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WORKING PAPER NO. 823

THE JUDGMENT PROOF OPPORTUNITY

by

Klaas T. van 't Veld

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1. INTRODUCTION

As policymaking in the areas of environmental preservation, consumer protection, and industrial safety turns increasingly to corporate liability for accidents as a way of inducing socially responsible behavior by firms, it is increasingly constrained by the so-called "judgment proof problem." This problem arises when a firm that engages in a hazardous activity may cause damages that exceed its wealth. It can then avoid legal liability for such damages through bankruptcy—thereby becoming "judgment proof"—and is, as a result, unlikely to choose a socially optimal level of care or scale of operating the hazardous activity. Starting with papers by Summers (1983) and Shavell (1986), this problem has been analyzed in a small but growing formal literature.

With very few exceptions, this literature has ignored the ability of individual firms, as well as entire hazardous industries, to respond strategically to liability. The canonical model in the literature considers only a single, representative firm (or at best two, if joint torts are at issue) with given revenues, given costs other than expenditures on safety, and given initial wealth.

The serious inadequacy of this model as a basis for analyzing the real-world judgment proof problem is illustrated by events surrounding the enactment of the U.S. Oil Pollution Act of 1990. As soon as it became clear that this law would effectively remove dollar limits on liability for oil spills, the ocean shippers and oil cargo owners targeted by the law began maneuvering to minimize their exposure to such liability. In July 1990, the *Wall Street Journal* reported that at least one (unidentified) international oil company had begun to park the title to some of its oil in transit with a company controlled by fugitive oil trader Mark Rich in Switzerland. Oil companies Royal Dutch/Shell and Elf Aquitaine announced that they would curtail U.S. oil shipments in vessels they owned or managed. Danish shipping giant A.P. Moeller, owner of one of the world's largest independent oil tanker fleets, soon followed suit. Oil refiners, too, started to shun legal ownership of oil until it arrived at their plants, instead of accepting it in the seller's port as used to be the practice.

> In fact, Texas oilman and trader Kyle McAlister says he's already canvassing U.S. refiners. For at least 10 cents a barrel, Mr. McAlister says, he is prepared to run the liability risk for them, buying their oil and holding possession of it until it reaches their refineries (*Wall Street Journal*, July 26, 1990, p. B1).

Meanwhile, some shippers were getting ready to subdivide their fleets into single-ship companies, each with minimal assets, to protect the rest of their operations from any one claim. Ketkar (1995) reports that in 1980, 29.6% of the ocean-going tankers trading in the U.S. were owned by single-ship companies, but that this figure had risen to 45.5% in 1991. Of course, even a single ocean-going tanker is a valuable asset to lose. Mid-1991, the *Wall Street Journal* reported that big shippers, rather than entering U.S. waters with such tankers, were transferring oil to small "ferry" ships in a move to further limit their liability exposure.

These events clearly show that firms view the ability to be judgment proof as an opportunity—one which they have many ways of exploiting. Moreover, competitive pressures in both capital and output markets ensure that firms will exploit this judgment proof opportunity to the hilt.

The model developed in this paper incorporates the firm's point of view by explicitly taking into account its optimal financial decisions in the face of liability, as well as the competitive environment in which it operates. Capital market competition, and the resulting endogeneity of firm wealth, is accounted for by decomposing wealth into those variables that determine how the capital market places a value on the firm's shares. Output market competition, and the resulting endogeneity of industry structure, is accounted for by allowing for entry and exit by firms of different scales, with arbitrary inherent advantages with respect to costs of production and care. These features distinguish the model from any in the existing literature.[1]

Section 2 of this paper lays out a version of the model that initially abstracts from scale economies in either production or accident prevention. This simplifies the analysis of Sections 3 and 4, which consider a firm of given scale and analyze the feedback effects—mediated by the capital market—between two sets of decisions made by the firm. On the one hand are the firm's decisions with respect to variables that directly impact welfare, namely its safety decision and, given market power, its choice of output price. On the other are the firm's purely financial decisions, whose direct effect is merely to allocate given revenues across time and across different stakeholders in the firm.

Section 3 focuses on the firm's optimal dividend policy in the face of liability. The analysis of this section clears up a persistent confusion in the existing literature concerning the care decision of potentially judgment-proof firms. Whereas some authors conclude that such firms take less than the socially optimal amount of care, a much-cited analysis by Beard (1990) suggests that, counterintuitively, they may exercise the right amount, or even *more* care than is socially optimal. It is shown here, however, that this result is counterintuitive for the wrong reason: it relies on an implicit assumption that the firms irrationally retain discretionary wealth, rather than immediately distributing such wealth as dividends.

Section 4 focuses on the financial decision that lies at the heart of the judgment proof problem, namely the decision whether or not to declare bankruptcy following an accident. Analysis of how this decision varies with the firm's scale and price leads to the main result of this section, which is that, in contrast to the well-known result by Buchanan (1969) for common pollution externalities, the externality created by the judgment proof problem is never fully offset by market power.

Section 5 expands the base model to allow for arbitrary scale economies in both production and accident prevention. The focus of this section is on how hazardous industries as a whole restructure in response to liability. A very simple condition on economies of scale in accident prevention is identified under which imposing liability on a hazardous industry is guaranteed to strictly improve welfare. In contrast, if this condition fails to hold, imposing liability may amount to a high-stakes gamble on welfare outcomes. The section concludes with a discussion of some empirical evidence on economies of scale in accident prevention.

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[1] A model by Boyd and Ingberman (1996), which is based on Boyd (1992) and used also by Boyd and Ingberman (1994) and Ingberman (1994) is exceptional in the existing literature in that it does treat both firm wealth and industry structure as endogenous to the imposition of liability. However, the model still treats wealth as an unexplained primitive, and imposes highly specific (and in several ways restrictive) assumptions on economies of scale. A related problem with one of Boyd and Ingberman's central results is briefly discussed in Section 5.

Section 6 summarizes the paper and discusses work on extensions of the model.

2. THE MODEL

Consider an industry in which all firms specialize in some hazardous activity h. The industry's technology for producing h is Leontief and is initially assumed to be constant returns to scale, with the cost of all inputs other than capital, k, absorbed in the output price m. Firms operate integer multiples of some smallest unit of capital: $k \in \{1, 2, 3, \dots\}$.[2] Exactly one unit of capital is required to produce one unit of output, so that the production function is simply $h = k$.

Unless specified otherwise, the industry's output market is assumed to be perfectly competitive. Aggregate demand $Q(m)$ is downward sloping and $Q(m) > 1$ at all relevant prices m.

The capital market is assumed to be perfect also, in the standard sense. Firms in the industry are all-equity financed and managed by their shareholders.[3] A firm that operates k units of capital (hereafter, "a firm of size k") has k shares outstanding. Each share has a face value of 1—the price of a unit of capital—and a market value of v. Firms immediately distribute all profits as dividends.[4] Shareholders are risk-neutral and face a constant opportunity cost of investment, equal to the return r on a riskless bond.

Each unit of capital operated by a firm gives rise to accidents at times generated by a Poisson process with mean q. These accidents do not damage the firm's capital, i.e., they are of the nature of chemical spills rather than explosions. Accidents happen independently across units of capital. As a result, the times at which a firm of size k experiences accidents can be treated as if they are generated by a Poisson process with mean kq. For brevity, q is hereafter referred to as simply "the accident rate" of the firm, and kq as the firm's "overall accident rate." The damages A from each such accident are drawn independently from a distribution with density $f(A)$ and support $[0, U]$, where $U > 1$. Given that an accident occurs, expected damages equal $\overline{A} \equiv \int_0^U Af(A)\,dA$.

Firms are able to influence q through expenditures on safety. Initially, it is assumed that there are no scale economies in accident prevention: to achieve a given expected accident rate q per unit of capital, any firm must spend the same amount $s(q)$ per unit of capital on safety, regardless of its size k. The care function $s(q)$ is downward sloping and convex: $s'(q) < 0,\ s''(q) > 0$. If a firm spends enough on safety, it can make

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[2] In the analysis of the model, k is treated as a continuous variable bounded below by 1.

[3] An argument along the lines of Brander and Spencer (1989) shows that firms strictly prefer equity over debt if creditors cannot verify expenditures on accident prevention.

[4] This is shown later to be the optimal financial policy for potentially judgment-proof firms.

q arbitrarily small, but never zero: $\lim_{q \to 0} s(q) = \infty$. If, on the other hand, the firm spends nothing at all on safety, q will equal some finite value $\overline{q}$. It is assumed that this upper limit is never strictly binding on the firm's optimal choice of q.

Firms are strictly liable for damages caused by accidents, but this liability is limited to the value of their assets. There are no transaction costs associated with bankruptcy, and firms have no liability insurance.[5] Under these assumptions, firms will file for bankruptcy as soon as they experience an accident of size $A > kv$, where kv is the total market value of the firm. Firms whose value falls short of U and therefore face the possibility of such a bankrupting accident are called *undercapitalized*; firms whose value exceeds U are called *fully capitalized*; firms whose value exactly equals U are called *borderline*, and are considered to be fully capitalized as well. For expositional simplicity, it is assumed that whenever a firm experiences an accident of size $A \leq kv$, i.e., a non-bankrupting accident, it simply collects the damage payments A directly from its shareholders in the form of cash contributions. Neither the number k nor the market value v of its outstanding shares is then affected by such an accident.[6] Given that an accident occurs, the expected loss incurred by the firm's shareholders equals $\ell(kv) \equiv \int_0^{kv} A f(A)\, dA + kv[1 - F(kv)]$.

Finally, for the purposes of calculating welfare W, equal weight is given to consumer surplus, firm profits, and accident damages not paid for by firms. The social discount rate is taken to equal r.

These assumptions imply the following expressions for welfare and firm value:

Lemma 1. *Welfare is given by*

$$W = \frac{\int_m^\infty Q(x)\, dx + Q(m)\left[m - r - s(q) - q\overline{A}\right]}{r}. \tag{1}$$

Firm value is given implicitly by

$$kv = \frac{k[m - s(q) - q\ell(kv)]}{r}. \tag{2}$$

Proof: All proofs are given in Appendix A.

Expression (1) is more intuitive when written as

$$W = \int_0^\infty e^{-rt} \left\{ \int_m^\infty Q(x)\, dx + Q(m)[m - r - s(q) - q\overline{A}] \right\} dt.$$

The expression in braces is just the expected increment in welfare over the course of a short time interval of length dt. The increment in consumer surplus equals dt times $\int_m^\infty Q(x)\, dx$ and the increment in firm profits, before accident damages, dt times $Q(m)[m - r - s(q)]$. For small enough dt, at most one accident will occur in the industry at the end of the interval, with probability approaching $Q(m)q\,dt$. The expected damages of that accident, if it occurs, equal $\overline{A}$.

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[5] Given that their shareholders are risk-neutral, firms have no incentive to buy insurance.

[6] The same result could be achieved in other, equivalent ways. The firm could issue k new shares, for example, announcing that it will use the proceeds to both pay off the damages A and buy up all k old shares at their post-accident market value of $v' = (kv - A)/k$.

A very useful way of rearranging expression (2) is

$$r\,kv = km - ks(q) - kq\ell(kv). \tag{3}$$

This equation can be interpreted as a no-arbitrage condition, which must hold across any time interval $(t, t + dt]$. Shareholders in a firm of size k face the following trade-off at t. If, on the one hand, they withdraw their capital kv from the firm and invest it in the riskless bond for the duration of the interval, they obtain return $r\,kv\,dt$. If, on the other hand, they keep their capital invested in the firm, their expected return will have two components. First, with k units of capital, the firm will produce and sell $k\,dt$ units of output, thereby earning revenues of $km\,dt$, all of which are by assumption distributed as dividends. Second, for small enough dt, at most one accident will occur at the end of the interval, with probability approaching $kq(k)\,dt$. If so, the shareholders incur an expected capital loss of $\ell(kv)$. In equilibrium, the shareholders must be indifferent between withdrawing their capital or keeping it invested in the firm.

3. FINANCIAL DECISIONS AND CARE

In this section, the model is used to investigate how firms' financial decisions affect their incentives to spend on safety. The care decision is of course central to the judgment proof problem, and the early literature on this problem, as exemplified by Shavell (1986), finds that undercapitalized firms unambiguously take less care than is socially optimal. As the next proposition show, the model of this paper is in agreement with this early literature:

Proposition 1. *Fully capitalized firms choose the socially optimal accident rate q^*, defined implicitly by condition*

$$-s'(q) = \overline{A}. \tag{4}$$

Undercapitalized firms choose a strictly higher accident rate $\hat{q}(\ell)$, defined implicitly by condition

$$-s'(q) = \ell(kv). \tag{5}$$

Both parts of the proposition appear very intuitive. Because fully capitalized firms fully internalize the cost of accidents, their marginal benefit of care expenditures is equal to society's, and so their privately optimal accident rate q^* is also socially optimal. In contrast, because undercapitalized firms can externalize part of the cost of accidents through bankruptcy, their marginal benefit of care expenditures is lower, and so they choose a privately optimal accident rate $\hat{q}(\ell)$ that is larger than q^*. This is exactly the result of Shavell.

This reasoning ignores, however, that undercapitalized firms' marginal cost of care expenditures may also be lower than society's. In particular, if the marginal dollar spent on care would be lost to the firm anyway in case of an accident, since the firm is then bankrupted and loses all its wealth, then the real opportunity cost of that dollar must be discounted by the probability of an accident. This is precisely the point made in a much-cited analysis by Beard (1990), which finds that undercapitalized firms may well take as much or even *more* care than is socially optimal.[7] It is shown below, however, that Beard's reasoning is flawed, at least for the case of all-equity financed firms.[8] What Beard ignores is that undercapitalized firms, if they act in the best interest of their shareholders, will never leave discretionary wealth exposed to loss in case of an accident. Instead, they will immediately distribute any such wealth to their shareholders, possibly after spending some of it on care. Every dollar spent on care then reduces the amount left to be distributed by one dollar, however, so that undercapitalized firms do face the full opportunity cost.

Although Beard does not make explicit how the firm obtains the discretionary wealth out of which it pays for safety expenditures, the most straightforward assumption is that this wealth consists of retained profits.[9] To replicate his argument using the model of this paper, a new variable x is therefore introduced, representing the rate at which the firm pays out dividends. Also, since all that is needed to replicate Beard's possibility result is an example, we can restrict ourselves to the simple case of an accident density $f(A)$ that is concentrated entirely on the single accident size U.[10] Undercapitalized firms are then bankrupted by the first accident they experience, and the function $\ell(kv)$ reduces to simply $\min(kv, U)$. Because all accidents are of size U, the average accident size, $\overline{A}$, is of course also equal to U. Lastly, for reasons that will become clear at the end of this section, it is necessary to adopt Beard's discrete-time framework with binomial accident probability. In other words, rather than considering a time interval of infinitesimal length dt, we consider a time interval of discrete length Δt, but still assume that at most one accident occurs at the end of that interval, with probability $kq\Delta t$. For notational convenience, we set $\Delta t = 1$.

With these changes, the no-arbitrage condition for shareholders in a fully capitalized firm becomes

$$r\,kv = kx + [km - ks(q) - kx] - kq\,U. \tag{6}$$

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[7] The same result has also been derived, apparently independently, by Craig and Thiel (1990) and Posey (1993).

[8] The result can be produced under two radically different assumptions on how an undercapitalized firm pays for care. The firm must either (1) pay for care up front, out of wealth that it otherwise loses in case of an accident, or (2) buy care on credit, on terms that do not depend on the level of care that it chooses. Beard (1988), which is the dissertation on which Beard (1990) is based, explicitly allows for both assumptions; Craig and Thiel (1990) explicitly make the second assumption; Posey (1993) is not explicit about either. Because the model of this paper assumes that firms are all-equity financed, it can only be used to analyze the reasoning based on the first assumption.

[9] The analysis of this section would be essentially identical if the firm paid for safety expenditures out of income from assets not associated with the production of h. As pointed out in Section 5 below, an undercapitalized firm would rationally divest such assets, however.

[10] Strictly speaking, doing so is inconsistent with the assumption made earlier that the density $f(A)$ has support $[0, U]$. A result showing that for some undercapitalized firm $\hat{q}(\ell)$ is *strictly* below q^* under the degenerate density should, by continuity, hold up also under some non-degenerate density, however.

The left-hand side can again be interpreted as the opportunity cost of keeping funds invested in the firm. The first term on the right-hand side represents one benefit of doing so, namely the dividends received. The second, bracketed term represents another benefit, namely the capital gain realized from profits retained by the firm. Because the firm is by assumption fully capitalized, this capital gain is realized regardless of whether the firm experiences an accident. If the firm does experience an accident, however, which happens with probability kq, then shareholders also incur a capital loss of U. Clearly, the two kx terms in (6) cancel, leaving

$$r\,kv = km - ks(q) - kq\,U.$$

This shows that for shareholders in a fully capitalized firm dividend policy is irrelevant: the value of their shares is the same regardless of the level of x.

The same is not true for an undercapitalized firm. Taking the simplest case of a firm that is liquidated at the end of the discrete time period regardless of whether an accident has occured (the only difference then being whether either shareholders or accident victims receive the proceeds) the no-arbitrage condition becomes

$$r\,kv = kx + [1 - kq][km - ks(q) - kx] - kq\,kv. \tag{7}$$

Because the undercapitalized firm goes bankrupt in case of an accident, shareholders realize the capital gain from retained profits only if the firm does *not* experience an accident, i.e., with probability $1 - kq$. As a result, the terms kx in (7) do not cancel. Suppose now that the undercapitalized firm retains all its profits and therefore sets $x = 0$. Condition (7) then reduces to

$$r\,kv = [1 - kq][km - ks(q)] - kq\,kv.$$

Differentiating kv with respect to q yields the following condition defining the firm's optimal accident rate:

$$-s'(q) = \frac{kv + km - ks(q)}{1 - kq}. \tag{8}$$

Suppose, instead, that the firm retains no profits at all, setting $x = m - s(q)$. Condition (7) then reduces to

$$r\,kv = km - ks(q) - kq\,kv,$$

and differentiating kv with respect to q yields

$$-s'(q) = kv. \tag{9}$$

Note that this is just condition (5) in Proposition 1, specialized to the degenerate accident distribution.

By inspection, the right-hand side of (8) is greater than that of (9), implying that a no-dividend firm will exercise more care than a maximum-dividend firm with the same value kv. First of all, the numerator $kv + km - ks(q)$ on the right-hand side of (8) is greater than kv. The difference is precisely the capital gain that shareholders in the no-dividend firm forgo in case of an accident. This added loss provides an added incentive to reduce q through expenditures on care. Second, the denominator $1 - kq$ of the right-hand side of (8) is less than unity. This is because any expenditures on care by the no-dividend firm reduce its capital gain dollar for dollar, but this capital gain is forgone anyway in case of an accident. The real opportunity cost of the marginal dollar spent on care is therefore just $1 - kq$ dollars, equal to the probability that no accident occurs.

It is this difference in the denominator that drives Beard's result. Note first that a no-dividend firm will file for bankruptcy after an accident only if its original value kv falls short of the accident damages U even after adding the capital gain $km - ks(q)$. Such a firm is therefore undercapitalized if

$$kv + km - ks(q) < U. \tag{10}$$

Note also from conditions (8) and (4) that a no-dividend firm's expenditure on care will equal or exceed the socially optimal level if the right-hand side of (8) equals or exceeds $\overline{A} = U$:

$$U \leq \frac{kv + km - ks(q)}{1 - kq}. \tag{11}$$

Clearly, it is possible for both inequalities (10) and (11) to hold simultaneously, and this, in essence, is Beard's result.

However, rearranging condition (7) to

$$r\,kv = km - ks(q) - kq[kv + \{km - ks(q) - kx\}],$$

shows equally clearly that it is optimal for an undercapitalized firm to distribute all its net earnings immediately. Retaining any part $km - ks(q) - kx$ of those earnings only adds to the shareholders' losses in case of an accident, with no offsetting benefits. This implies that Beard's result is counterintuitive for the wrong reason: it relies on an implicit assumption that undercapitalized firms act irrationally by retaining some of their profits. If, instead, these firms distribute all their net earnings as soon as they are earned, the intuitive result of Proposition 1 follows.

An additional, independent problem with Beard's result is that, even though the length Δt of the time period is essentially arbitrary, the result is not invariant to it. Writing Δt explicitly, condition (8) becomes

$$-s'(q) = \frac{kv + [km - ks(q)]\Delta t}{1 - kq\Delta t} \tag{12}$$

and inequalities (10) and (11) can be combined to

$$(1 - kq\Delta t)U \leq kv + [km - ks(q)]\Delta t < U.$$

Clearly, the range of values kv for which these inequalities can hold simultaneously is not independent of Δt and in fact vanishes in the continuous-time limit as $\Delta t \to 0$. In that same limit, condition (12) reduces to condition (9). This shows that Beard's result depends also on an implicit assumption that shareholders "lock in" their care decision for some discrete time interval Δt, rather than optimizing the decision continuously. The shorter this time interval (which, following Merton (1975), might appropriately be called the shareholders' "decision horizon"), the closer the perceived opportunity cost $(1 - kq\Delta t)$ of a marginal dollar spent on care will be to a full dollar, and therefore the less likely it is that shareholders will overinvest in care. It can be shown, moreover, that even if shareholders allow retained profits to cumulate in the absence of accidents (rather than liquidating the firm immediately at the end of Δt), equation (9)—and thereby Proposition 1—still holds in the limit as $\Delta t \to 0$.

4. FINANCIAL DECISIONS AND PRICE

In this section, the focus is on the financial decision that lies at the heart of the judgment proof problem, namely the firm's decision whether or not to declare bankruptcy after an accident has occurred. Although the existing literature treats this decision as simply a matter of determining whether or not the firm can "afford" to pay for a given accident, the reality is shown here to be considerably more complicated, and to depend importantly on the firm's size.

The context in which the bankruptcy decision is investigated is that of market power, defined here as the ability to raise price above average cost because of some barrier to entry. This context is of particular interest to an analysis of the judgment proof problem, because firms with market power are unambiguously wealthier, all else equal, than competitive firms. By Proposition 1, they will therefore also spend more on care. This then suggests a possibility similar to that demonstrated by Buchanan (1969) for common pollution externalities, namely that the externality created by the judgment proof problem might in some cases be fully offset by that created by market power. If so, a similar policy conclusion would follow, namely that measures aimed at remedying the judgment proof problem may, in the presence of market power, make matters only worse.

The following proposition shows, however, that the two externalities in fact never cancel.

Proposition 2. *If a firm with market power is fully capitalized and therefore exercises the socially optimal level of care, it must be charging a higher price than is socially optimal. If, conversely, the firm happens to charge the socially optimal price, it must be undercapitalized and will therefore exercise less care than is socially optimal.*

The result relies on the relationship between three prices, namely (1) the socially optimal price, denoted m^*; (2) the price at which a firm of size k is borderline (i.e., just barely fully capitalized), denoted $m^b(k)$; and (3) the price at which a firm of size k is just barely able to attract equity capital, denoted $ac(k)$.

To find the socially optimal price m^*, note from expression (1) that welfare W is strictly decreasing in what might be called the average social cost of production at a given accident rate q, defined by $asc(q) \equiv r + s(q) + q\overline{A}$. This cost includes the social opportunity cost r of the single unit of capital required to produce a unit of output, plus any safety expenditures $s(q)$ on that unit of capital, plus the full expected damages $q\overline{A}$ from operating the unit. The socially optimal price m^* minimizes $asc(q)$ with respect to q,

$$m^* \equiv \min_q [r + s(q) + q\overline{A}], \tag{13}$$

where the minimum is achieved at the socially optimal accident rate q^*.

To find the borderline price $m^b(k)$, set $kv = U$ in expression (2) and solve for m. Using that $\ell(U) = \overline{A}$, this yields

$$m^b(k) \equiv \min_q [r\frac{U}{k} + s(q) + q\overline{A}], \tag{14}$$

where the minimum is achieved at the firm's privately optimal accident rate $\hat{q}(\ell(U)) = q^*$. Since kv is strictly increasing in m, a firm of size k is fully capitalized at all prices above, and undercapitalized at all prices below $m^b(k)$. Note that, in addition to safety expenditure and expected liability, the price includes a return on capital rU/k that for firms of size $k < U$ is above the normal return r. The reason for this will become clear below.

To find the price $ac(k)$, note that if the firm is just barely able to attract equity capital, the market value v of its shares must equal the face value of 1. Setting $v = 1$ in expression (2) and solving for m yields

$$ac(k) \equiv \min_q [r + s(q) + q\ell(k)], \tag{15}$$

where the minimum is achieved at the firm's privately optimal accident rate $\hat{q}(\ell(k))$. The label $ac(k)$ is chosen because this price represents the effective average cost of the firm: no firm can survive in the industry unless the output price it receives covers at least a normal return r for its shareholders, after subtracting safety expenditures $s(q)$ and expected liability $q\ell(k)$.

To clarify the relationship between these three prices, Figure 1 plots, for each price, the expected per-share revenue stream of shareholders in a firm of size $k < U$. It does so for the simple case again of a degenerate accident distribution $f(A)$ concentrated on the single accident size U. In addition, the accident rate q is treated as exogenous, and expenditures on care therefore ignored.[11] The key areas to compare in

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[11] This figure (and others like it that follow) helps visualize the relationships between the variables of the model, but should not be treated as more than heuristic. In particular, the figures ignore any returns from reinvestment during the time intervals shown. This is as it should be, because the relationships between variables that in fact obtain—and which the figures merely present in a different way—are those in the no-arbitrage

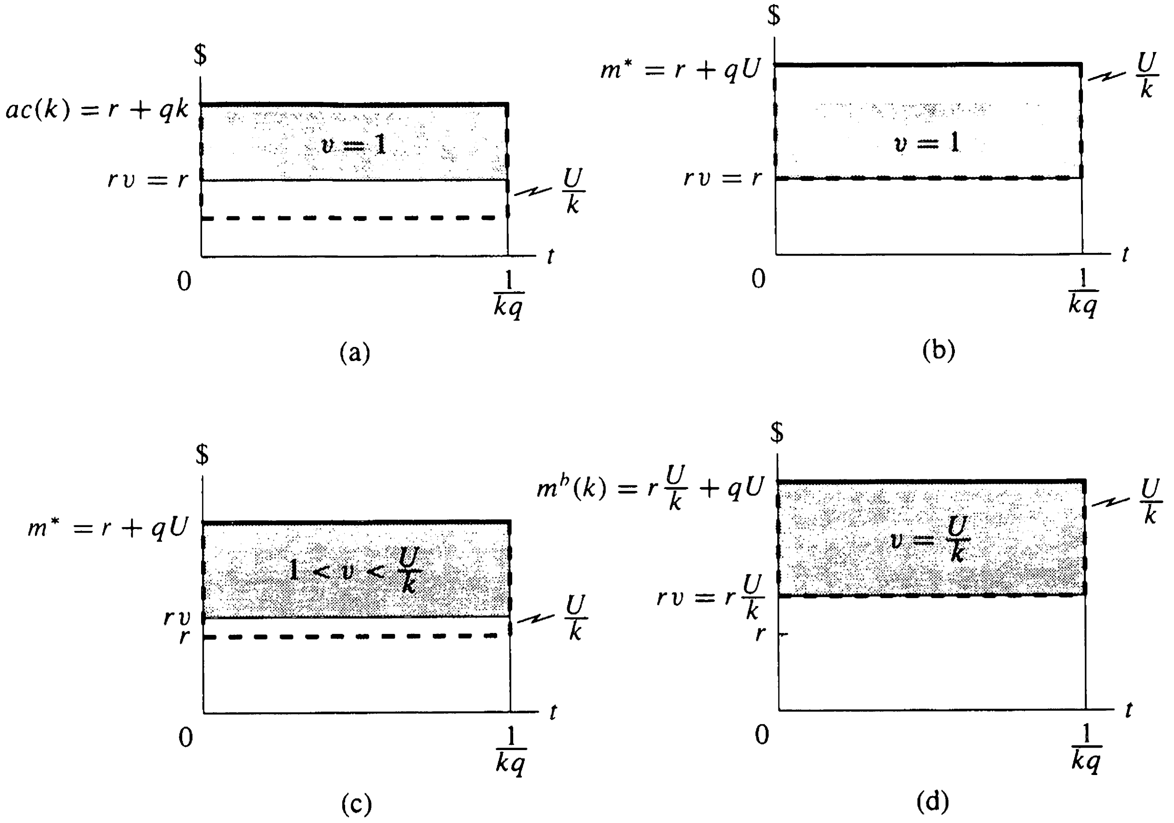

FIGURE 1. Relationship between average cost $ac(k)$, socially optimal price m^*, and borderline price $m^b(k)$ for a firm of size $k < U$.

each panel of the figure are the shaded rectangle, which represents the firm's share value v, the dashed box, which represents the per-share damages U/k caused by an accident, and the white rectangle just above the timeline, which represents the opportunity cost of owning the share. This opportunity cost is equal to the stream of returns that shareholders would receive if they sold the share and invested the proceeds in the riskless bond. The height of the white rectangle is therefore always equal to rv.

Panel (a) of the figure shows the expected per-share revenue stream if the firm receives a price just equal to its average cost $ac(k)$. Since the accident times of the firm are determined by a Poisson process with mean kq, the expected waiting time until its first accident equals $1/kq$. When the accident occurs, the firm files for bankruptcy, because the per-share capital loss of $v = 1$ that it incurs by doing so is less than the per-share loss of U/k that it would incur if it paid the accident damages. For shareholders to be prepared to invest in the firm despite this expectation of bankruptcy, their dividend m must in expectation make up for this capital loss of $v = 1$ and in addition cover the opportunity cost $rv = r$ of staying invested in the firm. As shown, these conditions will be met at output price $ac(k) = r + qk$. Note that nothing

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condition, across a time interval of infinitesimal length dt. Any proceeds from reinvestment in the course of such an interval are of second-order magnitude and therefore vanish as dt becomes small.

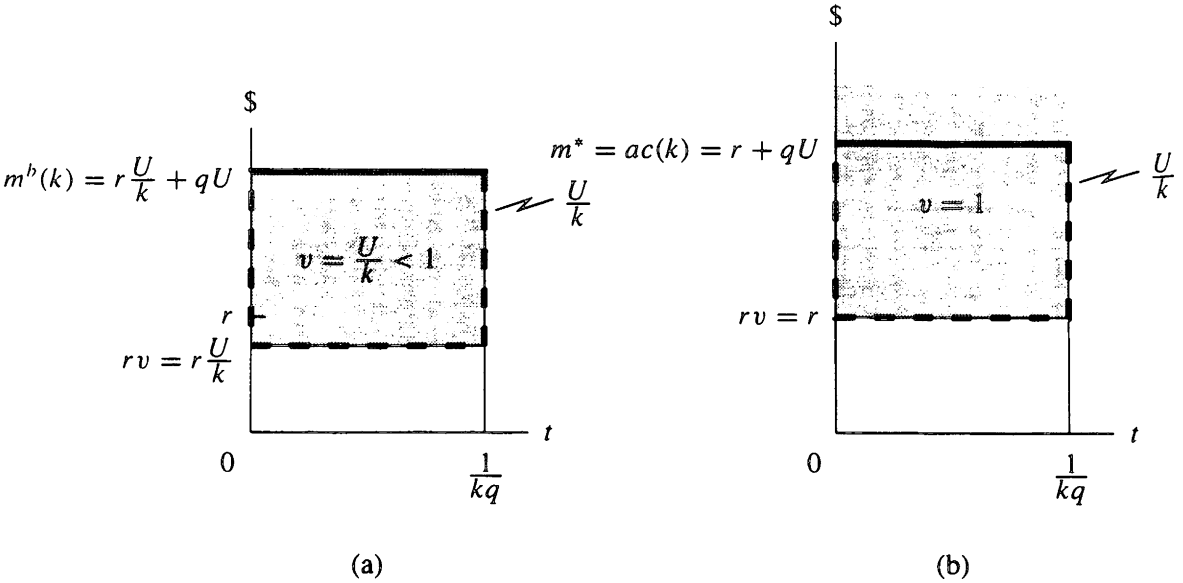

FIGURE 2. Relationship between average cost $ac(k)$, socially optimal price m^*, and borderline price $m^b(k)$ for a firm of size $k > U$.

prevents the shareholders from saving up the part of their dividends that corresponds to the shaded rectangle and investing these savings in an identical new firm as soon as the original firm goes bankrupt. Apart from a relabeling of the time origin, they would then again find themselves in exactly the situation of Panel (a). Repeating this behavior would, in expectation, yield the shareholders a normal return forever, just as if they had invested in the riskless bond.

Panels (b) and (c) both show the expected per-share revenue stream if the firm receives the socially optimal price m^*. Panel (b) shows the situation that would obtain under unlimited liability, however, whereas Panel (c) shows the situation that actually obtains under limited liability. The important point to note from Panel (b) is that, at m^*, shareholders could *afford* to pay the full per-share damages U/k of any accident and still expect to earn a normal return r on their investment. However, precisely because their return would be only normal, the value of their shares would just equal the face value of 1, exactly as at $ac(k)$. The shareholders will therefore not *want* to pay the damages U/k if liability is limited, preferring to file for bankruptcy instead. Interestingly, as Panel (c) shows, the very fact that, under limited liability, shareholders have this option of giving up their shares rather than paying the damages raises the value of their shares to $v > 1$, because it reduces the expected capital loss from accidents to below U/k. At the same time, this increase in v also raises the opportunity cost of staying invested in the firm to $rv > r$. In capital-market equilibrium, the return to shareholders that remains after the expected loss of v per share just covers this opportunity cost.

Panel (d), finally, shows the expected per-share revenue stream if the firm receives the borderline price $m^b(k)$. If, at this price, shareholders pay the per-share damages U/k from any accident, they are left not with a normal expected return, but with a higher return of rU/k. In capital-market equilibrium, the value v of their shares therefore just equals U/k, because this is the amount of money that, if invested in the riskless bond, would yield the same return of rU/k. As a result, shareholders will be exactly indifferent between either paying the damages or filing for bankruptcy and giving up their shares.

Figure 2 shows the very different situation for a firm of size $k > U$. If, as in Panel (a), such a firm were to receive the borderline price $m^b(k)$, its shareholders would still be exactly indifferent between paying the damages or losing the value $v = U/k$ of their shares. The firm would not be in business in the first place, however, because the expected return rU/k remaining after either course of action would be below-normal.

If, as in Panel (b), the firm receives a price equal to its average cost $ac(k)$, however, its shareholders will strictly prefer paying the per-share damages of U/k from any accident to giving up the firm and thereby incurring the larger per-share loss of $v = 1$. To attract investors, the firm's dividend therefore need only cover those expected per-share damages plus a normal return of r. As shown, this will be the case at output price $ac(k) = r + qU$, which is exactly the socially optimal price m^*.

Proposition 2 now follows for a firm of any size $k > U$ by noting that, if such a firm has market power, the price it charges will by definition lie above its average cost $ac(k)$, and thereby also above both $m^b(k)$ and m^*. The firm will therefore be fully capitalized, but the price it charges will be higher than is socially optimal. The same is true for a firm of size $k = U$, because it can be shown that for such a firm $m^b(k) = m^* = ac(k)$. In contrast, for a firm of any size $k < U$, the socially optimal price m^* lies strictly above $ac(k)$ and could therefore well be the privately optimal price for a firm with some degree of market power. The problem here is that m^* also lies strictly below $m^b(k)$ for such a firm. If, therefore, the firm is fully capitalized, it must be charging a higher price than is socially optimal. Conversely, if it charges the socially optimal price, it will be undercapitalized.

Allowing for a non-degenerate accident distribution and endogenous accident rates would considerably complicate Figures 1 and 2, but not affect the basic underlying relationships. Figure 3 plots the three prices against firm size k, for a non-degenerate accident distribution and endogenous accident rates. In terms of this figure, Proposition 2 follows simply because any point above $ac(k)$ lies either below $m^b(k)$ or else above m^*.

5. FINANCIAL DECISIONS AND SCALE

Whereas the foregoing sections focused on the individual firm's optimal financial decisions in the face of liability, this section investigates how hazardous industries as a whole restructure in response to liability. Important to the first result of this section is yet another financial decision of individual firms, however,

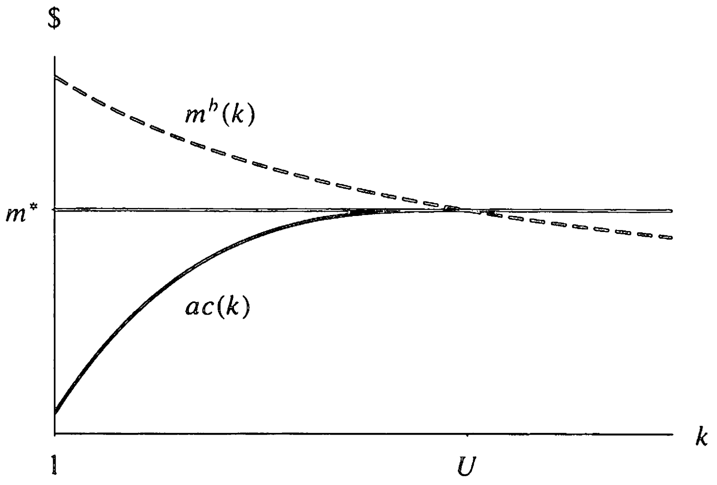

FIGURE 3. Relationship between average cost $ac(k)$, socially optimal price m^*, and borderline price $m^b(k)$ for a range of firm sizes k.

namely the decision to divest assets so as to legally segregate hazardous activities from non-hazardous ones. As several legal scholars have noted informally (e.g., Stone 1980, Kraakman 1984, Easterbrook and Fischel 1985, and Roe 1986), imposing liability gives horizontally or vertically integrated firms an incentive to shield assets not associated with a hazardous activity from liability. Such firms may, for example, create separate subsidiaries or spin-offs dedicated to the hazardous activity. Alternatively, they may abandon the hazardous activity altogether and, if it provides inputs required for their remaining activities, switch to buying those inputs from independent, specialized suppliers.

The anecdotal evidence discussed in the introduction suggests that divestment of this kind was an important part of the oil industry's response to the U.S. Oil Pollution Act. Statistical evidence pointing to the same phenomenon is provided in a much-cited empirical study by Ringleb and Wiggins (1990), which finds a strong positive association between the hazardousness of U.S. industries, as measured by the frequency of worker exposures to proven or suspected carcinogens, and the rate of entry by small firms into those industries following a sharp increase in potential liability for damage payments to workers. Ringleb and Wiggins suggest that large, integrated firms in these industries responded to the increased liability by divesting particularly hazardous steps of their production processes, and that the resulting increase in small, specialized firms accounts for their results.

The divestment decision itself is not modeled explicitly in this paper. That a firm may benefit from such divestment has already been demonstrated, in the abstract, by MacMinn and Brockett (1995). Moreover, any repercussions of divestment on the firm's other decisions are essentially identical to those of its dividend policy.[12] Instead, the divestment decision is taken into account implicitly, by assuming that all hazardous-industry firms are already specialized, and therefore have only assets directly associated with the hazardous activity.

By explicitly allowing for competition in the hazardous industry's output market between firms of different scales, the model does, however, point to an alternative explanation for Ringleb and Wiggins' empirical result, complementing their own explanation. The next proposition shows that, given constant returns to scale in both production and accident prevention, imposing liability on a hazardous industry will give small, undercapitalized firms a competitive edge over large, fully capitalized rivals, resulting in an industry equilibrium in which only the smallest firms survive.[13] The implication is that, even if all firms in the hazardous industries of Ringleb and Wiggins' sample were already specialized, so that divestment would not be an issue, one might still expect to observe small-firm entry into those industries following an increase in liability.[14]

Proposition 3. *In the competitive industry equilibrium that results after imposing liability only undercapitalized firms of size* $k = 1$ *survive.*

The result appears to have a very straightforward explanation, based on the following two intuitions. First, it seems obvious that firm size and wealth are positively related, so that being small and being undercapitalized go together. Second, it seems obvious that undercapitalized firms' ability to externalize damages should give them a competitive edge over fully capitalized firms, which fully absorb these damages. Putting the two intuitions together suggests that imposing liability will favor small, undercapitalized firms, exactly as Proposition 3 predicts. Closer examination reveals, however, that both intuitions ignore important countervailing effects, which appear as soon as even small deviations from constant returns to scale are allowed for.

In order to investigate the effects of such deviations, the model is modified in two ways. First, while retaining the assumption that the production function is Leontief, it is now assumed to be of the form $h = \min(k, f(x))$, where x is some input other than capital and $f(\cdot)$ is monotone increasing. Regardless of the relative price of k and x, a firm of size k will then optimally set $h = k = f(x)$. Per unit of capital, or, since $h = k$, per unit of output, it incurs expenditures on x equal to $c(k) \equiv wf^{-1}(k)/k$, where w is the unit price of x. With this modification of the model, positive (negative) economies of scale in production exist at any given k if $c'(k) < (>)\, 0$.

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[12] The divestment decision, too, has has been ignored in the formal literature on the judgment proof problem, however. Larson (1996), for example, in an analysis of how an undercapitalized firm will allocate its fixed initial wealth between a riskless investment and a hazardous production activity, assumes that wealth allocated to the former is left exposed to liability. As with Beard's analysis, this amounts to an assumption of irrational financial decisionmaking by the firm.

[13] An analogous result can be found in Boyd and Ingberman (1996), who show that imposing liability may result in lower levels of firm wealth and output after industry restructuring. They unnecessarily qualify this result, however, by suggesting that firms will be undercapitalized in equilibrium if and only if scale economies in production are such that firms would be undercapitalized even at the social optimum. Underlying this qualification is an implicit assumption that firms' average costs are a strictly convex function of their level of capitalization. There is no reason to suppose such convexity. In fact, as the model of this paper shows, under constant returns to scale in both production and accident prevention, firms' average costs will be a *concave* function of their level of capitalization.

[14] That this alternative explanation should be considered a complement to Ringleb and Wiggins' own explanation, rather than a substitute, is suggested by another empirical study, by Barney, Edwards and Ringleb (1992). Using the same data, but somewhat different specifications of the empirical model, these authors find more direct evidence of a reduction in vertical integration in the hazardous industries.

Second, firm size k is added as an argument to the care function, which therefore becomes $s(q, k)$. The original assumptions on $s(q)$ are retained, but conditioned on k. That is, for a firm of any given size k, the care function is downward-sloping and concave: $s_q(q, k) < 0$, $s_{qq}(q, k) > 0$; the firm can never reduce q to zero: $\lim_{q \to 0} s(q, k) = \infty$; and if the firm spends nothing on care, the accident rate equals some finite value $\overline{q}(k)$. It is again assumed that this upper limit is never strictly binding on the firm's optimal choice of q. Economies of scale in accident prevention are positive (negative) at given q and k if $s_k(q, k) < (>)\, 0$.

Note that, by conditioning the upper limit $\overline{q}$ on k, firms of different sizes are allowed to have different accident rates even if they spend nothing on safety. This is a reasonable assumption, because accident rates may differ purely as a byproduct of differences in firm organization. In large firms, for example, workers may be assigned to more specialized tasks, allowing them to become more skilled at performing those tasks safely. On the other hand, workers in large firms may also spend less time working with any given piece of equipment, making it more difficult for them to recognize impending equipment failures.

By a straightforward generalization of Lemma 1, the new expressions for welfare W and firm value kv become

$$W = \frac{\int_m^\infty Q(x)\, dx + Q(m)\left[m - r - c(k) - s(q, k) - q\overline{A}\right]}{r} \tag{16}$$

and

$$kv = \frac{k[m - c(k) - s(q, k) - q\ell(kv)]}{r}. \tag{17}$$

The firm's optimal accident rate, denoted $\hat{q}(k, \ell)$, is now determined by the first-order condition

$$-s_q(q, k) = \ell(kv). \tag{18}$$

The expressions for average cost $ac(k)$, average social cost $asc(k)$ and the borderline price $m^b(k)$ are modified in similar ways.

With these modifications of the model, we can now investigate the robustness of the first intuition discussed earlier, namely that firm wealth is positively related to firm size.

Proposition 4. *At any given price m, given a sufficiently low opportunity cost of capital r, arbitrarily small diseconomies of scale in either production or accident prevention can cause firm wealth to decline with firm size.*

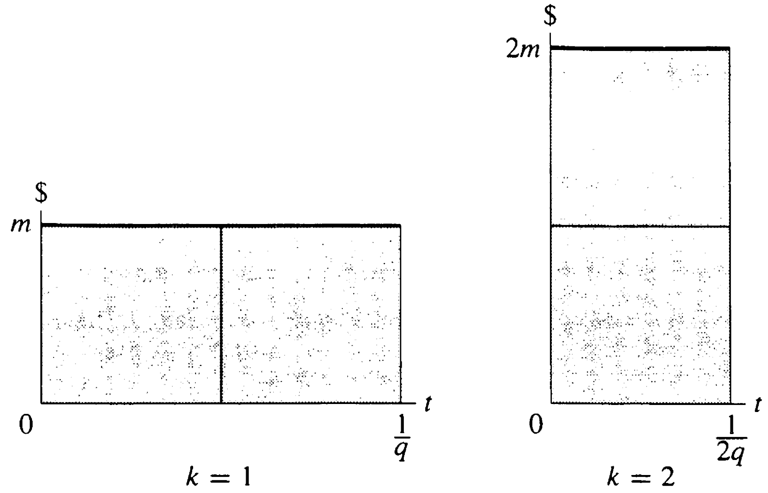

FIGURE 4. Total expected revenue streams for firms of sizes $k = 1$ and $k = 2$.

This result is again easiest to explain by considering the special case of an accident distribution concentrated on U. Assume also that there are no economies of scale in production, and that the accident rate is exogenous at the same value q for all firms, so that there are no economies of scale in accident prevention either. Figure 4 compares the total expected revenue streams under these assumptions for two undercapitalized firms of sizes $k = 1$ and $k = 2$, respectively. As shown, the larger firm earns revenues at a twice higher rate $2m$, but also expects to go bankrupt twice as soon, at time $1/2q$. Given zero discounting, the two size effects cancel, and the total expected revenue streams of both firms are exactly equal. By implication, the total value of both firms is exactly equal as well. Given positive discounting, this equality no longer holds: the larger firm's revenues are then worth slightly more in a present-value sense, because they are earned sooner. However, allowing for arbitrarily small diseconomies of scale in either production or accident prevention, just enough to offset the force of discounting, can reverse the ranking and make firm value decline with firm size.

It should be emphasized, however, that Proposition 4 assumes a given output price m at which firms of different sizes coexist. For large firms to be less wealthy than small ones at m, this price can never be equal to average cost for both. This follows because $v = 1$ at average cost, and firms' total value kv is therefore exactly equal to their size k. At average cost, in other words, the intuition that size and wealth are positively related is correct almost by definition. This implies that Proposition 4 only comes into play when, because of either market power or government intervention, price fails to be driven down to average cost. An example of a government policy that might have this effect would be a requirement that firms demonstrate assets worth at least U as a condition for operating in the industry.[15] If undercapitalized firms were only able to meet this requirement by raising their price (rather than by somehow acquiring outside assets), then $m^b(k)$ would become the effective average cost for such firms. Although $m^b(k)$ declines in k under constant returns to scale (given positive r), Proposition 4 implies that in the presence of even slight diseconomies of scale $m^b(k)$ may well become increasing in k. The industry equilibrium induced by the policy will then have only firms of size $k = 1$, because such firms will be able to meet the requirement most cheaply.

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[15] Both the U.S. Oil Pollution Act and the Resource Conservation and Recovery Act impose such requirements on firms that operate respectively oil facilities and hazardous-waste landfills. Both laws provide an alternative option of meeting a compulsory insurance requirement, however. Here, no such option is assumed.

The second intuition that appears to explain Proposition 3, namely that being undercapitalized should give firms a competitive edge over fully capitalized rivals, is shown by the following result to also require qualification:

Proposition 5. *Let $q(s, k)$ denote the accident rate achieved by a firm of size k if it spends an amount s per unit of capital on safety. Then, if economies of scale in accident prevention are such that*

$$\frac{\partial q}{\partial k}\frac{k}{q} \leq -1, \quad \forall k, \forall s, \tag{19}$$

imposing liability will confer a strict competitive advantage on large firms relative to small ones (and thereby on fully capitalized firms relative to undercapitalized ones) regardless of the distribution of accidents.

Underlying this result is the simple observation that what matters to a firm's competitiveness is not its total liability in case of an accident, but its liability per unit of output. For a firm of size k, the liability component of average cost is equal to kq, the probability of an accident, multiplied by $\ell(k)/k$, the expected liability per unit of output given that an accident occurs. For an undercapitalized firm of size $k^u < U$,

$$\frac{\ell(k^u)}{k^u} = \int_0^{k^u} \frac{A}{k^u} f(A)\, dA + \int_{k^u}^{U} f(A)\, dA.$$

For a fully capitalized firm of size $k^f > U$,

$$\frac{\ell(k^f)}{k^f} = \int_0^{k^u} \frac{A}{k^f} f(A)\, dA + \int_{k^u}^{U} \frac{A}{k^f} f(A)\, dA = \int_0^{U} \frac{A}{k^f} f(A)\, dA.$$

The first integral in both expressions represents the range of accident damages that both firms internalize. Because the fully capitalized firm is larger, however, it can spread these damages out over more units of output. As a result, the liability cost that it passes on in its price is lower. The second integral represents the range of accident damages that the undercapitalized firm partially externalizes through bankruptcy, but the fully capitalized firm absorbs in full. Per unit of output, and therefore also per unit of capital and per share, the undercapitalized firm incurs a loss of $v = 1$ from such accidents, because it gives up the entire value of its shares in bankruptcy. The loss for the fully capitalized firm, however, is only $A/k^f < 1$, since by assumption $k^f > U \geq A$. Per unit of output, the liability cost that the fully capitalized firm passes on in its price is therefore again lower, despite the ability of the undercapitalized firm to externalize part of the damages.

What this shows is that, for any given accident, the benefit that fully capitalized firms derive from being large, and therefore able to spread the accident damages thinly over many units of output, always strictly outweighs any benefit that undercapitalized firms derive from being able to externalize damages. That small, undercapitalized firms nevertheless have a competitive edge under constant returns to scale—as implied by Proposition 3—must then follow because they either have accidents less frequently, or else spend less per unit of capital on care.

To see how this observation leads to Proposition 5, define the liability-related component of average cost, including safety expenditures, as $alc(k) \equiv \min_q[s(q,k) + q\ell(k)] = \min_s[s + q(s,k)\ell(k)]$. This component is added to average production costs $apc(k) \equiv r + c(k)$ when liability is imposed, and any competitive advantage thereby conferred must therefore be due to differences in $alc(k)$ across firms. If condition (19) is met with equality, *overall* accident rates kq will be the same for firms of all sizes, provided they spend the same amount s per unit of capital on safety. If so, the only component of $alc(k)$ that will vary across firms is the average expected liability $\ell(k)/k$, which the discussion above shows to be strictly declining in firm size. If $alc(k)$ declines in k for any given level of s, however, it must also decline in k when firms choose their respective levels of s optimally.[16]

The result of Proposition 5 has two appealing policy implications, which the remainder of this section analyzes in more detail. First, it is shown that if condition (19)—hereafter, "the elasticity condition"—holds, imposing liability is guaranteed (under minor caveats) to result in a strict welfare improvement. Second, because the condition is independent of the accident distribution, some net welfare improvement is guaranteed to remain even after potential changes in this distribution over time. In contrast, welfare outcomes if the condition fails are potentially very sensitive to such changes. The latter consideration is important, because real-world accident distributions do change over time. Coastal development significantly affects the damages from oil spills, for example; changes in the population density around chemical plants affect the consequences of toxic releases; and even without changes in accident damages themselves, court damage awards may exhibit trends.

Of these two implications of Proposition 5, the most immediate is that if the elasticity condition holds, then imposing liability on the industry will induce it to restructure in the opposite direction of that implied by Proposition 3. Instead of inviting entry by small, undercapitalized firms, the policy will invite entry by large and possibly fully capitalized firms.[17] As the next proposition shows, provided such entry does not give rise to new problems associated with anti-competitive behavior, it guarantees that imposing liability is welfare-improving.

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[16] Suppose this were not the case. Then some larger firm could switch from its own optimal level s to the level s' chosen by some smaller firm and thereby strictly reduce its average costs. But then its original choice s cannot have been optimal after all.

[17] Strictly speaking, large-firm entry may in extreme cases be precluded, namely if the constraints $k \geq 1$ or $k \leq Q(m)$ bind respectively before or after imposing liability. In the former case, diseconomies of scale in production may everywhere outweigh the scale advantage induced by liability. If so, however, it is easy to show that an improvement in welfare is still guaranteed, because there can be no smaller-firm entry either.

Proposition 6. *If imposing liability induces any shift towards larger firms (without altering the competitiveness of the industry), the policy will strictly improve welfare.*

As was the case with Proposition 3, this result appears more straightforward than it is. Intuition suggests, correctly, that imposing liability on firms of any given size will improve welfare, by increasing those firms' internalization of damages. Intuition also suggests, however, that any subsequent entry by larger firms should increase welfare even further: not only does it exploit scale economies (or else it would presumably not occur, given the competitive advantage of small firms under constant returns to scale), but if it occurs at sizes $k < U$, it also further increases the equilibrium internalization of damages by firms.

Cases can be constructed, however (an example is provided in Appendix B) in which such a shift towards larger firms does both, and yet reduces welfare. Although in such cases imposing liability still improves welfare—consistent with Proposition 6—welfare would be even higher if no shift towards larger firm sizes occurred. In all cases like this, the counterintuitive outcome is driven by a negative cross effect s_{qk}, which implies that larger firms have to spend more to achieve any given marginal reduction in q. As a result, they may choose a higher accident rate despite internalizing more damages and despite having to spend less overall to achieve any given level of q.[18] The key to why Proposition 6 nevertheless holds even in these cases is that, if the larger firms indeed have to spend less to achieve any given level of q despite the negative cross effect, it must be the case that the upper limit on their accident rate lies below that for smaller firms. In other words, even if the larger firms spend nothing on safety after entering, their accident rate must still be lower than that of the smaller firms in the equilibrium before liability. As a result, there remains some net gain to society from lower expected damages even after the large-firm entry.

It is worth mentioning that the converse of the above intuition, namely that entry by small firms in the presence of positive scale economies must necessarily reduce welfare, can be shown to fail as well. Cases can be constructed in which entry by small, undercapitalized firms drives out larger, fully capitalized firms, and yet welfare improves. This may happen even under positive returns to scale in both production and

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In the latter case, market demand at the higher price induced by liability may simply be too small to allow a larger firm to enter. If so, however, the industry is unlikely to be competitive, so that the analysis of this section does not apply anyway.

[18] More specifically, these cases can arise because the difference in slope between average social cost and average cost equals

$$\frac{d\,asc(\hat{q},k)}{dk} - \frac{d\,ac(k)}{k} = \frac{d\hat{q}}{dk}[\overline{A} - \ell(k)] - \hat{q}[1 - F(k)], \tag{20}$$

where, from differentiating condition (18),

$$\frac{d\hat{q}}{dk} = -\frac{s_{qk} + [1 - F(k)]}{s_{qq}}.$$

If the cross effect s_{qk} is negative, $d\hat{q}/dk$ may be positive. The right-hand side of (20) may then be positive as well, and may be sufficiently positive to make average social cost increasing in k.

accident prevention, although, by Proposition 5, the latter must fall short of the elasticity condition. These cases, too, rely on a negative cross effect s_{qk}.

More important for the discussion of the elasticity condition is the following result, which complements Proposition 6:

Proposition 7. *If imposing liability induces any shift towards smaller, undercapitalized firms, it cannot be ruled out, without further assumptions, that the policy will strictly reduce welfare.*

This result follows directly from the assumptions of the model. Note first that welfare is fully determined by the combination of price, accident rate, and firm size. The model assumptions allow the market demand curve to be arbitrarily steep, however. In the limit as $Q'(m)$ goes to zero over the relevant range, any change in price induced by imposing liability becomes welfare-neutral, as it does not affect industry scale and otherwise just implies a transfer between consumers and producers. Similarly, the assumptions also allow firms' marginal cost $-s_q(q, k)$ of reducing their accident rate to be arbitrarily steep, i.e., s_{qq} may be arbitrarily high. In the limit as s_{qq} goes to infinity, no firm will spend on safety, and the optimal accident rate of all firms will be equal to the upper limit $\overline{q}(k)$. Finally, the assumptions allow for the possibility that this upper limit is the same for all firms, i.e., that $\overline{q}(k) = \overline{q}$ for all k. If so, imposing liability will have no effect on welfare via the accident rate either. That then leaves only the direct effect of the induced change in firm size on welfare, which operates via lost economies of scale. Because the smaller, undercapitalized entrants will have strictly lower average liability costs $alc(k) = \overline{q}\ell(k)$ than incumbent firms, their average production costs $apc(k) = r + c(k)$ may be strictly higher, while still leaving them with lower average costs overall. If so, their entry will strictly reduce welfare, because it raises average production costs in the industry with no offsetting benefits.

Although these assumptions on demand and on the care function are clearly extreme, they are required only to establish that *any* induced shift towards smaller, undercapitalized firms may make imposing liability welfare-reducing, no matter how small the shift is. More generally, if the induced shift is larger, a net welfare reduction may result even if demand is normally downward-sloping and the new entrants do spend on care.

The net reduction in welfare need not be caused by lost economies of scale in production. To see this, let average production costs $apc(k)$ be the same for all firms, but let the upper limit $\overline{q}(k)$ on firms' accident rates be decreasing in k (while retaining the extreme assumptions on $Q'(m)$ and s_{qq}). As long as the implied scale economies in accident prevention are not too large, average liability costs $\overline{q}(k)\ell(k)$ may still be strictly lower for small, undercapitalized firms, thereby allowing them to drive out larger firms. If so, however, the higher accident rates $\overline{q}(k)$ will strictly reduce welfare, again with no offsetting benefits.

As one would expect given the result of Proposition 5, the elasticity condition bounds the scale economies in accident prevention that may be lost in this manner. If this condition is met at $s = 0$, i.e., at the upper limit $\overline{q}(k) \equiv q(0, k)$ of firms' accident rates, then overall accident rates $k\overline{q}(k)$ at this upper limit will be

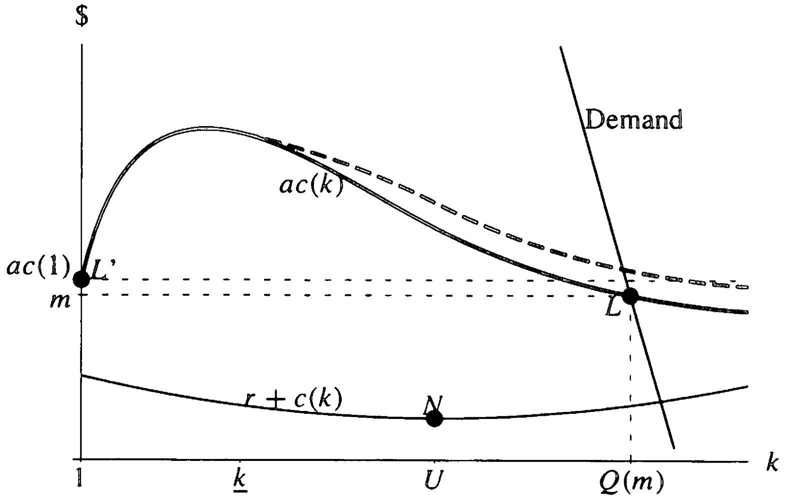

FIGURE 5. A slight shift in probability mass towards larger accidents changes the equilibrium from one in which a single firm of size $Q(m)$ serves the entire market to one with only firms of size $k = 1$.

non-increasing in k. At the same time, as established in the discussion following Proposition 5, the expected per-unit liability from any given accident, $\ell(k)/k$, is strictly decreasing in k. That then implies that the product $k\overline{q}(k) \,.\, \ell(k)/k = \overline{q}(k)\ell(k)$ must be strictly decreasing in k as well. Even if small, undercapitalized firms spend nothing on safety, their average liability costs will therefore always exceed those of larger firms, and no welfare-reducing entry can take place.

Clearly, then, in light of both Propositions 6 and 7, the elasticity condition is highly desirable, since it separates guaranteed positive welfare outcomes of imposing liability from potentially negative ones. That said, however, it should be noted that the condition is merely sufficient for the positive outcomes[19] not necessary. In fact, imposing liability may induce large-firm entry under arbitrarily small economies of scale in accident prevention, provided the advantage that small, undercapitalized firms derive from being potentially judgment proof is small enough. Locally, imposing liability will favor large firms over small ones if average liability cost $alc(k)$ declines in firm size, i.e. if $alc'(k) = s_k(q, k) + q[1 - F(k)] < 0$. At sizes $k \geq U$, i.e., for fully capitalized firms, this will be the case under any economies of scale $s_k < 0$ because for such firms the term $q[1 - F(k)]$ is zero. At sizes $k < U$, i.e., for undercapitalized firms, the required economies of scale become arbitrarily small as $q[1 - F(k)]$ goes to zero. Note, however, that $1 - F(k)$ represents the probability that an accident, if it occurs, will be bankrupting. If, therefore, either this probability or the accident rate q goes to zero, the judgment proof problem itself vanishes, and not surprisingly so does the competitive advantage of small, undercapitalized firms.

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[19] Barring the exceptional circumstances discussed in footnote 17.

More generally still, the possibility that $alc(k)$ is concave implies that $alc'(k)$ need not be negative everywhere for large firms to be favored on balance when liability is imposed. This is illustrated in Figure 5, which assumes that average production costs are minimized at $k = U$, so that the equilibrium before liability is at N. As drawn, the competitive advantage that small firms derive from being undercapitalized initially outweighs economies of scale in both production and accident prevention. As a result, $ac'(k)$ is positive at small firm sizes. Eventually, however, economies of scale in accident prevention dominate, and $ac'(k)$ becomes negative. In the particular case drawn, the industry equilibrium L after imposing liability has a single fully capitalized firm of size $k = Q(m)$ serving the entire market.

While it demonstrates that the elasticity condition is hardly necessary for liability to induce large-firm entry, the figure can also be used to illustrate why the condition nevertheless matters. Suppose that, given the same production technology and demand conditions, the industry's safety technology did meet the elasticity condition. An equilibrium similar to L might then be achieved, because $alc'(k)$ would be negative throughout. Because the condition is independent of the accident distribution, however, the competitive advantage conferred by liability on large firms would be robust to any changes in that distribution. Such changes would alter the equilibrium and might well shift the composition of the industry towards smaller firm sizes again. Any new equilibrium would always remain strictly to right of N, however. By Proposition 6, therefore, welfare would always remain higher than it was before liability. In contrast, if the elasticity condition is not met, welfare gains may easily turn into welfare losses. This is illustrated in Figure 5 by considering the effect of a slight shift in probability mass towards larger accident sizes, causing $F(A)$ to fall on some interval $[\underline{k}, \overline{k}]$ in the range $(1, U]$. As indicated by the dashed curve, the resulting increase in expected liability strictly raises average cost for firms of size $k > \underline{k}$, while not affecting smaller firms. As a result, the industry equilibrium shifts dramatically to a new equilibrium L' in which only small firms of size $k = 1$ survive. By the reasoning used earlier, welfare in this new equilibrium may well be lower than it was at N, before liability.

The elasticity condition might therefore appropriately be called the "no regret" condition. As long as it holds, policymakers will never have reason to regret imposing liability, because some net welfare gain will always remain from the policy. In contrast, if the condition fails to hold, the policy may amount to a high-stakes gamble on welfare outcomes.

This highly desirable property of the condition raises the empirical question whether it might in fact hold for the safety technology of any real-world hazardous industry. Unfortunately, the kind of data that would allow a direct test do not appear to be publicly available for any hazardous industry. Some indirect evidence can be deduced, however, from data on non-fatal occupational injury rates collected by the Bureau of Labor Statistics (BLS). Table 1 reports these for the year 1994, disaggregated by industry division and establishment size. Since no data are provided on safety expenditures, differences in injury rates across

	Establishment employment size (workers)				
Industry division	1 to 10	11 to 49	50 to 249	250 to 999	1000 or more
Private industry	3.3	6.8	9.9	8.9	8.2
Agriculture, forestry, and fishing	7.3	9.3	10.4	9.7	10.4
Mining	4.2	7.2	6.5	5.0	5.1
Construction	7.6	12.7	14.4	10.0	4.3
Manufacturing	5.3	10.4	12.2	9.7	9.7
Durable goods	6.9	12.6	13.6	10.4	10.7
Nondurable goods	2.9	7.0	10.5	8.9	7.7
Transportation and public utilities	3.4	7.8	10.5	8.9	10.1
Wholesale and retail trade	3.2	6.9	10.2	10.3	8.6
Wholesale trade	3.5	7.1	10.1	8.8	8.0
Retail trade	3.0	6.8	10.2	10.9	8.7
Finance, insurance, and real estate	1.9	2.4	3.1	2.4	1.5
Services	2.1	4.0	8.1	8.6	7.7

Source: Bureau of Labor Statistics news release USDL-95-508Y

TABLE 1. Nonfatal occupational injury incidence rates per 100 full-time workers, by industry division and employment size, 1994.

establishment sizes could in principle be explained entirely by differences in care. If we assume, however, that the cost of non-fatal worker injuries rarely, if ever, bankrupts firms, and that the other assumptions of the model hold to a good approximation as well, then all firms would optimally spend the same amount per worker if returns to scale in accident prevention were constant. Any differences in accident rates must then be accounted for by deviations from constant returns to scale.

The data do indicate significant such deviations, both positive and negative.[20] Specifically, in almost every row of the table, including that for private industry as a whole, the injury rate first increases with establishment size and then falls again. In the construction industry, for example, workers in establishments with 50 to 249 employees suffered an injury frequency rate that was roughly twice that of workers in establishments with only 1 to 10 employees, and roughly three times that of workers in establishments with 1000 or more employees. Oi (1974), using unpublished BLS data for the period 1968-70, studies the establishment size profile in more detail and finds that in a majority of U.S. industries the profile exhibits this inverted U-shape, while in a large minority of industries the profile declines monotonically.[21] Oi also finds significantly higher injury rate differentials in some industries. His data indicate that in the primary metals industry, for example, workers in establishments with 50 to 99 employees suffered an injury frequency rate almost 20 times that of workers in establishments with 2500 or more employees. Even this figure, however, falls short of the minimum of ~25 times (2500 divided by 99) that would be required to meet the elasticity condition. Moreover, this minimum assumes that all firms have the same number of establishments. In reality, the elasticity of establishment size with respect to firm size is probably well below unity, especially at large firm sizes, because large firms grow by adding new establishments rather than (or in addition to) increasing the size of their existing ones. If so, any elasticity of accident rates with respect to establishment size will exaggerate, in absolute terms, the corresponding elasticity with respect to firm size.[22] For the data to be consistent with the elasticity condition therefore, underlying safety expenditures per worker must be falling, possibly quite sharply, with establishment size. Under the assumptions of the model, this would be optimal for firms only if the safety technology exhibits a positive cross effect s_{qk}, implying that larger firms need to spend less to achieve any given marginal reduction in q. Barring a reason to reject the existence of such a cross effect out of hand, the evidence therefore remains inconclusive.

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[20] At the very least, therefore, they validate the concern of this section with modeling the potential consequences of such deviations.

[21] According to Oi, both the shape and dispersion of injury rate differentials are maintained when the data are disaggregated into finer industry classifications. He notes that the presumably close personal relationship between employer and employees in the smallest establishments may explain apparent initial diseconomies of scale in accident prevention, and that employment of safety specialists by large establishments may in part explain the eventual economies of scale.

6. SUMMARY AND EXTENSIONS

The model developed in this paper generalizes existing models of the judgment proof problem in two major respects. First, it takes full account of the firm's optimal financial decisions in the face of liability and the way in which, given these decisions, the capital market places a value on its shares. Second, it takes full account of how industry structure might change in response to liability, allowing for arbitrary scale economies in either production or accident prevention.

By analyzing the firm's dividend decision, the paper is able to clear up a persistent confusion in the existing literature concerning the safety decision of undercapitalized firms. Existing analyses suggesting that undercapitalized firms may well exercise the right amount, or even more care than is socially optimal are shown to rely—at least in the case of all-equity financed firms—on an implicit assumption that such firms retain discretionary wealth. This assumption is shown to be inconsistent, however, with rational financial decisionmaking by the firms: limiting dividend payments only increases firm losses in case of an accident, with no offsetting benefits.

By analyzing the firm's bankruptcy decision itself, the paper shows that, in contrast to common pollution externalities, the externality caused by the judgment proof problem is never fully offset by market power. The decision by a firm whether or not to declare bankruptcy following an accident is shown to turn

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[22] To show this, assume that there is a unique establishment size $e(k)$ associated with any firm size k. The data in Table 1 report $q(e(k), s)$. Assuming, as we have done, that s is invariant to firm size, we can write

$$\frac{\partial q}{\partial k}\frac{k}{q} = \frac{\partial q}{\partial e}\frac{e}{q}\,\frac{\partial e}{\partial k}\frac{k}{e}.$$

If the second elasticity on the right-hand side is positive but less than unity, elasticities of q with respect to e calculated from the table will be larger, in absolute terms, than the corresponding elasticities of q with respect to k.

not simply on whether the accident damages exceed what the firm can afford to pay. Limited liability allows firms to externalize even damages that they can afford, in the sense that after paying the damages they could still offer shareholders a normal return on their investment. This very fact, however, raises the price that firms must receive before they do choose to fully internalize all damages. This price is shown to always exceed the socially optimal price.

Finally, by analyzing how hazardous industries as a whole restructure in response to liability, the paper brings out the riskiness of policies that impose liability on hazardous industries, given real-world uncertainties about accident distributions and economies of scale. It is shown that seemingly robust intuitions about the relationship between firm size, wealth, and competitive advantage may in fact break down under even slight deviations from constant returns to scale in production or accident prevention. The same is true of intuitions about the welfare effects of firm entry into hazardous industries. Nevertheless, the paper is able to identify a very simple condition on scale economies in accident prevention under which a policy that imposes liability is guaranteed to improve welfare. In contrast, if the condition fails, slight, difficult-to-predict changes in the accident distribution may easily convert positive welfare outcomes of imposing liability into negative outcomes, and vice versa. Empirical evidence suggests, unfortunately, that the condition may not be met in real-world hazardous industries, although the evidence is far from conclusive.

More generally, the analysis of the paper implies that imposing liability on hazardous industries may give rise to significant nonconvexities in costs, making definite comparative-statics results difficult to obtain. The empirical evidence on scale economies in accident prevention suggests, for example, that these typically give rise to an inversely U-shaped accident rate profile when plotted against firm size. If so, the effect this has on average liability costs will be superimposed on that of inverse L-shaped internalization of accident damages, and typically U- or L-shaped average production costs. The sum of these effects may well yield a highly irregular average cost profile, with perhaps multiple and disjoint minima.

Nevertheless, current work on extending the model to investigate the likely impact of various potential remedies for the judgment proof problem shows that definite results can be obtained. A general finding is that industry restructuring induced by such remedies may either hinder or help the achievement of policy goals. For example, because of the adverse industry restructuring it induces, no purely price-based policy can ever achieve a first-best optimal industry equilibrium, regardless of assumptions about economies of scale. In contrast, because of the beneficial industry restructuring it induces, even a mild insurance requirement, covering far less than the full range of possible accident damages, may suffice to achieve a first-best optimum. Policy-relevant results such as these demonstrate the fruitfulness of the modeling approach laid out in the current paper.

APPENDIX A.

Proof of Lemma 1 The expected present value of consumer surplus is

$$\int_0^\infty e^{-rt} \int_m^\infty Q(x)\,dx\,dt = \frac{\int_m^\infty Q(x)\,dx}{r}.$$

The expected present value of profits gross of damage payments but net of safety expenditure and the social opportunity cost of capital is

$$\int_0^\infty e^{-rt}\, Q(m)[m - r - s(q)]\,dt = \frac{Q(m)[m-r]}{r}.$$

Damage payments by firms and damages borne by accident victims sum to total accident damages. Let $\sigma_1, \sigma_2, \ldots$ denote the random times at which accidents of respective sizes $A_1, A_2, \ldots$ occur. These times can be treated as if they were generated by a Poisson process with mean $Q(m)q$. The expected present value of accident damages then equals $E[\sum_{\sigma_i} X(\sigma_i)]$, where $X(\sigma_i) \equiv e^{-r\sigma_i} A_i$. Using Campbell's Theorem, this expectation evaluates to

$$\int_0^\infty \int_0^U e^{-rt} A Q(m) q f(A)\, dA\, dt = \frac{Q(m)q\overline{A}}{r}.$$

Combining the right-hand sides of these three expressions yields expression (1).

To derive the expression for firm value, consider a firm of size k, with value kv, and divide all accidents it might experience into non-bankrupting ones, of size $A \leq kv$, and bankrupting ones, of size $A > kv$. By the Coloring Theorem for Poisson processes, the times at which accidents of these two types occur can be treated as if they were generated by two independent Poisson processes with respective means $kqF(kv)$ and $kq[1-F(kv)]$. Let T denote the (random) lifetime of the firm, i.e., the waiting time until the first accident of size $A > kv$. Also, let $\tau_1, \tau_2, \ldots$ denote the times at which smaller accidents of respective sizes $A_1, A_2, \ldots$ occur. Then, for any realization of T, the present value of the firm's revenues net of safety expenditures until T is

$$\int_0^T e^{-rt}[km - ks(q)]\,dt = \frac{km - ks(q)]}{r}[1 - e^{-rT}].$$

The expected present value of the firm's damage payments until T equals $E[\sum_{\tau_i} Y(\tau_i)]$, where $Y(\tau_i) \equiv e^{-r\tau_i} A_i$ for $0 < \tau_i \leq T$ and $Y(\tau_i) \equiv 0$ otherwise. Using Campbell's Theorem, this expectation evaluates to

$$\int_0^T \int_0^{kv} e^{-rt} A\, kq F(kv) \frac{f(A)}{F(kv)}\, dA\, dt = \frac{kq \int_0^{kv} Af(A)\, dA}{r} [1 - e^{-rT}].$$

Subtracting expected damage payments from revenues yields

$$\frac{km - ks(q) - kq \int_0^{kv} Af(A)\, dA}{r} [1 - e^{-rT}].$$

The value of the firm is found by integrating these expected profits over all possible realizations of T, which yields

$$\begin{aligned} kv &= \frac{km - ks(q) - kq \int_0^{kv} Af(A)\, dA}{r} \int_0^{\infty} [1 - e^{-rT}] kq[1 - F(kv)] e^{-kq[1-F(kv)]T}\, dT \\ &= \frac{km - ks(q) - kq \int_0^{kv} Af(A)\, dA}{r + kq[1 - F(kv)]}. \end{aligned}$$

Rearranging and using the definition of $\ell(kv)$ yields expression (2). ☐

Proof of Proposition 1 Differentiating W with respect to q shows that the socially optimal accident rate, q^*, is defined implicitly by condition (4). Differentiating kv with respect to q (using the implicit function theorem) shows that the privately optimal accident rate, $\hat{q}(\ell)$, is defined implicitly by condition (5). Since $\ell(kv) = \overline{A}$ for $kv \geq U$, and since $s(q)$ is strictly concave, the two conditions combined imply that fully capitalized firms choose the socially optimal accident rate q^*. For undercapitalized firms, differentiating $\hat{q}(\ell(kv))$ with respect to kv yields

$$\frac{\partial \hat{q}(\ell(kv))}{\partial kv} = -\frac{1 - F(kv)}{s''(q)} \begin{cases} < 0, & kv < U, \\ = 0, & kv \geq U. \end{cases}$$

Given that $\hat{q}(\ell(kv))$ is continuous in kv, this implies that undercapitalized firms take strictly less care than is socially optimal. ☐

Proof of Proposition 2 For the first part of the proposition, note that for a firm of size $k \geq U$ the socially optimal price m^* as defined in (13) exactly equals average cost $ac(k)$ as defined in (15), since $\hat{q}(\ell(k))$ for such a firm equals q^*, and $\ell(k)$ equals $\overline{A}$. This immediately implies that if the firm is able to raise price above average cost because of some barrier to entry, that price will exceed m^*. For a firm of size

$k < U$, on the other hand, m^* strictly exceeds $ac(k)$. This follows because $ac(k)$ is a continuous function with, for $k < U$, strictly positive slope $ac'(k) = q[1 - F(k)]$. If, therefore, the firm has market power, it may well happen to charge the socially optimal price. At that price, the firm will be undercapitalized, however, because comparison of (14) with (13) shows that $m^b(k) > m^*$ for $k < U$. By Proposition 1, the firm will therefore take less care than is socially optimal. ☐

Proof of Proposition 3 Average costs are a continuous function of k, with derivative

$$ac'(k) = q[1 - F(k)]\begin{cases} > 0, & k < U, \\ = 0, & k \geq U. \end{cases}$$

Firms of the minimum size $k = 1 < U$ therefore have lower average costs than any other firms. Moreover, their total value in competitive equilibrium is $kv = 1 < U$, which implies that they are undercapitalized. ☐

Proof of Proposition 4 Substituting any positive constant X for kv in expression (17) and then solving for m yields

$$m(k; X) \equiv r\frac{X}{k} + c(k) + s(q, k) + q\ell(X),$$

where q is equal to $\hat{q}(k, \ell(X))$. Differentiating with respect to k shows that

$$\frac{dm(k; X)}{dk} = -r\frac{X}{k^2} + c'(k) + s_k(q, k).$$

This expression is positive at arbitrarily small diseconomies of scale $c' + s_k > 0$, provided r is small enough. If so, kv must be decreasing in k for given m, since along an iso-kv locus

$$\frac{dm}{dk} = -\frac{\frac{\partial kv}{\partial k}}{\frac{\partial kv}{\partial m}},$$

and $\partial kv/\partial m$ is positive. ☐

Proof of Proposition 5 The function $q(k, s)$ is just the inverse of $s(q, k)$ with respect to q. Average cost can therefore be written as

$$ac(s, k) = r + c(k) + s + q(s, k)\ell(k).$$

Let $alc(s, k)$ denote the component of average cost added when liability is imposed, equal to $s + q(s, k)\ell(k)$. The assumptions on $s(q, k)$ guarantee an interior minimum of this component with respect to s. By the envelope theorem, its total derivative with respect to k is therefore

$$\begin{aligned}\frac{d\,alc}{dk} = \frac{\partial alc}{\partial k} &= \frac{\partial q}{\partial k}\ell(k) + q\ell'(k) \\ &= \frac{q}{k}\left[\frac{\partial q}{\partial k}\frac{k}{q}\ell(k) + k\ell'(k)\right] \\ &\le \frac{q}{k}\left[-\ell(k) + k\ell'(k)\right] \\ &= \frac{q}{k}\left[\int_0^k [F(A) - F(k)]\,dA\right] \\ &< 0, \quad \forall k \ge 1,\end{aligned}$$

where the first inequality follows from condition (19) in the proposition. □

Proof of Proposition 6 Let $\tilde{k}$ be the size of a firm in the equilibrium before imposing liability, and $\hat{k}$ the size of a firm in the equilibrium after. We are to show that if $\hat{k} > \tilde{k}$ then welfare at $\hat{k}$ strictly exceeds welfare at $\tilde{k}$.

Note first that welfare can be written as

$$W = \int_m^\infty Q(x)\,dx + Q(m)[m - asc].$$

Totally differentiating this expression yields

$$dW = Q'(m)[m - asc]\,dm - Q(m)\,d\,asc.$$

A sufficient set of conditions for welfare to strictly increase is therefore that (1) $m < asc$, i.e., the initial price is strictly below average social cost; (2) $dm > 0$, i.e., price strictly increases; and (3) $d\,asc \le 0$, i.e., average social cost either falls or remains constant.

For condition (1), note that price equals average cost in any competitive equilibrium, and average cost in the equilibrium before liability is

$$ac^n(\tilde{k}) \equiv r + c(\tilde{k}).$$

Average social cost in the equilibrium before liability is

$$asc^n(\overline{q}, \tilde{k}) \equiv r + c(\tilde{k}) + \overline{qA}, \tag{A1}$$

where $\overline{q} = \overline{q}(\tilde{k})$. That $asc^n > ac^n$, as required for the condition, follows because $\overline{qA}$ is strictly positive.

For condition (2), note that average cost in the equilibrium after liability is

$$ac^l(\hat{k}) \equiv r + c(\hat{k}) + s(\hat{q}, \hat{k}) + \hat{q}\ell(\hat{k}),$$

where $\hat{q} = \hat{q}(\hat{k}, \ell(\hat{k}))$. Also, because average cost before liability is minimized at $\tilde{k}$, it must be the case that $c(\hat{k}) \geq c(\tilde{k})$. That $ac^l > ac^n$, as required for the condition, then follows because $s(\hat{q}, \hat{k}) + \hat{q}\ell(\hat{k})$ is strictly positive.

For condition (3), note that average social cost in the equilibrium after liability is

$$asc^l(\hat{q}, \hat{k}) \equiv r + c(\hat{k}) + s(\hat{q}, \hat{k}) + \hat{q}\overline{A}. \tag{A2}$$

Suppose, for a contradiction, that $asc^l > asc^n$, i.e., that average social cost strictly increases after imposing liability. Then, from (A1) and (A2),

$$r + c(\hat{k}) + s(\hat{q}, \hat{k}) + \hat{q}\overline{A} > r + c(\tilde{k}) + \overline{q}\overline{A}. \tag{A3}$$

Because average cost before liability is minimized at $\tilde{k}$, we have

$$r + c(\hat{k}) \geq r + c(\tilde{k}). \tag{A4}$$

Because average cost after liability is minimized at $\hat{k}$, we have

$$r + c(\hat{k}) + s(\hat{q}, \hat{k}) + \hat{q}\ell(\hat{k}) \leq r + c(\tilde{k}) + s(\tilde{q}, \tilde{k}) + \tilde{q}\ell(\tilde{k}). \tag{A5}$$

Subtracting (A4) from (A5) yields

$$s(\hat{q}, \hat{k}) + \hat{q}\ell(\hat{k}) \leq s(\tilde{q}, \tilde{k}) + \tilde{q}\ell(\tilde{k}). \tag{A6}$$

Subtracting (A5) from (A3) yields

$$\hat{q}[\overline{A} - \ell(\hat{k})] > \overline{q}\overline{A} - [s(\tilde{q}, \tilde{k}) + \tilde{q}\ell(\tilde{k})]. \tag{A7}$$

By revealed preference,

$$s(\tilde{q}, \tilde{k}) + \tilde{q}\ell(\tilde{k}) \leq \overline{q}\ell(\tilde{k}). \tag{A8}$$

Substituting this into (A7) yields

$$\hat{q}[\overline{A} - \ell(\hat{k})] > \overline{q}[\overline{A} - \ell(\tilde{k})]. \tag{A9}$$

By assumption, $\hat{k} > \tilde{k}$, which implies that $[\overline{A} - \ell(\hat{k})] \leq [\overline{A} - \ell(\tilde{k})]$. But then inequality (A9) can only hold if $\hat{q} > \overline{q}$. This in turn yields the string of inequalities

$$s(\hat{q}, \hat{k}) + \hat{q}\ell(\hat{k}) \geq \hat{q}\ell(\hat{k}) > \overline{q}\ell(\hat{k}) \geq \overline{q}\ell(\tilde{k}).$$

Combining the outer inequality with (A8) yields

$$s(\hat{q}, \hat{k}) + \hat{q}\ell(\hat{k}) > s(\tilde{q}, \tilde{k}) + \tilde{q}\ell(\tilde{k}). \tag{A10}$$

But this contradicts inequality (A6). Hence, inequality (A3), which directly implies (A10), cannot hold. This establishes condition (3) and thereby completes the proof. □

Proof of Proposition 7 Immediate from the text.

APPENDIX B.

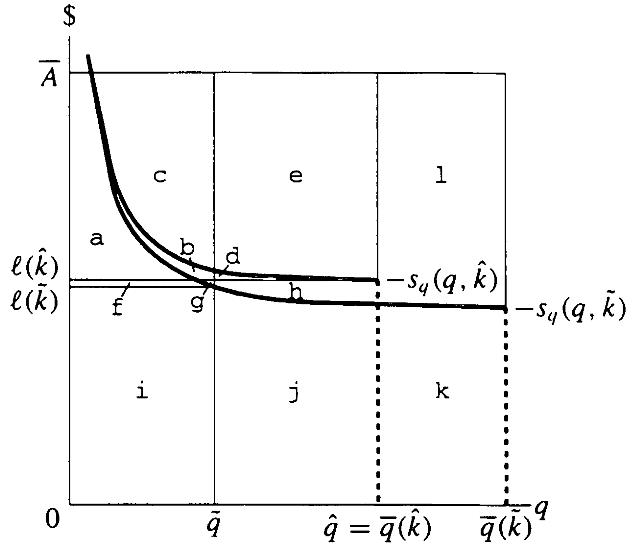

FIGURE B1. If the cross effect s_{qk} is negative, a shift towards larger firms may be welfare-reducing, despite the higher internalization of damages by such firms, and despite their ability to exploit economies of scale in accident prevention.

Initially only firms of size $\tilde{k}$ exist in the industry. When liability is imposed on these firms, they choose optimal accident rate $\tilde{q}$, where their marginal cost $-s_q(q,\tilde{k})$ of reducing q equals the marginal benefit $\ell(\tilde{k})$. This requires them to spend $s(\tilde{q},\tilde{k}) = j + k$ on safety, and leaves them with liability $\tilde{q}\ell(\tilde{k}) = i$. Subsequently, larger firms of size $\hat{k}$ enter and take over the industry. These firms optimally choose accident rate $\hat{q}$, which happens to require no expenditure on safety (since $\hat{q}$ happens to equal $\overline{q}(\hat{k})$) and leaves them with liability $\hat{q}\ell(\hat{k}) = f + g + h + i + j$. Assuming constant returns to scale in production, so that both types of firms have the same average production cost $r + c(k)$, the large firms' average cost is then lower, because $f + g + h < k$.

Because these firms are larger, they internalize more damages: $\ell(\hat{k}) > \ell(\tilde{k})$. Moreover, because the area $b + d + g + h$ between the two $-s_q$ curves is smaller than area k, the large firms also spend strictly less to achieve any given accident rate q than the small firms. Nevertheless, the large-firm entry reduces welfare, because average social cost increases. For the small firms, average social cost is equal to average cost plus externalized damages $\tilde{q}[\overline{A} - \ell(\tilde{k})] = a + b + c + f + g$. For the large firms, it is equal to average cost plus externalized damages $\hat{q}[\overline{A} - \ell(\hat{k})] = a + b + c + d + e$. Average social cost is therefore higher after entry by the large firms, because the increase $(d + e) - (f + g)$ in externalized damages is greater than the decrease $(f + g + h) - k$ in average costs.

Relative to the equilibrium before liability, however, average social cost is still lower by area l.

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