenhouse gases is given by

$$(19) \quad M^* = \frac{g(C^*) - H(K^*)}{\delta_M} \geq 0$$

M^* is zero when steady state emissions are exactly equal to steady state abatement and strictly positive when net emissions are positive and equal to stock decay. In either case the non-negativity constraint on net emissions is not binding. A similar argument holds when the constraint on consumption is binding.

However, if it is optimal to drive the steady state stock to zero then in the transition to steady state the non-negativity constraint will bind. Agents would prefer to emit negative amounts of greenhouse gases to drive the stock to zero as fast as possible. There is an irreversibility effect away

from the steady state. However, for this effect to hold it must be true that a drastic reduction of the stock is optimal or that agents begin with a large endowment of the stock of greenhouse gases.

To sum up, when emissions are restricted to be non-negative and the rate of decay is zero then there is an irreversibility effect associated with the stock of greenhouse gases. This effect goes away at steady state if irreversibility is defined by non-negative emissions alone. This analysis shows that the significance of the irreversibility effect here depends on the definition of an irreversible stock of greenhouse gases. It is essential to pick the definition carefully, especially when comparing opposing effects.

5. AVOIDABLE RISK

As noted earlier, previous models of global warming have ignored an important aspect of the decision-making environment—avoidable risk. If it is true that warming is being triggered by an increase in the stock of greenhouse gases then the threat can be mitigated by reducing the stock, that is, by economic agents changing their behavior. In other words, the risk of a catastrophe is avoidable. We now explore the implications of adding avoidable risk to the model for optimal rates of emissions of greenhouse gases.

5.1. Optimality Conditions. As with unavoidable risk, we begin by differentiating equation (6) with respect to the choice variable—consumption. This gives the following first order condition

$$(20) \quad U_1(C, M) - \lambda - V_1(K, M) + V_2(K, M)g_1(C) = 0$$

The steady state envelope conditions, obtained by differentiating equation (6) with respect to the state variables, K and M , and imposing steady state are

$$(21) \quad (r + \delta_K + p(M))V_1(K, M) = \lambda\Phi - V_2(K, M)H_1(K)$$

$$(22) \quad (r + \delta_M + p(M))V_2(K, M) = U_2(C, M) - V(K, M)p_1(M)$$

The probability of catastrophe, p , is now a function of the stock of greenhouse gases to allow for avoidable risk. Since equation (22) contains $V(K, M)$, to obtain the Euler equation we need an additional equation that relates the value function to the primitives of the economy. This additional equation is obtained by evaluating the Bellman-Hamilton-Jacobi equation (equation (6)) at the steady state. This gives the following relation

$$(23) \quad (r + p(M))V(K, M) = U(C, M) + \lambda(R + \Phi K - C)$$

Now equations (20), (21), (22) and (23) combine to give the Euler equation, which in turn with the steady state laws of motion gives a system of equations which determines the steady state level of

consumption, capital and stock of greenhouse gases

$$(24) \quad U_1 = \frac{(U + \lambda(R + \Phi K - C))p_1 - (r + p)U_2}{(r + p)(r + \delta_M + p)} \left(g_1 + \frac{H_1}{(r + \delta_K + p)} \right) + \lambda \left(\frac{\Phi}{(r + \delta_K + p)} + 1 \right)$$

$$(25) \quad K^* = \frac{R - C^*}{\delta_K}$$

$$(26) \quad M^* = \frac{g - H}{\delta_M}$$

Equation (24) is the Euler equation. When $\lambda = 0$ (the constraint on consumption is not binding) the Euler equation states that, along the optimal consumption path, net utility from an increase in consumption is zero. The gain comes from increased consumption utility while the loss comes from increased disutility from greenhouse gases caused in turn by the increase in consumption. The increase in the stock of greenhouse gases leads to a decrease in utility directly and indirectly by increasing the probability of a catastrophe. Equations (25) and (26) give arbitrage conditions for optimal stocks of capital and greenhouse gases. When the consumption constraint is binding ($\lambda > 0$), steady state consumption is once again equal to R . Furthermore, steady state capital is driven to zero and steady state stock of greenhouse gases satisfy the condition that decay is equal to gross emissions.

5.2. Irreversible Capital. With or without a binding constraint on consumption, Φ , the parameter governing the cost of converting capital to consumption, does not affect steady state consumption, capital or stock of greenhouse gases. Away from steady state too, Φ does not affect consumption decisions. If agents choose to consume all their endowment and devote nothing to investment then they will run down the capital stock at a rate dictated by Φ (quickly if Φ is high and slowly if Φ is low). However, Φ does not affect an agent's decision to invest nothing and consume all. As with unavoidable risk, the degree of irreversibility of abatement capital does not affect decisions to consume or emit greenhouse gases.

5.3. Durable Capital. Now let us consider the case where risk is avoidable and the rate of depreciation is used to capture capital irreversibility. Once again steady state capital can be expressed as a linearly decreasing function of steady state consumption (equation (25)) and steady state stock of greenhouse gases as an increasing function (equation (26)). This in turn implies that we can write the steady state Euler equation as a function of steady state consumption and some parameters.

To analyze the effect on the optimal rate of emissions of a change in the degree of durability of capital, we differentiate the Euler equation (equation (24)) with respect to the rate of depreciation. The differentiation yields a complicated expression with an ambiguous sign. Adding avoidable risk thus dilutes the result that durable capital leads to a decrease in investment. Individuals may

choose to increase investment in order to reduce the risk of the catastrophe. This counters the need to decrease investment as capital becomes more durable. Models that do not account for avoidable risk will find a stronger irreversibility effect for capital.

5.4. **Decay Rate.** With avoidable risk we use the same procedure as outlined in subsection 4.2.1 to get the following system of equations that determines the steady state level of consumption, capital and stock of greenhouse gases

$$(27) \quad U_1 = \frac{Up_1 - (r+p)U_2}{(r+p)(r+\delta_M+p)} \left(g_1(1-\hat{H}) + \frac{g\hat{H}_1}{r+\delta_K+p} \right)$$

$$(28) \quad K^* = \frac{R - C^*}{\delta_K}$$

$$(29) \quad M^* = \frac{(g(1-\hat{H}))}{\delta_M}$$

Here too equations (27), (28) and (29) give arbitrage conditions for steady state consumption, capital, and stock of greenhouse gases, and the Euler equation can be expressed as a function of steady state consumption and some parameters.

Now let us consider how consumption at the steady state changes with a change in the degree of reversibility of the stock of greenhouse gases.

Proposition 3. *If* $\frac{-\partial p}{\partial M} \frac{\partial M}{\partial \delta_M} \left(\frac{(r+\delta_M+p)}{(r+\delta_K+p)} + \frac{(2r+2p+\delta_M)}{(r+p)} \right) < 1$ *then* $\frac{dC^*}{d\delta_M} > 0$

In words, as the stock of greenhouse gases become irreversible, consumption decreases while investment increases if a reduction in the rate of decay leads to a relatively small increase in the probability of a catastrophe. Consequently, there is an irreversibility effect associated with irreversible stocks of greenhouse gases when the risk is avoidable so long as the increase in irreversibility does not cause a large increase in the probability of catastrophe. If risk does increase rapidly, then it may be optimal to increase consumption today rather than wait for a tomorrow that may never come.

5.5. **Non-negative Emissions.** Once again the non-negativity constraint does not bind at the steady state or during the transition to steady state unless the agents begin with a large stock of greenhouse gases that they want to reduce drastically. Consequently, defining irreversibility of the stock of greenhouse gases in terms of non-negative emissions weakens the irreversibility effect associated with the stock of greenhouse gases.

Thus far we have shown that whether or not irreversibility effects exist and what their magnitudes are depends on the definitions of irreversible capital and irreversible stocks of greenhouse gases. Results on the relative importance of the different irreversibility effects are sensitive to these

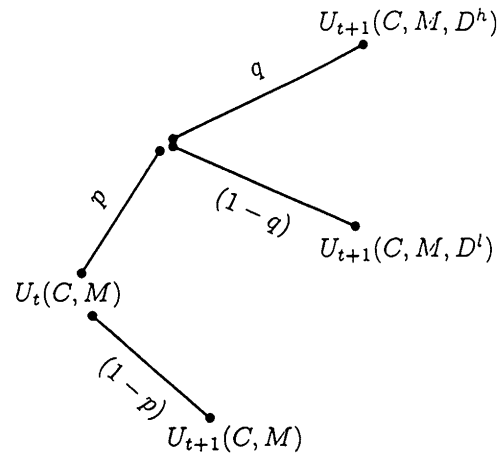
definitions, as we show also in the next section, a parameterization of the theoretical model which is more readily comparable to results of the multi-period simulations of Kolstad (1996a) and Ulph and Ulph (1997). It will also be possible to introduce learning into the simulation model, making our results still more readily comparable to those in the other simulations. Finally, we can determine whether the projected time paths appear to approach a steady state, consistent with our focus, in the analytical section, on behavior at the steady state.

6. NUMERICAL MODEL

In the theoretical model we have necessarily assumed that even though the event of a catastrophe is uncertain, the decision-maker knows what the damages will be should a catastrophe occur. Furthermore, since the world comes to an end after the catastrophe has occurred there is no scope for acting on the new information. By the time it is learned whether a catastrophe will occur it is too late to act. In this section we relax the assumption of known damages and allow the world to exist after the catastrophe has occurred. Once a catastrophe occurs, the decision-maker learns about the nature of the damages, which may be either high or low. These changes allow for learning, and give time to act on the new information.

We expect both stocks, capital and greenhouse gases, to yield irreversibility effects, though in opposite directions. Investment today locks the economy into a particular use of those resources which may turn out to be wasteful if tomorrow reveals that the damages due to global warming are small in magnitude. Consequently, one expects that investment in irreversible capital will be less than the investment that would be made if capital were reversible. With irreversible stocks of greenhouse gases, failure to invest today locks the economy into a level of damages which may be in future revealed as "too high." Consequently, to maintain the option of not having to bear these damages we should reduce emissions by increasing investment today. Avoidable risk should reinforce the effect of irreversible stocks of greenhouse gases and counter the effect of irreversible capital.

6.1. Primitives. With unknown damages and a catastrophe where the world does not come to an end, the specification of damages and the evolution of utility between any two periods changes compared to the previous theoretical model. The following event tree captures the evolution of utility.



Until the catastrophe occurs damages are zero and do not affect the utility function. Consequently, the utility function is given by $U_t(C, M)$. In any period $(t + 1)$ there is a probability p of a catastrophe occurring and should the catastrophe occur the nature of damages is fully revealed. Given that a catastrophe occurs, there is a probability q that damages will be high, leading to utility $U_{t+1}(C, M, D^h)$, and a probability $(1 - q)$ that damages will remain low, leading to utility $U_{t+1}(C, M, D^l)$.

6.2. Parameterization for the Simulations. To enable the numerical analysis we must assign functional forms for preferences, technology, endowments, system dynamics, information and constraints.

6.2.1. Preferences. Prior to a catastrophe agents have quadratic preferences over consumption (C_t) and the stock of greenhouse gases (M_t). Until a catastrophe occurs, damages do not affect the utility function.

$$(30) \quad U(C_t, M_t) = \frac{-1}{2} ((C_t - b)^2 + M_t^2)$$

where b is the bliss point.

Damages, once a catastrophe occurs, shift the utility function. Consequently, high or low damages change the utility function to

$$(31) \quad U(C_t, M_t, D) = \frac{-1}{2} ((C_t - b)^2 + (D^i M_t)^2)$$

where $i = l, h$ and D^l is a constant less than one and D^h is a constant greater than one.

6.2.2. *Technology.* Greenhouse gases are produced as a result of consumption with a linear technology

$$(32) \quad E_t = \sigma C_t$$

where E_t denotes emissions and σ is a constant denoting the emissions to consumption ratio. For both reversible and irreversible stocks of greenhouse gases we assume that only the flow of gases can be abated with capital. The abatement function is given by

$$(33) \quad \hat{A}_t = \hat{H}(K_t) = \frac{2}{1 + \exp(-\rho K_t)} - 1$$

where A_t is the amount of abatement in period t and ρ is the slope of the modified logistic function. We use a modified logistic function to limit abatement to be a fraction that lies between 0 and 1.

6.2.3. *Endowments.* Agents are given a fixed endowment, R , in every period to either consume or invest in abatement capital. Accordingly the budget constraint is given by

$$(34) \quad R = C_t + I_t$$

6.2.4. *Dynamics.* Capital stock changes from one period to the next due to depreciation and investment according to the law of motion

$$(35) \quad K_t = (1 - \delta_K)K_{t-1} + I_t$$

where δ_K is the rate of depreciation.

Abatement capital is allowed to vary continuously between reversible and non-reversible with the degree of irreversibility being captured by the parameter Φ . This amounts to an upper bound on consumption given by

$$(36) \quad C_t \leq R + \Phi K_{t-1}$$

If capital is irreversible then $\Phi = 0$ and consumption is constrained to be less than the fixed endowment R . Reversible capital amounts to $\Phi = 1$ so that consumption in a given period can be greater than R but less than $R + K_{t-1}$.

Similarly, the stock of greenhouse gases changes over time due to abatement and natural decay and follows the law of motion given by

$$(37) \quad M_t = (1 - \delta_M)M_{t-1} + \sigma C_t(1 - \hat{H}(K_t))$$

Whether or not the stock is reversible depends on the natural rate of decay. For a high rate the stock is reversible and irreversible for a low rate. As δ_M approaches zero the stock of greenhouse gases becomes irreversible.

In every period there is a possibility of catastrophe. This is captured by a damage function that follows a jump process.

$$(38) \quad D_t = \pi_t$$

with,

$$\pi_t = \begin{cases} a & \text{with probability } p_t, \\ 0 & \text{with probability } 1 - p_t. \end{cases}$$

where a is the magnitude of the jump and p_t is the probability of a catastrophic damage. We assume that risk is avoidable and express p as a function of the current stock of greenhouse gases.

$$(39) \quad p_t = \frac{2}{(1 + \exp(-\omega M_t))} - 1$$

where ω is a parameter that captures the sensitivity of p_t to the stock of greenhouse gases.

Prior to a catastrophe, both whether a catastrophe will occur and what the damages will be should it occur, are uncertain. A catastrophe reveals whether damages will be high or low. Agents believe that there is a q probability of high damages and a $(1 - q)$ probability of low damages. So a , the magnitude of the jump, is equal to D^h with probability q , and D^l with probability $(1 - q)$.

We solve for the value function and the policy function using MATLAB and NPSOL for non-linear optimization. Our simulation technique relies on the more efficient policy function iteration rather than value function iteration.

7. RESULTS

We are now in a position to consider how irreversible capital and an irreversible stock of greenhouse gases affect optimal consumption under avoidable and unavoidable risk, with and without learning. We conjecture that irreversible capital will reduce investment and thus increase consumption while irreversible stocks of greenhouse gases and avoidable risk will both lead to higher levels of investment and thus lower levels of consumption.

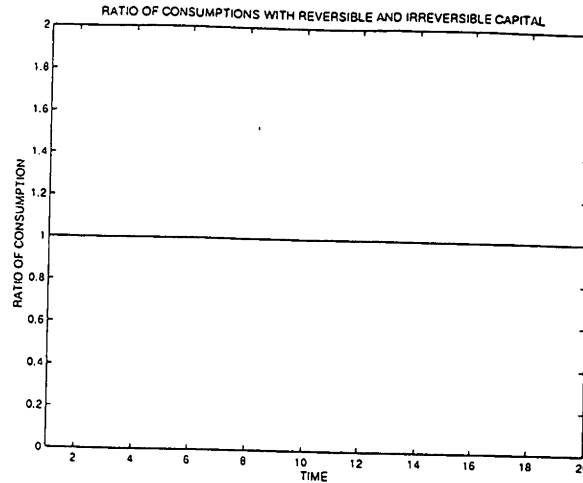


FIGURE 1. Ratio of Consumption under Irreversible and Reversible Capital when there is no Learning

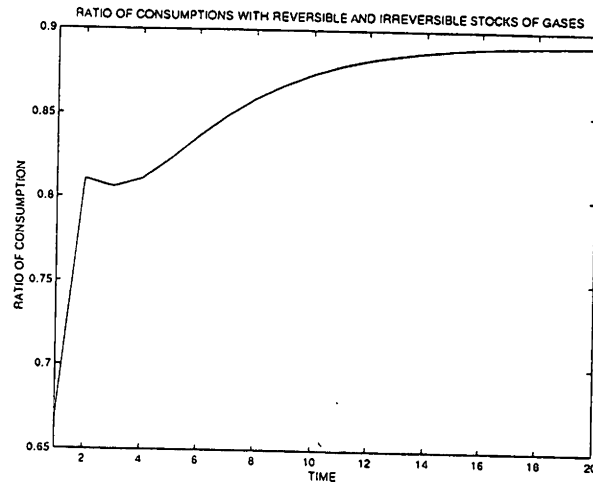


FIGURE 2. Ratio of Consumption under Irreversible and Reversible Stocks of Gases when there is no Learning

7.1. No learning. Our theoretical model predicts that in the steady state, if uncertainty is not resolved over time, that is, if agents do not learn about the nature of damages, then irreversible capital does not lead to an irreversibility effect but irreversible stocks of greenhouse gases do. These predictions are consistent with the numerical simulations of paths that appear to approach a steady state. Figure 1 shows that agents do not change consumption when capital becomes less reversible. The ratio of consumption under irreversible capital to consumption under reversible capital is one. On the other hand when the stock of greenhouse gases become more irreversible agents reduce consumption. The ratio of consumption under irreversible stocks of greenhouse gases to consumption under reversible stocks is less than one (see Figure 2). When both capital and the stock of greenhouse gases are irreversible the effect of the latter dominates.

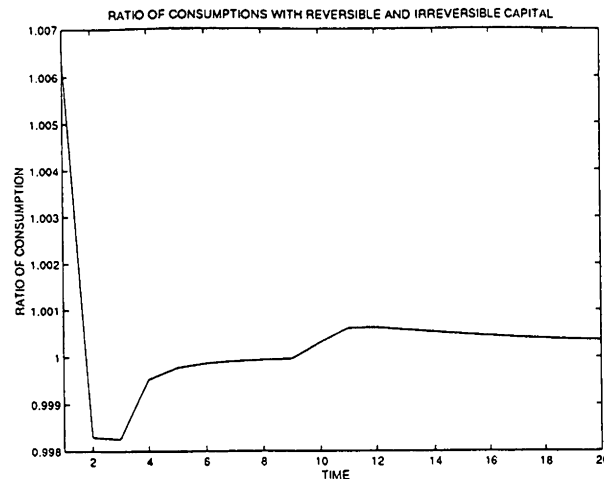


FIGURE 3. Ratio of Consumption under Irreversible and Reversible Capital when Risk is Unavoidable

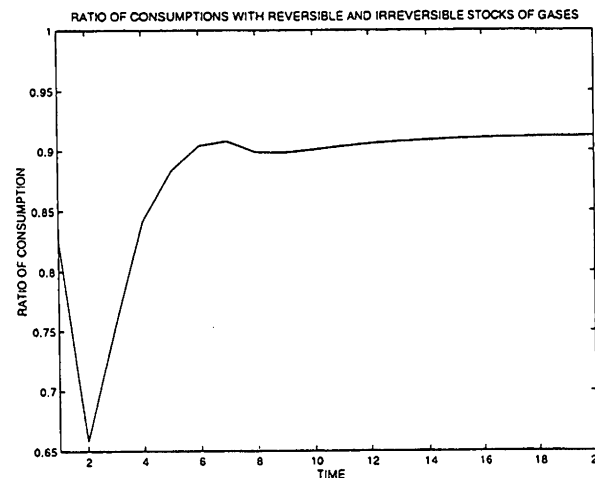


FIGURE 4. Ratio of Consumption under Irreversible and Reversible Stocks of Gases when Risk is Unavoidable

7.2. Learning and Unavoidable Risk. We now allow for uncertainty to be resolved over time so that agents learn about the nature of damages and have time to act on this information. We first consider the effect of irreversible capital on optimal consumption policy. The stock of greenhouse gases remains reversible. Figure 3 shows that when risk is unavoidable consumption mostly increases when capital becomes irreversible (the ratio of consumptions is mostly greater than one). This is Kolstad's (1996a) result, though obtained using our preferred definition of irreversibility. Next we change the degree of irreversibility of the stock of greenhouse gases but let abatement capital remain reversible. Once again we consider the ratio of consumption under irreversible and reversible stocks of greenhouse gases. As expected, Figure 4 shows that with unavoidable risk as the stock of greenhouse gases becomes irreversible consumption decreases. The consumption ratio is less than

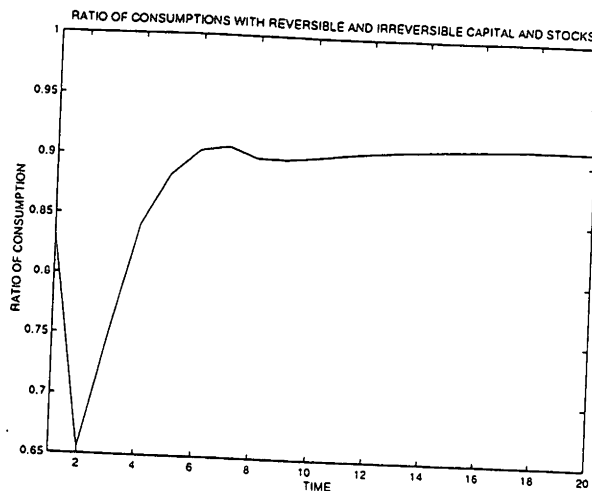


FIGURE 5. Ratio of Consumption when both Capital and the Stocks of Gases are Irreversible and Risk is Unavoidable

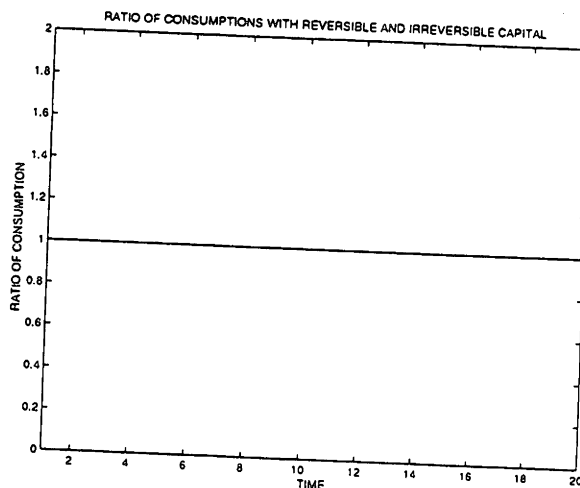


FIGURE 6. Ratio of Consumption under Irreversible and Reversible Capital when Risk is Avoidable

one. When both capital and the stock of greenhouse gases are irreversible the environmental impact dominates and the consumption ratio falls below one, as shown in Figure 5.

7.3. Learning and Avoidable Risk. Finally, we consider the effects of irreversible capital and stocks of greenhouse gases when risk is avoidable and there is learning. With only irreversible stocks of capital, Figure 6 shows that the irreversibility effect for capital no longer holds. Avoidable risk appears to fully counter this effect. Consumption does not increase as capital becomes irreversible and is in fact unaffected. On the other hand, avoidable risk reinforces the effect of irreversible stocks of greenhouse gases and consumption falls even further (Figure 7). Once again the effect

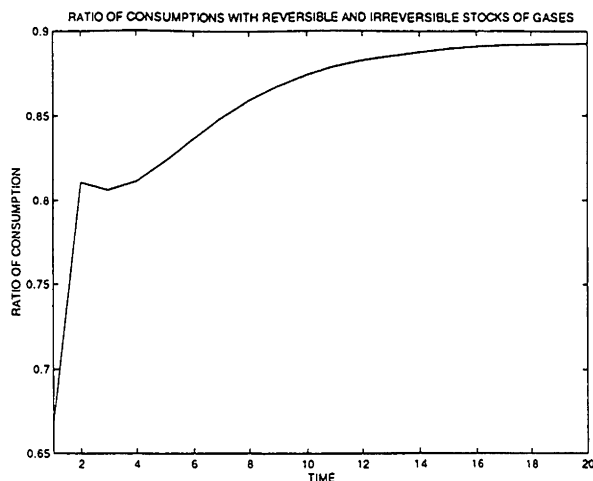


FIGURE 7. Ratio of Consumption under Irreversible and Reversible Stocks of Gases when Risk is Avoidable

of irreversible stocks of greenhouse gases dominates when both capital and the stocks of gases are irreversible.

8. SUMMARY AND CONCLUSIONS

In this paper we do three things. First, we provide an explanation for the somewhat counter-intuitive results on irreversibility in the literature on the economics of climate change, and suggest some alternative assumptions which we argue better reflect the nature of capital and environmental irreversibilities. Second, we introduce the realistic feature of avoidable risk into the decision framework. Third, we develop a numerical model that allows us to analyze the impact of our alternative assumptions and compare the results to the simulation results in the literature.

Previous studies have suggested that irreversible stocks of greenhouse gases do not lead to a reduction in emissions but irreversible capital does lead to an increase in emissions. We show that this result depends on the definition of irreversible capital—capital is irreversible if it does not decay. The result is weakened under what we suggest as a more intuitive definition for irreversibility—capital is irreversible if it cannot be converted into consumption (or other capital). Our definition of irreversible stocks of greenhouse gases, on the other hand, which requires that some part of the stock does not decay, along with nonnegative emissions, does lead to an irreversibility effect. Further, with avoidable risk there is an irreversibility effect associated with the stock of greenhouse gases but none with the steady state stock of capital. Finally, a numerical model that allows for unknown damages and learning yields an irreversibility effect both for capital and the stock of greenhouse gases. With unavoidable risk, irreversible capital leads to lower investment while irreversible stocks of greenhouse gases lead to higher investment. However, with avoidable risk only irreversible stocks

of greenhouse gases lead to a change (increase) in investment. There is no irreversibility effect associated with the stock of capital. Avoidable risk cancels the effect of irreversible capital.

These results clearly have policy implications. Results in the literature to date have implied a reduction in desired investment in abatement capital, since an irreversibility effect is found only for this investment. Our results suggest that this may not be an optimal policy. With avoidable risk, an irreversible stock of greenhouse gases leads to a lower level of desired emissions and thus greater investment in abatement. There is no irreversibility effect associated with capital. The optimal policy then is to reduce emissions by increasing investment in control today.

APPENDIX A. DERIVATION OF THE BELLMAN-HAMILTON-JACOBI EQUATION

An agent chooses a stream of consumption to maximize expected intertemporal utility subject to equations (1)-(4). The only source of uncertainty is whether or not a catastrophe will occur.

$$(40) \quad \max_C E_t \int_t^\infty U(C, M, \tau) d\tau$$

Let $J(K, M, D, t)$ denote the corresponding value function. To derive the appropriate Bellman-Hamilton-Jacobi equation we begin by splitting the dynamic program into two parts²⁰

$$(41) \quad J(K, M, D, t) = \max_C E_t \left[\int_t^{t+dt} U(C, M, \tau) d\tau + \int_{t+dt}^\infty U(C, M, \tau) d\tau \right]$$

Since

$$E_{t+dt} \int_{t+dt}^\infty U(C, M, \tau) d\tau = E_{t+dt} J(K + dK, M + dM, D + dD, t + dt)$$

equation (41) simplifies to

$$(42) \quad J(K, M, D, t) = \max_C \left[U(C, M, t)dt + J(K + dK, M + dM, D + a, t + dt)pdt \right. \\ \left. + J(K + dK, M + dM, D, t + dt)(1 - pdt) \right]$$

Next we take a first order Taylor series expansion of the last two terms on the right hand side of equation (42) around $dt = 0$. This gives the following expression

$$(43) \quad J(K, M, D, t) = \max_C \left[U(C, M, t)dt + J(K, M, D + a, t)pdt + J(K, M, D, t) \right. \\ \left. + J_1(K, M, D, t)(R - C - \delta_K K)dt \right. \\ \left. + J_2(K, M, D, t)(g(C) - H(K) - \delta_M M)dt \right. \\ \left. + J_4(K, M, D, t)dt - J(K, M, D, t)dt + h.o.t. \right]$$

where $J_1(K, M, D, t)$ is the derivative of the value function with respect to its first argument. $J_2(K, M, D, t)$ and $J_4(K, M, D, t)$ are similarly defined and h.o.t. denotes higher order terms in the Taylor expansion. Note that because damages take on integer values we do not differentiate the value function with respect to damages. Subtracting $J(K, M, D, t)$ from both sides, dividing

²⁰This derivation draws heavily on Mangel (1985) and Karp (1997).

through by dt and letting dt approach zero with the added assumption that $\lim_{dt \rightarrow 0} \frac{h.o.t.}{dt} = 0$ gives

$$(44) \quad 0 = \max_C \left[U(C, M, t) + (J(K, M, D + a, t) - J(K, M, D, t))p \right. \\ \left. + J_1(K, M, D, t)(R - C - \delta_K K) \right. \\ \left. + J_2(K, M, D, t)(g(C) - H(K) - \delta_M M) + J_4(K, M, D, t) \right]$$

For the autonomous problem the value function $J(K, M, D, t)$ can be written as $e^{-rt}W(K, M, D)$. Making this substitution into equation (44) and multiplying through by e^{rt} gives the following version of the Bellman-Hamilton-Jacobi equation

$$(45) \quad rW(K, M, D) = \max_C \left[U(C, M) + (W(K, M, D + a) - W(K, M, D))p \right. \\ \left. + W_1(K, M, D)(R - C - \delta_K K) \right. \\ \left. + W_2(K, M, D)(g(C) - H(K) - \delta_M M) \right]$$

Up until the time when the catastrophe occurs $D = 0$ and once the catastrophe has occurred utility goes to zero forever, or that, $W(M, a) = 0$. With these and the final simplification that $W(M, 0) = V(M)$ the Bellman-Hamilton-Jacobi equation can be written as

$$(46) \quad rV(K, M) = \max_C \left[U(C, M) + V_1(K, M)(R - C - \delta_K K) \right. \\ \left. + V_2(K, M)(g(C) - H(K) - \delta_M M) - V(K, M)p \right]$$

APPENDIX B. PROOFS FOR PROPOSITIONS AND COROLLARIES

Proof for Proposition 1

Proof. Differentiating the Euler equation with respect to δ_K yields the condition that

$$(47) \quad \frac{dC^*}{d\delta_K} < 0 \quad \text{if} \quad -H_{11}(K^*) \frac{K^*}{H_1(K^*)} \geq \frac{\delta_K}{(r + \delta_K + p)}$$

□

Corollary 1

Corollary 1. The effect of an increase in the durability of capital on steady state capital and stock of greenhouse gases is ambiguous.

Proof. First consider the effect on steady state capital. Totally differentiating equation (11) with respect to the rate of depreciation yields

$$(48) \quad \frac{dK^*}{d\delta_K} = \frac{-1}{\delta_K} \frac{dC^*}{d\delta_K} - \frac{(R-C)}{\delta_K^2}$$

Under the sufficient condition given in proposition 1 the first term on the right hand side of equation (48) is positive—an increase in consumption as a result of a decrease in the rate of depreciation decreases investment and thus decreases the capital stock. However, the second term is negative—a decrease in the rate of depreciation increases the capital stock. Similarly, from equation (12) the effect on steady state stock of greenhouse gases is given by,

$$(49) \quad \frac{dM^*}{d\delta_K} = \frac{\partial M^*}{\partial C^*} \frac{dC^*}{d\delta_K} + \frac{\partial M^*}{\partial \delta_K}$$

The first term on the right hand side of equation (49) is negative while the second term is positive. Under the sufficient condition, a decrease in the rate of depreciation increases the level of consumption which in turn increases the stock of greenhouse gases through an increase in emissions. However, a decrease in the rate of depreciation also increases the capital available for abatement which decreases the stock of greenhouse gases. The overall effect is ambiguous. \square

Proof for Proposition 2

Proof. Differentiate equation (16) with respect to δ_M . This expression is omitted here because of its complexity but it gives the result that $\frac{dC^*}{d\delta_M} > 0$. \square

Corollary 2

Corollary 2. *The effect of an increase in the irreversibility of the stock of greenhouse gases on the steady state stock is ambiguous.*

Proof. From equation (18)

$$(50) \quad \frac{dM^*}{d\delta_M} = \frac{\partial M^*}{\partial \delta_M} + \frac{\partial M^*}{\partial C^*} \frac{dC^*}{d\delta_M}$$

The first term on the right hand side of equation (50) is negative while the second term is positive. A decrease in the rate of natural decay (a decrease in δ_M) has two countervailing effects on the stock of greenhouse gases. A decrease in δ_M increases the stock of greenhouse gases directly but also decreases the stock indirectly by decreasing steady state consumption. \square

Proof for Proposition 3

Proof. Differentiate equation (27) with respect to δ_M . This expression is omitted here because of its complexity. Its denominator is negative from the second order condition while its numerator is

negative if

$$(51) \quad \frac{-\partial p}{\partial M} \frac{\partial M}{\partial \delta_M} \left(\frac{(r + \delta_M + p)}{(r + \delta_K + p)} + \frac{(2r + 2p + \delta_M)}{(r + p)} \right) < 1$$

□

APPENDIX C. PARAMETERS

Given below are the parameters we use to generate the simulations for the base case when damages are unknown.

| <i>Parameter</i> | <i>Values</i> | <i>Parameter</i> | <i>Values</i> |
|------------------|---------------|------------------|----------------|
| β | 0.95 | δ_K | 0.1 |
| <i>bliss</i> | 50 | δ_M | (0.5 or 0.3) |
| Φ | (0.5 or 1) | R | 10 |
| σ | 1 | γ | (0.08 or 0.05) |
| ω | 0.0005 | p | 0.2 |
| α^h | 2 | q | 0.002 |
| α^l | 0 | | |

TABLE 1. Parameter Values Used for the Simulations

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