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WORKING PAPER NO. 809

LEARNING, FORGETTING, AND THE DIFFUSION PROCESS OF FOOD AND AGRICULTURAL PRODUCTS

by

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Literature Survey

Identifying the Problems with Previous Models

Most of the previous models neglected the learning process needed to convert knowledge or even attitude toward purchasing behavior. According to the learning theory, a consumer needs some exposure with a product (multi-experience) in order to become a regular buyer. The sampling literature modeled and analyzed the utility of sampling as a one-shot event. The producer, the retailer, or both offered samples. The consumers who have tried the product during the product demonstration operation were counted by the researchers as adopters who became regular buyers in some implicit way. The empirical data we present in this paper provide us with evidence that this last assumption is not valid in all cases. In some cases consumers who tried the product and purchased it during the sampling operation did not repeat purchasing even though the time lag between the sampling period and the measurement period was significant.

The relationship between sampling and changes in purchasing behavior becomes even more complicated when the products are seasonal products, especially agricultural products. Seasonal products, because of its own nature, have a short presence in the marketplace, which makes the process of adoption more complicated, as visual stimuli are absent from the marketplace during this time. The absence of a product which was just introduced into the market requires adoption of new selling strategies by the seller.

The objectives of this paper are to: (1) empirically investigate the relationship between presence at the marketplace and purchasing behavior, and (2) develop a normative model of sampling which will take into account learning activity, absence from the marketplace, and sampling efforts of the seller.

The Model

General Overview

The product is consumable and belongs to the food category. It is seasonal and agricultural end users, such as fruits and vegetables, are excellent examples of this product category. The product is introduced in a market, characterized by the fact that large segments of the consumers are not familiar with its attributes and utility. The producer and the retailer should decide about the diffusion strategy where one of the decision variables is the sampling strategy. The decision variables of the sampling strategy are: Who will be offered, how much sampling should be offered, and what is the repeat rate of sampling? These questions are being answered using a model of sample and purchase relationship illustrated in Figure 1 in the next section. The product is presented to N potential consumers—α_1 percent of N tries the product if samples are offered, and α_2 percent of the population would try the product if it were introduced without sampling. It is natural to assume that $\alpha_1 > \alpha_2$; the empirical data support this assumption. β percent of the population that has tried the product purchase it (defined as adopters). In order to become a regular purchaser, defined as consumers who purchase the product regularly without the assistance of promotion efforts, consumers need to create a history of purchasing (see Bush and Hosteller, 1995; and Lilien, Kotler, and Moorthy, 1994, for a literature survey and model description).

During the process of building the experience stock, some consumers (γ percent) may decide that the product does not fit their needs and stop purchasing it. The others who continue to purchase the product and gain positive experience from it become, after purchases, regular buyers. The flow which describes the process of becoming regular buyers is described in Figure 1.

When the product is a seasonal product, the repurchasing activity is interrupted by the exogenic periodical life-cycle of the product. The discontinuous repurchasing behavior slows the diffusion of the products (the process of becoming a regular buyer) and implies

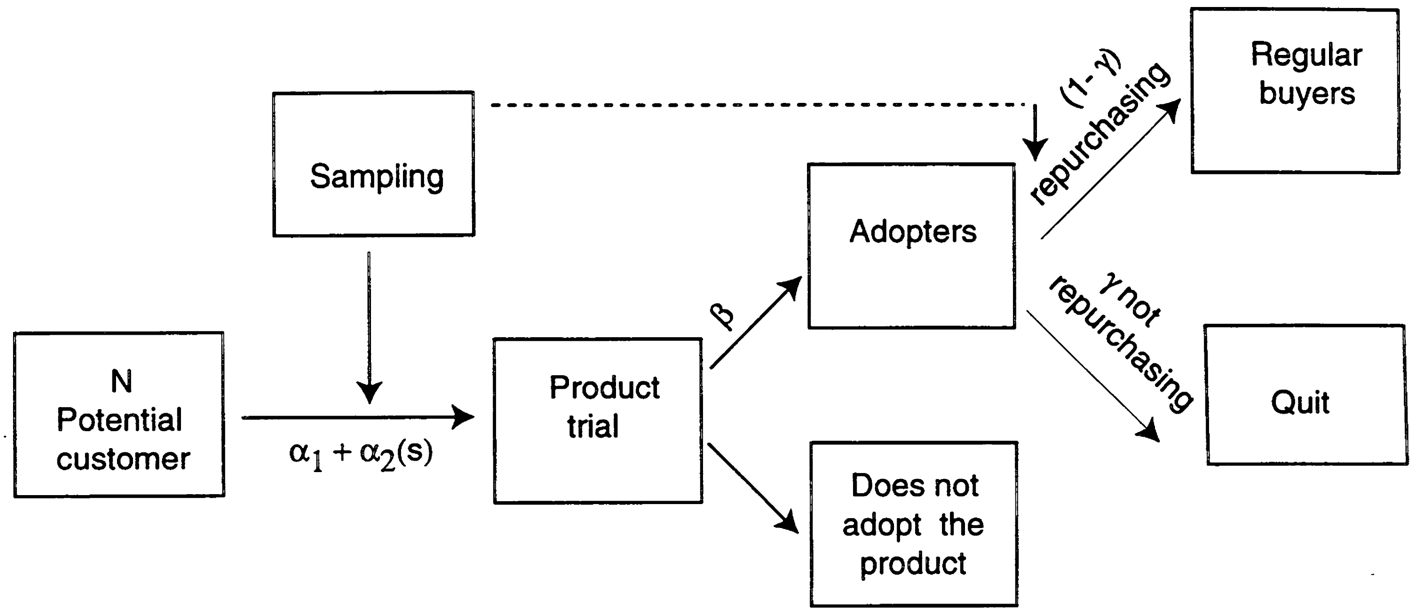

Figure 1

The dashed line describes the proposed influence of sampling on repurchasing that more purchasing opportunities are needed in order to become a regular buyer. The influence of seasonality on the number of buyers needed to convert adopters to regular buyers is modeled and presented in Figure 2.

In the next section we will model sampling activities taking into account the repeat purchasing and the seasonality effects on the diffusion of the new product.

Theoretical Model

Model assumptions are:

1. Any member of the target group will try the product.

2. From the population of previous adopters, K percent $1 \geq k > 0$, will become a regular buyer if they gain more experience with the product.

3. The first trial can be motivated by curiosity (self-motivation) or by the firm's actions, mainly sampling activity.

4. At any time, t, the probability of a first trial motivated by curiosity will be defined as a_0.

Model Building

Let $S(t)$ define the number of samples offered at period t. S will be normalized such that $S(t) \in [0,1]$.

N will define the market potential.

X will define the cumulative number of adopters (consumers who have tried the product).

S is the number of samples offered at time t.

a_1 is the efficient coefficient of demonstration in creating new trials.

a_2 is the efficient coefficient of demonstration in creating repeat purchase.

K will define the proportion of consumers who tried the product and are willing to repurchase it.

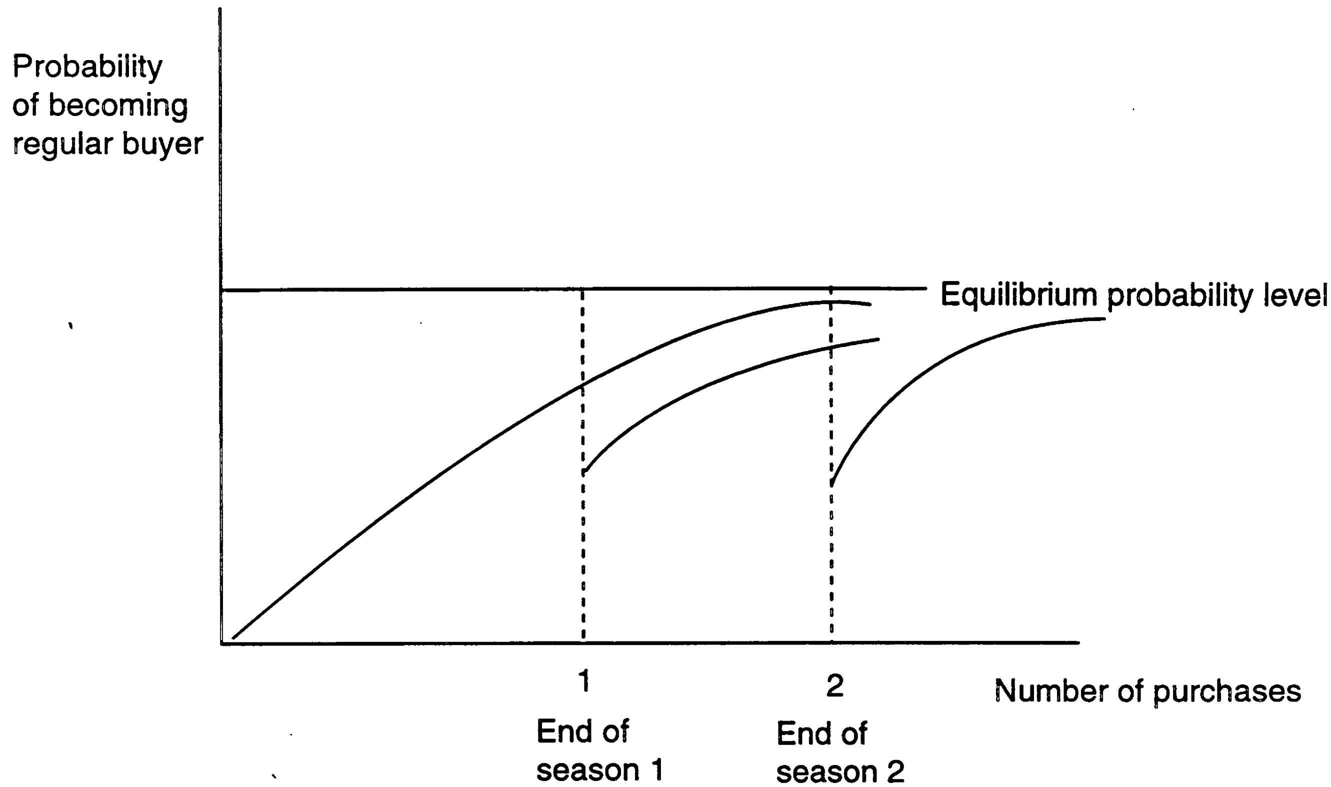
Probability
of becoming
regular buyer
Equilibrium probability level
1
2
Number of purchases
End of
season 1
End of
season 2

Figure 2

(1 - K) will define the proportion of consumers who tried the product and were not willing to repurchase it.

Next we assume that the probability of a first trial due to sampling in any period t decreases with the level of previous exposure to the product category and that the effectiveness of sampling in creating repeat purchases decreases as the time lag between trials increases. Sampling is an important opportunity for product trial before the consumers become regular buyers; therefore, we will assume that the longer the time lag between the sampling activities, the lower is the probability of repurchasing. We will define the time lag between sampling activity as time. Therefore, the probability of repurchasing will be written as

$$\left(\frac{S}{N}\right)^{a_2(1-\text{time})} . \tag{1}$$

The additional number of adopters at time t can be written as

$$\dot{X} = \left(a_0 + \left(\frac{S}{N}\right)^{a_1}\right)(N - X). \tag{2}$$

S is normalized to be in the range $[0, 1]$; $S \in [0,1]$

$$0 \le a_1 \le 0 \quad 0 \le a_2 \le 1, \quad 0 \le \text{time} \le 1.$$

We will assume that each adopter purchase, after the adoption decision has been made, one unit of the product; hence, the sales of the new adopter are given by

$$\text{sales of new adopters at } t = N\dot{S} = \left(a_0 + \left(\frac{S}{N}\right)^{a_1}\right)(N - X) = \dot{X}. \tag{3}$$

Repeat Purchase

We have argued that, after the adoption, k percent will become regular buyers given they had the opportunity to gain more experience with the product. We also have argued that the main stimuli for further trial will be sampling. Under these assumption, we can represent the repeat purchase at t, rS, by

$$r\dot{S} = k\left(\frac{S}{N}\right)^{a_2(1-\text{time})} \cdot X. \qquad (4)$$

<u>Sales at *t:*</u>

$\dot{Y} = N\dot{S} + r\dot{S}$, i.e.,

$$\dot{Y} = \left(a_0 + \left(\frac{S}{N}\right)^{a_1}\right)(N - X) + K\left(\frac{S}{N}\right)^{a_2(1-\text{time})} \cdot X. \qquad (5)$$

The firm solves the following maximization problem:

$$\pi = \int_0^T \left[\pi\left[\left(a_0 + \left(\frac{S}{N}\right)^{a_1}\right)[N - X] + K\left(\frac{S}{N}\right)^{a_2(1-\text{time})} \cdot X\right] - C \cdot S\right] e^{-rt} dt \qquad (6)$$

where $\pi = P - C \equiv$ profit from selling each unit of the product. For simplicity, we will assume that the prices, the cost variables of product, and the variable cost of sampling are constant. (Production decisions have been made before sampling decision, and sampling costs are estimated by the industry as fixed marginal cost.)

The firm solves the following maximization problem:

$$Max \int_0^T \left[\pi\left[\left(a_0 + \left(\frac{S}{N}\right)^{a_1}\right)(N - X) + K\left(\frac{S}{N}\right)^{a_2(1-\text{time})} \cdot X\right] - C \cdot S\right] e^{-rt} \qquad (7)$$

s.t.

$$\dot{y} = \left[a_0 + \left(\frac{S}{N}\right)^{a_1} \right](N - X) + KX\left(\frac{S}{N}\right)^{a_2(1-\text{time})}$$

We will use the Hamiltonian method to solve this maximization problem. The Hamiltonian H can be represented by

$$\begin{aligned} H = \pi\left[\left(a_0 + \left(\frac{S}{N}\right)^{a_1}\right)(N - X) + KX\left(\frac{S}{N}\right)^{a_2(1-\text{time})}\right] - CS \\ +\lambda\left[\left(a_0 + \left(\frac{S}{N}\right)^{a_1}\right)(N - K) + KX\left(\frac{S}{N}\right)^{a_2(1-\text{time})}\right] \end{aligned} \tag{8}$$

The first-order condition according to the sampling activity is:

$$(\pi + \lambda) = \frac{C \cdot N}{a_1\left(\frac{S}{N}\right)^{a_1 - 1}(N - X) + a_2 \cdot KX(1 - \text{time})\left(\frac{S}{N}\right)^{a_2(1-\text{time})-1}}. \tag{9}$$

The first-order condition according to the aggregate adopters X is

$$\frac{\partial H}{\partial X} = (\pi + \lambda)\left[-\left(a_0 + \left(\frac{S}{N}\right)^{a_1}\right) + K\left(\frac{S}{N}\right)^{a_2(1-\text{time})}\right]. \tag{10}$$

We know that

$$\dot{\lambda} = r\lambda - \frac{\partial H}{\partial X} = r\lambda + (\pi + \lambda)\left[\left(a_0 + \left(\frac{S}{N}\right)^{a_1}\right) - K\left(\frac{S}{N}\right)^{a_2(1-\text{time})}\right]. \tag{11}$$

By taking the derivative of (9) with respect to time, equalizing it to (11), and performing algebraic manipulation, we get the following (the detailed solution is presented in Appendix A):

$$r \cdot \left\{ \frac{CN}{\left[a_1 \left(\frac{S}{N} \right)^{a_1 - 1} (N - X) + KX a_2 (1 - \text{time}) \left(\frac{S}{N} \right)^{a_2 (1-\text{time}) - 1} \right]} - \pi \right\}$$

$$\frac{- CN \left\{ - \left(a_0 + \left(\frac{S}{N} \right)^{a_1} \right) + K \left(\frac{S}{N} \right)^{a_2 (1-\text{time})} \right\}}{\left[a_1 \left(\frac{S}{N} \right)^{a_1 - 1} (N - X) + KX \, a_2 (1 - \text{time}) \left(\frac{S}{N} \right)^{a_2 (1-\text{time}) - 1} \right]}$$

$$= \frac{- C \left[a_1 (a_1 - 1) \left(\frac{S}{N} \right)^{a_1 - 2} (N - X) + a_2 \, (1 - \text{time}) (a_2 \, (1 - \text{time}) - 1) \left(\frac{S}{N} \right)^{a_2 (1-\text{time}) - 2} \right] \dot{S}}{\left[a_1 \left(\frac{S}{N} \right)^{a_1 - 1} (N - X) + KX \, a_2 (1 - \text{time}) \left(\frac{S}{N} \right)^{a_2 (1-\text{time}) - 1} \right]^2}$$

$$\frac{- CN \left[-a_1 \left(\frac{S}{N} \right)^{a_1 - 1} + a_2 K (1 - \text{time}) \left(\frac{S}{N} \right)^{a_2 (1-\text{time}) - 1} \right] \left[\left(a_0 + \left(\frac{S}{N} \right)^{a_1} \right) [N - X] \right].}{\left[a_1 \left(\frac{S}{N} \right)^{a_1 - 1} (N - X) + KX \, a_2 (1 - \text{time}) \left(\frac{S}{N} \right)^{a_2 (1-\text{time}) - 1} \right]^2} \quad (12)$$

For $r = 0$, we get the following:

$$\dot{S} \left\{ a_1 (a_1 - 1) \left(\frac{S}{N} \right)^{a_1 - 2} (N - X) + KX a_2 \, (1 - \text{time}) \, (a_2 \, (1 - \text{time}) - 1) \left(\frac{S}{N} \right)^{a_2 (1-\text{time}) - 2} \right\}$$
$$= \left[- \left(a_0 + \left(\frac{S}{N} \right)^{a_1} \right) + K \left(\frac{S}{N} \right)^{a_2 (1-\text{time})} \right] \left[a_1 \left(\frac{S}{N} \right)^{a_1 - 1} (N - X) + KX a_2 (1 - \text{time}) \left(\frac{S}{N} \right)^{a_2 (1-\text{time}) - 1} \right] \quad (13)$$

$$-\left\{-a_1\left(\frac{S}{N}\right)^{a_1-1} + Ka_2\,(1-\text{time})\left(\frac{S}{N}\right)^{a_2(1-\text{time})-1}\right\}\dot{X}.$$

$\dot{S} > 0$ iff

a. $$\left(a_0 + \left(\frac{S}{N}\right)^{a_1}\right) > K\left(\frac{S}{N}\right)^{a_2(1-\text{time})}$$

and

b. $$a_1\left(\frac{S}{N}\right)^{a_1-1} < Ka_2(1-\text{time})\left(\frac{S}{N}\right)^{a_2(1-\text{time})-1},$$

that is, the marginal probability of repeat purchase due to increase in sampling is higher than the marginal probability of first purchase. However, the probability of first purchase is higher than the probability of repeat purchase (high self-motivated purchasers). The level of sampling increases over time as long as the marginal probability of the repeat purchase is higher than the marginal probability of the first purchase. For a mature product where

$$a_2\,K(1-\text{time})\left(\frac{S}{N}\right)^{a_2(1-\text{time})-1}$$

is small, then $\dot{S} < 0$.

If $r > 0$, then for large r, $\dot{S}$ monotonically decreases.

Comparative Statistics

For $r = 0$, if $\dot{S} > 0$, and

a. $$\frac{a_0 + \left(\frac{S}{N}\right)^{a_1}}{K\left(\frac{S}{N}\right)^{a_2(1-time)}} > 2\frac{X}{N}$$

b. $$\frac{-\left(2-\ln\left(\frac{S}{N}\right)\right)-\sqrt{4+\ln^2\left(\frac{S}{N}\right)}}{2\ln\left(\frac{S}{N}\right)} \le a_2(1-\text{time}) \le \frac{-\left(2-\ln\left(\frac{S}{N}\right)\right)+\sqrt{4+\ln^2\left(\frac{S}{N}\right)}}{2\ln\left(\frac{S}{N}\right)}.$$

Then

$$\frac{\partial \dot{S}}{\partial(1-\text{time})} < 0,$$

and, therefore,

$$\frac{\partial \dot{S}}{\partial \text{time}} > 0$$

(proof is available from the authors). The meaning of this finding is that, if the contribution of samples on repeat purchase is high relative to its contribution to the first sale, than the firm has to invest more in sampling when the seasonality of the product is high. We should remember that the marginal profit from repeat purchase as a function of sales should be always positive. At large seasonality (time) the marginal profit from additional unit of sampling is negative; therefore, the number of demonstrations will decrease. For $r > 0$, the effect of seasonality is even higher and $\dot{S}$ decreases. For very high seasonality effect where (1 - time) is close to 0, the firm faces new situations where, at the end of each season, $X = 0$ and $N - X = N$. Therefore, each season the firm solves the static model, and the number of samples is smaller than the number of samples at the dynamic model at the first year.

The additional sales in each period decreases as the time lag between the exposure to the product increases. Formally, $\frac{dg}{\partial \text{time}} < 0$. It is obvious that $\frac{\partial \dot{y}}{\partial(1-\text{time})} > 0$; hence, $\frac{\partial \dot{y}}{\partial \text{time}} < 0$.

Research Hypothesis and Empirical Findings

Research Hypothesis

In our theoretical model, we have predicted that (1) the additional annual sales decrease with the time lag between the exposure to the product and (2) if the seasonality effect is high, the firm actually faces a situation which is very close to a static (one-period) situation. That is, the effect of last year's samples will not influence the sales of this year if the seasonality effect is moderate or high. We should expect that there will be positive influence of last year's samples on this year's sales.

H_1: Product characterized by high seasonality.—There will be no effect of last year's samples on this year's samples (each year, the firm starts from the beginning).

H_2: Products characterized by low seasonality.—The sampling activity of last year will influence the sales of the following year.

H_3: Products characterized by low seasonality.—The contribution of sampling to sales increases each year at the first years of activity.

H_4: If samples, indeed, influence purchasing, then we shall not see a decrease in the effect of samples on sales. On the other hand, if samples do not influence repeat purchase, than we should expect decreasing efficiency of sampling over time.

Hypothesis Testing

In order to validate our research hypothesis, we have chosen two different fruits exported to Europe. The two fruits were each exported by one distributor who handled all the marketing activities. Both distributors allocated more than 30 percent of their marketing budget on sampling activities. They both agreed to share their results but under one condition: That absolute numbers and other details that could expose company secrets would remain confidential. Therefore, we will not specify the explicit product but, rather, describe some of its relevant attributes. In addition, we will present only results (ratios and coefficients).

The first product, Product A, is a subtropical fruit which is not widely used in Europe. The fruit is sweet and can be eaten as is or as a dessert. The important characteristic of this fruit is its short season (three months in a year). This fruit has been sampled during 1994 and 1995 seasons in one of the industrialized countries in Europe. The distributor who wanted to learn the effect of the samples have chosen (with the aid of a local marketing research company) 9 sites with similar demographic, socioeconomic, and post-purchasing behavior in 1994. In 1995, the sample activity was repeated in the same 9 sites; in addition, 11 new sites (again with similar characteristics) had been using samples.

Prices: The prices of the product are equal in terms of money adjusted to inflation. On average, we know that prices of other fruits have not changed. One should note that in fresh fruits there is a high supply sensitivity. That is, prices can change dramatically every week. Unfortunately, we do not take the relative prices or the prices of the competing product, and we should assume that they are constant.

Other marketing activity: During the sample activity, there were no other marketing activity of the rivals or of the distributor. The sampling activity has been published in the chain fliers.

Sampling Method

In each of the participating stores, samples were distributed to customers in the fruit and vegetable department. The demonstrators did not screen the customers who were interested in the samples according to past behavior. We had the sales data during the demonstration activity (which is also the sales period for this product). The number of people who received the samples in each group is 2,984 in the repeated sample group and 3,736 in the one-period sample.

Method

We should compare the ratio of direct investment in sampling to sales of the two groups: the group which received samples in the previous year, in the following year, and group B which received samples only in the second year.

If hypothesis No. 1 is supported, then the ratio of sampling to sales will not be significantly different in the two groups.

Results

The results are presented in Table 1. Columns 2 and 3 indicate whether the store participated in the sample activity one or twice. Column 4 indicates the return on samples which is calculated as the revenue over the cost of sampling.

The mean return on sales in the group which has received samples for two years is 6.22 (2.873 std). The average return on samples in the group which received samples only once is 7.14 (std. = 1.825). The Z value is 0.064; therefore, we cannot reject the null hypothesis that the two means are identical.

We can interpret this result as a support to our theoretical funding that, if a product has a short season, then the effect of past sampling on the following year's sales is very small. This result is very similar to one-period stochastic models where the memory of the consumer holds only for one period.

Rejecting Alternative Explanations

We have attempted to reject two alternative explanations to the following findings:

1. The customers in the store who received samples for two years are not the sample customers; that is, customers who have received samples last year did not receive it this year. Therefore, the customers are new customers and are not different than the customers of the second group.

TABLE 1

Comparison of Return of Samples Between Stores that Have Received Samples Twice and Stores Which Have Received Samples Once

Store	Demonstrations 93/94	Demonstrations 94/95	Return of samples 94/95
1	+	+	1.00
2	+	+	9.76
3	+	+	8.52
4	+	+	8.00
5	+	+	6.24
6	+	+	5.46
7	+	+	8.21
8	+	+	7.13
9	+	+	1.67 $\overline{X}$ = 6.22 (2.873)
10	-	+	8.29
11	-	+	3.37
12	-	+	6.56
13	-	+	4.58
14	-	+	5.52
15	-	+	6.16
16	-	+	3.63
17	-	+	8.10
18	-	+	6.51
19	-	+	9.35
20	-	+	7.14 $\overline{X}$ = 6.29 (1.825)

H_0: $\overline{X}_1 = \overline{X}_2$. $\overline{Z} = 0.064$ not significant. We cannot reject the null hypothesis.

2. Customers in the first group (the repeat samples) buy the product more frequently than the customers in the second group.

A survey of **500** customers was conducted in the same site where the customers had participated the previous year just before the sample activity of 1994-95; **54 percent** of the population reported remembering the previous year's free samples. This indicates that the customers in both groups are different, and we cannot treat the first group as unfamiliar customers as the customers in the second group.

Next the customers were asked about their past purchasing behavior; 98*j* and 96*i* of the respondents in the familiar and unfamiliar groups reported that the did not buy last year's product. Again, we can reject the alternative explanation of differences in regular purchasing behavior.

Next we have checked the multiperiod sampling of a fruit with a long season. We chose a fruit which is in the shelves almost all year. It is a sweet fruit which is widely purchased a Christmas and is considered to be a healthy (high energy) food. Its traditional customers are ethnic customers, and the distributor targets customers who are not familiar with this fruit to try free samples.

Sample Description

Industrialized countries in Western Europe with similar socioeconomic and past behavior of .25 retailers who participated in the free sample activity in the two-year period were chosen. Quantities, prices, and samples (unit? cost?) were collected.

The Model

We had two models:

$$\text{Quantity}_{95} = \text{constant} + \alpha_2 \text{ price}_{95} + \rho_2 \text{ sample}_{95}.$$

$$\text{Quantity}_{95} = \text{constant} + \alpha_2 \text{ price}_{94} + \rho_1 \text{ sample}_{94}.$$

if $\rho_2 > \rho_1$, then hypothesis H_1 is being supported.

Method

We estimated the model using an ordinary regression procedure. The results are presented in Table 2.

Results

$$\text{Quantity}_{95} = 125{,}423 - 14{,}877\ \text{price}_{95} + 1.197\ \text{sample}_{95}.$$

$$\text{Quantity}_{95} = 152{,}744 - 18{,}731\ \text{price}_{94} + 1.058\ \text{sample}_{94}.$$

The price coefficient is negative in both years, and the sampling coefficient is positive in both years which implies that the model is well behaved. The sample coefficient in 1995 (second year) is significantly ($Z = 3.64$; $\rho < 0.05$) higher than the coefficient in 1994.

The meaning of these results are interesting; the effect of samples at the second year is higher than it was in the first year implying that sample effects repeat purchases—not only first purchases (otherwise, we would find decreasing effect). The decline is customers' sensitivity to price might be interpreted as: The more customers become used to the product, the less their price sensitivity.

TABLE 2

Model: Quantity 95 = constant + price 95 + sampling 95

$N = 25$

$R^2 = 0.761$

adj $R = 0.743$

Variable	Coefficient	Stat. error	Stat. coeff.	T	p(2 tail)
Constant	125423.5	39500.9	0.00	3.175	0.004
Price 95	-14877.0	5195.5	-0.305	-2.863	0.009
Sample 95	1.197	0.143	0.894	8.396	0.000

F ratio	P
35.657	0.00

Model: Quantity 94 = constant + price 94 + sampling 94

$R^2 = 0.772$

adj $R^2 = 0.751$

Variable	Coefficient	Stat. error	Stat. coeff.	T	p(2 tail)
Constant	152743.64	42505.5	0.00	3.594	0.002
Price 94	-18731.041	6040.2	-0.32	-3.101	0.005
Sample 95	1.058	0.126	0.87	8.414	0.000

F ratio	P
37.147	0.00

Appendix A

Taking the derivatives of (9) with respect to time, we get

(12)

$$\dot{\lambda} = -\frac{CN \cdot \left[a_1(a_1 - 1)\left(\frac{1}{N}\right)\left(\frac{S}{N}\right)^{a_1 - 2}(N - X) + a_2(1 - \text{time})\,(a_2(1 - \text{time}) - 1)KX\left(\frac{S}{N}\right)^{a_2(1-\text{time})-2}\left(\frac{1}{N}\right)\right]}{\left[a^1\left(\frac{S}{N}\right)^{a_1 - 1}(N - X) + a_2 KX(1 - \text{time})\left(\frac{S}{N}\right)^{a_2(1-\text{time})-1}\right]^2}$$

$$-\frac{CN \cdot \left[-a_1\left(\frac{S}{N}\right)^{a_1 - 1} + a_2 k(1 - \text{time})\left(\frac{S}{N}\right)^{a_2(1-\text{time})^{-1}}\right]\dot{X}}{\left[a_1\left(\frac{S}{N}\right)^{a_1 - 1}(N - X) + a_2\ KX(1 - \text{time})\left(\frac{S}{N}\right)^{a_2(1-\text{time})^{-1}}\right]^2}.$$

Equating (11) with (12), we get:

$$r\lambda+(\pi+\lambda)\left[\left(a_0+\left(\frac{S}{N}\right)^{a_1}\right)-K\left(\frac{S}{N}\right)^{a_2(1-\text{time})}\right]=$$

$$-\frac{C\left[a_1(a_1-1)\left(\frac{S}{N}\right)^{a_1-2}(N-X)+a_2(1-\text{time})\left(a_2(1-\text{time})-1\right)KX\left(\frac{S}{N}\right)^{a_2(1-\text{time})^{-2}}\right]\dot{S}}{\left[a_1\left(\frac{S}{N}\right)^{a_1-1}(N-X)+a_2\;KX(1-\text{time})\left(\frac{S}{N}\right)^{a_2(1-\text{time})^{-1}}\right]^2}$$

$$-\frac{CN\left[-a_1\left(\frac{S}{N}\right)^{a_1-1}+a_2K(1-\text{time})\left(\frac{S}{N}\right)^{a_2(1-\text{time})^{-1}}\right]\dot{X}}{\left[a_1\left(\frac{S}{N}\right)^{a_1-1}(N-X)+a_2\;KX(1-\text{time})\left(\frac{S}{N}\right)^{a_2(1-\text{time})^{-1}}\right]^2}.$$

We will substitute λ with the λ derived from (9) and get:

$$r\cdot\left[\frac{CN}{a_1\left(\frac{S}{N}\right)^{a_1-1}(N-X)+a_2KX(1-\text{time})\left(\frac{S}{N}\right)^{a_2(1-\text{time})-1}}-(P-C)\right]$$

$$+\frac{CN}{a_1\left(\frac{S}{N}\right)^{a_1-1}(N-X)+a_2KX(1-\text{time})\left(\frac{S}{N}\right)^{a_2(1-\text{time})-1}}\left[\left(a_0+\left(\frac{S}{N}\right)^{a_1}\right)-K\left(\frac{S}{N}\right)^{a_2(1-time)}\right]$$