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GOVERNMENT MANAGEMENT OF VILLAGE COMMONS: EVALUATING JOINT FOREST MANAGEMENT

by

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1. Introduction

In India 77 million hectares of land are classified as forest lands however, at present only about half of these lands are actually forested. According to official accounts the country was losing 150,000 hectares per year up to the 1980s, and 47,000 hectares annually between 1980 and 1987. The government has been unable to reverse this trend.

One of the major causes of forest degradation is the nature of property rights over forests and their produce. While nominally owned by the state, a forest near a village tends to serve as a *de facto* commons for the villagers. Because the costs of acquiring firewood increase as the stock of the resource decreases, individual gatherers of firewood do not bear the full costs of their activity. The existence of this negative externality implies that the equilibrium outcome will differ from the efficient outcome, a traditional reason for advocating government intervention in markets.

What is interesting in this case is that there are reasons to expect that traditional government remedies to the problem of negative externalities are unlikely to be of much use in this situation. Levying a Pigovian tax is impractical, because of the difficulty of monitoring firewood collection over hundreds of millions of hectares of sometimes remote forest by hundreds of millions of people. Applying a Coasian solution of assigning property rights over the forest is impractical for a similar reason; it may be difficult to identify those who trespass against these rights.

The difficulties described above in implementing a Pigovian or Coasian scheme to eliminate the externality do not rest entirely on conjecture. Indeed, the forest departments of the various states of India have for years attempted to regulate private exploitation of the state forests. Forest guards monitor the resource, and a complicated system of fines tied to the market price of firewood implements something resembling a Pigovian tax. The chief problem with this scheme is hinted at

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above the forest guards are woefully inadequate in number and omniscience, so that the private returns to illicitly collecting firewood continue to greatly exceed the private costs.

In response to the failure of this scheme of taxation, the government in parts of India has proposed—and in places implemented—an incentive based scheme called Joint Forest Management (JFM). This scheme faces squarely the difficulties of state monitoring of firewood collection, and depends upon the reasonable supposition that the villagers themselves have better information about extractive behavior than the forest guards are likely to have. In this scheme, the forest department essentially turns over monitoring duties to the village as a whole for some fixed period of time (typically ten years). The village is given incentives meant to discourage extractive behavior in the form of a payment made at the termination date of the contract, which is an increasing function of the forest stock at that date. At that time the contract could presumably be renewed. JFM , though, is still too young an institution for us to have evaluated any such contract renewals.

We would like to establish conditions under which JFM really does improve upon the forest management policy in vogue prior to JFM, which we will call Status Quo (SQ). We have no reason, in advance, to suppose that either JFM or the status quo is a dominant policy in the sense that one or the other ought to be adopted for all villages in India; rather we expect that the superiority of one scheme over the other will depend on the local environment, and perhaps particularly on the degree to which villagers are able to cooperate in the management of their forest resources.

Unfortunately, most of the literature on the commons (see Ostrom, Gardner, and Walker (1994)) provides us little guidance in this endeavor. The literature's usual focus is on policies that rely on taxes or quota restrictions or privatization to achieve the socially optimal resource allocation to the neglect of incentive-compatible contracts between the owner (here the Indian government) and the users (here the villagers) of a natural resource (here the forests). At the same time, the literature on agency theory and mechanism design has yet to establish properties of an optimal contract where the evolution of the state variable (here the forests) is determined by the decisions of all the households.

In this paper we model the problem of managing a commons as a problem of efficient resource allocation in the face of some immutable externality. The externality in this story is taken as a primitive; something this paper does *not* do is to develop a more satisfactory approach relating the externality to specific features of the economic and informational environment;[1] this is left for future research. Another thing that this paper doesn't do is to evaluate the efficacy of either JFM or the status quo relative to other contracts; neither of the contractual forms is particularly flexible, and there seems to be good reason to suppose that some efficient contract exists which would dominate either.

We begin the paper by developing a dynamic household model of common property forest management. The model enables us to characterize the properties of the equilibrium for two polar cases: (i) households are able to overcome the externality completely; and (ii) households fail to overcome the externality. Thereafter, we introduce the state and we consider how JFM and SQ policies affect a household's behavior. We establish a locus of policy parameters that makes each household indifferent between JFM and SQ. In the remainder of the paper, we compare the value of JFM and SQ to the state and the villagers given that the state chooses policy parameters to maximize timber revenues while taking into account household behavior.

2. The Village

We think of a village as a set of m households, situated in or near a common property forest consisting of F trees, which grows at some fixed rate δ. Each household is endowed with preferences over firewood and perhaps some public amenities related to the level of the forest stock. These amenities might take many forms. For concreteness, we suppose that a chief amenity is that it is less costly to collect firewood when the forest stock is high. The single period utility of a household which consumes some quantity of firewood w when the size of the forest is F is given by some function $U(F,w)$. We assume that U is Gorman aggregable, strictly increasing, strictly concave, and twice continuously differentiable, with

$$U(0,0)=0, \quad \lim_{F\to 0} U_1(F,w)=+\infty, \quad \lim_{w\to 0} U_2(F,w)=+\infty,$$
$$\lim_{F\to\infty} U_1(F,w)=0, \quad \text{and} \quad \lim_{w\to\infty} U_2(F,w)=0$$

where $U_1(F,w)$ is the first derivative of the utility function with respect to its first argument and $U_2(F,w)$ is the first derivative of the utility function with respect to its second argument. Finally, we assume that the second derivatives of the utility function are bounded.

[1] The limited ability of the forest departments to monitor the behavior of the villagers coupled with limited incentives for the villagers to restrict their use of the forest is often stated as one of the causes of deforestation.

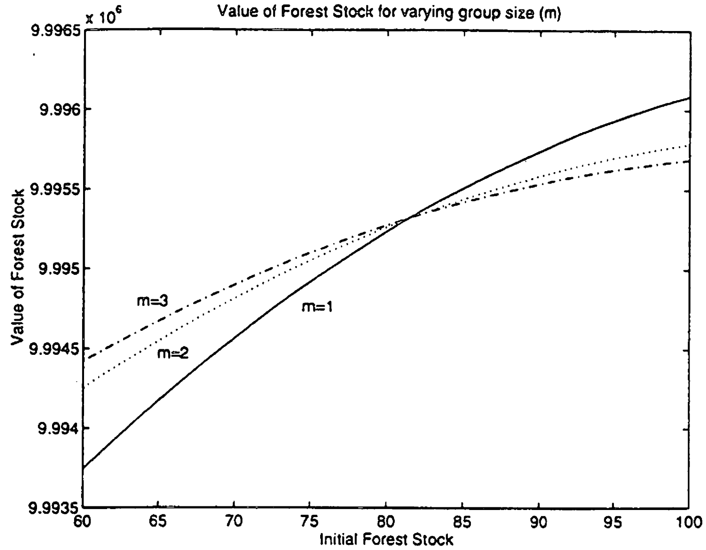


FIGURE 1. Value of the Initial Forest Stock for Varying Group Size

Each household derives utility from firewood and forest in every period over an infinite horizon. Future utility is discounted at a rate equal to $1/\beta - 1$, for some discount factor $\beta \in [0, 1)$. Accordingly, in the absence of any externalities, the value of the forest to a representative household may be recursively defined via Bellman's equation as

$$V(F) = \max_{w} U(F, w) + \beta V(F') \tag{1}$$

such that

$$F' = (1 + \delta)(F - mw) \tag{2}$$

where $V(F)$ denotes the value (in utils) of a forest stock F, and F' denotes tomorrow's level of forest stock (determined by the current stock, the quantity of firewood harvested today and the rate of growth of the forest).

Note that this representative household is effectively able to control the total quantity (mw) of the forest harvested by the entire village, guaranteeing an efficient outcome. One could say that the village was cooperating in the management of the common property forest.

A solution to this functional equation is some value function $V : \mathbb{R} \to \mathbb{R}$ which assigns value to the forest, and some policy function $g : \mathbb{R} \to \mathbb{R}$ which gives an optimal rule for the quantity to harvest given the current level of the forest stock, or $w = g(F)$. Standard arguments (Stokey and Lucas 1989) suffice to show that V will be increasing, concave, and continuously differentiable.

Figure 1 provides an illustration for an example economy.[2] This figure shows the value of different levels of initial forest stock for three different village populations, of 1, 2, and 3. Value is defined as the sum of utility derived from firewood and the stock of forests in each period over an infinite horizon. For all three population sizes the value of the forest is increasing in the initial level of the forest. The reason for this is that a higher initial level of the forest stock leads to a higher level of firewood and forest stock consumption[3] up to the point where the steady state level of forest stock is attained. Thereafter, the levels of consumption are independent of the initial level of the forest and are in fact equalized across all levels of the initial forest stock. This is a direct consequence of the fact that the steady state level of the forest is independent of the initial level of the forest. At very low levels of the forest stock, villagers' consumption may be zero or even negative (we interpret this latter possibility as amounting to investment).

Allowing for the possibility of negative consumption at low levels of initial forest stock leads to an interesting and somewhat counter-intuitive result, namely that, smaller groups do not necessarily achieve a higher level of intertemporal utility than larger groups for a given initial level of forest. In Figure 1 the value function for $m = 3$ lies above the value function for $m = 2$ or $m = 1$ so long as initial forest stock is less than 80. At low levels of initial forest stock larger groups do better than smaller ones because the benefit from being able to share the cost of investment, needed to build the forest stock to the steady state level, outweighs the loss from having to divide the benefits from consumption, once the villagers start extracting firewood from the forest. This result is reversed, that is, smaller groups achieve a higher intertemporal utility, if the initial level of the forest stock is relatively close to the steady state level and therefore the investment needed to attain the steady state level of forest stock is relatively small or even zero.

2.1. **A Tragedy of the Commons.** In order to think about the case in which the forest resource is a commons, we must admit the possibility that the resource is inefficiently exploited. This inefficiency may be thought of as an unexplained (in this model) failure to cooperate, which might have to do with private information, or limited commitment, or transactions costs, or something else entirely. For our present purposes we don't care about the source of the inefficiency, only that outcomes are inefficient and that this inefficiency can't be rectified by individual households.

[2]The example economy is detailed in the appendix.

[3]Again for concreteness, we can interpret a higher level of forest stock consumption to be a lower cost of firewood extraction.

The problem facing the household in this setting is quite similar to that given above, except that each household takes as given the behavior of the other $(m-1)$ households. In particular, consider the problem facing the ith household. Suppose that each of the remaining $(m-1)$ households take from the forest some quantity of firewood $\hat{w}$, so that the problem facing the household is to solve

$$V(F) = \max_{w} U(F, w) + \beta V(F') \tag{3}$$

such that

$$F' = (1+\delta)\Big(F - w - (m-1)\hat{w}\Big) \tag{4}$$

The solution to the household's problem is a pair of functions (V, g) where $w = g(F, \hat{w})$ gives the household's best response to a total harvest $(m-1)\hat{w}$ by each of the other households, given that the current level of the forest stock is F.

Since, in addition to others' behavior, strategies depend only on the current realization of the state variable, and because we assume that households have no means of precommitting to some particular sequence of harvests, the natural equilibrium concept in this setting is Markov Perfect Equilibrium (MPE). This equilibrium notion has previously been applied to games involving a common resource by Levhari and Mirman (1980), Hansen, Epple, and Roberds (1985), Reinganum and Stokey (1985), and Karp (1992). In addition, we require strategies to be continuously differentiable functions of the state, and restrict our attention to symmetric strategies. Accordingly, a symmetric equilibrium is some particular function $g(F, \hat{w})$ such that (i) $g(F, \hat{w})$ solves the household's problem; and (ii) $w = g(F, w)$.

In general it seems difficult to establish the existence of an MPE in pure strategies and continuous state spaces, such as we have here, and we avoid the issue in this paper simply by assuming existence. However, existence has been established in environments closely related to the one we consider here, and general results are available for special cases of the environment we consider. In particular, Dutta and Sundaram (1992) establish existence in lower semi-continuous strategies in an environment very similar to our own. Their results are, unfortunately, not applicable to the case we consider because we require strategies to be continuously differentiable. In the special case of our problem in which preferences are specified to be linear-quadratic functions of the state and action, the game we discuss here becomes a linear-quadratic one. Existence in this class of games is assured under some standard regularity conditions (Papavassilopoulos and et al 1979); furthermore,

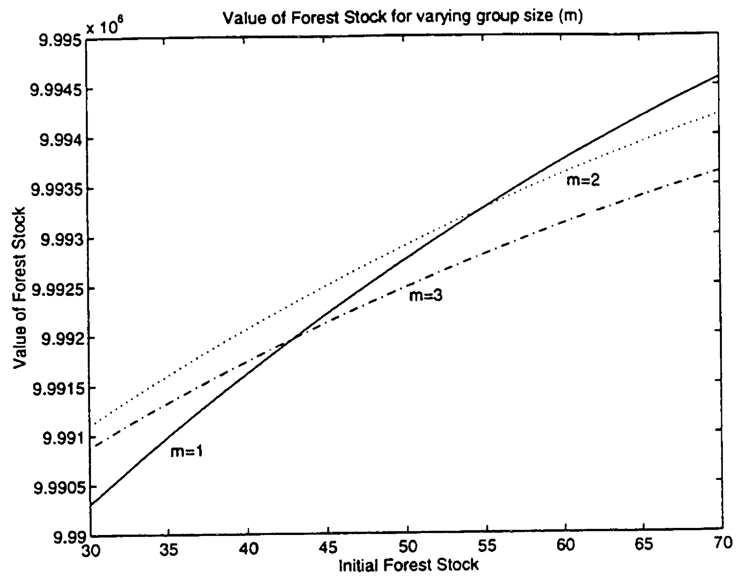


FIGURE 2. Value of the Forest Stock for Varying Group Size

there exists a unique MPE so long as the horizon is finite (Papavassilopoulos and Cruz 1979). We exploit these latter results in the computed examples of this paper by specifying a linear-quadratic structure (see Appendix), and selecting an equilibrium to the infinite horizon game by taking the limit as the terminal period approaches infinity.[4]

Figure 2 shows that even when the group is not cooperating larger groups can achieve higher intertemporal utility for low levels of initial forest stock because the benefit from dividing the cost of investment outweighs the cost of dividing the forest produce.

The "tragic" consequences of the commons can be seen in Figure 3, Figure 4 and Figure 5. The first two of these figures show that the value of the forest stock is higher under the cooperative regime than under the non-cooperative regime for all group sizes. Figure 3 is drawn for the case when the villagers start with an initial forest stock of 100 which is greater than the steady state level of forest stock for all group sizes. No investment is needed to regenerate the forest stock and consequently, the per capita value of the forest stock decreases in group size. In Figure 4 the initial forest stock is 0 which is significantly below the steady state level. Contrary to the usual intuition regarding commons, here the benefit from dividing the cost of investment outweighs the loss from having to share the forest produce and therefore the per capita value of the forest stock increases

[4]In general, there are problems with the procedure: the limit strategies may not be optimal in the infinite horizon game (Magierou 1976), and the limit may not exist (Papavassilopoulos and Olsder 1984). Our computed examples suffer from neither of these defects.

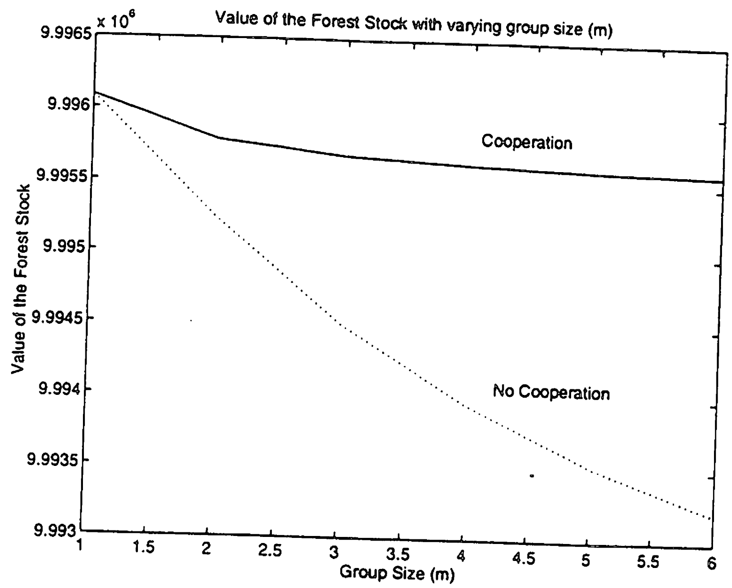


FIGURE 3. Value of an Initial Forest Stock of 100 as a Function of m

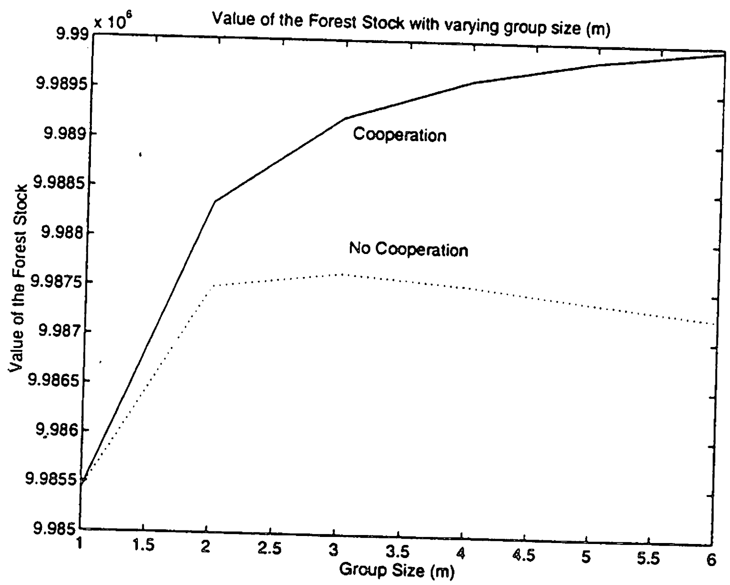


FIGURE 4. Value of an Initial Forest Stock of 0 as a Function of m

with group size. Let the *cost of non-cooperation* be defined as the difference between the value of the forest stock under cooperation and non-cooperation. Then Figure 3 and Figure 4 show that the cost of non-cooperation is an increasing function of group size. That is, the tragedy of the commons increases with the size of the group. Furthermore, when there is a single household ($m = 1$) there is no inefficiency and the value of the initial forest stock under the two regimes is equalized.

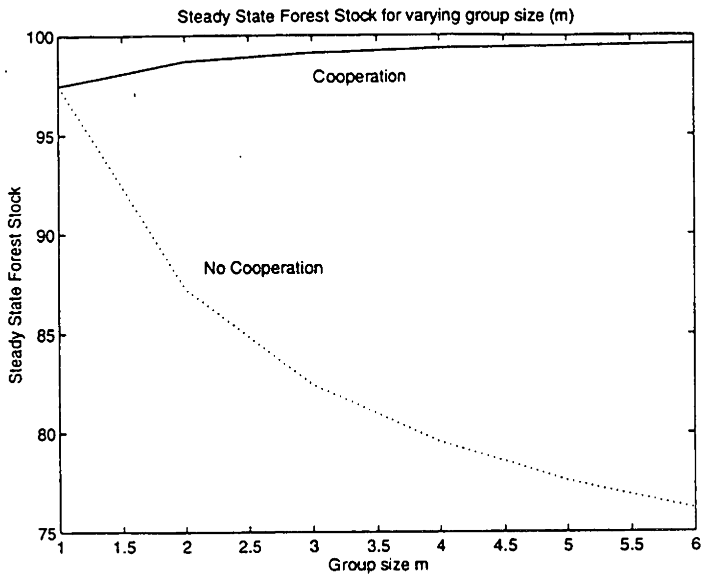


FIGURE 5. Steady State of F as a Function of m

Figure 5 shows another interesting feature of the village economy, namely, that the steady state level of forest stock is an increasing function of group size when the group is cooperating and a decreasing function of group size when the group is not cooperating. Under both the cooperative and the non-cooperative regimes each household chooses the amount of firewood to extract in each period by equating the loss in today's utility to the gain in tomorrow's utility caused by a unit reduction in firewood consumption today. We assert, and will show below, that when households are cooperating the gain in utility tomorrow (from a unit reduction in consumption today) increases with the size of the group. This causes cooperating households to reduce current firewood consumption and thereby to increase next period's forest stock and eventually to increase the steady state level of forest stock as the group size increases. However, if households are not cooperating then the gain in utility tomorrow instead decreases as the group becomes larger. As a result, non-cooperating households increase current firewood consumption and thereby eventually decrease the steady state forest stock.

In order to explain the differences in the relationship between group size and the steady state level of the forest stock when households are cooperating versus not we turn to the appropriate Euler equations. The Euler equations are derived from the first order and envelope condition associated with the problem being solved by each household while cooperating (as outlined in section 2) or while playing strategically (as outlined in section 2.1). By equating intertemporal marginal utilities, the Euler equation (along with the equilibrium condition when households are not cooperating) defines the optimal mapping from the current level of the forest stock to the harvest quantity.

Lets first consider the Euler equation for the efficient problem. The rule equating intertemporal marginal utilities is,

$$U_2(F, w) = m\beta(1+\delta)U_1(F', w') + \beta(1+\delta)U_2(F', w') \tag{5}$$

where U_1 is the marginal utility of the forest stock and U_2 is the marginal utility of firewood. The gain in tomorrow's utility (the right hand side of the Euler equation) from a unit decrease in firewood consumption today has two components: (i) an increase in the consumption of the forest stock (the first term on the right hand side of the Euler equation); and (ii) an increase in the consumption of firewood (the second term on the right hand side of the Euler equation). The increase in the consumption of the forest stock is an increasing function of group size because each household includes the increase in consumption for all other households (again for concreteness, each household includes the reduction in the cost of extraction for all other households) when measuring the total gain from a unit increase in the forest stock. While, consumption tomorrow is independent of group size. Consequently, under cooperation the total gain in utility tomorrow increases with group size.

When households are not cooperating the Euler equation changes to

$$\begin{aligned} U_2(F, w) &= \beta(1+\delta)U_1(F', w') \\ &\quad + \beta(1+\delta)\left(1 - (m-1)\frac{\partial \hat{w}'}{\partial F'}\right)U_2(F', w') \end{aligned} \tag{6}$$

As under the cooperative regime, when households are not cooperating the gain in tomorrow's utility is made up of a gain from an increase in the consumption of the forest stock and a gain from an increase in the consumption of firewood. The former gain is independent of the group size. However, consumption of firewood tomorrow is a decreasing function of group size because an increase in the stock of forests causes all other households to play strategically and to increase the amount of firewood they extract in order to induce other households to reduce their rate of extraction since the rate of extraction is an increasing function of the level of the forest stock. This behavior reduces the amount of firewood available for consumption tomorrow. Consequently, if the households are not cooperating then the total gain in utility tomorrow decreases with an increase in group size.

The ensuing proposition formalizes this relationship between the steady state level of forest stock and the group size when households are either cooperating or not cooperating.

Proposition 1. *If there exists a unique steady state level of the forest stock, $F^\star$, and if $U_{12}(F, w) = 0$, then there exists some group size $\bar{m}$ such that for all $m > \bar{m}$:*

i) *If the households are cooperating then $\frac{\partial F^\star}{\partial m} > 0$.*

ii) *If the households are not cooperating then $\frac{\partial F^\star}{\partial m} < 0$.*

Proof. See Appendix D.1 □

2.2. **Protecting the Commons.** Although the sub-game perfect equilibrium discussed above has rather grim properties, we have ruled out in that game any possibility of the households designing institutions or participating in markets which might ameliorate the 'commons externality' which they jointly face. Modeling the design of such institutional arrangements would require us to be more precise about the properties of the environment, and in particular to spell out reasons why a fully efficient arrangement could not be recovered. This problem is beyond the scope of this paper. Nonetheless, we wish to consider the consequences of households overcoming the externality they face, either partially or completely. To this end we parameterize the externality faced by the households, referring to the size of the externality as some number ξ which lies between zero (fully efficient) and one (which delivers the equilibrium characterized above).

In order to capture this behavior, we rewrite the law of motion for the forest stock as

$$F' = (1+\delta)\Big(F - w - (m-1)\big((1-\xi)w + \xi\hat{w}\big)\Big) \tag{7}$$

From this equation it seems clear that the individual household is, by cooperating with other households, in some sense able to control a larger portion of the forest harvest. When ξ is equal to 0, we recover the fully efficient problem discussed above; when ξ is equal to one, then we retrieve the 'tragic' problem. This holds for the Euler equation as well. The Euler equation using the parameter ξ is

$$\begin{aligned} U_2(F, w) &= \beta(1+\delta)\Big(1 + (m-1)(1-\xi)\Big)U_1(F', w') \\ &\quad + \beta(1+\delta)\Big(1 - (m-1)\xi\frac{\partial \hat{w}'}{\partial F'}\Big)U_2(F', w') \end{aligned} \tag{8}$$

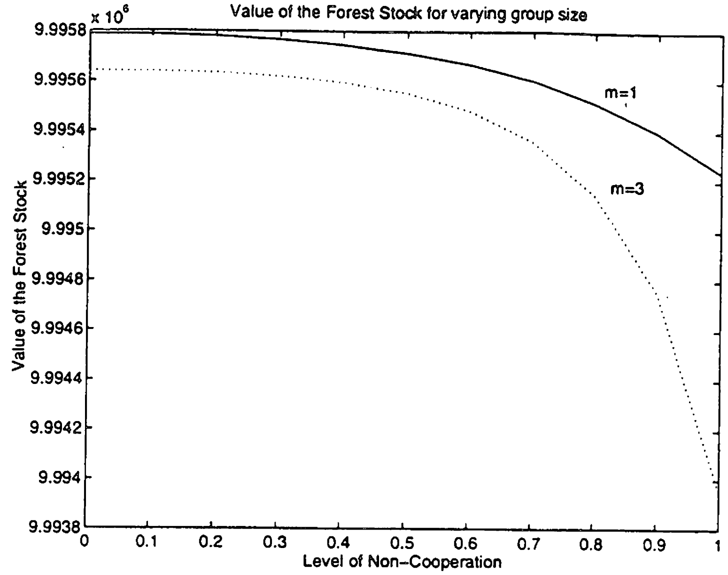


FIGURE 6. Value and Externality

When $\xi = 0$, equation (8) is the same as equation (5), that is, we recover the Euler equation associated with the efficient problem. Similarly, if $\xi = 1$ then equation (8) is the same as equation (6) and we recover the Euler equation for the tragic problem.

For intermediate values of ξ, we impose the same equilibrium notion as that developed in subsection 2.1. Equilibrium behavior as a function of the externality is illustrated in Figure 6. Intertemporal utility is an increasing function of the level of cooperation or alternatively a decreasing function of ξ. Similarly, the steady state level of forest stock also decreases as the level of non-cooperation increases.

Note that m does not capture the extent of the externality in our model. In fact, for $\xi = 0$ the value of m does not effect efficiency at all (all outcomes remain efficient) and equation (8) coincides with the efficient Euler equation (equation (5)). However, $m = 1$ *is* sufficient, though not necessary, for an efficient outcome (equations (8) and (5) coincide when $m = 1$).

We would like to find a consistent way to think about the welfare consequences of non-cooperation and population. Let $V(F|\xi, m)$ denote the value of some forest stock F given externality ξ and population m. Define $c(F, \xi, m)$ to be the cost of non-cooperation; that is, $c(F, \xi, m) = V(F|0, m) - V(F|\xi, m)$. The following proposition collects and extends results described less formally above.

Proposition 2. *If a Markov perfect equilibrium exists, with equilibrium strategy $g(F)$ and a unique steady state level of the forest stock, F^*, then*

i) *if* $U_{12}(F, w) > 0$ *for all* (F, w), *then next period's stock* F' *is a non-increasing function of* F; *conversely,*

ii) *if* $U_{12}(F, w) < 0$ *for all* (F, w), *then next period's stock* F' *is a non-decreasing function of* F.

iii) *If either* $U_{12}(F, w) > 0$ *for all* (F, w) *or* $U_{12}(F, w) < 0$ *for all* (F, w), *then for any initial forest stock* F, *forest stock converges monotonically to some unique steady-state level of stock* F^*.

iv) *If* $U_{12}(F, w) = 0$, *then there exists some group size* $\bar{m}$ *such that for all* $m > \bar{m}$, $\frac{\partial F^*}{\partial \xi} < 0$.

Proof. See Appendix D.2 □

3. Effects of Government Harvest

To this point we have not explicitly introduced the government to our model. In fact the government plays a key role in the evolution of the forest stock through logging operations. Later in the paper we will permit the government to make optimizing decisions; for now we simply ask how each of two possible government policies—status quo and joint forest management—effects village and household behavior.

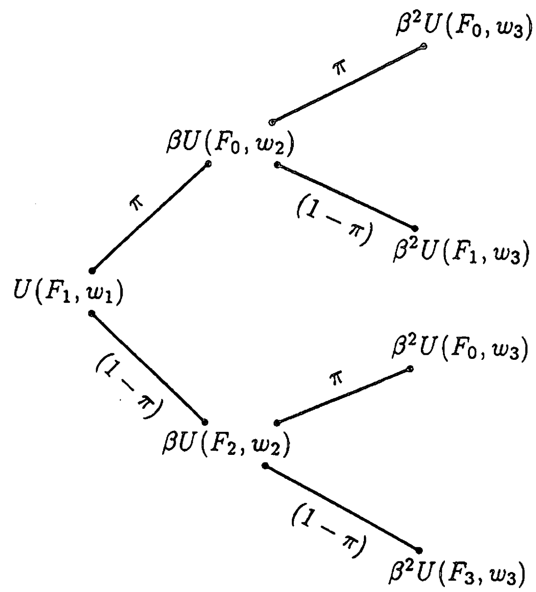
3.1. The Status Quo: Random Government. Although there is a wide variety of arrangements governing the use of forests by households and villages, there is a somewhat typical set of arrangements which we will stylize and call the status quo.

Under the status quo, the state holds title to all forest lands and grants villagers the right to harvest some stream of forest products.[5] The state, however, reserves the right to cut timber in the forest. The state also bears the entire burden of managing the forest; local households have no formal involvement in forest management.

In every period villagers harvest some firewood knowing that there is some fixed non-negative probability (denoted by π) that the state will cut the forest in the next period. For example, let F_i denote forest stock that is i years old and w_t denote an action taken by a household at time t.

[5] In the jargon of the forest department, villagers are granted a set of "rights," "concessions" and "privileges" over forest products. We draw a distinction only in this footnote. If a right is violated by the state or forest department the villagers can seek judicial recourse. However, this is does not hold for privileges or concessions. Privileges are granted by the forest department while concessions are granted by the state government. In some districts of the state of Gujarat in India, households have the right to graze cattle in the forests and the privilege to collect fallen wood, though not timber. Households living less than 5 km from the forest are granted a concession to purchase 10 per cent of the timber harvested by the forest department at 20 per cent of the market price while households living less than 10 km from the forests can purchase the timber at 60 per cent of the market price.

The events faced by the villagers over three years (only three years are shown in order to conserve trees) can be captured by the following event tree,



In the first period the stock of forest is F_1 and the household chooses a decision w_1 to maximize his or her utility. This gives the household a utility of $U(F_1, w_1)$. In the second period there is a positive probability given by π that the state will cut the forest in which case the stock of forest will be given by F_0 and the optimal decision w_2 will give the household a discounted utility of $\beta U(F_0, w_2)$. If the state does not cut the forest (there being a $(1 - \pi)$ probability of this event occurring) then the household gets a discounted utility of $\beta U(F_2, w_2)$ (the forest stock is two years old). In the third period if the state cuts the forest again after having cut it in the second period then the household gets a discounted utility of $\beta^2 U(F_0, w_3)$ by choosing an optimal decision given by w_3. If after having cut the forest in the second period the state lets the forests grow in the third period the household gets a utility of $\beta^2 U(F_1, w_3)$. Finally, after having allowed the forests to grow in the second period if the state cuts the forests in the third period the households get $\beta^2 U(F_0, w_3)$ and if the state lets the forests grow then the households get $\beta^2 U(F_3, w_3)$ (the forests are now three years old).

This is a reasonably general formulation of the problem, save for the fact that governmental behavior (from the villagers' perspective) is summarized by a single number, π, which in particular is not permitted to be a function of the level of the forest stock. This assumption is necessary to maintain analytical tractability.

For an infinite horizon the problem faced by the household can be written as the following dynamic program,

$$V_\pi(F) = \max_w U(F, w) + \beta\left[\pi V_\pi(0) + (1-\pi)V_\pi(F')\right] \tag{9}$$

such that

$$F' = (1+\delta)\Big(F - w - (m-1)\big((1-\xi)w + \xi\hat{w}\big)\Big) \tag{10}$$

where $V_\pi(F)$ is the discounted expected utility that each household receives from an initial forest stock F over the infinite horizon when there is a π probability in each period of the forests being cut by the state. $V_\pi(F)$ has three components, utility received in the first period, $U(F, w)$, sum of discounted benefits after the forest stock has been cut, $\pi\beta V_\pi(0)$ and a stream of discounted benefits when the state allows the forest stock to grow $\beta(1-\pi)V_\pi(F')$.

In order to solve this program, note that, since $\beta\pi V_\pi(0)$ is just some unknown constant (because it does not depend on w_t) we can just subtract it from the return function without loss of generality, and so recover a problem which is formally equivalent to a problem without uncertainty.

We proceed by defining $U_\pi(F, w) = U(F, w) + \beta\pi V_\pi(0)$, and rewrite Bellman's equation as

$$V_\pi(F) = \max_w U_\pi(F, w) + \beta(1-\pi)V_\pi(F') \tag{11}$$

such that equation (10) holds. Since this has precisely the same form as the program in earlier sections, we can use the same methods to solve for the equilibrium. There is only one detail which needs to be resolved; we must find $U_\pi(F, w)$ first. In order to do so, we employ a two step algorithm.

i) Solve $V_\pi^0(F) = \max_w U(F, w) + \beta(1-\pi)V_\pi^0(F')$ subject to equation (10). Call the resulting optimal policy $g_\pi(F)$; note that this policy remains optimal even if we add an arbitrary finite constant to the return function.

ii) Find V_π as the unique fixed point of the contraction mapping Γ defined by

$$(\Gamma V_\pi)(F) = U(F, g_\pi(F)) + \beta\pi V_\pi(0) + \beta(1-\pi)V_\pi\left((1+\delta)F - mg_\pi(F)\right) \tag{12}$$

With V_π in hand, U_π is a known function.

We are now left in a position to address the questions which interest us in this section, namely, (i) how welfare depends on the probability of government harvest, and (ii) how government policy

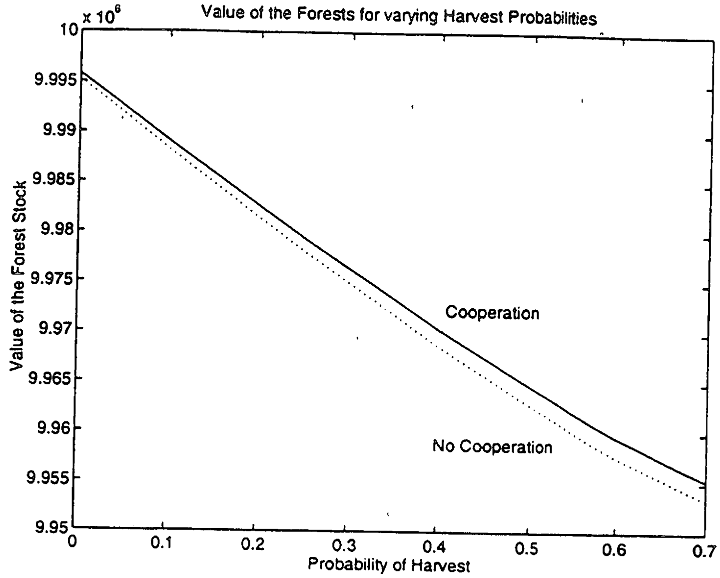


FIGURE 7. Value of Forest for Varying Probabilities of Harvest

and village cooperation interact to affect the optimal harvest policy and the steady state forest stock.

So long as the equilibrium path of F is nonnegative, it's obvious from the transformed problem above that village welfare is declining in π, since increases in π have the same formal effect as increases in the rate of future discounting. We formalize this argument in the following proposition,

Proposition 3. *Let $F_t \geq 0$ for all $t \in \{0, 1, \ldots\}$ and $\pi \geq \pi'$ then $V_\pi(F) \leq V_\pi'(F)$.*

Proof. See Appendix D.3 □

Proposition 3 is illustrated in Figure 7 for two extreme levels of cooperation ($\xi \in \{0, 1\}$). The next proposition establishes how the probability of harvest and the level of cooperation interact of effect the amount households harvest in a given period as well as the steady state level of the forest stock.

Proposition 4. *If a Markov perfect equilibrium exists, with equilibrium strategy $g_\pi(F)$ and a unique steady state level of the forest stock, F^*_π,[6] and if $U_{12}(F, w) = 0$, then*

i) *The amount harvested in a period, $w = g_\pi(F)$, is an increasing function of π, the probability of government harvest, and of ξ, the level of non-cooperation.*

[6] *Steady state level of forest stock under status quo is defined as the stock of forest that will be attained if the state never fells the forest.*

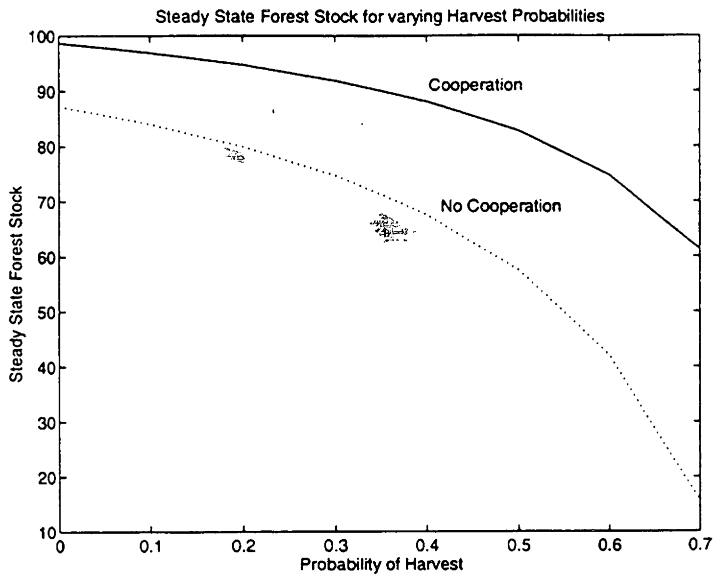


FIGURE 8. Steady State Level of Forest Stock for Varying Probabilities of Harvest

ii) *There exists some group size* $\bar{m}$ *such that for all* $m > \bar{m}$*: (i)* $\frac{\partial F^\star_\pi}{\partial \pi} < 0$*; and (ii)* $\frac{\partial F^\star_\pi}{\partial \xi} < 0$.

Proof. See Appendix D.4 □

Figure 8 shows that the steady state level of the forest stock decreases with an increase in the probability of harvest tomorrow irrespective of the level of cooperation.

3.2. **Joint Forest Management.** In an effort to improve the management of forests state governments have promoted a new form of forest management — joint forest management. Joint forest management recognizes the difficulties that the forest department has in monitoring individual behavior and thus contracts with villages in order to give the village leadership an incentive to regulate the behavior of individual households within the village.

A typical JFM contract specifies the date at which the forests adjoining the village will be harvested, and promises to the village a share of the proceeds from the harvest at that date. The village now bears the responsibility of regulating the use of the forests until the date of harvest.

The value to the village of a single JFM contract that extends over T periods is

$$J^1_T(F) = \max_{w_t} \sum_{t=1}^{T} \beta^{t-1} U(F_t, w_t) + \beta^{T+1} \varrho(F_{T+1}) \tag{13}$$

subject to the usual law of motion for the forest stock, and where $F_1 = F$, and $\varrho(F_{T+1})$ is the time $T+1$ payoff (denominated in utils) given that the forest stock is F_{T+1}.

If we assume that JFM is repeated infinitely then the value of JFM over an infinite horizon is given by the expression

$$J_T^{\infty}(F) = \max_{w_t} J_T^1(F) + \sum_{j=1}^{\infty} \beta^{(T+2)j} J_T^1(0) \tag{14}$$

which for $0 \leq \beta < 1$ simplifies to

$$J_T^{\infty}(F) = \max_{w_t} J_T^1(F) + \frac{\beta^{(T+2)}}{1-\beta^{(T+2)}} J_T^1(0) \tag{15}$$

We would like to understand how a change in the time horizon of JFM changes the value of JFM to the village. The villagers do not unambiguously prefer a larger T, since extending the length of the contract also postpones the date at which the villagers receive the terminal payoff from the government. We are able to make only two rather weak claims in general. Both these claims hold irrespective of whether the villagers cooperate or not.

Before we state these claims we simplify the problem by restricting the terminal share to be an affine function of the forest stock. That is, we restrict $\varrho(F_{T+1})$ to be $U(\Psi, \rho F_{t+1})$ where Ψ is the forest stock that drives the marginal utility from the forest stock to zero (or alternatively, Ψ drives the harvesting costs to zero) for the household[7] and ρ is a constant denoting the share of the terminal harvest given to each household. Although ρ is fixed for the villagers we allow ρ to vary across villages. Specifically, $\rho = r(F_0, \xi)$ where r is some function which gives the share of the terminal harvest given the initial level of forest and level of externality in the village.

The first of the claims is that, for ρ sufficiently small, household welfare is increasing in T. This claim is illustrated in Figure 9 where $\rho = 0.5$. We formalize this claim in the next proposition for a single implementation of the contract.

Proposition 5. *There exists a $\hat{\rho}^1 > 0$ such that, for any $\rho < \hat{\rho}^1$ and any fixed $F \geq 0$, if $T \geq T'$ then $J_T^1(F) \geq J_{T'}^1(F)$.*

Proof. See Appendix D.5 □

If the contract is repeated then in the first few periods right after harvest the households have to invest in the forest to build the forest stock up to the steady state level. This imposes a cost on households that they would prefer to avoid or at least delay incurring. Extending the length of

[7]Under the JFM contract the state bears the cost of harvesting the timber once the contract is completed.

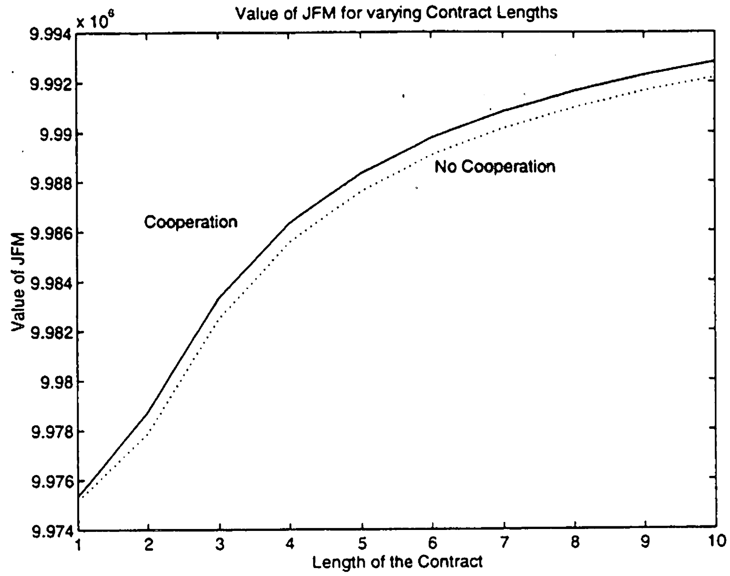


FIGURE 9. Value of JFM for varying Lengths of the Contract

the contract enables them to delay this investment. This implies that if the households prefer a longer one time contract then they will prefer a longer repeated contract. Another way of saying this is that given a relatively small terminal payoff villagers prefer to consume a smooth stream of firewood rather than have their firewood consumption fluctuate between high—when the terminal payoff is granted—and low—when the villagers need to invest to rebuild the forest stock to the steady state level.

The second claim is that, for a high enough ρ and a low cost of forest regeneration, it is possible that the terminal payoff becomes important enough that the household welfare is decreasing in T. Any $\rho > \hat{\rho}^1$ establishes this claim.

Also of interest is the time path of the forest stock as T varies. There are two countervailing forces at work here. Suppose that the initial stock is quite low. There are two different levels of forest stock that the village would like to achieve, given enough time; one of these the steady state level of the stock, at which the benefits or costs of the forest externality are balanced against the optimum level of extraction for the village; the second point is the optimum level of the forest stock just before the government harvest. Figure 10 illustrates this point; with $T = 10$, there is time for the forest to reach a 'steady state'; however, before the government harvest, the village acts in order to reduce the stock to its optimal "preharvest" level. Over shorter time periods, of course,

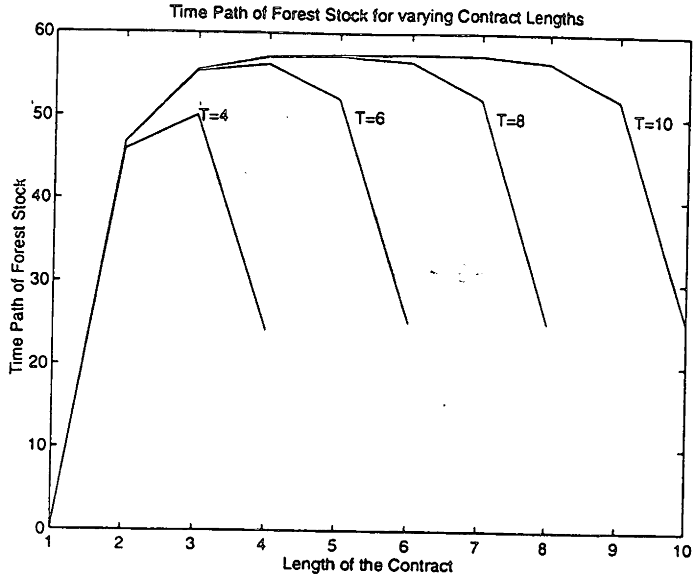


FIGURE 10. Time Path of the Forest Stock for Varying Contract Lengths

there may not be time to reach the steady state (or indeed, the optimal preharvest) level of the stock.

Finally, we are interested in determining how the "preharvest" level of forest stock varies with (i) the share of the terminal stock offered to the villagers (ρ); and (ii) the level of externality present in the village (ξ). Let φ be a function that gives the terminal forest stock[8] for a given ρ, T, F_0 and ξ, that is, $F_{t+1} = \varphi(\rho, T|F_0, \xi)$. F_{t+1} is determined by the villagers' collective reaction to the imposition of a particular JFM contract (ρ, T), which is in turn the solution to equation (15). Also, let α denote the Arrow-Pratt measure of relative risk aversion[9]

Proposition 6. *If a Markov perfect equilibrium exists, then*

i) *If a household's coefficient of relative risk aversion is less than or equal to one ($\alpha \leq 1$) then the terminal stock is a non-decreasing function of the harvest share,* $\frac{\partial\varphi(\rho,T|F_0,\xi)}{\partial\rho} \geq 0$.

ii) *If a household's coefficient of relative risk aversion is greater than or equal to one ($\alpha \geq 1$) then the terminal stock is a non-increasing function of the harvest share (ρ),* $\frac{\partial\varphi(\rho,T|F_0,\xi)}{\partial\rho} \leq 0$.

iii) *The terminal stock, $\varphi(\rho, T|F_0, \xi)$, is a decreasing function of the level of non-cooperation, that is, if $\xi > \xi'$ then $\varphi(\rho, T|F_0, \xi) < \varphi(\rho, T|F_0, \xi')$.*

[8]Preharvest forest stock and terminal forest stock are directly related with the terminal stock being equal $(1 + \delta)$ times the preharvest stock. We use these terms interchangeably.

[9]We are using the Arrow-Pratt measure of relative risk aversion to refer to the elasticity of the marginal utility of consumption. This is a slight abuse of notation since the proof of the proposition does not involve uncertainty.

iv) *If* $\frac{\partial^2 w_1}{\partial \rho^2} > 0$[10] *and* $\frac{\rho}{\varphi(\rho,T|F_0,\xi)}\frac{\partial \varphi(\rho,T|F_0,\xi)}{\partial \rho}\alpha > \frac{1}{2}$, *then the cross derivative of the terminal stock with respect to the harvest share and the level of the externality is negative,* $\frac{\partial^2 \varphi(\rho,T|F_0,\xi)}{\partial \xi \partial \rho} < 0$.

Proof. See Appendix D.6 □

Parts (i) and (ii) of proposition 6 establish the relationship between the "preharvest" level of the forest stock and the share of the terminal harvest offered to the villagers. In particular they help us to understand whether the state can slow down the rate of deforestation, prior to state felling, by offering the villagers a larger share of the felled timber. According to the proposition the answer to this query is, maybe. If a villager's coefficient of relative risk aversion is less than (greater than) 1 then the government can induce forest conservation, prior to government felling, by increasing (decreasing) the share of the terminal harvest granted to the villagers. In fact, with $\alpha > 1$, an increase in ρ rather than inducing conservation causes the villagers to further reduce the "pre-harvest" level of forest stock.

Part (iii) of the proposition establishes a relationship we would expect to hold, namely that, the terminal stock of forests is a decreasing function of the level of externality in the village. Non-cooperating villagers typically run down the "pre-harvest" level of forest stock more than cooperating villagers.

Finally, part (iv) establishes the conditions under which we can sign the cross derivative of the terminal harvest function with respect to ρ and ξ. We are in particular interested in establishing conditions under which this cross derivative is negative because a negative cross derivative ($\frac{\partial^2 \varphi(\rho,T|F_0,\xi)}{\partial \xi \partial \rho} < 0$) implies that an increase in the share of the terminal harvest allotted to the villagers slows down the rate at which firewood is harvested by non-cooperating villagers. If so then the state may be able to curb the tragedy of the commons by increasing the share of the terminal harvest given to non-cooperating villages.

In words part (iv) of proposition 6 states that by offering the villagers a higher share of the timber felled, the state can slow down the rate at which non-cooperating villagers exploit the forest if (i) the villagers extract firewood at an increasing rate; and (ii) if the product of the Arrow-Pratt measure of relative risk aversion and the elasticity of the terminal stock with respect to the share is greater than $\frac{1}{2}$. In the appendix D we establish that for the villagers to extract firewood at an increasing rate (that is, for the second derivative of the policy function with respect to ρ to be

[10] *We establish conditions under which this result holds in the appendix.*

positive) one of the conditions needed is that α, the relative risk aversion coefficient, be less than one. This in turn implies by proposition 6, also implies that as the state offers the villagers a higher share of the timber harvest, the villagers will decrease the amount of firewood they harvest from the forest. In essence the first condition needed to slow down the rate of exploitation by non-cooperating villagers can be re-stated to be that as the state offers the villagers a higher share of the timber felled, the villagers decrease their harvest of firewood at an increasing rate. Furthermore, if $\alpha < 1$ then the second condition, represented by equation (54), implies thatthe elasticity of the terminal stock with respect to $\rho \to 5$ as $\alpha \to 0$ and this elasticity $\to 1$ as $\alpha \to 1$. That is, the second condition needed to decreases the rate of exploitation is that the villager's coefficient of relative risk aversion be less than 1 and that the terminal stock be share elastic.

We conclude this section by exploring the sensitivity of our results to some of the assumptions we have made while developing the model for household behavior under the JFM policy regime to maintain tractability.

The first assumption we tackled is the one governing the nature of the terminal reward granted to the villagers once the forests have been felled by the state. We have assumed that the state fells the entire forest first and then awards the villagers their share of the timber. This assumption implies that a household's decision in period $T+2$ is independent of its decisions up to and including period T. This has the effect of breaking the infinite horizon problem into a sequence of finite horizon problems. Clear felling necessitates high cost of forest regeneration which the villagers want to delay and this biases the villager's preferences towards a relatively longer contract. The question is whether this assumption also causes the villagers to harvest a larger share of the forest stock just before the state fells the forest? Alternatively, if the state was to grant the villagers' share in terms of standing stock[11] would the villagers' tendency to increase harvest in the period prior to state felling be checked? We claim that it would not. Villagers increase harvest because of the share they are being granted not because of the form of payment. Consequently, our results hold even if we change the form of the payment. The same argument holds if we relax the assumption currently being made which prevents the villagers from storing firewood they get from the terminal reward. The villagers have to consume the entire reward in the period that it is granted.

[11] At present there is a debate in India as to whether the terminal share should be granted as standing stock or as felled timber. This debate makes the present discussion relevant.

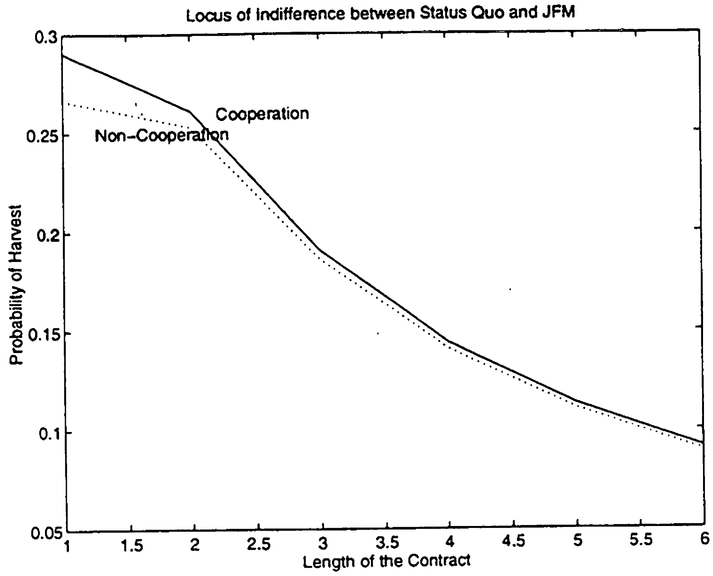


FIGURE 11. Locus of T and π for which the Households are Indifferent Between JFM and SQ

Finally, if instead of requiring the villagers to make all the investment necessary to regenerate the forests we instead assume that the state subsidizes the investment then do our results still hold? Here too we claim that they do. An investment subsidy enables the villagers to lower the cost of forest regeneration but it does not give them the incentive to curb harvest in the period prior to felling by the state. It does not change the problem faced by each household in the finite horizon.

3.3. **Comparing JFM and the Status Quo.** Suppose that we fix the initial level of the forest stock at some number F and the share of the timber given to villagers at some number ρ. We can now imagine tracing out the locus of points (π, T) at which the value of JFM and status quo are equal. The following proposition establishes some properties of this locus.

Proposition 7. *There exists a $\hat{\rho} > 0$ such that, for any $\rho < \hat{\rho}$ and any fixed $F \geq 0$: (i) for any $T \in \{0, 1, \ldots\}$ there exists a unique π such that $J_T^{\infty}(F) = V_\pi(F)$; (ii) For any $T' > T$, and π' and π satisfying $J_T^{\infty}(F) = V_\pi(F)$ and $J_{T'}^{\infty}(F) = V_{\pi'}(F)$, π' must be weakly less than π; and (iii) $J_{\infty}^{\infty}(F) = V_0(F)$ for all F.*

Proof. See Appendix D.7 □

The locus is illustrated in Figure 11. The main content of proposition 7 is that so long as the terminal payment is sufficiently small, the locus will be downward sloping in T. This proposition holds true whether the villagers are cooperating or not cooperating. Furthermore, the villagers prefer JFM for any (π, T) pair which occurs to the right of the locus. This in turn implies that from the villagers point of view, neither policy dominates the other—for some parameter values status quo is preferred to JFM and for others JFM is preferred to status quo.

Having established the shape of the locus we would like to establish conditions under which the locus for the cooperative regimes lies above or below the locus for the non-cooperative regime. In cases where the locus for the non-cooperating villages lies below the locus for the cooperating villagers non-cooperating villagers are more likely to prefer JFM. To establish this result we turn to the next proposition.

Proposition 8. *For any: (i) $F \geq 0$; (ii) T and π combination such that $J_T^{\infty}(F)_{\xi=0} = V_{\pi}(F)_{\xi=0}$; and (iii) T and $\hat{\pi}$ combination such that $J_T^{\infty}(F)_{\xi=1} = V_{\hat{\pi}}(F)_{\xi=1}$, if $(J_T^{\infty}(F)_{\xi=0} - J_T^{\infty}(F)_{\xi=1}) \gtreqless (V_{\pi}(F)_{\xi=0} - V_{\pi}(F)_{\xi=1})$ then $\pi \lesseqgtr \hat{\pi}$.*

At a given T, the locus for the non-cooperative regime lies under the locus for the cooperative regime if $\pi > \hat{\pi}$ and above if $\pi < \hat{\pi}$. The loci may cross at some T if $\pi = \hat{\pi}$.

Put into words proposition 8 states that for a non-negative forest stock, and for any T and π combination that equates the value of the forest stock under status quo and JFM when the villagers are cooperating and for the same T but different $\hat{\pi}$ combination that equates the value of the forest stock under status quo and JFM when the villagers are *not* cooperating if the cost of non-cooperation under JFM (evaluated at T) is less than the cost of non-cooperation under status quo (evaluated at π) then at that T the locus for the non-cooperating villages lies below the locus for the cooperating villages. Equivalently, whenever the cost of non-cooperation under JFM is less than the cost of non-cooperation under status quo $\hat{\pi} < \pi$. With the length of the contract remaining constant $\hat{\pi} < \pi$ implies that the locus for non-cooperating villages must lie below the locus for cooperating villages. As stated in a previous section the cost of non-cooperation is the difference in the value of the forest stock under cooperation and non-cooperation.

Proof. See Appendix D.8 □

The question is whether the cost of non-cooperation is given exogenously or whether the state, through appropriate policy changes, can control the cost of non-cooperation, be it under JFM or under status quo. For example, in proposition 6 (subsection (iv)) we outlined conditions under which the state, by offering a higher share of the felled timber to the villagers, could slow down the rate of exploitation, and thereby reduce the cost of non-cooperation under JFM. In such a case it is possible that the locus for the non-cooperating villages may lie under the locus for the cooperating villages. Consequently, non-cooperating villagers are more likely to prefer JFM than cooperating ones and thus the state is well advised to initiate JFM in villages that find it hard to cooperate and maintain status quo in villages that do.

We conclude this section by making concrete the idea that if for all combinations of T and π the locus for the cooperating villages lies above the locus for the non-cooperating villages then non-cooperating villagers are more likely to prefer JFM than cooperating ones. This follows directly from the position of the locus and the fact that villagers prefer JFM over status quo for all T and π combinations that lie to the right of the locus. We state this result as a corollary (a similar corollary holds when the positions of the loci are reversed).

Corollary 1. *(i) There exist combinations of π and T such that $J_T^{\infty}(F)_{\xi=1} < V_\pi(F)_{\xi=1}$ and $J_T^{\infty}(F)_{\xi=0} > V_\pi(F)_{\xi=0}$; (ii) For any π and T if $J_T^{\infty}(F)_{\xi=0} < V_\pi(F)_{\xi=0}$ then $J_T^{\infty}(F)_{\xi=1} < V_\pi(F)_{\xi=1}$; and (iii) for any π and T if $J_T^{\infty}(F)_{\xi=1} > V_\pi(F)_{\xi=1}$ then $J_T^{\infty}(F)_{\xi=0} > V_\pi(F)_{\xi=0}$.*

If the locus for the cooperative regime lies above the locus for the non-cooperative regime then for all combinations of T and π for which non-cooperating households prefer status quo cooperating households will also prefer status quo and for all combinations of T and π for which cooperating households prefer JFM non-cooperating households will also prefer JFM. Furthermore, there exit combinations of T and π for which cooperating households prefer status quo and non-cooperating households prefer JFM but there are no combinations of T and π for which the reverse is true.

4. The State

Up to this point we have permitted the government no volition. We will right this wrong in this section, relaxing our assumption that the government is a shock. However, we will find it convenient to continue to indulge our cynicism to a different extent; we will imagine that the government will choose whether to apply JFM or to maintain the status quo in order to maximize its expected revenues from timber.

If the government chooses to implement JFM it has to then choose the villager's share ρ and the length of the contract; the pair (ρ, T) completely specifies a JFM contract. Alternatively, if it chooses to maintain the status quo then it must choose the probability of harvest, which is completely determined by the single probability π. In making this decision, the government will, in general, take into account the circumstances of the village it proposes to contract with. In this very stylized model, villages vary only according to the initial level of forest stock, F_0; and the externality villagers face, ξ.

We first consider status quo where the government chooses to harvest timber with some probability π. Let

$$F_t = h(\pi, t, F_0 | n, \xi) \tag{16}$$

denote the equation of motion for forest stock as a function of the probability of government harvest, π, the time since last harvest (or the initiation of the harvesting policy), t, and the initial forest stock F_0. This function depends on the rate of growth of the forest of course, but more importantly depends on the behavior of the villagers, given the government's policy.

We require the government to choose some π once and for all.[12] In order to formulate the government's problem, let us begin by considering the problem faced by the government given that the current forest stock is zero. In this case, we can write an equation expressing the surplus for the government as a function of the probability of harvest. Call this function $S(0,\pi)$; it is given by

$$\begin{aligned} S(0,\pi) &= \pi(h(\pi,0,0) + \beta S(0,\pi)) \\ &\quad + \beta(1-\pi)[\pi(h(\pi,1,0) + \beta S(0,\pi)) \\ &\quad + \beta(1-\pi)[\pi(h(\pi,2,0) + \beta S(0,\pi)) \\ &\quad + \quad \vdots \end{aligned}$$

which, with a bit of rearrangement, becomes

$$S(0,\pi) = \pi \left[\sum_{t=0}^{\infty} (\beta(1-\pi))^t h(\pi,t,0) + \frac{\beta}{1-\beta(1-\pi)} S(0,\pi) \right]$$

[12] If the government were to vary π as a function of the present level of stock, then in the absence of strategic behavior on the part of the villagers, the linearity of the government's return function would give rise to a "bang-bang," or stopping-time rule—the government would harvest with probability one if the forest stock was above some critical level, and wouldn't harvest at all for values of the forest stock below this level. However, if the government were to employ such a rule, the villagers would presumably never allow the forest stock to reach the critical level, and the government's surplus would be zero. Hence the unchanging, stochastic rule we propose dominates a simple stopping time rule. Whether or not more complicated strategies for the government could lead to superior outcomes is an open question.

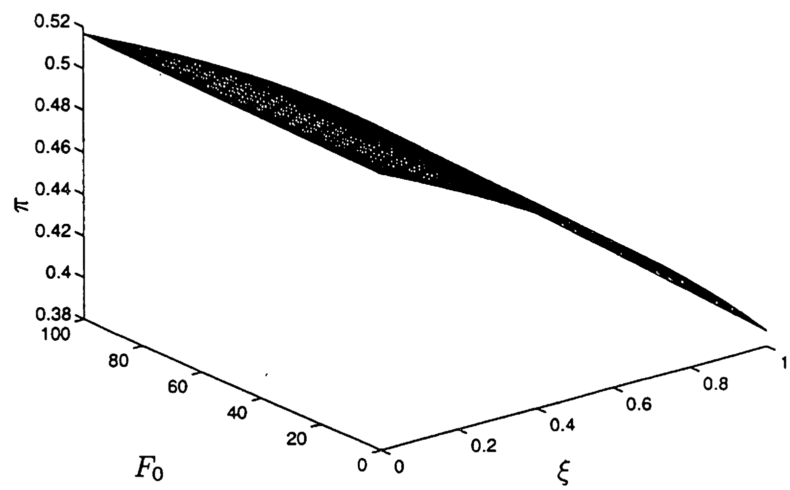


FIGURE 12. Optimal Harvest Probability

Thus, solving for $S(0,\pi)$, we have

$$S(0,\pi) = \frac{\pi[1-\beta(1-\pi)]}{1-\beta(1-\pi)-\pi}\sum_{t=0}^{\infty}(\beta(1-\pi))^t h(\pi,t,0) \tag{17}$$

Next we'd like to consider the case in which the initial forest isn't necessarily zero; we'll also assume that the government chooses π optimally. This gives us

$$\begin{aligned} S(F_0) = \max_{\pi} & \, \pi(h(\pi,0,F_0) + \beta S(0,\pi)) \\ & + \beta(1-\pi)[\pi(h(\pi,1,F_0) + \beta S(0,\pi)) \\ & + \beta(1-\pi)[\pi(h(\pi,2,F_0) + \beta S(0,\pi)) \\ & + \ \vdots \end{aligned}$$

which, as above, can be rearranged to give

$$S(F_0) = \max_{\pi} \pi \left[\sum_{t=0}^{\infty}(\beta(1-\pi))^t h(\pi,t,F_0) + \frac{\beta}{1-\beta(1-\pi)}S(0,\pi)\right]$$

or

$$S(F_0) = \max_{\pi} \pi [\sum_{t=0}^{\infty}(\beta(1-\pi))^t h(\pi,t,F_0) + \frac{\pi\beta}{1-\beta(1-\pi)-\pi}\sum_{t=0}^{\infty}(\beta(1-\pi))^t h(\pi,t,0)] \tag{18}$$

Solutions to this problem for the government are illustrated in Figure 12, which shows the optimal probability of harvest as a function of the initial forest stock and the village externality, and in

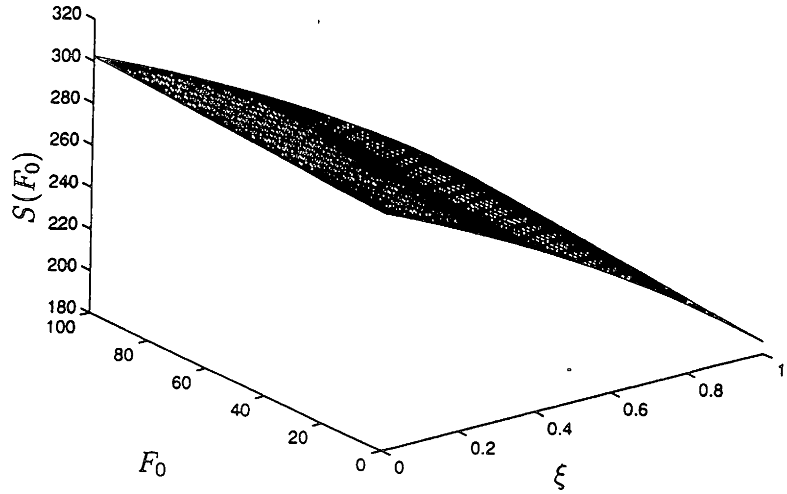


FIGURE 13. Government's Surplus

Figure 13, which shows the surplus derived by the government under this optimal policy. Villagers increase harvest with both an increase in the externality (ξ) and an increase in the probability of harvest (π). By reducing π the government can reduce the amount the villagers harvest as ξ increases. A larger reduction in the probability of harvest in the next period is required when the initial stock is high than when it is low.

We next consider JFM, the alternative policy measure available to the government. The problem facing the government then is to choose T and ρ to maximize its expected timber revenues from an infinite repetition of JFM. Under such a repeated JFM regime, the forest stock is reduced to zero every $(T+1)$ periods. This has the effect of breaking the infinite horizon problem up into a sequence of finite horizon problems. Note that only the first government harvest is related to the forest stock at the time JFM is initiated. Thus, the government's problem can be stated

$$\max_{T,\rho}(1-\rho)\beta^{(T+1)}\varphi(\rho,T|F_0,\xi)+(1-\rho)\beta^{T+2}\frac{\varphi(\rho,T|0,\xi)}{1-\beta^{T+2}}. \tag{19}$$

The function $\varphi(\rho,T|F_0,\xi)$ gives the forest stock at T under a JFM contract (ρ,T), given an initial forest stock F_0, and with an externality measured by ξ. Thus the first term gives the present value of the first harvest to the government. After the first harvest, JFM is repeated *ad infinitum*, and the initial forest stock is zero. Figure 14 gives the government's optimal choice of contract length, given some initial forest stock F_0 and some village level externality ξ. It is clear from this

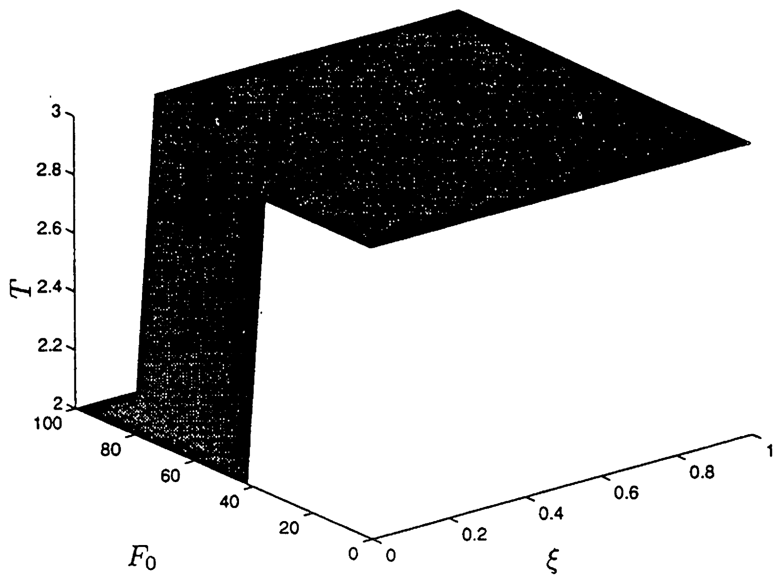


FIGURE 14. Optimal T for Government Under JFM

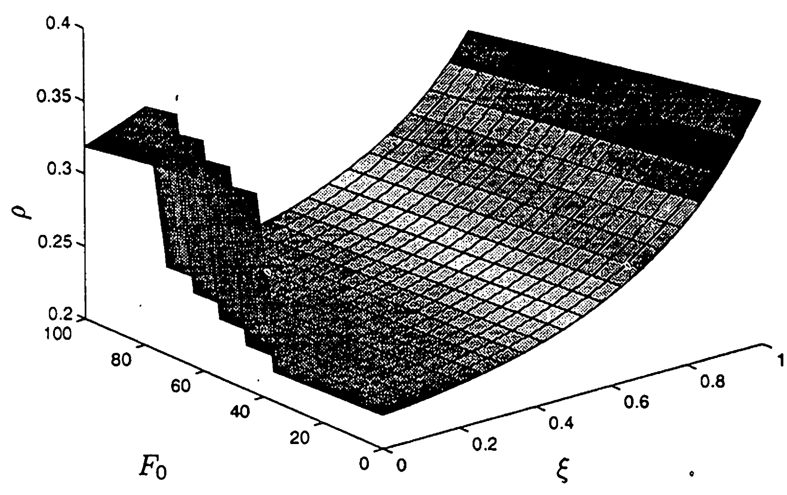


FIGURE 15. Optimal ρ for Government Under JFM

picture that the choice of contract length may be a fairly blunt instrument, at least so long as T is required to be an integer. For all but the highest levels of forest stock and the lowest levels of externality, the government chooses a contract length of three years in our model. This allows the forest stock to reach the maximum preharvest level prior to government felling. Additionally, once the forest stock has the maximum preharvest level there is no gain in waiting any further to harvest the timber.

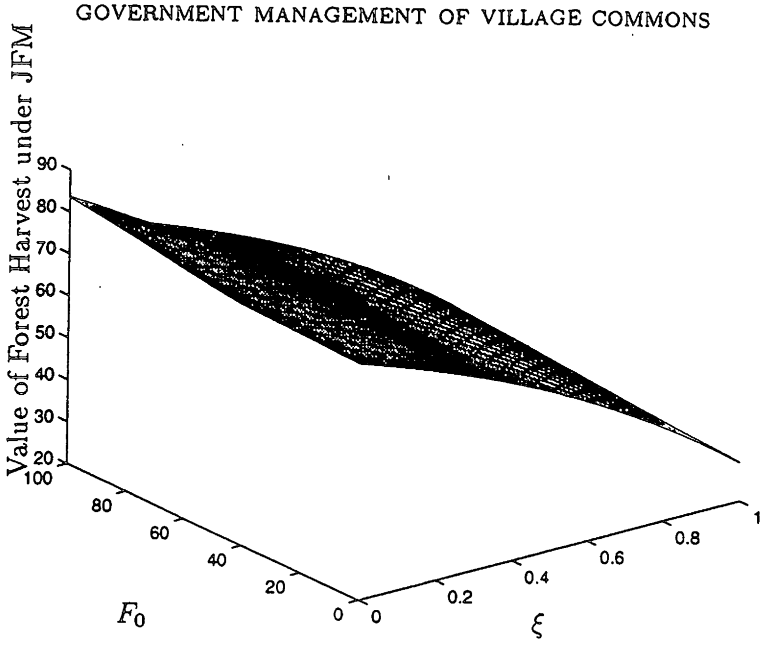


FIGURE 16. Value of JFM to Government

The sharp shift in optimal contract length is mirrored by a sharp change in the share offered to different villages; this is shown in Figure 15. At the same values of initial forest stock and externality which lead to an increase in the length T of the contract, the government finds it advantageous to sharply reduce the share of the final harvest offered to the village. The optimal share granted to the villagers increases with a decline in the level of cooperation to provide the villagers with the incentive not to run down the forest stock prior to harvest. We can make the following claim about the optimal share offered to the villagers,

Proposition 9. *Let the length of the contract be fixed at T, if $\rho \leq 1$, if the household's coefficient of relative risk aversion is less than one ($\alpha \leq 1$), if households extract firewood at an increasing rate as the harvest share increases ($\frac{\partial^2 w_t}{\partial \rho^2} > 0$), and if an increase in the harvest share decreases the rate at which non-cooperating villagers extract firewood then the optimal harvest share offered by the state is an increasing function of the level of non-cooperation, that is, $\frac{\partial \rho}{\partial \xi} > 0$.*

Proof. See Appendix D.9 □

The combination of the shift in the optimal contract length and optimal share actually smoothes out the government's surplus from forest harvest, as shown in Figure 16. This surplus is, of course, increasing in the level of the initial forest stock. It may be less obvious that the government's value is decreasing as the externality, ξ, grows large. However, this follows from the fact that large

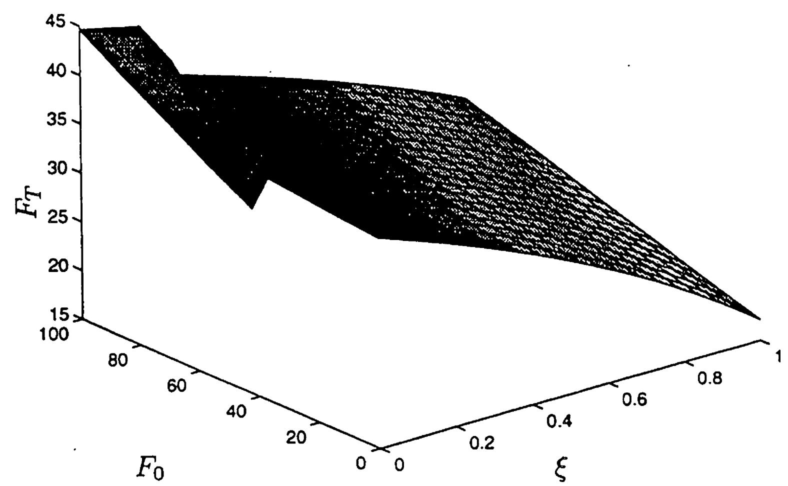


FIGURE 17. Pre-harvest Stocks

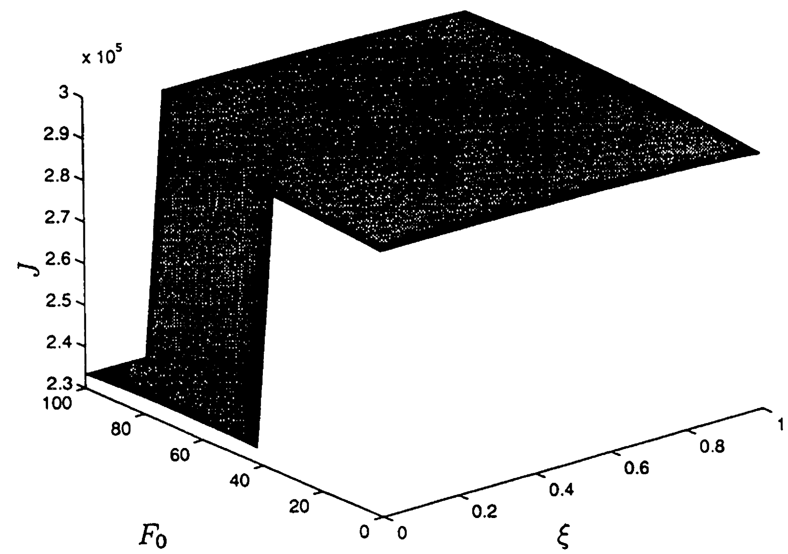


FIGURE 18. Value of JFM to the Village

externalities lead to lower rates of forest growth over the period of the JFM contract, since villagers will continue to harvest more than is socially optimal when ξ is greater than zero. Furthermore, the state has to part with a greater share of the timber harvest when the households are not cooperating to dampen the tendency to over-harvest.

Though we have already examined the village response to particular JFM contracts above, it may be interesting to think about village response to the particular values of (ρ, T) which are optimal

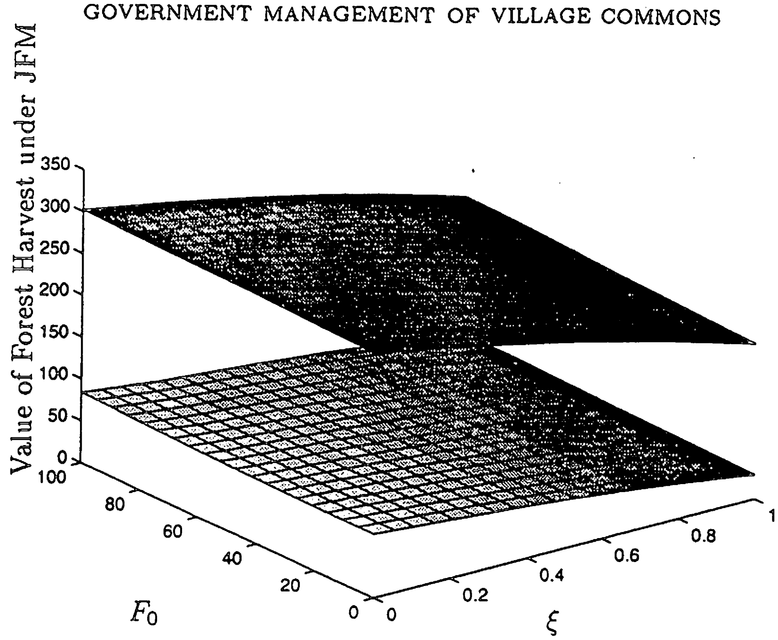


FIGURE 19. Optimal Locus for the state

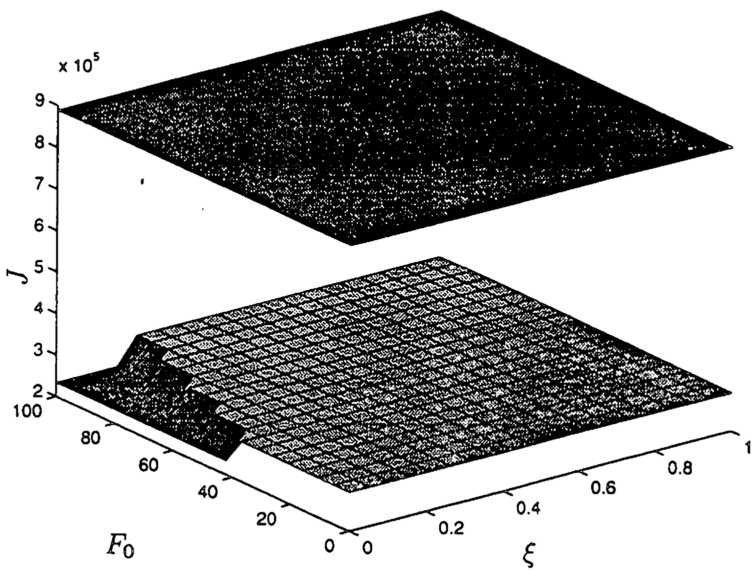


FIGURE 20. Optimal Locus for the village

from the government's point of view, given the exogenous variables ξ and F_0. Figure 17 shows the preharvest forest stock—that is, the stock available for harvest by the government—given the initial forest stock and the externality ξ. Figure 18 shows the discounted utility derived by the villagers given that the government chooses an optimal (ρ, T) pair. Note that the villager's utility is actually decreasing for sufficiently large levels of the initial forest stock; if the forest stock is large, the government will wish to harvest it soon.

We are now in a position to compare the value of SQ and JFM to the state. We claim and intend to establish conditions under which the state can do at least as well under SQ as under JFM. Figure 19 illustrates the cases when the government is always better off with SQ as compared to JFM.

We begin by restricting ourselves to a model of SQ where the world ends after the second period. Each household solves the problem,

$$\max_{w_1} U(F_0, w_1) + \beta(1-\pi)U(F_1, w_2) \tag{20}$$

where $F_1 = (1+\delta)\left(F_0 - w_1 - (m-1)((1-\xi)w_1 - \xi\hat{w}_1)\right)$ and $w_2 = \frac{F_1}{m}$. The first order condition defines the optimal harvest in the first period for a given initial forest stock and a given probability of harvest by the state. Formally,

$$U_2(F_0, w_1) = \beta(1-\pi)(1+\delta)m^\star\left(\frac{U_2(F_1, w_2)}{m} + U_1(F_1, w_2)\right) \tag{21}$$

gives $w_1 = h(F_0, \pi)$.

Let $w_1 = g(F_0, \Psi, \rho)$ denote the optimal harvest function when a single period JFM contract is implemented once. The state bears the harvesting cost in the second period and allots the villagers a share ρ of the harvest. The function g is obtained as a solution to the problem,

$$\max_{w_1} U(F_0, w_1) + \beta U(\Psi, \rho(1+\delta)(F_0 - f_1 - (m-1)((1-\xi)f_1 - \xi\hat{w}_1))) \tag{22}$$

where the first order condition is given by,

$$U_2(F_0, w_1) = \beta(1+\delta)m^\star\left(\rho U_2(\Psi, w_2) + U_1(\Psi, w_2)\right) \tag{23}$$

We claim that for all $F < \Psi$, $g(F_0, F, \rho) < g(F_0, \Psi, \rho)$. With a forest stock of Ψ the villagers bear no cost of harvesting in the second period. A positive harvesting cost ($F < \Psi$) implies that the villagers will reduce the amount of firewood harvested in the first period to balance the marginal utility of firewood consumption today with the marginal utility of firewood consumption tomorrow and the marginal cost of firewood harvest tomorrow.

The state wants to maximize expected timber revenues and therefore it will choose π under SQ and ρ under JFM[13] to minimize the amount of firewood harvested in the first period. We claim

[13] To keep the proof manageable we assume that the length of the contract is fixed.

that the state can do at least as well under SQ as it can under JFM if the group is sufficiently large and for every ρ there exists a π such that $h(F_0, \pi) = g(F_0, F_1, \rho) < g(F_0, \bar{F}, \rho)$. These conditions imply that the expected timber revenue of the state under SQ, $\pi(F_0 - mh(F_0, \pi)$, is at least as large as the state's expected revenue under JFM, $(1-\rho)(F_0 - mg(F_0, F_1, \rho))$.

Proposition 10. *If the households' coefficient of relative risk aversion is less than one and $U_{12}(F, w) = 0$ then there exists some group size $\bar{m}$ such that for all $m > \bar{m}$ the state's expected timber revenue under SQ ($\pi(F_0 - mh(F_0, \pi))$) is at least as large as its expected timber revenue under JFM ($(1-\rho)(F_0 - mg(F_0, F_1, \rho))$).*

Proof. See Appendix D.10 □

5. Conclusion

In section 1, the introduction, we set ourselves the task of evaluating the new forest management policy being promoted at present by the Indian government—Joint Forest Management (JFM)—and of establishing conditions under which JFM really does improve upon the forest management policy in vogue prior to JFM, which we have called Status Quo (SQ). We are particularly interested in answering two questions: (*i*) Which of the two policies do the villagers prefer and how are a villager's preferences over the two policies affected by the level of cooperation in the village?; and (*ii*) Given that the state wants to maximize its expected timber revenues, which of the two policies should the state choose to implement in the villages?

In section 3, after establishing some aspects of village behavior under the two policy regimes, we have proved that the locus of policy parameters (the length of the contract offered to the villagers under JFM and the probability of harvest by the state under S), at which the villagers are indifferent between JFM and SQ, is downward sloping. This in turn implies that from the villagers' point of view neither policy dominates the other and that for some parameter values status quo is preferred to JFM and for others JFM is preferred to SQ. Furthermore, in the same section we have proved that if the cost of non-cooperation (which we have defined as the difference in the value of the forest stock under cooperation and non-cooperation) under JFM is less than the cost of non-cooperation under SQ then non-cooperating villagers are more likely to prefer JFM over SQ as compared to villagers who have succeeded in cooperating.

Section 4 addresses the second question where we have proved that the state's expected timber revenue from SQ is *at* least as large as its expected timber revenue from JFM if: (*i*) the villagers' coefficient of relative risk aversion is less than one; (*ii*) the cross derivative of the utility function with respect to the forest stock and firewood consumption is zero; and (*iii*) the size of the village is sufficiently large. We have found sufficient conditions under which the state can do at least as well by implementing SQ than by implementing JFM!

More generally, by identifying sufficient conditions that make the state as well as the villagers prefer SQ to JFM we have attempted to show that different village level circumstances will in general make different policies suitable. And that one can determine the suitability of a given policy by careful modeling of the local environment and understanding how villagers react to the imposition of a given policy, an exercise often overlooked by policy makers.

Appendix A. Solution Method for the Infinite Horizon Problem

Throughout the paper we have supplemented our analytical results with graphical illustrations to enhance understanding. We now outline the numerical methods we have used to solve for both the value of a particular stock of forests and for the optimal path of firewood extraction. We restrict our attention to a linear-quadratic framework where the utility function is quadratic and the technology is linear. The the utility function governing preferences is,

$$U(F_t, w_t) = -\frac{1}{2}((w_t - b)^2 + (\psi - \phi F_t)^2) \tag{24}$$

where w_t is the amount of firewood harvested in period t, F_t is the stock of forests in period t, b is the bliss point and ψ and ϕ are parameters that govern the relationship between the cost of extraction and the forest stock.

The stock of forests evolve by a linear rule, which is,

$$F_{t+1} = (1+\delta)\,(F_t - mw_t) \tag{25}$$

where δ is the growth rate of the forest stock.

The numerical solution relies on a formulation of the Riccati equation that nests the planned and distorted economy. This in turn requires that we re-write the utility maximization problem as an optimal linear control problem. To this end we write the return function in matrix notation as,

$$U(F, w) = y'Ry + x'Qx + y'Wx \tag{26}$$

where y is the vector of control variables and x is the vector of state variables. For the optimization problem that we are considering $y = w$, that is, the amount of firewood consumed is the only control variable. There are three state variables—F, the stock of forests, b, the bliss point and ψ, the parameter governing the relationship between the cost of harvesting and the stock of forests. b and ψ are defined as state variables so that they can be accommodated into the optimal linear control framework. The matrices, R, Q and W are defined as follows,

$$R = \begin{bmatrix} -\frac{1}{2} \end{bmatrix} \quad Q = -\frac{1}{2}\begin{bmatrix} \phi^2 & 0 & -\phi \\ 0 & 1 & 0 \\ -\phi & 0 & 1 \end{bmatrix}$$

$$W = -\frac{1}{2}\begin{bmatrix} 0 & -2 & 0 \end{bmatrix}$$

Finally, the equation of motion for the state variables in matrix notation can be written as,

$$x_{t+1} = Ax_t + By_t + C\hat{y}_t \tag{27}$$

where $\hat{y}$ is the amount of firewood extracted by the other households that the household maximizing utility treats as a constant. The time subscript has been introduced for expositional clarity. Elsewhere we have dropped the time subscript because we are considering a stationary problem. The matrices, A, B and C are defined as follows,

$$A = \begin{bmatrix} (1+\delta) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} -(1+(m-1)(1-\xi)) \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} -(m-1)\xi \\ 0 \\ 0 \end{bmatrix}$$

Now the Bellman equation, given that the value function is a quadratic function of the state variables, can be written as,

$$x'Vx = \max_y x'Qx + y'Ry + y'Wx + \beta(Ax + By + C\hat{y})'V(Ax + By + C\hat{y}) \tag{28}$$

This is an optimal linear control problem. Differentiating the Bellman equation with respect to the control, y and then imposing the equilibrium condition $\hat{y} = y$ gives the optimal policy function, that is, a function between the control and the state variables.

$$y = -2(R' + \beta B'V(B+C))^{-1}(W + 2\beta B'VA)x \tag{29}$$

To get the Riccati equation associated with this problem, which we will call the Pseudo-Riccati equation, we substitute the optimal policy function into the Bellman equation along with the equilibrium condition. Dropping the x' and x on either side of the equation gives us the following Pseudo-Riccati equation.

$$V = Q + r_1'r_2'Rr_2r_1 - r_1'r_2'W + \beta(A-(B+C)r_2r_1)'V(A-(B+C)r_2r_1) \tag{30}$$

where, $y = r_2r_1x$ which implies that $r_1 = (W + 2\beta B'VA)$ and $r_2 = -2(R' + \beta B'V(B+C))^{-1}$. The Pseudo-Riccati equation is solved for V, the value function, just like the Riccati equation, that is, by iteration.

Appendix B. Solution Method for the Finite Horizon Problem

When solving numerically for the value of the forest stock and for the optimal path of firewood extraction for a finite horizon we again formulate the problem as a optimal linear regulator's problem. The major difference though is that we now use backward induction rather than iteration to find the relevant solutions. Furthermore, for the finite horizon case we need to define a terminal condition which is governed by the reward given to the households once the forests are felled by the state.

Appendix C. Parameter Specification for Simulations

Unless stated otherwise, the figures in the text of the paper are drawn using the following parameter specifications:

Parameter	*Values*	*Parameter*	*Values*
β	0.9	δ	0.02
b	30	m	2
ϕ	1	F_0	100
ψ	20.4		

Table 1. Parameter Values Used for the Simulations

Appendix D. Proofs for the Lemma and Propositions

D.1. Proof for Proposition 1.

i) The steady state level of the forest stock and the steady state level of firewood consumption are obtained by simultaneously solving the Euler equation (equation (5)) and law of motion for the forest stock (equation (2)), both being evaluated at the steady state. Furthermore, if we substitute the steady state law of motion (which is in fact equal to $w^\star = \frac{\delta F^\star}{m}$, where $w^\star$ is the steady state level of firewood consumption) into the steady state Euler equation we get

$$U_2\left(F^\star, \frac{\delta F^\star}{m}\right) = \frac{\beta(1+\delta)}{1-\beta(1+\delta)} m U_1\left(F^\star, \frac{\delta F^\star}{m}\right) \tag{31}$$

The solution to this equation is some function Φ which gives the steady state level of the forest stock for a given β, δ and m, that is, $F^\star = \Phi(\beta, \delta, m)$. Note that the steady state level of the forest stock goes to infinity if $\beta(1+\delta) \geq 1$. We will restrict our attention to the case where the forest stock at steady state is finite and thus we assume that $\beta(1+\delta) < 1$. The next step is to differentiate $\Phi(\beta, \delta, m)$ (implicitly given by equation (31)) with respect to the group size holding the discount rate and the growth rate of the forest stock constant. With the added assumption that $U_{12}(F, w) = 0$ we get,

$$\frac{\partial \Phi(\beta, \delta, m)}{\partial m} = \frac{\frac{\delta F^\star}{m^2} U_{22}\left(F^\star, \frac{\delta F^\star}{m}\right) + \gamma_1 U_1\left(F^\star, \frac{\delta F^\star}{m}\right)}{\frac{\delta}{m} U_{22}\left(F^\star, \frac{\delta F^\star}{m}\right) - \gamma_1 m U_{11}\left(F^\star, \frac{\delta F^\star}{m}\right)} \tag{32}$$

where, $\gamma_1 = \frac{\beta(1+\delta)}{1-\beta(1+\delta)} > 0$ if $\beta(1+\delta) < 1$. For a sufficiently large group size, m, the numerator and the denominator are both positive and therefore the steady state level of the forest stock increases as the group becomes bigger. The steady state level of firewood consumption, $w^\star = \frac{\delta F^\star}{m}$, increases if the elasticity of the forest stock with respect to the group size is greater than one and decreases if the reverse is true. To see this note that,

$$\frac{\partial w^\star}{m} = \frac{m\delta \frac{\partial F^\star}{\partial m} - \delta F^\star}{m^2} \tag{33}$$

and that $\frac{\partial w^\star}{m} \gtreqless 0$ if $\frac{m}{F^\star}\frac{\partial F^\star}{\partial m} \gtreqless 1$.

ii) When households are not cooperating the steady state Euler equation is,

$$U_2(F^\star, w^\star) = \beta(1+\delta)\left(U_1(F^\star, w^\star) + \left(1 - (m-1)\frac{\partial \hat{w}^\star}{\partial F^\star}\right) U_2(F^\star, w^\star)\right) \tag{34}$$

The steady state equilibrium law of motion implies that $w^\star = \hat{w}^\star = \frac{\delta F^\star}{m}$. Consequently, $\frac{\partial \hat{w}^\star}{\partial F^\star} = \frac{\delta}{m}$ and equation (34) can be written as,

$$U_2\left(F^\star, \frac{\delta F^\star}{m}\right) = \frac{m\beta(1+\delta)}{m - \beta(1+\delta)(m-(m-1)\delta)} U_1\left(F^\star, \frac{\delta F^\star}{m}\right) \tag{35}$$

The solution to equation (35) is some function ϕ which assigns to each combination of β, δ and m a steady state level of the forest stock, or, $F^\star = \phi(\beta, \delta, m)$. Differentiating the function

ϕ with respect to the group size under the assumption that $U_{12}(F, w) = 0$ yields,

$$\frac{\partial \phi(\beta, \delta, m)}{\partial m} = \frac{\gamma_3 U_1\left(F^*, \frac{\delta F^*}{m}\right) + \frac{\delta F^*}{m^2} U_{22}\left(F^*, \frac{\delta F^*}{m}\right)}{\frac{\delta}{m} U_{22}\left(F^*, \frac{\delta F^*}{m}\right) - \gamma_2 U_{11}\left(F^*, \frac{\delta F^*}{m}\right)} \tag{36}$$

where, $\gamma_2 = \frac{m\beta(1+\delta)}{m-\beta(1+\delta)(m-(m-1)\delta)} > 0$ and $\gamma_3 = \frac{-m\beta^2(1+\delta)^2\delta}{\left(m-\beta(1+\delta)(1-(m-1)\delta)\right)^2} < 0$ if $\beta(1+\delta) < 1$. Consequently, for a sufficiently large m, the denominator of equation (36) is positive while its numerator is always negative. Thus if the households are not cooperating, then, for a sufficiently large group size the steady state level of forest stock decreases as the group size increases. Furthermore, the steady state level of firewood consumption also decreases as the group becomes bigger.

D.2. **Proof for Proposition 2.**

i) Rewrite the representative household's problem given above, but using next period's forest stock F' as the choice variable. The new return function can be written

$$h(F, F') = U\left(F, \frac{F - F'/(1+\delta) - (m-1)\xi\hat{w}}{1 + (m-1)(1-\xi)}\right) = U(F, w),$$

so that the resulting objective function is

$$h(F, F') + \beta V(F').$$

Fix two forest stocks, F and $\tilde{F}$, with $\tilde{F} < F$. Let $F' = g(F)$, and $\tilde{F}' = g(\tilde{F})$. Next, note that $U_{12}(F, w) > 0$ implies that $h_{12}(F, F') \leq 0$. By definition,

$$\frac{\partial^2 h(F, F')}{\partial F \partial F'} = \lim_{\tilde{F} \to F} \lim_{\tilde{F}' \to F'} \frac{1}{(F - \tilde{F})(F' - \tilde{F}')} \left[h(F, F') - h(\tilde{F}, F') - h(F, \tilde{F}') + h(\tilde{F}, \tilde{F}')\right].$$

Equilibrium requires that each households' action be a best response, or, for all $(F, \tilde{F})$,

$$h(F, F') + \beta V(F') \geq h(F, \tilde{F}') + \beta V(\tilde{F}')$$

and

$$h(\tilde{F}, \tilde{F}') + \beta V(\tilde{F}') \geq h(\tilde{F}, F') + \beta V(F').$$

Note that adding this pair of incentive compatibility constraints implies that

$$h(F, F') - h(\tilde{F}, F') - h(F, \tilde{F}') + h(\tilde{F}, \tilde{F}') \geq 0.$$

But then the sign of the cross-partial derivative depends only on the sign of $(F' - \tilde{F}')$. Since we know that $h_{12} < 0$, it must be the case that $\tilde{F}' \geq F'$.

ii) Proof proceeds as for (i).

iii) Since $g(F)$ is continuous, and since F is an element of the compact interval $[0, \Psi]$, we can immediately apply Brouwer's fixed point theorem to establish the existence of a steady-state. Because the forest stock evolves monotonically under the conditions given in parts (i) and (ii) of the proposition, the steady state is unique.

iv) At the steady state and after substituting in for the steady state equilibrium law of motion, equation (8) simplifies to,

$$U_2\left(F^*, \frac{\delta F^*}{m}\right) = \frac{\beta(1+\delta)m^*}{1-\beta(1+\delta)\left(1-\frac{(m-1)\xi\delta}{m}\right)} U_1\left(F^*, \frac{\delta F^*}{m}\right) \tag{37}$$

where $m^* = (1+(m-1)(1-\xi))$. The solution to this equation is some function Φ which maps combinations of (β, δ, m and ξ) into the steady state level of the forest stock. Specifically, $F^* = \Phi(\beta, \delta, m, \xi)$. Differentiating the function Φ (given implicitly by equation (37)) with respect to ξ holding all other parameters constant gives,

$$\frac{\partial \Phi(\beta,\delta,m,\xi)}{\partial \xi} = \frac{-\beta(1+\delta)(m-1)\left(mU_1(F^*, \frac{\delta F^*}{m}) + \delta U_2(F^*, \frac{\delta F^*}{m})\right)}{\delta\left(1-\beta(1+\delta)(1-\frac{(m-1)\xi\delta}{m})\right)U_{22}(F^*, \frac{\delta F^*}{m}) - mm^*\beta(1+\delta)U_{11}(F^*, \frac{\delta F^*}{m})} \tag{38}$$

The numerator is negative for all m while for sufficiently large m the denominator is positive and thus $\frac{\partial F^*}{\partial \xi} < 0$.

D.3. **Proof for Proposition 3.** For a well behaved dynamic programming problem,[14] $V_\pi(F)$ is strictly increasing and strictly concave. Given that $V_\pi(F)$ is strictly increasing, then for any $\pi \geq \pi'$ and $F_t \geq 0$ the following relationship must hold,

$$\pi V_\pi(0) + (1-\pi)V_\pi(F_t) \leq \pi' V_\pi(0) + (1-\pi')V_\pi(F_t) \tag{39}$$

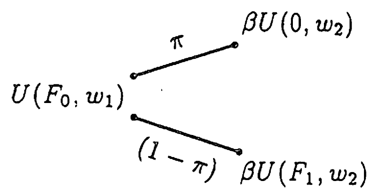
Next consider a utility possibility frontier (UPF) which maps the maximum utility that can be attained in the future $(\beta(\pi V_\pi(0)+(1-\pi)V_\pi(F_{t+1})))$ for every level of utility attained today $(U(F_t, w_t))$.

[14]For a dynamic programming problem to be well behaved the following must be true: (*i*) the set of all possible values for the state variable must be a convex subset of R_+; (*ii*) the correspondence describing the feasibility constraint must be non-empty, compact-valued, continuous, monotone, and convex; (*iii*) The return function must be bounded, continuous, strictly increasing in the state variable and strictly concave; (*iv*) the discount factor must be greater then zero and less than one; and (*v*) the problem has a unique solution.

Concavity of the utility and the value function insure that the utility possibility frontier will be concave. Furthermore, equation (39) implies that an increase in the probability of harvest decreases the maximum utility that can be attained in the future for every level of utility attained today (the UPF shifts downwards). Consequently, the total value of a given stock of forest, $V_\pi(F_t)$, decreases as the probability of harvest, π, increases.

D.4. **Proof for Proposition 4.**

i) We begin by restricting ourselves to a model of SQ where the world ends after the second period and consider how the households behave under such a situation. The events faced by the households over these two years can be captured by the event tree,



Working backwards from the second period we note that if the state were to cut the forests after the first period (so that we are at the top right hand corner of the event tree) then given that the world ends after the second period the households will have no incentive to invest in the forest in the second period, that is, $w_2 = 0$. To simplify the problem and without any loss of generality we assume that $U(0,0) = 0$. On the other hand, if the state does not cut the forests in the second period then we expect the households to do so because there is no tomorrow to make conservation beneficial. We assume that in such a situation so long as $U_{12}(F, w) = 0$ the households will divide the forest equally between themselves, that is, $w_2 = \frac{(1+\delta)\left(F_0 - w_1 - (m-1)((1-\xi)w_1 - \xi\hat{w}_1)\right)}{m}$. With this expected behavior in the second period the problem faced by each household in the first period simplifies to,

$$\max_{w_1} U(F_0, w_1) + \beta(1-\pi)U(F_1, w_2) \tag{40}$$

where $F_1 = (1+\delta)\left(F_0 - w_1 - (m-1)((1-\xi)w_1 - \xi\hat{w}_1)\right)$ and $w_2 = \frac{F_1}{m}$. The first order condition corresponding to equation (40) gives the optimal harvest in the first period for a given initial forest stock, a given level of non-cooperation and a given probability of harvest by the state. Formally, the first order condition gives $w_1 = h(F_0, \xi, \pi)$. To establish our first claim we need to show that $\frac{\partial h(F_0, \xi, \pi)}{\partial \pi} > 0$. We begin by differentiating the first order condition with respect to π. This gives us the derivative of function h with respect to π which turns out to be an ugly expression and is thus omitted here. The numerator of this expression is negative as is the denominator if,

$$\frac{-U_{12}(F_0, w_1)}{U_{11}(F_0, w_1)} \leq m \leq \frac{-U_{22}(F_1, w_2)}{U_{12}(F_0, w_1)} \tag{41}$$

If $U_{12}(F, w) = 0$ then the condition stipulated in (41) holds. Consequently, the amount of firewood harvested increases with the level of the externality. Now to establish the second claim we differentiate the first order condition with respect to ξ. Imposing the equilibrium condition that $w = \hat{w}$ gives,

$$\frac{\partial h(F_0, \xi, \pi)}{\partial \xi} = \frac{-\beta(1-\pi)(1+\delta)(m-1)\left(U_1(F_1, w_2) + \frac{U_2(F_1, w_2)}{m}\right)}{U_{22}(F_1, w_2) + \beta(1-\pi)(1+\delta)^2 m^{\star 2}\left(U_{11}(F_1, w_2) + \frac{U_{22}(F_1, w_2)}{m}\right)} \tag{42}$$

The numerator and denominator of this equation (42) are negative.

ii) The Euler equation corresponding to the problem faced by a household over an infinite horizon (equation (9)) is,

$$U_2(F, w) = \beta(1-\pi)(1+\delta)\left(m^\star U_1(F', w') + \left(1 - (m-1)\xi\frac{\partial \hat{w}'}{\partial F'}\right)U_2(F', w')\right) \tag{43}$$

Imposing steady state at the equilibrium and substituting in for the equilibrium steady state law of motion (equation (10) at the steady state) defines a function Φ_π which gives the steady state level of the forest stock associated with a given combination of (π, ξ, m, F_0). Differentiating this function Φ_π with respect to π (under the additional condition that $U_{12}(F, w) = 0$ and holding all other parameters constant) gives,

(44)

$$\frac{\partial \Phi_\pi(\pi|\xi, m, F_0)}{\partial \pi} = \frac{-\beta(1+\delta)\left(m^\star U_1\left(F_\pi^\star, \frac{\delta F_\pi^\star}{m}\right) + \left(1 - \frac{(m-1)\xi\delta}{m}\right)U_2\left(F_\pi^\star, \frac{\delta F_\pi^\star}{m}\right)\right)}{\frac{\delta - \delta\beta(1-\pi)(1+\delta)\left(1 - \frac{(m-1)\xi\delta}{m}\right)}{m} U_{22}\left(F_\pi^\star, \frac{\delta F_\pi^\star}{m}\right) - \beta(1-\pi)(1+\delta)m^\star U_{11}\left(F_\pi^\star, \frac{\delta F_\pi^\star}{m}\right)}$$

The numerator is negative and remains so as m increases. However, the sign to the denominator is ambiguous for small values of m but positive for a sufficiently large m. Consequently, for a sufficiently large group the steady state level of the forest stock decreases as the probability of harvest in the next period increases. The second claim follows from proposition 2 (subpart (iv)) once we note that the Euler equation when there is no government intervention (equation (8)) differs from the Euler equation when status quo is imposed (equation (43)) only by the factor $(1-\pi)$ and that this difference does not affect the proof of the proposition.

D.5. **Proof for Proposition 5.** Consider a utility possibility frontier which maps the maximum utility that can be attained in periods $T+1$ and $T+2$ for a given level of total utility attained between the beginning of the contract and period T. With a joint forest management contract of length T, utility attained up to and including period T is equal to $\sum_{t=0}^{T}\beta^t U(F_t, w_t)$ and utility attained in periods $T+1$ and $T+2$ is given by $\beta^{(T+1)}U(\Psi, \rho F_{T+1})$. While with a contract of length $T+1$ the utility up to and including period T is the same and the utility in the last two periods is $\beta^{(T+1)}U(F_{T+1}, w_{T+1}) + \beta^{(T+2)}U(\Psi, \rho F_{T+1})$. For a small ρ, $U(\Psi, \rho F_{T+1}) < U(F_{T+1}, w_{T+1}) + \beta U(\Psi, \rho F_{T+2}$. This implies that an increase in the length of the contract increases the maximum utility that can be attained in period $T+1$ for every level of utility attained up to and including period T. In other words, an increase in the length of the contract increases the total value of a given level of forest stock over $T+1$ periods.

D.6. **Proof for Proposition 6.**

i) Consider a one period contract. The villagers inherit a stock of forests denoted by F_0 at the beginning of the first period and in the same period they extract firewood from the forest. In the next period the state comes in and cuts down the forest and gives the village a share ρ of the terminal forest stock. The villagers choose an optimal level of harvest under the knowledge that the amount they harvest today is inversely proportional to the reward given to them tomorrow. Formally, the problem facing the villagers is to

$$\max_{w_1} U(F_0, w_1) + \beta U(\Psi, w_2) \tag{45}$$

where $w_2 = \rho(1+\delta)(F_0 - (1+(m-1)(1-\xi))w_1 - (m-1)\xi\hat{w}_1)$, w_1 is the amount of firewood harvested in the first period and $\hat{w}_1$ is the amount of firewood harvested by the other households that the maximizing household takes as a given. The first order condition for equation (45)is,

$$U_2(F_0, w_1) - \beta\rho(1+\delta)m^* U_2(\Psi, w_2) = 0 \tag{46}$$

which implicitly defines w_1, as a function of ρ and ξ. Next, differentiating the optimal policy function with respect to ρ gives,

$$\frac{\partial g(\rho,\xi)}{\partial \rho} = \frac{\beta(1+\delta)m^*\Big(U_2(\Psi,w_2)+w_2U_{22}(\Psi,w_2)\Big)}{U_{22}(F_0,w_1)+\beta\rho^2(1+\delta)^2m^{*2}U_{22}(\Psi,w_2)} \tag{47}$$

Diminishing marginal utility ensures that the denominator of equation (47) is negative. The numerator will be non-negative if and only if

$$U_2(\Psi,w_2)+w_2U_{22}(\Psi,w_2)\geq 0 \tag{48}$$

If we divide equation (48) by $U_2(\Psi,w_2)$ and rearrange the terms we get the condition that the numerator is non-negative if

$$\alpha \leq 1 \tag{49}$$

where α is the Arrow-Pratt measure of relative risk aversion. So far we have established that

$$\frac{\partial g(\rho,\xi)}{\partial \rho} \leq 0 \quad \text{if} \quad \alpha \leq 1 \tag{50}$$

Since $\varphi(\rho,T|F_0,\xi)=(1+\delta)(F_0-mw_1)$, it follows that $\frac{\partial\varphi(\rho,T|F_0,\xi)}{\partial\rho} = -m(1+\delta)\frac{\partial g(\rho,\xi)}{\partial\rho}$. This along with the equation (50) establishes the proposition.

ii) Proceed as for (i).

iii) Differentiate the optimal policy function for the two period model outlined in (i), $g(\rho,\xi)$, with respect to ξ. This yields

$$\frac{\partial g(\rho,\xi)}{\partial \xi} = \frac{-\rho(1+\delta)\Big((m-1)\beta U_2(\Psi,w_2)+m^*U_{22}(\Psi,w_2)(w_1-\hat{w}_1)\Big)}{U_{22}(F_0,w_1)+\beta\rho^2(1+\delta)^2m^{*2}U_{22}(\Psi,w_2)} \tag{51}$$

The denominator is negative by virtue of diminishing marginal utility and so is the numerator so long as $w_1 \geq \hat{w}_1$. Since we are only considering the equilibrium outcomes where w_1 is equal to $\hat{w}_1$ this condition is met. Consequently, $\frac{\partial w_1}{\partial\xi} > 0$. Furthermore since an increase in the firewood harvest in the first period reduces the amount of terminal stock available in the second period it follows that $\frac{\partial\varphi(\rho,T|F_0,\xi)}{\partial\xi} < 0$.

iv) Take the cross derivative of the optimal policy function, $w_1 = g(\xi,\rho)$, with respect to ρ and ξ. The derivative turns out to be an ugly expression and is therefore omitted here. Here too

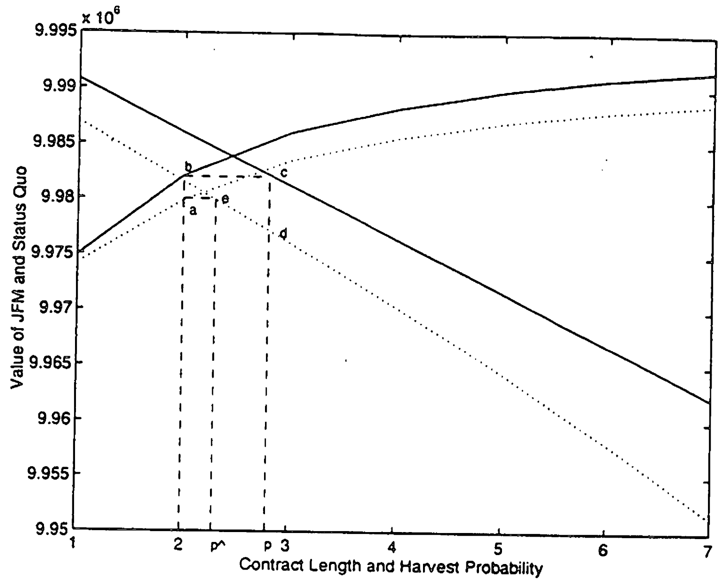


FIGURE 21. Proof for Proposition 8

the denominator is negative by virtue of diminishing marginal utility and the numerator is positive if $\frac{\partial^2 w_1}{\partial \rho^2} > 0$ holds and,

$$U_2(\Psi, w_2) < 2\rho^2 m^*(1+\delta) g_1(\rho, \xi) U_{22}(\Psi, w_2) \tag{52}$$

Rearranging the terms after dividing both sides by $U_2(\Psi, w_2)$ and multiplying and dividing the right hand side by w_2 gives,

$$\frac{1}{2} < -\frac{\rho^2 m^*(1+\delta)}{w_2} g_1(\rho, \xi)\alpha \tag{53}$$

This in turn simplifies to

$$\frac{1}{2} < \frac{\rho}{\varphi(\rho, T|F_0, \xi)} \frac{\partial \varphi(\rho, T|F_0, \xi)}{\partial \rho}\alpha \tag{54}$$

since $w_2 = \rho\varphi(\rho, T|F_0, \xi)$ and $\varphi_1(\rho, T|F_0, \xi) = -m^*(1+\delta)g_1(\rho, \xi)$. We then establish the proposition by appealing to the inverse relationship between the amount of firewood harvested in the first period and the terminal stock of firewood.

D.7. **Proof for Proposition 7.** Parts (i) and (ii) of this proposition follow from proposition 3 and 5 where we establish that $V_\pi(F)$ is monotonically decreasing in π and that for any $\rho < \bar{\rho}$, $J_T^\infty(F)$ is monotonically increasing in T. Part (iii) holds because for $T = \infty$ and $\pi = 0$ the households face the identical problem under JFM and status quo.

D.8. **Proof for Proposition 8.** Consider Figure 21. The solid lines denote the value of the forest stock when the villagers are cooperating while the dotted lines indicate the value when there is no cooperation. Also, the upward sloping curves capture the value of the stock under JFM and the downward sloping curves the value under status quo. Consider a JFM contract of length $T = 2$. A p probability of harvest equates the value of the forest stock under JFM and status quo when the villagers are cooperating while a $\hat{p}$ probability equates the value of the forest when the villagers are not cooperating. The cost of non-cooperation under JFM with a contract of length $T = 2$ is $(b - a)$ while the cost of non-cooperation under status quo with a p probability of harvest is $(c - d)$. Clearly whenever the latter dominates the former $\hat{p}$ will be less than p. With the length of the contract remaining constant, $\hat{p} < p$ implies that the locus for non-cooperation lies below the locus for cooperation.

D.9. **Proof for Proposition 9.** If we take the derivative of the equation (19) with respect to ρ and set it to zero we get the optimal policy function for ρ as a function of ξ and F_0, that is, $\rho = r(\xi, F_0)$. Specifically,

$$-\varphi(\rho, T|F_0, \xi) + (1-\rho)\varphi_1(\rho, T|F_0, \xi) - \frac{\beta^{T+2}}{1-\beta^{T+2}}\left(\varphi(\rho, T|0, \xi) + (1-\rho)\varphi_1(\rho, T|0, \xi)\right) = 0 \tag{55}$$

gives $\rho = r(\xi, F_0)$. Next take the total derivative of the optimal policy function with respect to ξ. This gives $\frac{\partial r(\xi, F_0)}{\partial \xi}$. The numerator of which is given by,

$$\begin{aligned} &-\varphi_4(\rho, T|F_0, \xi) + \frac{\beta^T}{1-\beta^{T+2}}\varphi_4(\rho, T|0, \xi) \\ &+(1-\rho)\left(\varphi_{14}(\rho, T|F_0, \xi) + \frac{\beta^T}{1-\beta^{T+2}}\varphi_{14}(\rho, T|0, \xi)\right) \end{aligned} \tag{56}$$

while the denominator is,

$$\begin{aligned} &-2\varphi_1(\rho, T|F_0, \xi) - 2\frac{\beta^T}{1-\beta^{T+2}}\varphi_1(\rho, T|0, \xi) \\ &+(1-\rho)\left(\varphi_{11}(\rho, T|F_0, \xi) + \frac{\beta^T}{1-\beta^{T+2}}\varphi_{11}(\rho, T|0, \xi)\right). \end{aligned} \tag{57}$$

The numerator is positive if $\rho < 1$, the terminal harvest is a decreasing function of the level of the externality and an increase in the harvest share decreases the rate at which non-cooperating villagers extract firewood from the forest. While the denominator is positive if $\rho < 1$, $\frac{\partial^2 w_1}{\partial \rho^2} > 0$ and if $\alpha < 1$. The proposition is thus established, that is, $\frac{\partial \rho}{\partial \xi} > 0$.

D.10. **Proof for Proposition 10.** We begin by establishing that for any ρ there exits a π such that households harvest the same amount of firewood under SQ as compared to JFM when the households bear the cost of harvesting firewood in the last period under both policy measures. When $\rho = 0$ and $\pi = 1$ the households face identical problems under SQ and JFM. Under both policies the households know that they will get nothing from the forest in the next period and hence we expect $h(F_0, 1) = g(F_0, F_0, 0)$. Similarly, we expect $h(F_0, 0) = g(F_0, F_0, 1)$. If we can show that functions g and h are both monotonic over the interval $(0, 1)$ then we would have succeeded in showing that for any ρ there exists a π such that households harvest the same amount of firewood under both policies, all else being equal. Proposition 4 establishes that h behaves monotonically over the interval $(0, 1)$ if $U_{12}(F, w) = 0$ and so we are only left with function g. To establish the conditions under which g is monotonic over the interval $(0, 1)$ we proceed as before by differentiating the first order condition (equation (23) with the modification that the households bear the cost of harvesting in the last period) with respect to ρ. This yields the derivative of function g with respect to ρ. The numerator of which simplifies to,

$$U_2(F_1, w_2) + \rho(1+\delta)(F_0 - mw_1)U_{22}(F_1, w_2) + (1+\delta)(F_0 - mw_1)U_{12}(F_0, w_1) \tag{58}$$

If $U_{12}(F_0, w_1) = 0$ then equation (58) further simplifies to,

$$U_2(F_1, w_2) + w_2 U_{22}(F_1, w_2) \tag{59}$$

Once we divide equation (59) by $U_2(F_1, w_2)$ we get the expression $1 - \alpha$, where α is the Arrow-Pratt measure of relative risk aversion. Consequently, if $\alpha < 1$ then the numerator is positive. The denominator turns out to be negative if,

$$\frac{-U_{22}(F_1, w_2)}{U_{12}(F_1, w_2)} \geq \rho \geq \frac{-U_{12}(F_1, w_2)}{U_{12}(F_1, w_2)} \tag{60}$$

If $U_{12}(F, w) = 0$ then this condition holds. To re-capitulate, if $\alpha < 1$ and $U_{12}(F, w) = 0$ then function g is monotonically decreasing over the interval $\rho \in (0, 1)$. All that remains for us to do to complete the proof is to show that $(1-\rho)(F_0 - mg(F_0, \Psi, \rho)) \leq \pi(F_0 - mh(F_0, \pi))$. The condition that the state's expected revenue be at least as large under SQ as under JFM can be re-written as,

$$(1-\rho)g(F_0, \Psi, \rho) - \pi h(F_0, \pi) \geq \frac{F_0(1-\rho-\pi)}{m} \tag{61}$$

For a sufficiently large m the right hand side of the equation goes to zero and the condition simplifies to $(1-\rho)g(F_0, \Psi, \rho) \geq \pi h(F_0, \pi)$. If we can show that $(1-\rho) \geq \pi$ then we are through with the proof.

We have already shown that for any ρ in the interval $(0,1)$ there exists a π such that households harvest the same amount of firewood under SQ as compared to JFM when the households bear the cost of harvesting in the last period. Let $\bar{w}$ be the amount of firewood harvested under both JFM and SQ. For $\pi \leq (1-\rho)$ it must be true that function h lies above function g[15] over the unit interval or for a fixed point on the unit interval, say ϵ,

$$(62) \quad U_2(F_0,\bar{w}) - \beta(1+\delta)m^\star\left(\frac{\epsilon}{m}U_2(F_1,\frac{\epsilon F_1}{m}) + U_1(F_1,\frac{\epsilon F_1}{m})\right) \leq U_2(F_0,\bar{w}) - \beta(1-\epsilon)(1+\delta)m^\star\left(\frac{U_2(F_1,\frac{F_1}{m})}{m} + U_1(F_1,\frac{F_1}{m})\right)$$

This simplifies to,

$$(63) \quad \frac{\epsilon}{m}U_2(F_1,\frac{\epsilon F_1}{m}) + U_1(F_1,\frac{\epsilon F_1}{m}) \geq \frac{(1-\epsilon)}{m}U_2(F_1,\frac{F_1}{m}) + (1-\epsilon)U_1(F_1,\frac{F_1}{m})$$

If $U_{12}(F,w) = 0$ then it must be true that $U_1(F_1,\frac{\epsilon F_1}{m}) = U_1(F_1,\frac{F_1}{m})$. With this simplification and for a sufficiently large m equation (63) will be true and $\pi \leq (1-\rho)$. Note that the right hand side of equation (61) goes to zero for a smaller group size the smaller the initial stock of forest and the closer ρ and π are to each other.

D.11. Lemma.

Lemma 1. *If* $\alpha < (>)1$, $\frac{\partial^3 U(F_0,w_1)}{\partial w_1^3} < (>)0$ *and* $\rho \geq (\leq)\frac{\beta^{-3}}{m^\star(1+\delta)}(\frac{U_{222}(F_0,w_1)}{U_{222}(\Psi,w_2)})^{-3}$ *then* $\frac{\partial^2 w_1}{\partial \rho^2} > (<)0$

Note that as m increases $\frac{\beta^{-3}}{m^\star(1+\delta)}(\frac{U_{222}(F_0,w_1)}{U_{222}(\Psi,w_2)})^{-3}$ approaches 0 and thus the restriction changes to $\rho \geq (\leq)0$.

Proof. Take the second derivative of the optimal policy function, $g(\rho,\xi)$, with respect to ρ. The derivative is a complicated expression and is therefore omitted here. It turns out that the denominator is positive by virtue of diminishing marginal utility and that the numerator is positive (negative) if the following equation is positive (negative), namely if,

$$(64) \quad U_{222}(F_0,w_1)g_1(\rho,\xi)^2 > (<)U_{222}(\Psi,w_2)\beta\rho m^\star(1+\delta)\left(\frac{w_2}{\rho} - \rho(1+\delta)m^\star g_1(\rho,\xi)\right)^2$$

(64) is positive (negative) if the third derivative of the utility function with respect to firewood consumption, $\frac{\partial^3 U(F_0,w_1)}{\partial w_1^3}$, is negative (positive), $\alpha < (>)1$ and $\rho > (<)\frac{\beta^{-3}}{m^\star(1+\delta)}(\frac{U_{222}(F_0,w_1)}{U_{222}(\Psi,w_2)})^{-3}$. □

[15] To make the comparison simpler we are defining function g in terms of $\frac{(1-\rho)}{m}$ rather than ρ because then both functions (h and g) will be increasing in the interval from 0 to 1 and consumption is explicitly in per capita terms.

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