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INVESTMENT UNDER UNCERTAINTY AND OPTION VALUE IN ENVIRONMENTAL ECONOMICS

by

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1. Introduction

Following the recent appearance of the volume on investment under uncertainty by Dixit and Pindyck (1994), and perhaps also the earlier articles by Pindyck (1991) and Dixit (1992) on implications of irreversibility for the investment decision, a question has arisen concerning the relationship of the Dixit-Pindyck approach, and specifically the identification of a positive option value under conditions of uncertainty and irreversibility, to the concept of option value, or quasi-option value, developed in the literature on environmental preservation (Arrow and Fisher, 1974, and Henry, 1974). What Dixit and Pindyck show is that, if an investment is irreversible, future returns are uncertain and the uncertainty will be resolved by waiting, say for one period, then the expected present value of the investment opportunity in the first period will be greater than the expected present value of the investment in the first period. The difference can be interpreted as the value of an option to postpone the investment decision. The formulation of the problem in the environmental literature is somewhat different, leading to an option value, or quasi-option value, that has been interpreted somewhat differently. As shown by Hanemann (1989), option value in this setting can be interpreted as a conditional value of information, a value of information about future returns (net of environmental damages) conditional on refraining, in the first period, from making an investment that would entail uncertain future environmental damages.

The concept of option value due to Arrow and Fisher, Henry, and Hanemann (AFHH), in the environmental literature, is what was originally called quasi-option value by Arrow and Fisher to distinguish it from another concept of option value first developed by Cicchetti and Freeman (1971) and later discussed in detail by many other authors, summarized in Hanemann (1989). This concept of option value is essentially static, related to risk aversion, and, it is now generally agreed, can be either positive or negative. The AFHH option value, on the other hand, is, like the Dixit-Pindyck measure, dynamic, not dependent on risk aversion, and nonnegative. In the years since the original formulation, it has sometimes been called quasi-option value, and sometimes just option value, especially as it has gained favor at the expense of the other concept of option value. In this paper I am referring only to the AFHH option value and, for simplicity, will call it option value, not quasi-option value.

In the next section the most basic formulation of the AFHH analysis is presented. In section 3 a similarly basic version of the Dixit-Pindyck model of the investment decision under uncertainty and irreversibility is presented. The heart of the paper is section 4, in which I show that the models of sections 2 and 3 are formally equivalent, and discuss what we can however learn from the differing formulations, and the somewhat different interpretations that have emerged. Part of the benefit of this exercise is a contribution to the intellectual history of environmental economics and its relation to the broader discipline. Perhaps more important is the "license" given to environmental and resource economists working in the AFHH tradition to adapt results from the much more extensive body of work set out in Dixit and Pindyck, including the theory of call options in finance, which they show is in turn equivalent to their theory of investment under uncertainty.

2. Option Value in Environmental Economics

Consider the problem of choosing whether to preserve or develop a tract of land in each of two periods, present and future. The development, we assume, is irreversible. Future benefits of development and preservation are uncertain, but we learn about them with the passage of time. In this simplest case, we assume that the uncertainty about future benefits is resolved at the start of the second period.

Let the benefit from first-period development, net of environmental costs (the benefits of preservation), be $B_1(d_1)$, where d_1, the level of development in period 1, can be zero or one. The present value of the benefit from second-period development is $B_2(d_1 + d_2, \theta)$, where d_2 can be zero or one and θ is a random variable. Note that, if $d_1 = 1, d_2 = 0$.

We want the first-period decision to be consistent with maximization of expected benefits over both periods. If benefits are measured in utility units, then this is equivalent to expected-utility maximization. But we can allow benefits to be measured in money units so that the results we shall obtain do not depend on risk aversion.

Let $\hat{V}(d_1)$ be the expected value over both periods as a function of the choice of first-period development ($d_1 = 0$ or $d_1 = 1$) given that d_2 is chosen to maximize benefits in the second period. Then, we have, for $d_1 = 0$,

$$\hat{V}(0) = B_1(0) + E\left[\max_{d_2}\left\{B_2(0,\theta), B_2(1,\theta)\right\}\right]. \tag{1}$$

Second-period development, d_2, is chosen at the start of the second period when we learn whether or not $d_2 = 0$ or $d_2 = 1$ yields greater benefits. At the start of the first period, when d_1 must be chosen, we have only an expectation, $E[\cdot]$, of the maximum.

If $d_1 = 1$, we have

$$\hat{V}(1) = B_1(1) + E\left[B_2(1,\theta)\right]. \tag{2}$$

With development in the first period, we are locked into development in the second ($d_1 = 1 \Rightarrow (d_1 + d_2) = 1$).

To get the decision rule for the first period, compare (1) and (2):

(3) $$\hat{V}(0) - \hat{V}(1) = B_1(0) - B_1(1) + E\left[\max_{d_2}\{B_2(0,\theta), B_2(1,\theta)\}\right] - E[B_2(1,\theta)]$$

and choose

(4) $$\hat{d}_1 = \begin{cases} 0 \text{ if } \hat{V}(0) - \hat{V}(1) \geq 0 \\ 1 \text{ if } \hat{V}(0) - \hat{V}(1) < 0 \end{cases}.$$

Now, let us suppose that, instead of waiting for the resolution of uncertainty about future benefits before choosing d_2, we simply replace the uncertain future benefits by their expected value. This may seem irrational, but that has not prevented generations of benefit/cost analysts, including the present author, from doing so. In this case, the expected value over both periods, for $d_1 = 0$, is

(5) $$V^*(0) = B_1(0) + \max_{d_2}\{E[B_2(0,\theta)], E[B_2(1,\theta)]\}.$$

Second-period development, d_2, is in effect chosen in the first period, to maximize expected benefits in the second period, because we do not assume that further information about second-period benefits will be forthcoming before the start of the second period.

For $d_1 = 1$,

(6) $$V^*(1) = B_1(1) + E[B_2(1,\theta)].$$

As before, development in the first period locks in development in the second.

Comparing (5) and (6),

(7) $$V^*(0) - V^*(1) = B_1(0) - B_1(1) + \max_{d_2}\{E[B_2(0,\theta)], E[B_2(1,\theta)]\} - E[B_2(1,\theta)]$$

and

(8) $$d_1^* = \begin{cases} 0 \text{ if } V^*(0) - V^*(1) \geq 0 \\ 1 \text{ if } V^*(0) - V^*(1) < 0 \end{cases}.$$

How do the decision rules in (4) and (8) compare? First, notice that

(9) $$\left[\hat{V}(0) - \hat{V}(1)\right] - \left[V^*(0) - V^*(1)\right] = \hat{V}(0) - V^*(0)$$

since $\hat{V}(1) = V^*(1)$. Then,

(10) $$\hat{V}(0) - V^*(0) = E\left[\max_{d_2}\{B_2(0,\theta), B_2(1,\theta)\}\right] - \max_{d_2}\{E[B_2(0,\theta)], E[B_2(1,\theta)]\}.$$

Finally,

(11) $$\hat{V}(0) - V^*(0) \geq 0$$

from the convexity of the maximum function and Jensen's Inequality, which states that the expected value of a convex function of a random variable is greater than or equal to the convex function of the expected value of the random variable.

It is this difference, $\hat{V}(0) - V^*(0)$, that has been interpreted as option value in the environmental literature. It is more properly considered a (conditional) value of information, however: The value of information about future benefits is *conditional* on retaining the option to preserve or develop in the future ($d_1 = 0$).

3. Investment Under Uncertainty

Now, let us consider the general problem, as set out in Dixit and Pindyck, of investment under uncertainty. A firm faces a decision of whether or not to make an investment, with a sunk cost of I, in a factory that will produce one widget per period forever. The current price of widgets is P_0 and, in the second period and thereafter, it will be either $(1 + u) P_0$, with probability q, or $(1 - d) P_0$, with probability $(1 - q)$. The expected present value of the return to the investment is then

$$V_0 = P_0 + \left[q(1+u) P_0 + (1-q)(1-d) P_0\right]/r, \tag{12}$$

where r is the discount rate. If $V_0 > I$, the investment will be made; otherwise, it will not. Letting Ω_0 denote the net payoff, we have

$$\Omega_0 = \max\{V_0 - I, 0\}. \tag{13}$$

This is the standard present-value criterion and, as we shall see, is in fact equivalent to the second decision rule [equation (8)] in the last section's model of the decision on environmental preservation.

Implicit in equation (13) is that the investment is considered only for the first period. Now, suppose that the opportunity will be available in the second period if it is not taken in the first. The present value of the return to the second-period investment is

$$V_1 = \begin{cases} (1+u) P_0 + \dfrac{(1+u) P_0}{r} & \text{if price} = (1+u) P_0 \\ (1-d) P_0 + \dfrac{(1-d) P_0}{r} & \text{if price} = (1-d) P_0 \end{cases} \tag{14}$$

The net payoff, the outcome of a future optimal decision, called the continuation value, is

$$F_1 = \max\{V_1 - I, 0\}. \tag{15}$$

What is the implication for the first-period decision? Notice that, although the second-period decision is made under certainty [by the start of the second period, the firm knows whether price is $(1 + u)\, P_0$ or $(1 - d)\, P_0$ and optimizes accordingly], from the perspective of the first period, V_1 and F_1 are uncertain. Then, the expected continuation value, from the perspective of the first period, is

$$
\text{(16)} \qquad \begin{aligned} E_0[F_1] = {} & q \max\left\{(1+u)\, P_0 + \frac{(1+u)\, P_0}{r} - I, 0\right\} \\ & + (1-q) \max\left\{(1-d)\, P_0 + \frac{(1-d)\, P_0}{r} - I, 0\right\}. \end{aligned}
$$

The net payoff to the investment opportunity presented in the first period, optimally taken (in the first period or the second), is

$$
\text{(17)} \qquad F_0 = \max\left\{V_0 - I, \frac{1}{1+r} E_0[F_1]\right\},
$$

where $V_0 - I$ is the expected present value of the investment made in the first period and $(1)/(1 + r)\, E_0[F_1]$ is the (discounted) expected continuation value—what the firm gets if it does not make the investment in the first period.

The difference, $F_0 - \Omega_0$, can be interpreted as option value: the value of the option to postpone the investment decision. As Dixit and Pindyck point out, the investment opportunity is analogous to a call option on a share of stock. It confers the right to exercise an option to invest at a given price (cost of the investment) to receive an asset (the widget factory) that will yield a stream of uncertain future returns. This is somewhat different from the interpretation of option value in the environmental literature as a conditional value of information. In the next section, I show the equivalence of the analytical formulations and reconcile the conflicting interpretations.

4. The Equivalence of Option Values

The demonstration of equivalence is immediate, as is the reconciliation of the different interpretations. We work with the expressions for Ω_0, equation (13), F_1, equation (15), and F_0, equation (17), substituting the development benefits from the AFHH model of section 2 for the investment returns specified in equations (13), (15), and (17).

Thus, we have

$$(13') \qquad \Omega_0 = \max\{B_1(1) + E[B_2(1, \theta)], B_1(0) + \max\{E[B_2(0, \theta)], E[B_2(1, \theta)]\}\}.$$

The expression, $B_1(1) + E[B_2(1, \theta)]$, corresponds to $V_0 - I$ in the Dixit-Pindyck framework: the net present value of the investment, or the project, undertaken in the first period. The rather lengthy expression, $B_1(0) + \max\{E[B_2(0, \theta)], E[B_2(1, \theta)]\}$, corresponds to zero. This requires a little explanation. In the Dixit-Pindyck framework, the alternative to making the investment is just to do nothing, with a net present value of zero. In the AFHH framework, not undertaking the project in the first period yields an alternative stream of returns with two components: the net benefit from not having the project during the first period, $B_1(0)$, which, given the environmental setting, may be positive, and the expected benefit, $E[B_2(\cdot)]$, from either having the project in the future or not having it, whichever is greater. In this formulation, a decision not to undertake the project in the first period does not preclude having it in place in the future—though the decision would have to be made solely on the basis of the expected future benefits.

Proceeding, we have

$$(15') \qquad F_1 = \max\{B_2(1, \theta), B_2(0, \theta)\},$$

where $B_2(1, \theta)$ corresponds to $V_1 - I$, and $B_2(0, \theta)$ corresponds to zero, and

(17') $$F_0 = \max\left\{B_1(1) + E[B_2(1,\theta)], B_1(0) + E\left[\max\{B_2(1,\theta), B_2(0,\theta)\}\right]\right\}.$$

The expression, $B_1(1) + E[B_2(1, \theta)]$, is, as before, $V_0 - I$ in the Dixit-Pindyck framework. The expression, $E[\max\{B_2(1, \theta), B_2(0, \theta)\}]$, corresponds to $E[F_1]$, from (15').

Now, we take the difference, $F_0 - \Omega_0$, the value of the option to postpone the investment in the Dixit-Pindyck framework, equation (17') – equation (13'), and obtain

(18) $$F_0 - \Omega_0 = E\left[\max\{B_2(1,\theta), B_2(0,\theta)\} - \max\{E[B_2(0,\theta)], E[B_2(1,\theta)]\}\right].$$

Notice that the other terms in the expressions for F_0 and Ω_0 are identical and therefore vanish when we take the difference.

The Dixit-Pindyck option value is thus of the form, $E[\max\{\cdot\}] - \max\{E[\cdot]\}$, exactly like the AFHH option value, or value of information conditional on retaining the option to preserve or develop, shown to be nonnegative. To reconcile the interpretations, we need only recognize that the option to postpone the investment has value only because the decision maker is assumed to learn about future returns by waiting. If this were not the case, nothing would be gained by postponing a decision to exercise the option. And the information (about future benefits of a development project, net of the environmental costs) is valuable only because the decision maker is assumed to have the flexibility to postpone the project. The interpretations are consistent, as one would expect them to be, given the formal equivalence. The comparison, and reconciliation of what might appear to be different concepts, can perhaps lead to a deeper understanding of option value.

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