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WELFARE ANALYSIS WITH DISCRETE CHOICE MODELS

by

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1. INTRODUCTION

A major accomplishment in recent years has been the development of statistical models suitable for the analysis of discrete dependent variables. This has enabled economists to study behavioral relationships involving purely qualitative variables which are not amenable to conventional regression techniques. In Amemiya's [1981] terminology, the multiresponse qualitative response (MRQR) model involves a dependent variable taking N distinct values, $\tilde{y} = 1, 2, \ldots,$ or N, which is related to vectors of independent variables, W_j, and parameters, β_j, by some functions of the general form[1]

$$(1.1) \qquad \pi_j \equiv \Pr\{\tilde{y} = j\} = H_j(W_1\beta_1, \ldots, W_N\beta_N) \qquad j = 1, \ldots, N.$$

Specific examples are the polychotomous probit model (Daganzo [1979]),

$$(1.2)\ \pi_1 = \int_{-\infty}^{\infty} \int_{-\infty}^{W_1\beta_1 - W_2\beta_2 + \varepsilon_1} \ldots \int_{-\infty}^{W_1\beta_1 - W_N\beta_N + \varepsilon_1} n(\varepsilon_1, \ldots, \varepsilon_N; 0, \Sigma)\, d\varepsilon_1, \ldots, d\varepsilon_N,$$

where $n(\cdot)$ is a multivariate normal density with zero mean and covariance matrix Σ, and the generalized logit (GEV) model (McFadden (1978, 1981)],

$$(1.3)\ \ \pi_j = e^{W_j\beta_j} G_j(e^{W_1\beta_1}, \ldots, e^{W_N\beta_N})\, G(e^{W_1\beta_1}, \ldots, e^{W_N\beta_N})^{-1} \qquad j = 1, \ldots, N,$$

where G is a positive, linear homogeneous function, and G_j denotes its partial derivative with respect to the $\underline{j}$th argument.[2]

These statistical models have been used to analyze many types of economic behavior. Aitchinson and Bennett [1970] and McFadden [1974] have offered a theoretical derivation of these models which applies whenever the events whose probabilities are given by (1.1) represent the outcome of a decision by a maximizing agent. Suppose an agent is choosing among N courses of action and π_j = Pr{jth act chosen}. Assume that the payoff or utility associated with the jth act, $\tilde{u}_j$, is a random variable with mean $W_j\beta_j$. Equivalently, $\tilde{u}_j = W_j\beta_j + \tilde{\varepsilon}_j$, where $\tilde{\varepsilon}_j$ is a random variable with zero mean. The agent chooses that act which has the highest utility. This yields a MRQR model of the form (1.1):

$$(1.4) \qquad \pi_j = \Pr\{W_j\beta_j + \tilde{\varepsilon}_j \geq W_i\beta_i + \tilde{\varepsilon}_i, \text{ all } i\} \equiv H_j(W_1\beta_1, \ldots, W_N\beta_N) \qquad j = 1, \ldots, N.$$

Let $\tilde{\eta}_{(j)} = (\tilde{\eta}_{1j}, \ldots, \tilde{\eta}_{j-1,j}, \tilde{\eta}_{j+1,j}, \ldots, \tilde{\eta}_{Nj})$ where $\tilde{\eta}_{ij} \equiv \tilde{\varepsilon}_i - \tilde{\varepsilon}_j$. It follows from (1.4) that:

$$(1.5) \quad H_j(W_1\beta_1, \ldots, W_N\beta_N) = F_{(j)}(W_j\beta_j - W_1\beta_1, \ldots, W_j\beta_j - W_N\beta_N) \qquad j = 1, \ldots, N,$$

where $F_{(j)}$ is an (N - 1) dimensional joint cumulative distribution function associated with the random vector $\tilde{\eta}_{(j)}$. As Daly and Zachary (1970) have shown, the converse is also true. Any MRQR model (1.1) in which the probability functions $H_j(\circ)$ can be cast in the form of an (N - 1) dimensional joint cumulative distribution function as in (1.5) is derivable from a utility maximization choice model such as (1.4). For this reason, a MRQR model satisfying (1.5) is said to be a random utility maximization (RUM) model.

This link between statistical models for discrete dependent variables and the economic concept of utility maximization is potentially very valuable because it raises the possibility of applying the conventional apparatus of welfare theory to empirical models of purely qualitative choice. Suppose the statistical model satisfies (1.5) and some subset of the variables in W_j represents attributes of the jth discrete choice. Can one derive from the fitted model an estimate of the effect on the agent's welfare of a change in these attributes analogous to the compensating and equivalent variation measures of conventional utility theory?

This issue was first raised in connection with RUM models of transportation mode choice by Domenich and McFadden [1975], Williams [1977], and Daly and Zachary [1978] but, until recently, it has received relatively little attention in other branches of applied economics. An exception is the papers by McFadden [1981] and Small and Rosen [1982] which explore the relationship between RUM models and conventional deterministic models of consumer behavior. However, both of these papers impose special restrictions on the underlying random utility function which have the effect that the discrete choice probabilities are independent of the consumer's income. Not only does this limit the applicability of their analysis, but it also obscures some important distinctions between alternative approaches to welfare measurement in the random utility context that happen to vanish when there are no income effects. When income effects <u>are</u> present, there are at least three distinct ways to formulate measures of compensating variation for RUM models (and three ways to formulate measures of equivalent variation) that can differ significantly in numerical value. In this paper I explain these different approaches to welfare measurement and analyze the relationships among them. I also provide formulas for computing the welfare measures, together with some numerical examples. Furthermore, I show that the same approaches to welfare measurement carry over to RUM models involving mixed discrete/continuous choices of the type analyzed by Dubin and McFadden [1984] and Hanemann [1984a].

The paper is organized as follows. Sections 2 and 3 focus on the most common type of logit and probit models involving what I will call an additively random utility function and purely discrete, budget-constrained choices. In section 2 I analyze the relationship between this type of RUM model and the more conventional, deterministic model of consumer choice. In section 3 I explain the alternative approaches to measuring welfare changes in the random utility setting and investigate the relationships among them. In section 4 this analysis is extended to other forms of RUM models including those with a more general stochastic structure and those involving mixed discrete/continuous choices. Section 5 deals with the practical problems of calculating the welfare measures and analyzes their properties in the case of some simple price/quality changes. The conclusions are summarized in section 6.

2. BUDGET-CONSTRAINED DISCRETE CHOICE

2.1. Deterministic Utility Models. The general setup of a purely discrete choice model is as follows. An individual consumer has a quasi-concave, increasing utility function defined over the commodities $x_1, \ldots x_N$, and z, where z is taken as the numeraire. In addition, the individual's utility may depend on some other variables, $q_1, \ldots, q_N$, which he takes as exogenous; these are, for example, quality attributes of the nonnumeraire goods.[3] He chooses (x, z) so as to maximize

(2.1) $$u = u(x_1, \ldots, x_N, q_1, \ldots, q_N, z)$$

subject to a budget constraint,

(2.2) $$\Sigma p_j x_j + z = y,$$

and two other constraints which introduce an element of discreteness into his choice. First, for logical or institutional reasons, the x_j's are mutually exclusive in consumption,

(2.3) $$x_i x_j = 0 \quad \text{all } i \neq j.$$

Secondly, the x_j's can only be purchased in fixed quantities,

(2.4) $$x_j = \bar{x}_j \text{ or } 0 \quad j = 1, \ldots, N.$$

An example might be where the x_j's are different brands of an indivisible durable good, and the consumer needs only one of these brands. Since the quantities of the x_j's are limited by (2.4), the choice among them is a qualitative choice. Moreover, although the numeraire is inherently a divisible good, once one of the x_j's has been selected the quantity of z is fixed by the budget constraint (2.2).[4] Thus, the model (2.1)-(2.4) represents a purely discrete utility-maximizing choice.

To obtain the demand functions implied by this model, first suppose that the individual has selected good j. His utility conditional on this decision, denoted by u_j, is

(2.5) $$\begin{aligned} u_j &= u(0, \ldots, 0, \bar{x}_j, 0, \ldots, 0, q_1, \ldots, q_N, y - p_j\bar{x}_j) \\ &\equiv v_j(q_1, \ldots, q_N, y - p_j\bar{x}_j) \end{aligned}$$

where v_j is increasing in $(y - p_j\bar{x}_j)$. I will refer to the $v_j(\circ)$'s as conditional indirect utility functions. At this point it is common to make an additional assumption about the utility function (2.1) that the consumer does not care about the attributes of a good unless he actually consumes that good, i.e.,

$$(2.6) \qquad x_j = 0 \Rightarrow \frac{\partial u}{\partial q_j} = 0 \qquad j = 1, \ldots, N.$$

This assumption was introduced by Maler [1974], who named it "weak complementarity." Given (2.6), the conditional indirect utility functions (2.5) take the special form[5]

$$(2.7) \qquad u_j = v_j(q_j, y - p_j\bar{x}_j) \qquad j = 1, \ldots, N.$$

The solution to the consumer's problem can be represented by a set of binary-valued indices, $\delta_1, \ldots, \delta_N$, where $\delta_j \equiv 1$ if $x_j > 0$ and $\delta_j \equiv 0$ if $x_j = 0$. These indices are related to the conditional indirect utility functions by

$$(2.8) \qquad \delta_j(p, q, y) = \begin{cases} 1 & \text{if } v_j(q_j, y - p_j\bar{x}_j) \geq v_i(q_i, y - p_i\bar{x}_i) \text{ all } i \\ 0 & \text{otherwise.} \end{cases}$$

Accordingly, the unconditional ordinary demand functions associated with the utility model (2.1)-(2.4) can be expressed as

$$(2.9) \qquad x_j(p, q, y) = \delta_j(p, q, y)\,\bar{x}_j \qquad j = 1, \ldots, N.$$

Substitution of these demand functions into the direct utility function (2.1) yields the unconditional indirect utility function,

$$(2.10) \qquad v(p, q, y) = \max\,[v_1(q_1, y - p_1\bar{x}_1), \ldots, v_N(q_N, y - p_N\bar{x}_N)].$$

This purely discrete choice model may be compared with the conventional utility maximization model where (2.1) is maximized subject only to the budget constraint (2.2) and a nonnegativity constraint on x and z that is assumed not to be binding. The point to be emphasized is that all the constructs of conventional, continuous choice models--the ordinary demand functions, the indirect utility function, and consumer's surplus--carry over to the discrete choice model. Duality relationships also carry over, including Roy's Identity (see Small and Rosen [1981]) and the duality between expenditure minimization and utility maximization (see below). The discrete choice model serves to provide a theoretical underpinning for the statistical MRQR model. However, in order to generate the statistical model, it is necessary to add a stochastic element and introduce the notion of random utility.

2.2. Random Utility Models. A random utility model arises when one assumes that, although an individual's utility function is deterministic for him, it contains some components which are unobservable to the econometric investigator and are treated by the investigator as random variables. This combines two notions which have a long history in economics--the idea of a variation in tastes among individuals in a population and the idea of unobserved variables in econometric models. These components of the utility function will be denoted by the random vector $\tilde{\varepsilon}$, and the utility function will be written $\tilde{u} = u(x, q, z, \tilde{e})$. More specifically, throughout the remainder of this section I assume that the random elements enter additively as follows:[6]

$$(2.1') \qquad u(x, q, z, \tilde{\varepsilon}) = u(x, q, z) + \Sigma\zeta(x_j)\, \tilde{\varepsilon}_j,$$

where $\zeta(x_j) = 1$ if $x_j > 0$ and $\zeta(x_j) = 0$ otherwise. For the individual consumer $\tilde{\varepsilon}_j, \ldots, \tilde{\varepsilon}_N$ is a set of fixed constants (or functions); but for the investigator, it is a set of random variables with some joint cumulative distribution function, $F_\varepsilon(\varepsilon_1, \ldots, \varepsilon_N)$, which induces a distribution on $\tilde{u}$.[7]

In the budget-constrained random utility discrete choice model, the individual is assumed to maximize (2.1') subject to the constraints (2.2)-(2.4). In addition, I will assume that the nonstochastic component of (2.1') satisfies (2.6). This maximization yields a set of ordinary demand functions and an indirect utility function which parallel those developed above except that they now involve a random component from the point of view of the econometric investigator. Suppose that the individual has selected good j. Conditional on this decision, his utility is $\tilde{u}_j$ where, from (2.1'), (2.2), (2.3), (2.4), and (2.6),

$$(2.7') \qquad \tilde{u}_j = v_j(q_j, y - p_j\bar{x}_j) + \tilde{\varepsilon}_j \qquad j = 1, \ldots, N,$$

the nonstochastic component being identical to (2.7). The discrete choice indices,

$$(2.8') \quad \tilde{\delta}_j = \delta_j(p, q, y, \tilde{\varepsilon}) = \begin{cases} 1 \text{ if } v_j(q_j, y - p_j\bar{x}_j) + \tilde{\varepsilon}_j \geq v_i(q_i, y - p_i\bar{x}_i) + \tilde{\varepsilon}_i \text{ all } i \\ 0 \text{ otherwise,} \end{cases}$$

are now random variables. Their mean, $\{\delta j\} \equiv \pi j$, is given by

$$\begin{aligned}(2.11)\quad \pi_j &= \Pr\{v_j(q_j, y - p_j\bar{x}_j) + \tilde{\varepsilon}_j > v_i(q_i, y - p_i\bar{x}_i) + \tilde{\varepsilon}_i \text{ all } i\} \\ &= F_{(j)}[v_j(q_j, y - p_j\bar{x}_j) - v_1(q_1, y - p_1\bar{x}_1), \ldots, v_j(q_j, y - p_j\bar{x}_j) \\ &\quad - v_N(q_N, y - p_N\bar{x}_N)],\end{aligned}$$

where $F_{(j)}$ is the joint cumulative distribution function of the (N - 1) differences $\tilde{\eta}_{ij} = \tilde{\varepsilon}_i - \tilde{\varepsilon}_j$. When v (•) can be cast in the form $v_j = W_j\beta_j$, (2.11) constitutes a RUM as defined in (1.5). I refer to it as a budget-constrained discrete choice RUM because of the restrictions on the regressors W_j and coefficients β_j implied by (2.7'), namely, that the variables y and p_j enter in the form $(y - p_j\bar{x}_j)$ and that v_j is increasing in this term.[8]

The requirement that the arguments of F(j) in (2.11) take the form of utility differences may be regarded as the analog of the integrability conditions in conventional demand theory. It provides a criterion for determining whether a given statistical MRQR model is compatible with the economic hypothesis of utility maximization. In addition, it offers a practical procedure for specifying a statistical model in empirical applications: First postulate some parametric function for $v_j(q_j, y - p_jx_j)$, j - 1, ..., N, and then form the differences $v_j - v_1, \ldots, v_j - v_N$ and substitute them into F_j. Another analog with conventional demand theory is worth mentioning. Suppose that the utility function (2.1') is replaced by some monotonic transformation, $\hat{u}(x, q, z, \tilde{\varepsilon}) = T[u(x, q, z) + \Sigma\zeta_j\tilde{\varepsilon}_j]$, $T' > 0$. The discrete choices indices (2.8') and, hence, the discrete choice probabilities (2.11) are invariant with respect to this transformation since

$$(2.12)\quad v_j(q_j, y - p_j\bar{x}_x) + \tilde{\varepsilon}_j \geq v_i(q_i, y - p_i\bar{x}_i) + \tilde{\varepsilon}_i \Leftrightarrow T[v_j(q_j, y - p_j\bar{x}_j) + \tilde{\varepsilon}_j]$$
$$\geq T[v_i(q_i, y - p_i\bar{x}_i) + \tilde{\varepsilon}_i].$$

Thus, when one estimates the MRQR model (2.11), he recovers the underlying utility function (2.1') only up to an arbitrary monotonic, increasing transformation.[9]

The unconditional ordinary demand functions associated with the budget-constrained discrete choice RUM model are

$$(2.9')\qquad \tilde{x}_j = x_j(p, q, y, \tilde{\varepsilon}) = \delta_j(p, q, y, \tilde{\varepsilon})\, \bar{x}_j \qquad j = 1, \ldots, N,$$

and the expected quantity demanded is $\{\tilde{x}_j\} = \pi_j\bar{x}_j$. Substituting the demand functions (2.9') into the discrete utility function (2.1') yields the unconditional indirect utility function

$$(2.10')\qquad \tilde{u} = v(p, q, y, \tilde{\varepsilon}) \equiv \max\,[v_1(q_1, \bar{y} - p_1x_1) + \tilde{\varepsilon}_1, \ldots,$$
$$v_N(q_N, y - p_N\bar{x}_N) + \tilde{\varepsilon}_N].$$

Recall that $v(\cdot)$ gives the utility attained by the individual maximizing consumer when confronted with the choice set (p, q, y). This is a known number for the consumer; but for the econometric investigator, it is a random variable with a cumulative distribution function $F_v(\omega) \equiv \Pr\{v(p, q, y, \varepsilon) \leq \omega\}$ derived from the assumed distribution $F_\varepsilon(\circ)$ by a change of variables

$$(2.13)\qquad F_v(\omega) = F_\varepsilon(\omega - v_1, \ldots, \omega - v_N).$$

In section 3 I show how the unconditional utility function is used to measure the welfare effects of a change in p or q. But first I identify a special family of utility models in which this welfare analysis is considerably simplified.

2.3. <u>The Case of No Income Effects</u>. Dual to the above utility maximization is an expenditure minimization problem: minimize $\Sigma p_i x_i + z$ subject to (2.1'), (2.3), and (2.4). This generates a set of compensated demand functions and an expenditure function which, like the ordinary demand functions and the indirect utility function, involve a random component from the econometrician's viewpoint. Suppose that the individual has selected good j. Assuming that his utility function satisfies the weak complementarity condition (2.6), his expenditure conditional on this decision is $\tilde{e}_j = g_j(q_j, u - \tilde{\varepsilon}_j) + p_j\bar{x}_j$, where $g(\cdot)$ is the inverse of $v_j(\cdot)$ in (2.7'), i.e., $g_j[q_j, v_j(q_j, t)] \equiv t$. The unconditional compensated demand functions can be written as $x_j(p, q, u, \varepsilon) = \delta_j(p, q, u; \varepsilon)\, \bar{x}_j$, where

$$(2.14) \quad \delta_j(p, q, u, \tilde{\varepsilon}) = \begin{cases} 1 \text{ if } g_j(q_j, u - \tilde{\varepsilon}_j) + p_j\bar{x}_j \leq g_i(q_i, u - \tilde{\varepsilon}_i) + p_i\bar{x}_i \text{ all } i \\ 0 \text{ otherwise,} \end{cases}$$

and the unconditional expenditure function is

$$(2.15) \qquad \tilde{e} = e(p, q, u, \tilde{\varepsilon}) = \min\, [g_1(q_1, u - \tilde{\varepsilon}_1) + p_1\bar{x}_1, \ldots, g_N(q_N, u - \tilde{\varepsilon}_N) + p_N\bar{x}_N].$$

An important class of utility models, to which Small and Rosen [1982] and McFadden [1981] have drawn attention, is that for which the unconditional ordinary and compensated demand functions coincide. In the Appendix the following result characterizing this class of utility models is proved:

PROPOSITION. <u>The unconditional ordinary and conditional demand functions coincide iff the direct utility function (2.1') is some monotonic transformation of</u>

$$\tilde{u} = h(x,\ q) + \gamma z + \Sigma\zeta(x_j)\ \tilde{\varepsilon}_j \tag{2.16a}$$

<u>for some function h(•) and positive constant γ. Assuming that h(•) satisfies (2.6), the corresponding form of the conditional indirect utility function is</u>

$$\tilde{u}_j = h_j(q_j) + \gamma y - \gamma p_j \bar{x}_j + \tilde{\varepsilon}_j \qquad j = 1,\ \ldots,\ N, \tag{2.16b}$$

<u>where $h_j(q_j) \equiv h(0,\ \ldots,\ 0,\ \bar{x}_j,\ 0,\ \ldots,\ 0,\ q)$</u>.

In order to motivate the proof of this proposition, it is useful to introduce an alternative method of representing the unconditional ordinary and compensated demand functions. Consider the demand for the first good. Given $(p_2,\ \ldots,\ p_N,\ q,\ y)$, one can write the ordinary demand function as a step function

$$x_1(p,\ q,\ y,\ \tilde{\varepsilon}) = \begin{cases} 0 & \text{if } p_1 \geq \tilde{p}_1^* \\ \bar{x}_1 & \text{otherwise,} \end{cases} \tag{2.17}$$

where the switch price, $\tilde{p}_1^*$, is a function of $(p_2, \ldots, p_N, q, y, \tilde{\varepsilon})$. Suppose that the actual price of the good is p_1^0; accordingly, the utility attained by the consumer is $\tilde{u}^0 = v(p_1^0, p_2, \ldots, p_N, q, y, \tilde{\varepsilon})$. The compensated demand function evaluated at $\tilde{u}^0$ is also a step function

$$(2.18) \qquad x_1(p, q, \tilde{u}^0, \tilde{\varepsilon}) = \begin{cases} 0 & \text{if } p_1 \geq \tilde{p}_1^{**} \\ \bar{x}_1 & \text{otherwise,} \end{cases}$$

where the switch price, $\tilde{p}_1^{**}$, is a function of $(p_2, \ldots, p_N, q, \tilde{u}^0, \tilde{\varepsilon})$. By construction, $x_1(p_1^0, p_2, \ldots, p_N, q, y, \tilde{\varepsilon}) \equiv x_1(p_1^0, p_2, \ldots, p_N, q, \tilde{u}^0, \tilde{\varepsilon})$. However, the entire graphs of the two demand functions coincide, $x_1(p_1, p_2, \ldots p_N, q, y, \tilde{\varepsilon}) \equiv x_1(p_1, p_2, \ldots, p_N, q, \tilde{u}^0, \tilde{\varepsilon})$, for <u>all</u> p_1 if and only if $\tilde{p}_1^* = \tilde{p}_1^{**}$. In the appendix I show that this occurs nontrivially only when the direct utility function takes the form in (2.16a). The assertion about the conditional indirect utility functions (2.16b) follows directly from (2.16a) by application of (2.5).

There is an important corollary to this proposition which enables one to test whether an empirical MRQR model satisfies (2.16). Observe from (2.15a) that the income variable drops out of the utility differences

$$(2.19) \qquad \tilde{u}_j - \tilde{u}_i = h_j(q_j) - h_i(q_i) - \gamma(p_j\bar{x}_j - p_i\bar{x}_i) + \tilde{\varepsilon}_j - \tilde{\varepsilon}_i .$$

Since it is these utility differences that enter into the formula for the discrete choice probabilities (2.11), it follows that the choice probabilities are <u>independent</u> of the consumer's income when the utility function satisfies (2.16)--there are no income effects.[10]

The utility function in (2.16a) satisfies the quasilinearity property that one finds when there are no income effects in conventional, continuous choice models--for example, see Katzner [1970, p. 93]. As will be shown in the next section, it has the same implications for welfare analysis in discrete choice RUM models as in conventional, continuous choice models, namely, that the compensating and equivalent variations concide and can be measured by areas under ordinary demand functions.

3. COMPENSATION MEASURES

In this section I show how one can perform welfare evaluations with statistical MRQR models that satisfy the integrability condition (2.11) and, hence, are derivable from the utility maximization model (2.1')-(2.4). Suppose that the set of prices and qualities available to the individual changes from (p^0, q^0) to (p^1, q^1). Thus his utility changes from $\tilde{u}^0 \equiv v(p^0, q^0, y, \tilde{\varepsilon})$ to $\tilde{u}^1 \equiv v(p^1, q^1, y, \tilde{\varepsilon})$. By analogy with welfare analysis in conventional, continuous choice models, this utility change could be measured in money units by the quantity $\tilde{C}$ which satisfies

$$(3.1) \qquad v(p^1, q^1, y - \tilde{C}, \tilde{\varepsilon}) = v(p^0, q^0, y, \tilde{\varepsilon})$$

or the quantity $\tilde{E}$ which satisfies

$$(3.2) \qquad v(p^1, q^1, y, \tilde{\varepsilon}) = v(p^0, q^0, y + \tilde{E}, \tilde{\varepsilon}).$$

The problem in the RUM context is that $\tilde{C}$ and $\tilde{E}$ are random variables since they depend on $\tilde{\varepsilon}$. Although the compensation required to offset the price/quality change is a fixed number for the individual consumer, for the econometric investigator it is a random variable since the individual's utility function is known only up to a random component. How then to obtain a single number representing the compensating or equivalent variation for the price/quality change?

In fact, the existing literature contains hints of up to three different approaches to welfare evaluation in the random utility context, but the conceptual distinction between these approaches does not appear to have been recognized. One approach is to derive the probability distribution of the quantity $\tilde{C}$ and calculate its mean, $C^{+} \equiv \{C\}$. As shown below, this calculation is sometimes difficult because of the complexity of the distribution of $\tilde{C}$. A second approach is to employ the expectation of the individual's indirect utility function, $V(p, q, y) \equiv \{v(p, q, y, \tilde{\varepsilon})\}$ and define the compensating variation in terms of this function (Table I).[11] The resulting welfare measure $C^{\bullet}$ satisfies

$$(3.3) \qquad V(p^1, q^1, y - C^{\bullet}) = V(p^o, q^o, y).$$

The distinction between C^{+} and $C^{\bullet}$ is subtle but important. C^{+} is the observer's expectation of the maximum amount of money that the individual could pay after the change and still be as well off as he was before it. By contrast, $C^{\bullet}$ is the maximum amount of money that the individual could pay after the change and still be as well off, in terms of the observer's expectation of his utility, as he was before it. The third welfare measure is derived as follows. One might want to know the amount of money such that the individual is just at the point of indifference between paying the money and securing the change or paying nothing and foregoing the change. For the observer, this could be taken as the quantity C^* such that

TABLE 1

Formulas For $V \equiv \{\max [v_1 + \tilde{\varepsilon}_1, \ldots, v_N + \tilde{\varepsilon}_N]\}$

1. Generalized extreme value

$$F_\varepsilon(\varepsilon_1, \ldots, \varepsilon_N) = \exp \quad -G \quad e^{-\varepsilon_1}, \ldots, e^{-\varepsilon_N}$$

$$V = \ln G \quad e^{v_1}, \ldots, e^{v_N} \quad + 0.57722$$

2. Independent logit

$$F_\varepsilon(\varepsilon_1, \ldots, \varepsilon_N) = \exp \quad -\Sigma e^{-\varepsilon_j}$$

$$V = \ln \Sigma e^{v_j} + 0.57722$$

3. Probit[a]

$$F_\varepsilon(\varepsilon_1, \ldots, \varepsilon_N) = N(0, \Sigma), \; \Sigma = \{\sigma_{ij}^2\}$$

a. Binary probit, N = 2

$$V_2 = (v_1 - v_2)\,\Phi\left(\frac{v_1 - v_2}{\kappa_2}\right) + v_2 + \kappa_2\,\phi\left(\frac{v_1 - v_2}{\kappa_2}\right),$$

$$\kappa_2 \equiv \left(\sigma_{11}^2 + \sigma_{22}^2 - 2\sigma_{12}^2\right)^{1/2}$$

(Continued on next page.)

Table 1--continued.

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b. Trichotomous probit, N = 3

$$V_3 \simeq (V_2 - v_3)\,\Phi \quad \frac{V_2 - v_3}{\kappa_3} \quad + v_3 + \kappa_3\,\phi \quad \frac{V_2 - v_3}{\kappa_3}$$

$$\kappa_3 \equiv \quad \sigma_{33}^2 + S_2^2 - 2S_{2,3}^2 \quad ^{1/2}$$

$$S_{2,3}^2 \equiv \sigma_{23}^2 + \quad \sigma_{13}^2 - \sigma_{23}^2 \quad \Phi \quad \frac{v_1 - v_2}{\kappa_2}$$

$$S_2^2 \equiv v_2^2 + \sigma_{22}^2 + \quad v_1^2 + \sigma_{11}^2 - v_2 - \sigma_{22}^2 \quad \Phi \quad \frac{v_1 - v_2}{\kappa_2}$$

$$+ (v_1 + v_2)\,\kappa_2\,\phi \quad \frac{v_1 - v_2}{\kappa_2} \quad - v_2^2$$

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[a]ϕ and Φ are, respectively, the standard univariate normal probability density function (p.d.f.) and cumulative distribution function (c.d.f.).

$$\text{(3.4)} \qquad \Pr\{v(p^1, q^1, y - C^*, \tilde{\varepsilon}) \geq v(p^0, q^0, y, \tilde{\varepsilon}\} = 0.5,$$

i.e., there is no more than a 50:50 chance that the individual would be willing to pay C* for the change.

Although these three welfare measures are conceptually distinct, several relationships can be established among them. First, it is simple to show that, while C^+ is the <u>mean</u> of the distribution of the true but random compensation $\tilde{C}$, C* is the <u>median</u> of this distribution.[12] Thus, if the distribution were symmetric, C^+ and C* would coincide. In practice, however, this may not occur: the distribution of $\tilde{C}$ may be highly skewed, and its mean, C^+, may be an order of magnitude different from its median, C*. Some circumstances in which this can occur are described in section 5.

The second point is that, whereas C^+ and C* are both invariant with respect to a transformation of the utility function, the welfare measure C° is <u>not</u> invariant. As noted earlier, the statistical MRQR model allows one to recover the underlying utility function (2.1') only up to an arbitrary monotone transformation. Consider the transformation $\hat{u}(x, q, z, \tilde{\varepsilon}) \equiv T[u(x, q, z, \tilde{\varepsilon})]$, $T' > 0$, introduced in connection with (2.12), and let $\hat{v}(p, q, y, \tilde{\varepsilon}) \equiv T[v(p, q, y, \tilde{\varepsilon})]$. Then

$$\text{(3.5)} \qquad v(p^1, q^1, y - \tilde{C}, \tilde{\varepsilon}) = v(p^0, q^0, y, \tilde{\varepsilon}) \Leftrightarrow \hat{v}(p^1, q^1, y - \tilde{C}, \tilde{\varepsilon}) = \hat{v}(p^0, q^0, y, \tilde{\varepsilon}).$$

It follows that $\tilde{C}$ and, therefore, both C^+ and C* are unaffected by the utility transformation. This is not true for C° because, if one defines $\hat{C}^\circ$ by

$$\text{(3.6)} \qquad E\{\hat{v}(p^1, q^1, y - \hat{C}^\circ, \tilde{\varepsilon})\} = E\{\hat{v}(q^0, q^0, y, \tilde{e})\},$$

in general $\hat{C}^{\bullet}$ does not also satisfy (3.3). Thus, $\hat{C}^{\bullet} \neq C^{\bullet}$. In effect, the welfare measure $C^{\bullet}$ implies a cardinal concept of utility.

This general result notwithstanding, there <u>are</u> some circumstances in which $C^{\bullet}$ is invariant with respect to a utility transformation. The most important is when there are no income effects. In this case, from (2.16b) the unconditional indirect utility function takes the following form:

$$v(p, q, y, \tilde{\varepsilon}) = \gamma y + \max\, [h_1(q_1) - \gamma p_1 \bar{x}_1 + \tilde{\varepsilon}_1, \ldots, h_N(q_N) - \gamma p_N \bar{x}_N + \tilde{\varepsilon}_N] \tag{3.7}$$

$$\equiv \gamma y + s(p, q, \tilde{\varepsilon}).$$

Hence

$$V(p, q, y) = \gamma y + \mathcal{E}\{s(p, q, \tilde{\varepsilon})\} \equiv \gamma y + S(p, q), \tag{3.8}$$

and, from (3.3)[13]

$$C^{\bullet} = \frac{1}{\gamma}\, [S(p^1, q^1] - S(p^o, q^o)]. \tag{3.9}$$

However, on substituting (3.7) into (3.1), one obtains

$$\tilde{C} = [s(p^1, q^1, \tilde{\varepsilon}) - s(p^o, q^o, \tilde{\varepsilon})]/\gamma. \tag{3.10}$$

It follows, therefore, that when there are no income effects[14]

$$C^{+} \equiv \mathcal{E}\{\tilde{C}\} = C^{\bullet}. \tag{3.11}$$

What about measures of equivalent variation? By working with (3.2) rather than (3.1), one obtains three alternative measures of equivalent variation, which I denote E^{+}, E^{*}, and $E^{\bullet}$.[15] These are related to one another in the same way as C^{+}, C^{*}, and C°. Moreover, it follows directly from (3.7) and (3.8) that, when there are no income effects, $E^{+} = C^{+}$, $E^{*} = C^{*}$, and $E^{\circ} = C^{\circ}$. When there are income effects, however, the corresponding equivalent and compensating variations differ. The similarity with welfare analysis in conventional, continuous choice models is evident.

Another result that carried over from coventional, continuous choice models is the relationship beween compensation measures and areas under ordinary demand curves when there are no income effects. To show this, I need to employ the following result about $V(\cdot)$, which applies regardless of whether or not there are income effects[16]

$$(3.12) \qquad \frac{\partial V}{\partial v_j} \equiv \frac{\partial \{\max [v_1 + \tilde{\varepsilon}_1, \ldots, v_N + \tilde{\varepsilon}_N]\}}{\partial v_j} = \pi_j \qquad j = 1, N.$$

Now suppose that there are no income effects and, for simplicity, that the only change is in p_1 and q_1, with $p_2, \ldots, p_N$ and $q_2, \ldots, q_N$ remaining constant. In this case, using (3.12),

$$(3.13) \qquad C^{\bullet} = \frac{1}{\gamma} [V(p^1, q^1, y) - V(p^o, q^o, y)]$$

$$= \frac{1}{\gamma} \int_{v_1^o}^{v_1^1} \frac{\partial V}{\partial v_1} \, dv_1$$

$$(3.14) \qquad = \frac{1}{\gamma} \int_{v_1^o}^{v_1^1} \pi_1(v_1, \ldots, v_N) \, dv_1.$$

In particular, if only p_1 changes, (3.14) becomes[17]

$$C^{\bullet} = \frac{1}{\gamma} \int_{p_1^o}^{p_1^1} \frac{\partial V}{\partial v_1} \frac{\partial v_1}{\partial p_1} \, dp_1$$

$$(3.15) \qquad = - \bar{x}_1 \int_{p_1^o}^{p_1^1} \pi_1(p_1) \, dp_1$$

$$= - \int_{p_1^o}^{p_1^1} \mathcal{E}\{x_1(p, q, y, \tilde{\varepsilon})\} \, dp_1 .$$

Thus, when there are no income effects, the expected compensating variation for a price change is given by the area under the expected ordinary demand function.

It may be useful to relate the foregoing analysis to the papers by McFadden [1981] and Small and Rosen [1982], which also deal with welfare evaluations in RUM models. Both of these papers focus on the case where there are no income effects and employ the welfare measure $C^{\bullet}$.[18] Thus they do not consider the distinction between $C^{\bullet}$ and the other two welfare measures introduced above. McFadden derives the formula for $C^{\bullet}$ in (3.13) from the utility function (2.16), and in his Theorem 5.1 he proves the converse: if the formula for $C^{\bullet}$ is given by (3.13), the utility function is (2.16).[19] Small and Rosen obtain the formula for $C^{\bullet}$ in (3.15) but with some additional assumptions. However, their analysis appears to be defective: given the additively random utility specification, the no-income-effects utility function (2.16) is both necessary and sufficient for (3.15) to hold.[20]

4. OTHER RANDOM UTILITY MAXIMIZATION MODELS

4.1. Random Coefficients Models. In the discrete choice model studied in sections 2 and 3, the random element representing differences in tastes among individuals and/or unobserved variables was introduced in a very specific way, namely, additively as in (2.1'). In some circumstances, however, this may seem unduly restrictive, and one may prefer to introduce the random element in a different manner. For example, one may wish to specify the no-income-effects utility model (2.16a, b) as

$$u(x, q, z, \tilde{\varepsilon}) = h(x, q) + \tilde{\gamma} z + \Sigma \xi(x_j) \tilde{\varepsilon}_j \tag{4.1a}$$

$$\tilde{u}_j = h_j(q_j) + \tilde{\gamma} y - \tilde{\gamma} p_j \bar{x}_j + \tilde{\varepsilon}_j, \tag{4.1b}$$

where $\tilde{\gamma}$ is now a random variable, uncorrelated with $\tilde{\varepsilon}_1, \ldots, \tilde{\varepsilon}_N$, with a mean of $\bar{\gamma}$ and a variance of σ_γ^2. Equivalently, $\tilde{\gamma} = \bar{\gamma} + \tilde{\varepsilon}_0$, where $\mathcal{E}\{\tilde{\varepsilon}_0\} = 0$ and var $\{\tilde{\varepsilon}_0\} = \sigma_\gamma^2$. An interpretation of this formulation could be that consumers vary in the weight they place on the numeraire good, z, relative to the x's; in addition, because of (our) errors of measurement or observation in the attributes of the discrete choices, consumers appear to vary in their preferences for individual x's. I will refer to any RUM model such as (4.1) where the random element enters nonadditively via the slope coefficients as a "random coefficients" model. This type of model was introduced into the MRQR literature by Hausman and Wise [1978].[21]

Much of the analysis in sections 2 and 3 carries over to random coefficients models. Given some direct utility function $u(x, q, z, \tilde{e})$, the conditional indirect utility functions are

$$(4.2)\quad \tilde{u}_j = u(0, \ldots, 0, \bar{x}_j, 0, \ldots, 0, q, y - p_j\bar{x}_j, \tilde{\varepsilon}) \equiv v_j(q_j, y - p_j\bar{x}_j, \tilde{\varepsilon}).$$

The discrete choice indices are

$$(4.3)\quad \tilde{\delta}_j = \delta_j(p, q, y, \tilde{\varepsilon}) = \begin{cases} 1 \text{ if } v_j(q_j, y - p_j\bar{x}_j, \tilde{\varepsilon}) \geq (q_i, y - p_i\bar{x}_i, \tilde{\varepsilon}) \text{ all } i \\ 0 \text{ otherwise,} \end{cases}$$

and the discrete choice probabilities are

$$(4.4)\quad \pi_j = \Pr\{v_j(q_j, y - p_j\bar{x}_j, \tilde{\varepsilon}) \geq v_i(q_i, y - p_i\bar{x}_i, \tilde{e}) \text{ all } i\}.$$

Similarly, the unconditional indirect utility function is

$$(4.5)\quad v(p, q, y, \tilde{\varepsilon}) = \max\,[v_1(q_1, y - p_1\bar{x}_1, \tilde{\varepsilon}), \ldots, v_N(q_N, y - p_N\bar{x}_N, \tilde{\varepsilon})].$$

Using this function, the welfare measures C^+, C^*, and $C^\bullet$ or E^+, E^*, and $E^\bullet$ can be constructed along the lines indicated above for the additively random utility model.

However, depending on the precise form of the random coefficients specification, some of the relationships among these welfare measures may no longer hold. In particular, it is not necessarily true that $C^+ = C^\bullet$ when there are no income effects. In the case of the model (4.1), the discrete choice probabilities are independent of the consumer's income since they take the form

$$(4.6)\quad \pi_j = \Pr\{h_j(q_j) - \bar{\gamma}p_j\bar{x}_j + \tilde{\omega}_j \geq h_i(q_i) - \bar{\gamma}p_i\bar{x}_i + \tilde{\omega}_i \text{ all } i\}$$

where $\tilde{\omega}_j \equiv \tilde{\varepsilon}_j - \tilde{\varepsilon}_0 p_j\bar{x}_j$, $j = 1, \ldots, N$. But, from (4.1b),

$$(4.7) \qquad v(p, q, y, \tilde{\varepsilon}) = (\bar{\gamma} + \tilde{\varepsilon}_0)\, y + \max\, [h_1(q_1) - \bar{\gamma} p_1 \bar{x}_1 + \tilde{\omega}_1, \ldots, h_N(q_N) - \bar{\gamma} p_N \bar{x}_N + \tilde{\omega}_N]$$
$$\equiv (\bar{\gamma} + \tilde{\varepsilon}_0)\, y + s(p, q, \tilde{\omega});$$

hence,

$$(4.8) \qquad C^+ = \mathcal{E}\left\{\frac{s(p^1, q^1, \tilde{\omega}) - s(p^o, q^o, \tilde{\omega})}{\bar{\gamma} + \tilde{\varepsilon}_0}\right\}$$

while

$$(4.9) \qquad C^{\bullet} = \frac{\mathcal{E}\{s(p^1, q^1, \tilde{\omega}) - s(p^o, q^o, \tilde{\omega})\}}{\mathcal{E}\{\bar{\gamma} + \tilde{\varepsilon}_0\}}.$$

Thus, $C^+ \neq C^{\bullet}$. Similarly, although the relationships in (3.13) and (3.14) still apply to $C^{\bullet}$, the relationship in (3.15) no longer holds.[22] Nevertheless, it still follows from (4.7) that $C^+ = E^+$, $C^* = E^*$ and $C^{\bullet} = E^{\bullet}$ in the random coefficients, no-income-effects model.

4.2. Nonbudget-Constrained and Mixed Discrete/Continuous Choices. The budget-constrained discrete choice RUM model implies that the conditional indirect utility functions have the form given in (2.7') or (4.2). This imposes substantive restrictions on the manner in which the price and income variables enter the formula for the discrete choice probabilities. However, the literature contains many empirical examples of logit or probit models of consumer choices that violate these restrictions. For example, one finds MRQR models based on conditional indirect utility functions of the form

(4.10) $\bar{u}_j = h_j(q_j) - \beta_j p_j + \gamma_j y + \tilde{\varepsilon}_j, \; \beta_j \neq \gamma_j \qquad j = 1, \ldots, N$

or

(4.11) $\tilde{u}_j = h_j(q_j) - \beta p_j + \gamma y p_j + \tilde{\varepsilon}_j \qquad j = 1, \ldots, N,$

which are clearly inconsistent with (2.7') or (4.2). How can such models occur?

One possible explanation is that the consumer is not actually making a purely discrete choice but rather what might be called a "mixed discrete/continuous" choice. In this case, the utility maximization is not constrained by (2.4); instead, the x's can vary continuously, subject to a nonnegativity constraint. However, there is an element of discreteness in the consumer's choices which arises either because the x's are mutually exclusive--i.e., the constraint (2.3) applies--or because the consumer's preferences force a corner solution in which some of the x's are not consumed (in effect, the various x's are perfect substitutes). Thus, the consumer faces both a discrete choice--which of the x's to select--and a continuous choice--how much to consume if he selects x_j. The discrete choice may lead to a statistical MRQR model which satisfies (1.5), but the structure of the conditional indirect utility functions is now different; they no longer satisfy (2.7') or (4.2).

Since these models are described in detail in Hanemann [1984], my discussion here will be brief. They typically involve a random coefficients specification of the utility function rather than the additive formulation in (2.1'). Suppose the consumer has selected good j. Maximization of $u_j(x_j, q_j, z, \tilde{\varepsilon}) \equiv u(0, \ldots, 0, x_j, 0, \ldots, 0, q, z, \tilde{\varepsilon})$ with respect to x_j

(now freely variable) and z subject to a budget constraint, $p_j \tilde{x}_j + z = y$, yields a conditional ordinary demand function, $\tilde{x}_j(p_j, q_j, y, \varepsilon)$, and a conditional indirect utility function, $v_j(p_j, q_j, y, \varepsilon)$. The latter is quasi-convex and decreasing in p_j and increasing in y, but it does not have the same structure as (4.2)--the coefficient of p_j is no longer equal to minus the coefficient of y. Allowing for this difference, the consumer's discrete choice indices are defined as in (4.3), and the discrete choice probabilities are defined as in (4.4). Instead of (2.9'), the unconditional ordinary demand function for the jth good takes the form: $x_j(p, q, y, \tilde{\varepsilon}) = \delta_j(p, q, y, \tilde{\varepsilon})\, \tilde{x}_j(p_j, q_j, y, \tilde{\varepsilon})$. Thus, the probability that one observes an individual who selects, say, the first brand and consumes three units is

$$\Pr\left\{\begin{matrix} x_1(p,q,y,\tilde{\varepsilon}) = 3 \text{ and} \\ x_i(p,q,y,\tilde{\varepsilon}) = 0, \text{ all } i \geq 2 \end{matrix}\right\} = \Pr\left\{\tilde{x}_1(p_1,q_1,y,\tilde{\varepsilon}) = 3 \,\middle|\, v_1(p_1,q_1,y,\tilde{\varepsilon}) \right.$$

$$(4.12)\qquad \left. \geq v_i(p_i,q_i,y,\tilde{\varepsilon}) \text{ all } i \right\} \cdot \Pr\left\{v_1(p_1,q_1,y,\tilde{\varepsilon}) \geq v_i(p_i,q_i,y,\tilde{\varepsilon}) \text{ all } i\right\}.$$

Substituting the unconditional ordinary demand functions into the direct utility function yields the unconditional direct utility function which also can be defined as in (4.5). From this, the welfare measures C^+, C^*, or $C^{\bullet}$ can be constructed in the same manner as for purely discrete choices.[23]

Thus, mixed discrete/continuous choices can give rise to formulas for the discrete choice probabilities involving conditional indirect utility functions that violate the restrictions implied in (2.7') or (4.2)--c.f., the second probability statement on the right-hand side of (4.12). However, precisely because there is also a continuous choice in these models, it is inefficient to estimate the parameters of the utility model from data on the discrete choices alone: the continuous choices contain information about the individual's preferences that should not be overlooked. Accordingly, if one really is dealing with a mixed discrete/continuous choice, the estimation should be based on (4.12) rather than on (4.2) as in conventional MRQR models. Once the model has been estimated, the three approaches to welfare evaluation described in section 3 carry over directly.

Another explanation for MRQR models which violate the restrictions in (2.7') or (4.2) is that the individual genuinely faces a purely discrete choice but one that is not bound by the budget constraint (2.2). An example where this occurs is discrete choices among actions with uncertain consequences by a von Neumann-Morgenstern expected-utility-maximizing individual. Suppose an individual has wealth y and a utility-of-wealth function whose non-stochastic component is denoted by $\psi(y)$. The individual must choose among N actions whose consequences depend on the state of the world, $s = 1, \ldots, S$. Associated with act j are a vector of state probabilities, $\rho_j = (\rho_{j1}, \ldots, \rho_{jS})$, and a vector of monetary consequences, $z_j = (z_{j1}, \ldots, z_{jS})$. Using an additively random formulation, the individual's utility conditional on the choice of act j is

$$\tilde{u}_j = \sum_s \rho_{js} \psi(y + z_{js}) + \tilde{\varepsilon}_j, \tag{4.13}$$

and the discrete choice probabilities are

$$\pi_j = \Pr \{\Sigma\rho_{js} \psi(y + z_{js}) + \tilde{\varepsilon}_j \geq \Sigma \rho_{is} \psi(y + z_{is}) + \tilde{\varepsilon}_i \text{ all } i\} \qquad j = 1, \ldots, N, \tag{4.14}$$

which is a statistical MRQR model that differs from (2.7'). Given that the individual has chosen optimally, his utility is $v(\rho, z, y, \tilde{\varepsilon}) \equiv \max[\tilde{u}_1, \ldots, \tilde{u}_N]$. Suppose that the state probabilities and/or payoffs change from (ρ^0, z^0) to (ρ^1, z^1). In order to measure the welfare effects of this change, the quantities C^+, C^*, and C° or E^+, E^*, and E° can be constructed from $v(p, z, y, \tilde{e})$ along the lines indicated above. For example, $C^+ = \mathcal{E}\{\tilde{C}\}$, where $\tilde{C}$ satisfies $v(\rho^1, z^1, y - \tilde{C}, \tilde{\varepsilon}) = v(\rho^0, z^0, y, \tilde{\varepsilon})$ and, similarly, with the other welfare measures.[24]

5. ECONOMETRIC APPLICATIONS

In this section I show how one actually computes the welfare measures once the parameters of the RUM model have been estimated. For simplicity, I deal with measures of compensating variation; but, with appropriate changes, everything carries over to measures of equivalent variation. I will concentrate mainly on the calculation of C^+ and C^*: the formulas in Table 1 should usually suffice for calculating the expected indirect utility function, $V(\cdot)$, from which $C^\bullet$ can be obtained via (3.3). If there are no income effects, one obtains a closed-form expression for $C^\bullet$ [see (3.12) and (4.9)]. If there are income effects, however, numerical techniques, such as Newton's method, will be required to solve (3.3).[25]

In order to cover both additively random and random coefficient specifications, I write the conditional indirect utility functions as $v_j(p_j, q_j, y, \varepsilon)$, $j = 1, \ldots, N$, where ε is a vector of <u>all</u> the random elements in the model, with joint density function $f_\varepsilon(\cdot)$. I focus on the special case where there is a change in the prices and/or quality attributes of only <u>one</u> good, say, x_1. Furthermore, I assume that the change is unambiguously an improvement, i.e.,

$\tilde{u}_1^1 \equiv v_1(p_1^1, q_1^1, y, \tilde{\varepsilon}) > \tilde{u}_1^o \equiv v_1(p_1^o, q_1^o, y, \tilde{\varepsilon})$. In addition to presenting computational formulas, I will develop some bounds on the magnitudes of C^+ and C^* and identify the circumstances in which $C^+ \gtrless C^*$. When there are more complex price/quality changes, the analysis becomes more complicated, but it follows the same basic logic as that presented here.

To simplify the exposition, it is convenient to present the formulas for the case when N = 3; however, with appropriate changes everything carries over to the case of an arbitrary $N \geq 2$. Define $\tilde{u}_i \equiv v_i(p_i, q_i, y, \tilde{\varepsilon})$, i = 2, 3, $\tilde{u}^o \equiv \max\,[\tilde{u}_1^o, u_2, \tilde{u}_3]$, and $\tilde{u}_1^1 \equiv \max\,[\tilde{u}_1^1, \tilde{u}_2, \tilde{u}_3]$. The trick in computing C^+ in this case is to recognize that there are five possible events which partition the domain of $f_\varepsilon(\cdot)$ into five disjoint regions. I denote these events (1/1), (2/1), (2/2), (3/1), and (3/3) and the corresponding regions A(1/1), A(2/1), etc. The events are as follows. The first event (1/1) is that the individual originally chose good 1; since good 1 improves but there is no change in goods 2 and 3, it follows that he continues to choose good 1. Alternatively, the individual originally chose good 2 and, after the change, he either still prefers good 2 (2/2) or switches to good 1 (2/1). The last two events are that the individual originally chose good 3 and either still prefers that good after the change (3/3) or switches to good 1 (3/1). The corresponding regions of ε-space are

$$A(1/1) = \{\tilde{\varepsilon} \mid \tilde{u}_i \leq \tilde{u}_1^o, \qquad i = 2, 3\}$$

$$A(2/2) = \{\tilde{\varepsilon} \mid \tilde{u}_1^1 \leq \tilde{u}_2 \text{ and } \tilde{u}_3 \leq \tilde{u}_2\} \qquad A(2/1) = \{\tilde{\varepsilon} \mid \tilde{u}_3 \leq \tilde{u}_2 \text{ and } \tilde{u}_1^o \leq \tilde{u}_2 \leq \tilde{u}_1^1\}$$

$$A(3/3) = \{\tilde{\varepsilon} \mid \tilde{u}_1^1 \leq \tilde{u}_3 \text{ and } \tilde{u}_2 \leq \tilde{u}_3\} \qquad A(3/1) = \{\tilde{\varepsilon} \mid \tilde{u}_2 \leq \tilde{u}_3 \text{ and } \tilde{u}_1^o \leq \tilde{u}_3 \leq \tilde{u}_1^1\}.$$

The probabilities of the events are

$$\Pr\{1/1\} = \pi_1^o$$

(5.1) $$\Pr\{2/2\} = \pi_2^1 \qquad \Pr\{2/1\} = \pi_2^o - \pi_2^1$$

$$\Pr\{3/3\} = \pi_3^1 \qquad \Pr\{3/1\} = \pi_3^o - \pi_3^1$$

where π_i^t is the probability that the individual chooses the <u>ith</u> good either before the change ($t = 0$) or after it ($t = 1$).

Observe that, if events (2/2) or (3/3) occur, the individual does <u>not</u> gain from the improvement in good 1 because it is still dominated by some other good; if events (1/1), (2/1), or (3/1) occur, he <u>does</u> gain and the improvement in his welfare can be measured in money by the quantities $\tilde{C}(1/1)$, $C(2/1)$, or $C(3/1)$ where

(5.2a) $$v_1[p_1^1,\ q_1^1,\ y - \tilde{C}(1/1),\ \tilde{\varepsilon}] = \tilde{u}_1^o$$

(5.2b) $$v_1[p_1^1,\ q_1^1,\ y - \tilde{C}(2/1),\ \tilde{\varepsilon}] = \tilde{u}_2$$

(5.2c) $$v_1[p_1^1,\ q_1^1,\ y - \tilde{C}(3/1),\ \varepsilon] = \tilde{u}_3.$$

Thus, the compensation $\tilde{C}$ defined in (3.1) is given by

(5.3) $$\tilde{C} = \begin{cases} 0 & \text{if } \tilde{\varepsilon}\ \ A(2/2) \text{ or } \tilde{\varepsilon}\ \ A(3/3) \\ \tilde{C}(1/1) & \text{if } \tilde{\varepsilon}\ \ A(1/1) \\ \tilde{C}(2/1) & \text{if } \tilde{\varepsilon}\ \ A(2/1) \\ \tilde{C}(3/1) & \text{if } \tilde{\varepsilon}\ \ A(3/1). \end{cases}$$

Hence,

(5.4) $$C^+ = \int_{A(1/1)} \tilde{C}(1/1)\, f_\varepsilon(\varepsilon)\, d\varepsilon + \int_{A(2/1)} \tilde{C}(2/1)\, f_\varepsilon(e)\, d\varepsilon + \int_{A(3/1)} \tilde{C}(3/1)\, f_\varepsilon(\varepsilon)\, d\varepsilon.$$

By virtue of the assumption that the change in (p_1, q_1) is unambiguously an improvement,

$$\tilde{C}(1/1) > 0. \tag{5.5}$$

When the event (2/1) occurs, since $\hat{u}_1^o \leq \tilde{u}_2 \leq \tilde{u}_1^1$, from (5.2) one has

$$v_1[p_1^1, q_1^1, y - \tilde{C}(1/1), \tilde{\varepsilon}] \leq v_1[p_1^1, q_1^1, y - \tilde{C}(2/1), \tilde{\varepsilon}] \leq v_1(p_1^1, q_1^1, y, \tilde{\varepsilon}).$$

Because $v_1(\cdot)$ is increasing in y, this implies that, over the region where $\tilde{C} = \tilde{C}(2/1)$, $0 \leq \tilde{C}(2/1) \leq \tilde{C}(1/1)$. Similarly, over the region where $\tilde{C} = \tilde{C}(3/1)$, $0 \leq \tilde{C}(3/1) \leq \tilde{C}(1/1)$. Hence, from (5.3),

$$0 \leq \tilde{C} \leq \tilde{C}(1/1). \tag{5.6}$$

Since $\tilde{C} > 0$ with positive probability (as long as $\pi_1^o > 0$), and also $\tilde{C} < \tilde{C}(1/1)$ with positive probability (as long as $\pi_2^1 + \pi_3^1 > 0$), it may be deduced that

$$0 < C^+ \equiv \mathcal{E}\{\tilde{C}\} < \mathcal{E}\{\tilde{C}(1/1)\}. \tag{5.7}$$

What about the welfare measure C*? It follows from (3.2), (5.1), and (5.3) that if $\pi_2^1 + \pi_3^1 = (1 - \pi_1^1) \geq 0.5$, i.e., if $\pi_1^1 \leq 0.5$, then $C^* = 0$. If $\pi_1^1 > 0.5$, C* can be determined in the following manner. Given any constant C, define $\tilde{u}_1^*(C) \equiv v_1(p_1^1, q_1^1, y - C, \tilde{\varepsilon})$, $\tilde{u}_i^*(C) \equiv v_i(p_i, q_i, y - C, \tilde{\varepsilon})$, $i = 2, 3$, $\tilde{u}^*(C) \equiv \max\, [\tilde{u}_1^*(C), \tilde{u}_2^*(C), \tilde{u}_3^*(C)]$, and $\pi^*(C) \equiv \Pr\,\{\tilde{u}^*(C) \geq \tilde{u}^o\}$. Then the welfare measure C* solves

$$
\begin{aligned}
0.5 &= \pi^*(C^*) \\
&= \Pr\{\tilde{u}^*(C^*) \geq \tilde{u}^o \text{ and } \tilde{u}_1^*(C^*) \geq \tilde{u}_1^o\} \\
&= \Pr\{\tilde{u}_2 \leq \tilde{u}_1^o, \tilde{u}_3 \leq \tilde{u}_1^o \text{ and } \tilde{u}_1^o \leq \tilde{u}_1^*(C^*)\} \qquad (5.8) \\
&\quad + \Pr\{\tilde{u}_3 \leq \tilde{u}_2 \text{ and } \tilde{u}_1^o \leq \tilde{u}_2 \leq \tilde{u}_1^*(C^*)\} \\
&\quad + \Pr\{\tilde{u}_2 \leq \tilde{u}_3 \text{ and } \tilde{u}_1^o \leq \tilde{u}_3 < \tilde{u}_1^*(C^*)\}.
\end{aligned}
$$

These results apply to <u>any</u> RUM model. They can sharpen somewhat if one focuses specifically on additively ramdon models in which $v_j(p_j, q_j, y, \tilde{\varepsilon}) = v_j(p_j, q_j, y) + \tilde{\varepsilon}_j$, j = 1, 2, and 3. In that case (5.2a) becomes $v_1[p_1^1, q_1^1, y - \tilde{C}(1/1)] + \varepsilon_1 = v_1(p_1^o, q_1^o, y) + \varepsilon_1$, or, canceling out ε_1, $v_1[p_1^1, q_1^1, y - C(1/1)] = v_1(p_1^o, q_1^o, y)$, i.e., C(1/1) is nonstochastic.[26] Accordingly (5.4) becomes

$$
(5.4') \qquad C^+ = C(1/1)\,\pi_1^o + \int_{A(2/1)} \tilde{C}(2/1)\, f_\varepsilon(\varepsilon)\, d\varepsilon + \int_{A(3/1)} \tilde{C}(3/1)\, f_\varepsilon(\varepsilon)\, d\varepsilon.
$$

Now, the quantity C(1/1) is the compensation measure that one might calculate if he disregarded the random elements in the utility function, and it has been employed by several authors. For example, Feenberg and Mills [1980] used C(1/1) to measure the benefits from an improvement in the quality of a site after they estimated an additively random logit model of discrete choices among recreation sites. If we knew for sure that an individual would select good (site), then C(1/1) would indeed be the appropriate welfare measure. In the random utility context, however, two adjustments must be made: C(1/1) must be multiplied by $\pi_1^o < 1$, and the other terms on the right-hand side of (5.4') must be added which measure the gain to the individual if he originally selected some other good (site) and then switched to good 1. The net effect is that C(1/1) <u>overestimates</u> the value of C^+ since, with C(1/1) nonstochastic, (5.7) yields[27]

$$(5.7') \qquad 0 < C^+ < C(1/1).$$

As for C*, it was already noted that, if $\pi_1^1 \leq 0.5$, $C^* = 0$. Similarly, from (5.2) and (5.3), if $\pi_1^o \geq 0.5$, then $C^* = C(1/1)$. If $\pi_1^o < 0.5 < \pi_1^1$, C* can be obtained by saving (5.8) which, in this case, may be simplified to

$$(5.8') \qquad \begin{aligned} 0.5 &= \pi^*(C^*) \\ &= \Pr\{\tilde{u}_2 \leq \tilde{u}_1^o \text{ and } \tilde{u}_3 \leq \tilde{u}_1^o\} + \Pr\{\tilde{u}_3 \leq \tilde{u}_2 \text{ and } \tilde{u}_1^o \leq \tilde{u}_2 \leq \tilde{u}_1^*(C^*)\} \\ &\quad + \Pr\{\tilde{u}_2 \leq \tilde{u}_3 \text{ and } \tilde{u}_1^o \leq \tilde{u}_3 \leq \tilde{u}_1^*(C^*)\} \\ &= \pi_1^o + (\pi_2^o - \pi_2^*) + (\pi_3^o - \pi_3^*) \\ &= 1 - (\pi_2^* + \pi_3^*) \\ &= \pi_1^* \end{aligned}$$

where $\pi_1^* \equiv \Pr\{\tilde{u}_2 \leq \tilde{u}_1^*(C^*) \text{ and } \tilde{u}_3 \leq \tilde{u}_1^*(C^*)\}$, $\pi_2^* = \Pr\{\tilde{u}_1^*(C^*) \leq \tilde{u}_2$ and $\tilde{u}_3 \leq \tilde{u}_2\}$ and $\pi_3^* \equiv \Pr\{\tilde{u}_1^*(C^*) \leq \tilde{u}_3 \text{ and } \tilde{u}_2 \leq \tilde{u}_3\}$.

As an illustration, consider the additively random model derived from the conditional indirect utility functions.

$$(5.9) \qquad \tilde{u}_j = \psi_j(p_j, q_j) + \gamma_j(p_j, q_j)\, y + \tilde{\varepsilon}_j \equiv v_j + \tilde{\varepsilon}_j \qquad j = 1, 2, 3,$$

where $\gamma_j(\cdot) > 0$, which is a generalization of (4.10) and (4.11). Applying (5.2a-c), one obtains

$$(5.10a) \qquad C(1/1) = \frac{[\psi_1^1 - \psi_1^o + y(\gamma_1^1 - \gamma_1^o)]}{\gamma_1^1} \equiv \frac{v_1^1 - v_1^o}{\gamma_1^1}$$

$$(5.10b) \qquad \tilde{C}(2/1) = \frac{[\psi_1^1 - \psi_2 + y(\gamma_1^1 - \gamma_2) + \tilde{\varepsilon}_1 - \tilde{\varepsilon}_2]}{\gamma_1^1} \equiv \frac{v_1^1 - v_2 + \tilde{\varepsilon}_1 - \tilde{\varepsilon}_2}{\gamma_1^1}$$

$$(5.10c) \qquad \tilde{C}(3/1) = \frac{[\psi_1^1 - \psi_3 + y(\gamma_1^1 - \gamma_3) + \tilde{\varepsilon}_1 - \tilde{\varepsilon}_3]}{\gamma_1^1} \equiv \frac{v_1^1 - v_3 + \tilde{\varepsilon}_1 - \tilde{\varepsilon}_3}{\gamma_1^1}$$

where $\psi_1^t \equiv \psi_1(p_j^t, q_j^t)$, $\gamma_1^t \equiv \gamma_1(p_1^t, q_1^t)$, and $v_1^t \equiv \psi_1^t + \gamma_1^t y$, $t = 0, 1$. By assumption, $v_1^1 > v_1^o$. Then, C^+ is given by (5.4') where, for $i = 2, 3$

$$(5.11) \qquad \int_{A(i/1)} \tilde{C}(i/1) f(\varepsilon)\, d\varepsilon = \frac{v_1^1 - v_i}{\gamma_1^1} (\pi_i^o - \pi_i^1) + \int_{v_i - v_i^1}^{v_i - v_1^o} \int_{-\infty}^{v_i - v_j} \frac{\eta_1}{\gamma_1^1} f_\eta(\eta_1, \eta_2)\, d\eta_1\, d\eta_2$$

where $\eta_1 \equiv \varepsilon_1 - \varepsilon_i$, $\eta_2 \equiv \varepsilon_j - \varepsilon_i$, $j \neq 1$, $j \neq i$, and $f_\eta(\cdot)$ is the bivariate density of $(\tilde{\eta}_1, \tilde{\eta}_2)$. Similarly, assuming that $\pi_1^o < 0.5 < \pi_1^1$, C^* solves

$$(5.12) \qquad 0.5 = \int_{-\infty}^{\infty} \int_{-\infty}^{v_1^1 - \gamma_1^1 C^* - v_2 + \varepsilon_1} \int_{-\infty}^{v_1^1 - \gamma_1^1 C^* - v_3 + \varepsilon_1} f(\varepsilon_1, \varepsilon_2, \varepsilon_3)\, d\varepsilon_1\, d\varepsilon_2\, d\varepsilon_3.$$

Suppose, specifically, that $f_\varepsilon(\cdot)$ is the extreme value density so that this is a standard logit model. The integral in (5.11) can readily be evaluated and, on substituting into (5.4') and simplifying, one obtains

$$C^+ = \frac{1}{\gamma_1^1} \ln \left(\frac{e^{v_1^1} + e^{v_2} + e^{v_3}}{e^{v_1^o} + e^{v_2} + e^{v_3}} \right) . \tag{5.13}$$

The corresponding formula for C* is

$$C^* = \begin{cases} 0 & \text{if } v_1^1 \le \ln\left(e^{v_2} + e^{v_3}\right) \\ \dfrac{v_1^1 - \ln\left(e^{v_2} + e^{v_3}\right)}{\gamma_1^1} & \text{if } v_1^o < \ln\left(e^{v_2} + e^{v_3}\right) < v_1^1 \\ \dfrac{v_1^1 - v_1^o}{\gamma_1^1} & \text{if } v_1^o \ge \ln\left(e^{v_2} + e^{v_3}\right) \end{cases} \tag{5.14}$$

Hence,

$$C^* \sim C^+ \text{ as } \frac{v_1^1 + v_1^o}{2} \sim \ln\left(e^{v_2} + e^{v_3}\right) . \tag{5.15}$$

Observe from (5.13) that C^+ satisfies

$$\ln\left(e^{v_1^1 - \gamma_1^1 C^+} + e^{v_2 - \gamma_1^1 C^+} + e^{v_3 - \gamma_1^1 C^+}\right) = \ln\left(e^{v_1^o} + e^{v_2} + e^{v_3}\right) . \tag{5.16}$$

By contrast, using (3.3) and the formula in Table 1, the welfare measure $C^{\bullet}$ satisfies

$$\ln\left(e^{v_1^1 - \gamma_1^1 C^{\bullet}} + e^{v_2 - \gamma_2 C^{\bullet}} + e^{v_3 - \gamma_3 C^{\bullet}}\right) = \ln\left(e^{v_1^o} + e^{v_2} + e^{v_3}\right) . \tag{5.17}$$

Thus, $C^{\bullet} < C^+$ if $\gamma_1^1 < \min(\gamma_2, \gamma_3)$ and $C^{\bullet} > C^+$ if $\gamma_1^1 > \max(\gamma_2, \gamma_3)$, while $C^{\bullet} = C^+$ if $\gamma_1^1 = \gamma_2 = \gamma_3$; the last case corresponds to the no-income effects utility model (2.16).

In order to get a feel for these formulas, it may be helpful to resort to a numerical example. Suppose that $v_2 = v_3 = 0$, $\gamma_1^1 = 1$, $\gamma_2 = 0.5$, and $\gamma_3 = 1.5$. I consider three sets of values for v_1^o and v_1^1: (i) $v_1^o = -2$, $v_1^1 = 0$; (ii) $v_1^o = 0$, $v_1^1 = 2$; and (iii) $v_1^o = 2$, $v_1^1 = 4$. Thus, in each case, $C(1/1) = 2$. In the first case, $\pi_1^1 < 0.5$, so that $C^* = 0$, while in the third case $\pi_1^o > 0.5$ so that $C^* = C(1/1)$. The corresponding values of C^+ and $C^\bullet$ are presented in Table 2. It will be seen that C^+ and $C^\bullet$ are both close in value but differ from C^*. As one would expect, in the first two cases the quantity $C(1/1)$ significantly overestimates all three welfare measures. The last column in the table gives the values of $\pi_1^o \cdot C(1/1)$, the first term in the formula for C^+ (5.4'). It can be seen that this yields a very crude approximation of the value of C^+, the quality of the approximation being worse the lower π_1^o.

To what extent can these formulas be generalized? If $N > 3$ in the logit model (5.9), the term $(e^{v_2} + e^{v_3})$ in (5.13)-(5.17) is replaced by

$$\sum_2^N e^{v_j} .$$

This is when the change is restricted to good 1: When one is dealing with a more complex change, the formulas are different; but they can readily be developed by retracing the steps leading to (5.13)-(5.17). For example, if there is an improvement in good 1, combined with a deterioration in good 2, there are now <u>six</u> possible events which partition ε-space--(1/1), (2/2), (2/1), (3/3), (3/1), and (2/3), and

TABLE 2

Welfare Calculations for the Logit Model (5.9)

Case	π_1^o	π_1^1	C(1/1)	C*	C^+	$C^\bullet$	$\pi_1^o \cdot C(1/1)$
i	0.06338	0.33333	2	0	0.33999	0.35018	0.12676
ii	0.33333	0.78699	2	1.31	1.14093	1.17827	0.66667
iii	0.78699	0.96466	2	2	1.79643	1.81183	1.57397

$$\tilde{C} \begin{cases} < 0 & \text{if } \tilde{\varepsilon} \in A(2/2) \\ = 0 & \text{if } \tilde{\varepsilon} \in A(3/3) \\ > 0 & \text{otherwise.} \end{cases}$$

Suppose the only change is in good 1 and the utility function is given by (5.9), but this is a GEV (generalized logit) or multivariate probit model. In the GEV case, the appropriate formulas are a straightforward extension of (5.13)-(5.17). In the probit case, however, numerical techniques would be required to evaluate the integrals in the formulas for C^+ and C^*, (5.11) and (5.12). If the RUM model is additively random but not linear in y, unlike (5.9), this affects the formulas for C(1/1), $\tilde{C}(2.11)$, and $\tilde{C}(3/1)$ in (5.10) as well as (5.11) and (5.12). Finally, if the RUM model is not additively random, one has to work directly with (5.2), (5.4), and (5.8) and numerical evaluation may well be required.

APPENDIX

Proof of Proposition

Here I prove that the consumer's preferences have the form given in (2.16a) iff the switch prices $\tilde{p}_1^*$ in (2.17) and $\tilde{p}_1^{**}$ in (2.18) coincide. With no loss of generality, I shall assume that N = 2 and $\bar{x}_1 = \bar{x}_2 = 1$. The switch price $\tilde{p}_1 = p_1^* \, (p_2, q_1, q_2, y, \tilde{\varepsilon})$ is defined implicitly by

$$u(1, 0, q_1, q_2, y - \tilde{p}_1^*) + \tilde{\varepsilon}_1 = u(0, 1, q_1, q_2, y - p_2) + \tilde{\varepsilon}_2. \tag{A.1}$$

Suppose that the actual price of good 1 is p_1^o. By virtue of (A.1), one can write

$$\tilde{p}_1^* = p_1^o - \tilde{A}^*, \tag{A.2}$$

where $\tilde{A}^*$ is defined by

$$u(1, 0, q_1, q_2, y - p_1^o + \tilde{A}^*) + \tilde{\varepsilon}_1 = u(0, 1, q_1, q_2, y - p_2) + \tilde{\varepsilon}_2. \tag{A.3}$$

The switch price $\tilde{p}_1^{**} = p_1^{**}(p_2, q_1, q_2, \tilde{u}\ , \tilde{\varepsilon})$ is defined by

$$g_1(q_1, \tilde{u}^o - \tilde{\varepsilon}_1) + \tilde{p}_1^{**} = g_2(q_2, \tilde{u}^o - \tilde{\varepsilon}_2) + p_2 \tag{A.4}$$

or

$$\tilde{p}_1^{**} = p_2 + u^{-1}(\tilde{u}^o - \tilde{\varepsilon}_2 \,|\, 0, 1, q_1, q_2) - u^{-1}(\tilde{u}^o - \tilde{\varepsilon}_1 \,|\, 1, 0, q_1, q_2), \tag{A.5}$$

where $u^{-1}(u \,|\, x_1, x_2, q_1, q_2)$ is the inverse of $u(x_1, x_2, q_1, q_2, z)$ with respect to its last argument.

Observe that $\tilde{p}_1^* = \tilde{p}_1^{**}$ trivially when $p_1^o \geq \tilde{p}_1^*$; then $x_1 = 0$ from (2.17), $\tilde{u}^o = u(0, 1, q_1, q_2, y - p_2) + \tilde{\varepsilon}_2$, and so

$$\text{(A.6)} \qquad u^{-1}(\tilde{u}^o - \tilde{\varepsilon}_2 | 0, 1, q_1, q_2) = y - p_2.$$

Substituting this into (A.5) yields

$$\text{(A.7)} \qquad \tilde{p}_1^{**} = y - u^{-1}(\tilde{u}^0 - \tilde{\varepsilon}_1 | 1, 0, q_1, q_2).$$

The last two equations together imply

$$\text{(A.8)} \qquad u(1, 0, q_1, q_2, y - \tilde{p}_1^{**}) + \tilde{\varepsilon}_1 = u(0, 1, q_1, q_2, y - p_2) + \tilde{\varepsilon}_2,$$

and a comparison with (A.1) shows that $\tilde{p}_1^* = \tilde{p}_1^{**}$.

Accordingly, I focus on the nontrivial case where $p_1^o < \tilde{p}_1^*$. In this case, $\tilde{u}^o = u(1, 0, q_1, q_2, y - p_1^o) + \tilde{\varepsilon}_1$ and, in general, $\tilde{p}_1^* \neq \tilde{p}_1^{**}$. Since

$$\text{(A.9)} \qquad u^{-1}(\tilde{u}^o - \tilde{\varepsilon}_1 | 1, 0, q_1, q_2) = y - p_1^o,$$

(A.5) may be written as

$$\text{(A.10)} \qquad \tilde{p}_1^{**} = p_1^o - \tilde{A}^{**},$$

where

$$\text{(A.11)} \qquad \tilde{A}^{**} \equiv y - p_2 - u^{-1}(\tilde{u}^o - \tilde{\varepsilon}_2 | 0, 1, q_1, q_2).$$

It follows from (A.2) and (A.10) that $\tilde{p}_1^* = \tilde{p}_1^{**}$ iff $\tilde{A}^* = \tilde{A}^{**}$. However, (A.1) implies that

$$\text{(A.12)} \qquad u(1, 0, q_1, q_2, y - p_1^o) + \tilde{\varepsilon}_1 = u(0, 1, q_1, q_2, y - p_2 + \tilde{A}^{**}) + \tilde{\varepsilon}_2.$$

From (A.3) and (A.12), $\tilde{A}^* = \tilde{A}^{**}$ independently of $(p_1^o, p_2, q_1, q_2, y)$ if and only if the utility function has the quasilinear form given in (2.16a).

FOOTNOTES

[1]Throughout the paper a tilde will be used to denote random variables.

[2]The standard independent logit model (McFadden [1974]) is a special case where $G(t_1, \ldots, t_N) \equiv \Sigma t_j$ and $\pi_j = e^{W_j\beta_j}\left[\Sigma e^{W_i\beta_i}\right]^{-1}$.

[3]For simplicty, I treat the q_j's as scalars, but they could be vectors.

[4]I assume that $y \geq \max\,[p_i\bar{x}_i]$ so that $z \geq 0$.

[5]If $u(\cdot)$ is increasing in q_j, $v_j(\cdot)$ in (2.5) and (2.7) is increasing in q_j.

[6]This additive specification is employed by Domenich and McFadden [1975], Williams [1977], Daly and Zachary [1978], McFadden [1981], Small and Rosen [1982], and many others. More general formulations of $u(x, q, z, \tilde{\varepsilon})$ will be considered in section 4.1.

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[8]With no loss of generality, I assume that $\xi\{\varepsilon_j\} = 0$, all j.

[9]Another restriction follows from the weak complementarity assumption, (2.6), namely, that the elements of W_j include the attributes and price of good j but not those of the other goods. Without this assumption, $\tilde{u}_j = v_j(q_1, \ldots, q_N, y - p\bar{x}_j) + \tilde{\varepsilon}_j$, and the vector W_j includes q_i, $i \neq j$. In the case of the independent logit model where the $\tilde{\varepsilon}_j$'s are independent extreme value variables, the resulting discrete choice possibilities,

$$\pi_j = e^{v_j(q_1,\ldots,q_N,y-p_j\bar{x}_j)}\left[\Sigma e^{v_i(q_1,\ldots,q_N,y-p_i\bar{x}_i)}\right]^{-1},$$

do <u>not</u> possess the Independence of Irrelevant Alternatives (IIA) property. (This is a version of what McFadden [1981] calls the "universal logit" model.) Thus, there is some connection between weak complementarity and the IIA property.

[10]The marginal utility of income, γ, <u>can</u> still be estimated because it appears as the coefficient of the price difference term in (2.19). The point is that income itself cannot appear as an explicit variable in a MRQR model satisfying (2.16).

[11]Table 1 provides formulas for calculating $V(\cdot)$ for the GEV model, (1.3), the independent logit model, and binary and trichotomous probit models.

[12]The median of the distribution of $\tilde{C}$, C_M, has the property that $\Pr\{\tilde{C} \leq C_M\} = 0.5$. But, since $v(p, q, y, \tilde{\varepsilon})$ is increasing in y,

$$\tilde{C} \leq C_M \Rightarrow v(p^1, q^1, y - C_M, \tilde{\varepsilon}) \geq v(p^1, q^1, y - \tilde{C}, \tilde{\varepsilon}) = v(p^o, q^o, y, \tilde{\varepsilon}).$$

From (3.4), $C^* = C_M$.

[13]For example, in the independent logit model, (3.9) becomes

$$\Sigma v_j(q_j^1, y - C^* - p_j\bar{x}_j) = \Sigma v_j(q_j^o, y - p_j^o\bar{x}_j).$$

[14]$S(p, q)$ can be constructed from the formulas given in Table 1. For example, with the GEV model one obtains

$$C = \frac{1}{\gamma} \ln G\ e^{v_1^o}, \ldots, e^{v_N^o} - \ln G\ e^{v_1^1}, \ldots, e^{v_N^1}$$

while, with the binary independent probit model where Σ is diagonalized and normalized so that $\sigma = 1$, one obtains

$$C = \frac{1}{\gamma}[\Delta^o\phi(\Delta^o) + v_2^o + \phi(\Delta^o) - \Delta^1\phi(\Delta^1) - v_2^1 - \phi(\Delta^1)],$$

where $v_j^t \equiv v_j(q_j^t, y - p_j^t\bar{x}_j) = h_j(q_j^t) + \gamma(y - p_j^t\bar{x}_j)$ and $\Delta^t \equiv v_1^t - v_2^t$, $t = 0, 1$.

[15]When there are no income effects, C^+ satisfies: $A\{s(p^1, q^1, \tilde{\varepsilon}) - s(p^o, q^o, \tilde{\varepsilon}) \geq \gamma C^+\} = 0.5$.

[16]The equivalent variation for a change from (p^a, q^a) to (p^b, q^b) is equal to the negative of the corresponding compensating variation measure for the change from (p^b, q^b) to (p^a, q^a).

[17]This is proved in Williams [1977], Daly and Zachary [1978], and Sheffs and Daganzo [1979].

[18]The second line follows from the fact that $\partial v_1/\partial p_1 = -\gamma\bar{x}$ using (2.16b). The third line follows from the fact that $\xi\{\bar{x}_1\} = \bar{x}_1\pi_1$.

[19]McFadden [1981] and Small and Rosen [1982] interpret $V(\cdot)$ as the average indirect utility function over a population of individuals and $C^{\bullet}$ as the average compensation. I interpret $V(\cdot)$ and $C^{\bullet}$ as the observer's expectation of a single individual's utility function and compensation. I would calculate $C^{\bullet}$ (or C^{+} or $C^{\bullet}$) for each individual separately and then aggregate over the entire population, perhaps using weights derived from some social welfare function along the lines in Muellbauer [ ].

[20]McFadden [1981] actually derived (3.14) for a more general RUM model involving continuous as well as discrete choices. This type of model is discussed further in section 4.2.

[21]In addition to assuming that there are no income effects, Small and Rosen [1981] make two additional assumptions: (1) $\partial v_j/\partial y$ is independent of p_j and q_j and (2) $\partial v_j/\partial q_j \to 0$ as $p_j \to \infty$. It can be shown that (2.16) implies (1) but precludes (2).

[22]This has generally been restricted to probit rather than logit models because the normal distribution is closed under addition unlike the extreme value distribution. This is less of a consideration if the discrete alternative-specific random terms, $\varepsilon_1, \ldots, \varepsilon_N$, are omitted from the model leaving only the random slope coefficient(s); for an example, see Hanemann [1984c].

[23]From (4.7) one obtains

$$C^{\bullet} = [V(p^1, q^1, y) - V(p^o, q^o, y)]/\,\bar{\gamma} = -\,[(\bar{\gamma} + \tilde{\varepsilon}_0)/\,\bar{\gamma}] \int_{p_1^o}^{p_1^1} \xi\{x_1(p, q, y, \tilde{\varepsilon})\}\, dp_1 .$$

It should be emphasized that these anomalies arise from the particular form of the random coefficients specification in (4.1). They would vanish if the coefficient of income were nonstochastic and any other slope coefficient in (4.1b) were random--e.g., a coefficient in $h_j(\cdot)$.

[24] An example of an application of welfare analysis in a mixed discrete/continuous choice RUM model is given in Hanemann [1982].

[25]In Hanemann [ ] this type of discrete choice model is employed to infer the value of life (i.e., the value of changes in mortality probabilities from data on individual risk-taking behavior.

[26]Some formulas for approximating $C^{\bullet}$ were presented in Hanemann [1983b] and in an earlier draft of this paper (Hanemann [1982]).

[27]Equivalently, C(1/1) satisfies

$$\xi\{v_1(p_1^1, q_1^1, y - C(1/1)] + \tilde{\varepsilon}_1]\} = \xi\{v_1(p_1^o, q_1^o, y) + \tilde{\varepsilon}\}.$$