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ADJUSTING TO CLIMATE CHANGE: IMPLICATIONS OF INCREASED VARIABILITY AND ASYMMETRIC ADJUSTMENT COSTS FOR INVESTMENT IN WATER

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Anthony C. Fisher and Santiago J. Rubio

DEPARTMENT OF AGRICULTURAL AND RESOURCE ECONOMICS

BERKELEY

California Agricultural Experiment Station

University of California

DEPARTMENT OF AGRICULTURAL AND RESOURCE ECONOMICS
DIVISION OF AGRICULTURE AND NATURAL RESOURCES
UNIVERSITY OF CALIFORNIA AT BERKELEY

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Adjusting to Climate Change: Implications of Increased Variability and Asymmetric Adjustment Costs for Investment in Water Reserves

Anthony C. Fisher*

Department of Agricultural & Resource Economics

University of California, Berkeley, USA

Santiago J. Rubio

Department of Economic Analysis

University of Valencia, Spain

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Abstract

In this paper we study the determination of optimal water storage capacity in a region, taking into account that the supply of the resource, the flow into the reserve, is uncertain, that building the capacity is costly, and that the commercial development of water resources may entail also *environmental costs*. We find that water storage capacity in the long run is *positively* related to increases in uncertainty if the marginal benefit of water withdrawal is *convex*, and that, for the case of costly reversibility of investment, a range of inaction for investment appears, and the stability of water storage capacity with respect to changes in variance increases.

Key words: water resource infrastructure, water reserves management, stochastic control

JEL Classification: Q25, D81, D90

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1 Introduction

Global climate change is likely to bring an increase in the uncertainty attached to water supply in many regions (Karl, Knight and Plummer, [11]). Part of this is an increase in the variability of precipitation. But it is also related to a more subtle shift in patterns of precipitation. In regions like California, for instance, it is expected that more precipitation will fall as rain than as snow, and that the snowpack will melt earlier in the spring (Gleick, [8]). Both of these changes will in effect increase the variability of water flows since a late-melting snowpack evens out water flow over the course of a year. The question considered in this paper is, what are the implications for optimal water storage of an increase in uncertainty, in the sense of an increase in the variance of water supply?

Although there is a substantial literature on uncertainty in water resource management, we have not found anything of immediate relevance to this question. The literature is concerned primarily with the impact of stochastic surface water flows on the value of additional stocks, either surface reservoirs or groundwater aquifers, and on optimal withdrawals from these stocks. Tsur and Graham-Tomasi [17] show that the buffer value of groundwater (to mitigate impacts from fluctuations in surface flow) is positive and, in an application to wheat farming in the northern Israeli Negev region, significant in magnitude. Knapp and Olson [12] consider the relationship between socially optimal and common property withdrawals from a groundwater stock. Relatively small gains from optimal management are found in an application to Kern County, California. Tsur and Zemel [18] study the impact of an uncertain irreversible event, such as pollution, that may render a groundwater resource unusable. A key finding is that it does not pay to extract in excess of recharge, even though this would be beneficial under certainty.

Not in the context of water resources, storage has been related to analysis of the firm's decisions under uncertainty. The standard inventory control problem has focused on planning production under demand uncertainty (Bertsekas, [5]). More recently, Scheinkman and Schechtman [15] and Stokey and Lucas [16] have presented a competitive, partial equilibrium model with storage under supply uncertainty that reflects approximately the production conditions of agricultural commodities which are not traded internationally.

For nonrandom demand, they study the market equilibrium of a storable commodity whose output each period depends on previous period effort on production and a realization of a shock that affects all producers equally.

Although we too focus on supply uncertainty, our model presents two new features. First, we connect the analysis with the theory of investment, considering in an explicit way the cost of building the stock. We assume that there exists some kind of *complementarity* between *water storage capacity* and a *capital stock* of dams and canals, and that, therefore, a larger reserve in the long run requires a larger capital stock. A second novel feature of our model is the specification of possibly irreversible environmental impacts associated with the investment in water resources infrastructure. Here we develop a more general approach that that of the earlier literature that introduced the notion of irreversibility of environmental impacts (Arrow and Fisher [4], Henry [10]). We consider the possibility of (costly) recovery of the natural environment, studying the case of a reversible development where the cost of disinvestment in development is the cost of removing water infrastructure and restoring something like the original environment.

Our main results are: (1) With symmetric linear adjustment costs an increase in uncertainty implies an increase in long-run capital stock if the marginal benefit function associated with water withdrawal is convex. We find that with *convex* marginal benefits the net marginal value of the capital stock is *positively* related to the instantaneous variance rate which characterizes water flow as a stochastic process. Then an increase in variance shifts upward the net marginal value function and leads to an increase in the optimal capital stock. (2) The existence of asymmetric linear adjustment costs reduces the variability of optimal investment in water infrastructure. The asymmetry defines a range of inaction and increases the stability of the long-run capital stock with respect to changes in variance. (3) If there is no market for water resource infrastructure, and if in addition environmental restoration is costly, changes in variance again do not affect the optimal level of reserves. In this case, the range of inaction is larger, suggesting an interpretation of the earlier literature on project investment with irreversible environmental impacts. Irreversibility can be considered an economic phenomenon, related to the cost of disinvestment: the range of inaction is increasing with the cost of disinvestment.

The paper is organized as follows. In Section 2 we set up the model as a stochastic control problem of maximization of the expected present value of net social welfare from water reserves management and investment in storage capacity. In Section 3 we obtain the expected dynamics of water consumption which allows us to characterize the long-run equilibrium capital stock, and in Section 4 we discuss the effects of an increase in uncertainty on the equilibrium. In this first part of the paper we assume a symmetric adjustment cost function in order to focus on the effects of uncertainty. In the second part, Section 5, we incorporate into the model more realistic assumptions about the reversibility of investment, looking first at asymmetric purchase and sale costs of capital, and then at costly reversibility of investment. Conclusions are restated in Section 6.

2 The model

We now present a formal description of the model: let W and w represent stochastic water resources and water withdrawal respectively. We assume that W is an Îto process, which evolves through time according to the stochastic differential equation[1]:

$$dW = a(W,t)dt + b(W,t)dz, \quad W(0) = W_0 > 0 \tag{1}$$

Then the difference between the stochastic water resources and the control variable, water withdrawal, determines the increment in water reserves, S, but reserves cannot increase above the storage capacity defined by the physical capital stock, K. In symbols,

$$dS = \begin{cases} (W-w)dt & if \quad S < \alpha K \\ & \qquad\qquad\qquad S(0) = S_0 > 0 \\ 0 & if \quad S = \alpha K \end{cases} \tag{2}$$

where α is a conversion factor that gives storage capacity as a function of the capital stock[2].

Utility depends on water withdrawal $U(w)$, with $U_w > 0$ and $U_{ww} < 0$[3]. There are of course costs of withdrawal, the costs of conveying water to its

[1] An Ito process is a generalization of a simple Brownian motion with drift where dz is the increment of a Wiener process, and $a(W,t)$ and $b(W,t)$ are, respectively, the expected instantaneous *drift rate* and the instantaneous *variance rate*. See Dixit and Pindyck [6, Ch. 3] for a good introduction to the subject of stochastic processes.

[2] We assume that the conversion factor α is calculated taking into account, among other things, the reduction in water storage capacity caused by the mud sedimentation on reservoirs' floor.

[3] As we are interested in the socially optimal management of water reserves the utility

place of final consumption by farms, municipalities and firms, $C(w)$, with $C_w > 0$ and $C_{ww} \leq 0$. The concavity of this function is explained by the existence of increasing returns to scale in the technology because pipeline volume increases more rapidly than built surface.

Finally, the capital stock K evolves according to

$$\dot{K} = (I - \delta K), \qquad K(0) = K_0 > 0 \tag{3}$$

where I is investment and δ is the rate of depreciation. We assume that the built storage carries environmental costs, $H(K)$, with $H_K > 0$ and $H_{KK} > 0$. Environmental costs are included because, in its natural state, the environment yields some benefits.

Then, for a risk neutral authority, the problem is to choose w and I to maximize the expected present value of net social benefits:

$$\max_{\{w,I\}} E_0 \int_0^\infty [U(w) - C(w) - H(K) - cI]\, e^{-rt} dt \tag{4}$$

subject to differential equations (1), (2) and (3) where r is the social discount rate.

function must be interpreted as a monetary measure of consumer welfare from water consumption. This measure could be the gross consumers' surplus so that the price of water would be the marginal utility of water consumption.

Notice that assuming a risk neutral authority is compatible with a concave utility function if we distinguish between the utility that consumers obtain from water consumption and the (net) welfare generated by management of water reserves and investment in storage capacity. For this formulation social welfare is defined as the difference between the consumers' surplus from water consumption net of withdrawal costs, and the environmental and adjustment costs of the capital stock, as given by the expression in brackets in equation (4)[4].

3 Investment under uncertainty

Let J be the value of water reserves and capital stock assuming w is chosen optimally, so that

$$J(S, K, W) = \max_{\{w,I\}} E_t \int_t^{\infty} B(\tau)\, e^{-r\tau} d\tau$$

where $B(\tau) = U(w) - C(w) - H(K) - cI = P(w) - H(K) - cI$ is the social welfare and $P(\cdot)$ is the (net) consumers' surplus or benefit from water

[4] To analyze the effects of different attitudes toward risk we could introduce explicitly a water management authority's utility function whose argument would be the social welfare as it has been defined above. In our model we implicitly assume that this utility function is linear (risk neutrality).

consumption.

Because time appears in the maximand only through the discount factor, the Bellman equation for this problem can be written as

$$rJ = \max_{\{w,I\}}[P(w) - H(K) - cI + \lambda(\alpha K - S) + (1/dt)E_t dJ]. \tag{5}$$

where λ is the multiplier associated with the restriction $S \leq \alpha K$; λ is positive if $S = \alpha K$, and zero otherwise.

Since W is a stochastic process, we can use Itô's Lemma to write

$$dJ = J_S dS + J_K dK + J_W dW + \frac{1}{2} J_{WW}(dW)^2. \tag{6}$$

substituting for dS, dK, dW and $(dW)^2$ we obtain

$$\begin{aligned} dJ &= (W-w)J_S dt + (I - \delta K)J_K dt + a(W,t)J_W dt + b(W,t)J_W dz \\ &\quad + \frac{1}{2}a^2(W,t)J_{WW}(dt)^2 + a(W,t)b(W,t)J_{WW}dtdz + \frac{1}{2}b^2(W,t)J_{WW}(dz)^2, \end{aligned}$$

which reduces to

$$\begin{aligned} dJ &= (W-w)J_S dt + (I - \delta K)J_K dt + a(W,t)J_W dt \\ &\quad + b(W,t)J_W dz + \frac{1}{2}b^2(W,t)J_{WW}dt \end{aligned} \tag{7}$$

since $(dz)^2$ is equal to dt, from the definition of a Wiener process, and $dtdz$, which is of order $(dt)^{\frac{3}{2}}$, and $(dt)^2$ both go to zero faster than dt as dt becomes infinitesimally small, and can therefore be neglected[5]. Applying the

[5]See Dixit and Pindyck [6, p.71].

differential operator $(1/dt)E_t$ to (7) and considering that $E_t[dz] = 0$, again from the definition of a Wiener process, the Bellman equation can be written as

$$rJ = \max_{\{w,I\}} [P(w) - H(K) - cI + \lambda(\alpha K - S) + (W - w)J_S + (I - \delta K)J_K + a(W,t)J_W + \frac{1}{2}b^2(W,t)J_{WW}]. \tag{8}$$

Maximizing with respect to w and I we have the optimality conditions:

$$U_w = C_w + J_S \tag{9}$$

$$c = J_K \tag{10}$$

The first condition states that the marginal utility of water consumption must be equal to marginal cost, which has in turn two components, the marginal withdrawal cost C_w and what we might call the marginal user cost J_S. The user cost arises because water consumption today reduces the availability of water for tomorrow's uses[6]. The second condition establishes that the marginal adjustment cost of capital must be equal to its marginal user cost J_K.

[6]The second order condition requires a decreasing marginal utility. However, this is a necessary condition but not sufficient to satisfy the second order condition because of the assumed concavity of the cost function for withdrawing and distributing water, $C(w)$. See the appendix for an evaluation of the scope of this problem.

To examine how uncertainty affects long-run capital stock, we shall need to derive an expression for the expected dynamics of water consumption and investment, thus making the transition from the Bellman equation to solution of the stochastic control problem defined in Section 2 in terms of one stochastic differential equation for the control variables, water consumption and investment, and the differential equations (1), (2) and (3). It will then be possible to use that system of equations to characterize a long-run (steady state) stochastic equilibrium and evaluate the effects of changes in variance on long-run capital stock or storage capacity[7].

Differentiating equation (8) with respect to S, and taking into account that the optimal value of w is given by equation (9), we obtain

$$\begin{aligned} rJ_S &= (P_w - J_S)\frac{\partial w}{\partial S} - \lambda + (W - w)J_{SS} + (I - \delta K)J_{KS} \\ &\quad + a(W,t)J_{WS} + \frac{1}{2}b^2(W,t)J_{WWS}. \end{aligned} \tag{11}$$

Since the terms in the first set of parentheses sum to zero, J_S is given by (9),

[7]This approach is applied by Pindyck [13] to study the optimal investment of the firm under uncertainty with adjustment costs, and Rubio [14] to analyze optimal investment in an extractive industry. Basically, it is an extension of the comparative statics analysis used in deterministic control theory to evaluate changes in steady state values caused by variations in the model's parameters.

and $(1/dt)E_t dJ_S = (W-w)J_{SS}+(I-\delta K)J_{KS}+a(W,t)J_{WS}+\frac{1}{2}b^2(W,t)J_{WWS}$ by Itô's Lemma, equation (11) can be written as

$$(1/dt)E_t dJ_S = rP_w + \lambda. \tag{12}$$

where $\lambda = 0$ if $S < \alpha K$. Applying the differential operator to (9) and equating it to (12), we get

$$(1/dt)E_t dP_w = rP_w + \lambda \tag{13}$$

Developing the left-hand side of (13), we can obtain the expected dynamics of water consumption. From Itô's Lemma

$$dP_w = P_{ww}dw + \frac{1}{2}P_{www}(dw)^2. \tag{14}$$

Considering that $w^* = w(S,K,W)$ along the optimal path, using Itô's Lemma again,

$$dw = \frac{\partial w}{\partial S}dS + \frac{\partial w}{\partial K}dK + \frac{\partial w}{\partial W}dW + \frac{1}{2}\frac{\partial^2 w}{\partial W^2}(dW)^2, \tag{15}$$

and by substitution of dS, dK, dW and $(dW)^2$,

$$\begin{aligned} dw \;&=\; (W-w)\frac{\partial w}{\partial S}dt + (I-\delta K)\frac{\partial w}{\partial K}dt + a(W,t)\frac{\partial w}{\partial W}dt \\ &\quad + b(W,t)\frac{\partial w}{\partial W}dz + \frac{1}{2}b^2(W,t)\frac{\partial^2 w}{\partial W^2}dt. \end{aligned} \tag{16}$$

The implied expression for $(dw)^2$ is greatly simplified by neglect of terms in higher powers of dt as dt goes to zero, so that we are left with $(dw)^2 =$

$\left(\frac{\partial w}{\partial W}\right)^2 b^2(W,t)dt$, from which equation (14) can be written

$$dP_w = P_{ww}dw + \frac{1}{2}P_{www}\left(\frac{\partial w}{\partial W}\right)^2 b^2(W,t)dt, \tag{17}$$

and the differential operator of (17) is

$$(1/dt)E_t dP_w = P_{ww}(1/dt)E_t dw + \frac{1}{2}P_{www}\left(\frac{\partial w}{\partial W}\right)^2 b^2(W,t). \tag{18}$$

Equating (13) and (18) and ordering terms, we obtain the desired expression for the expected dynamics of water consumption

$$P_{ww}(1/dt)E_t dw = rP_w + \lambda - \frac{1}{2}P_{www}\left(\frac{\partial w}{\partial W}\right)^2 b^2(W,t) \tag{19}$$

Now differentiating equation (8) with respect to K, and taking into account that the optimal value of w is given by equation (9), we obtain

$$\begin{aligned} rJ_K &= (P_w - J_S)\frac{\partial w}{\partial K} - H_K + \lambda\alpha - \delta J_K + (W-w)J_{SK} \\ &\quad +(I-\delta K)J_{KK} + a(W,t)J_{WK} + \frac{1}{2}b^2(W,t)J_{WWK} \end{aligned} \tag{20}$$

Since the terms in the first set of parentheses sum to zero, J_K is given by (10), and $(1/dt)E_t dJ_K = (W-w)J_{SK} + (I-\delta K)J_{KK} + a(W,t)J_{WK} + \frac{1}{2}b^2(W,t)J_{WWK}$ by Itô's Lemma, equation (20) can be written as

$$(1/dt)E_t dJ_K = (r+\delta)c + H_K - \lambda\alpha \tag{21}$$

Applying the differential operator to (10) and equating it to (21), we get

$$\lambda = \frac{1}{\alpha}[(r+\delta)c + H_K] \tag{22}$$

since c is a constant. This condition establishes that investment in new capacity will take place only when reserves reach capacity. The shadow price of the constraint,λ, is equated to $(r+\delta)c$, the opportunity cost of capital plus H_K, the environmental costs and $1/\alpha$ is a conversion factor, the capital requirement to store an additional unit of water. In that case (19) becomes

$$P_{ww}(1/dt)E_t dw = rP_w + \frac{1}{\alpha}[(r+\delta)c + H_K] - \frac{1}{2}P_{www}\left(\frac{\partial w}{\partial W}\right)^2 b^2(W,t) \tag{23}$$

Equation (23), together with $(1/dt)E_t dS$ and (3) describes the expected dynamics of w, S and K. Notice that as the marginal cost of adjustment of the capital stock is constant the adjustment is instantaneous. If a stochastic equilibrium exists, in the sense of a convergence to a long-run (steady state) distribution for w, S and K, the distribution has to satisfy the conditions $(1/dt)E_t dw = (1/dt)E_t dS = (1/dt)E_t dK = 0$. At the long-run equilibrium the expected value of variations in water consumption, water reserves and capital stock is zero, and the expected values of control, w, and state variables, S and K, must satisfy these conditions. Then we can use the steady state conditions to study how the expected value of water reserves, S, (long-run equilibrium) when $\lambda = 0$ and the expected value of capital stock, K, and storage capacity, αK, (long-run equilibrium) when $\lambda > 0$ will be affected by changes in the variance of water flow in a way dependent on the properties of the (net) benefit function, $P(w)$, associated with water consumption[8].

4 Effects of uncertainty

In order to obtain the long-run equilibrium for capital stock and consequently for storage capacity we are going to analyze the (steady state) optimality condition for water withdrawal defined by $(1/dt)E_t dw = 0$ and equation (23), assuming $\lambda > 0$. As $\lambda > 0$ implies $S = \alpha K$, equation (23) can be written as

$$U_w(w)+\frac{1}{\alpha r}[(r+\delta)c+H_K(K)] = C_w(w)+\frac{1}{2r}P_{www}(w)\left(\frac{\partial w(W,K)}{\partial W}\right)^2 b^2(W,t), \tag{24}$$

[8] As Pindyck [13, p.421] has pointed out, we can also use these steady state conditions to determine how, given a current realization of W, the values of w, S and K that satisfy the steady state conditions will change with variance, so that we can determine the effects of uncertainty (for a given value of water resources) even if in the long-run there is no stationary distribution for this variable. Basically, it is an extension of the comparative statics analysis used in deterministic control theory to evaluate changes in steady state values caused by variations in the model's parameters.

where the number of state variables has been reduced to two: W, water resources, and K, capital stock[9]. The marginal benefit, on the left-hand side of this condition, presents two components, the marginal utility and the present value of a flow of benefits originated when a marginal unit of water is devoted to consumption instead of being stored. This second term appears because when we are defining the optimal long-run equilibrium for the capital stock, an increment in consumption turns into a reduction in storage costs equal to the opportunity cost of capital, $(r + \delta)c$, plus the environmental costs, H_K, a reduction that must be taken into account to correctly define the optimality condition for water withdrawal. Notice that when a unit of water is devoted to consumption the reduction in capital is given by the inverse of the conversion factor: $1/\alpha$. On the other side, the marginal cost of water consumption incorporates a term related to the instantaneous variance rate whose sign depends on the convexity of the marginal net benefit function, P_w, for water consumption.

To interpret this last component, let us assume that the marginal bene-

[9]If $\lambda = 0$ $(S < \alpha K)$ condition (23) reduces to $rP_w = 1/2 \; P_{www} \left(\frac{\partial w}{\partial W}\right)^2 \delta^2(W,t)$ which defines the long-run equilibrium water reserves for K, capital stock or storage capacity, at its optimal value.

fit is convex, i.e. that $P_{www} > 0$. Then for variations of water withdrawal around its expected value, as P_w (marginal net benefit) defines the marginal valuation of water withdrawal by consumers, a reduction in consumption has a larger impact on consumers' welfare than an increase, and there is an *incentive* to reduce water withdrawal, to store water to avoid low consumption due to low realizations of W. If $P_{www} < 0$, i.e., if the marginal benefit function is concave, a reduction in consumption has a lower impact on consumers' welfare than an increase and then the incentive is to increase water withdrawal and reduce water reserves. We shall have more to say about the sign of P_{www} shortly.

In any event, equation (24) allows us to determine the effect of an increase in variance on optimal capital (or storage capacity) for the resource flow W equal to its expected value. When $(1/dt)E_t dS = 0$, $E_t(W - w) = 0$, so $E_t(W) = E_t(w) = \bar{W}$, or in other words, the expected value of water consumption at the stochastic steady state is going to be the expected value of water resources, $\bar{W}$. Then equation (24) can be rewritten as

$$U_w(\bar{W}) + \frac{1}{\alpha r}[(r+\delta)c + H_K(K)] = C_w(\bar{W}) + \frac{1}{2r}P_{www}(\bar{W})\left(\frac{\partial w(\bar{W},K)}{\partial W}\right)^2 b^2(\bar{W},t), \tag{25}$$

which can be used to establish the relation between optimal capital stock,

K, and the variance rate, $b^2(\bar{W}, t)$, when water resources are equal to their expected value[10].

To develop this analysis we rewrite equation (25) as

$$(r + \delta)c = \alpha A(\bar{W}, b^2, K) - H_K(K), \tag{26}$$

where the first term on the right-hand side is given by

$$A(\bar{W}, b^2, K) = \frac{1}{2} P_{www}(\bar{W}) \left(\frac{\partial w(\bar{W}, K)}{\partial W} \right)^2 b^2(\bar{W}, t) - rP_w(\bar{W})$$

Equation (26) can be interpreted as the optimality condition for the long-run equilibrium capital stock where $(r + \delta)c$ is the marginal opportunity cost of capital stock and $\alpha A(\bar{W}, b^2, K) - H_K(K)$ is the (net) marginal value of capital stock (NMV). Before using this condition to determine the effects of an increase in uncertainty about future water resources, we need to verify that a solution exists. To do this, it is useful to present the following definition:

[10] When there exists a steady state distribution for the capital stock the optimal value is just the expected value, given that the expected values of w and K have to satisfy the steady state conditions. In the other case (non existence), the optimal value is the value that satisfies the steady state conditions when the current value of water resources is its expected value. Obviously the definition of a true steady state implies that the expected value, $\bar{W}$, be constant and the instantaneous variance rate independent of time.

Definition. *If* $\frac{\partial^2 w}{\partial W \partial K} > (<)\ 0$, *then* K *and* W *are* ***complements*** ***(substitutes)*** *with respect to the optimal policy function* $w = w(W, K)$.

Although we have developed the analysis presented in this Section for both cases, K and W complements as well as substitutes, it seems plausible that only the substitute case is relevant. W and K substitutes means that an increase in the capital stock, K, and consequently in storage capacity, weakens the link between flow, W, and withdrawals, w; a decrease in capital stock strengthens the relationship between flow and withdrawals. In other words, we assume that the cross partial derivative $\partial^2 w/\partial W \partial K$ is negative. In that case we obtain the following result:

Result 1. *If K and W are substitutes and the marginal benefit is convex then $(r+\delta)c < \alpha A(\bar{W}, b^2, 0) - H_K(0)$ is a necessary and sufficient condition to have a unique optimal value for the capital stock.*

Assuming that optimal water consumption responds positively to water resources, $\left(\frac{\partial w}{\partial W} > 0\right)$, when the state variables are substitutes the net marginal value function is (monotonically) decreasing, since

$$\frac{\partial NMV}{\partial K} = \alpha P_{www} \frac{\partial w}{\partial W} \frac{\partial^2 w}{\partial W \partial K} b^2 - H_{KK} < 0, \quad for \ \frac{\partial^2 w}{\partial W \partial K} < 0$$

Then if the marginal value for zero capital stock is higher than the constant marginal cost, equation (26) has a unique solution, as illustrated in Figure 1.

FIG.1

Notice that as the marginal value of the capital stock is positively related to the variance of water resources it could happen that for a low variance it does not pay to build a stock because the cost is greater than the benefit of the first unit built. In the case of complementary state variables the previous condition is neither necessary nor sufficient since the slope of the marginal revenue function is not determined. When the state variables are complementary, neither existence nor uniqueness are guaranteed[11].

We are now ready to determine the effect on the capital stock of a change in variance. Totally differentiating optimality condition (26), we obtain

$$0 = \alpha P_{www} \frac{\partial w}{\partial W} \frac{\partial^2 w}{\partial W \partial K} b^2 dK + \frac{\alpha}{2} P_{www} \left(\frac{\partial w}{\partial W} \right)^2 db^2 - H_{KK} dK$$

Reordering terms,

$$\frac{dK}{db^2} = \frac{\frac{\alpha}{2} P_{www} \left(\frac{\partial w}{\partial W} \right)^2}{H_{KK} - \alpha P_{www} \frac{\partial w}{\partial W} \frac{\partial^2 w}{\partial W \partial K} b^2} > 0, \quad for \ \frac{\partial^2 w}{\partial W \partial K} < 0 \qquad (27)$$

With substitute state variables an increase in variance *increases* the optimal level of capital. When K and W are substitutes the second derivative of the optimal policy function that appears in the denominator of the right hand side of (27) is negative , and the sign for the effect of an increase in variance on the optimal capital stock is unambiguous for a convex marginal benefit function. This conclusion allows us to present the following result:

[11] However, if the state variables are *complementary* and the marginal benefit function is *concave* the condition in Result 1 is necessary and sufficient to have a unique optimal value for the capital stock. Thus, the existence and uniqueness of an optimal value for capital stock depends crucially on the relationship between the complementarity/substitutability of the state variables and the concavity/convexity of the marginal net benefit function.

Result 2. *If the state variables K and W are* **substitutes**, *the marginal benefit function is* **convex**, *and there exists an optimal value for the capital stock, an increase in variance implies a higher capital stock for water flow at its expected value.*

When K and W are substitutes and the marginal benefit function is convex, an increase in the variance rate implies that more capital is required to avoid or reduce damages from the realization of low values of water resources given that now the probability of these extreme events is higher. If the marginal benefit function is concave the effect is ambiguous. With a concave marginal benefit, the (net) marginal value of the capital stock can be increasing or decreasing. With a decreasing marginal value an increment in variance would imply lower capital stock[12]. This somewhat counter-intuitive

[12]When the marginal benefit function is concave an increase in variance shifts downward the (net) marginal value function. Then with constant marginal cost and a decreasing marginal value that movement would cause a reduction in the optimal capital stock. The effect for an increasing marginal value is not determined because the long-run equilibrium for the capital stock is not well defined. On the other hand, when the state variables are complementary, an increase in variance implies a lower capital stock for water flow at its

result is another example of the mischief caused by nonconvexity, here the presence of increasing returns in the water distribution technology. Without this, the marginal net benefit function could safely be assumed convex, as marginal utility is convex for a concave utility function.

5 Asymmetric adjustment costs: the range of inaction

In the standard neoclassical theory of investment it is well known that asymmetric adjustment costs produce a range of inaction for investment (Hayashi [9], Abel [1,2,3]). The result is based on Tobin's q model which says that if there exists an asymmetry between the purchase and sale prices of capital goods, two critical levels q_m and q^m can be defined such that if q, the increase in the value of the firm that would result if the capital stock were increased by one unit, lies between these critical levels, zero investment is optimal. Here we obtain the same result, but using what we have called the (net) marginal value of capital,$\alpha\ A(\bar{W}, b^2, K) - H_K(K)$, instead of Tobin's q, which is more

expected value if the marginal benefit function is concave and has an undetermined effect if is convex.

difficult to calculate, requiring the solution of a partial differential equation like our earlier equation (8).

5.1 Asymmetric capital purchase and sale costs

Letting c_p equal the purchase price of capital goods and c_s the sale price, we can rewrite optimality condition (26) as

$$\begin{aligned} (r+\delta)c_p &= \alpha A(\bar{W}, b^2, K) - H_K(K) \\ (r+\delta)c_s &= \alpha A(\bar{W}, b^2, K) - H_K(K) \end{aligned} \tag{28}$$

Condition (28) allows us to define a range of inaction for the variance of the random variable W, i.e., an interval $[(b^2)_m, (b^2)^m]$ such that if changes in variance stay within that interval, effects on the optimal stock are null and no investment takes place. This is shown in Fig.2 and stated as Result 3 [13].

$\boxed{\text{FIG. 2}}$

[13] The results presented in this Section only work for a decreasing (net) marginal value of capital because for an increasing (net) marginal value function the long-run equilibrium is not well defined. See footnote 11. We analyze the range of inaction only for a convex marginal benefit function. The extension to a concave marginal benefit function is straightforward and presents minimal changes.

Result 3. *If there exists asymmetry in adjustment costs, changes in variance do not affect the optimal capital stock in the range defined by the interval* $[(b^2)_m, (b^2)^m]$. *The limits of the interval are calculated for each value of* K *from condition (28)*

$$(r+\delta)c_p = \alpha A(\bar{W}, b^2, K) - H_K(K) \;\rightarrow\; F^m[(b^2)^m, K] = 0$$

$$(r+\delta)c_s = \alpha A(\bar{W}, b^2, K) - H_K(K) \;\rightarrow\; F_m[(b^2)_m, K] = 0$$

The $F(\cdot,\cdot)$ functions are implicit functions in b^2 and K defined by the optimality condition for the two prices when the water resources are equal to their expected value. In other words, given that $(r+\delta)$ and $\bar{W}$ are fixed, the functions $F(\cdot,\cdot)$ associate a K with a given b^2 or conversely. Clearly $(b^2)_m < (b^2)^m$ because the marginal value function is increasing with respect to the variance and $c_s < c_p$.The existence of a range of inaction means that, given any level of capital stock and storage capacity, it will not be optimal to disinvest at least until the variance falls below the critical value $(b^2)_m$, and it will not be optimal to invest again until the variance rises above the critical value $(b^2)^m$. The distance between the critical values depends on the difference between the purchase and sale prices given that the elasticity of $A(\bar{W}, b^2, K)$ with respect to the variance is equal to one (since A is an increasing, linear function of b^2), so a big difference between these prices implies long periods of inaction for investment in water resource infrastructure.

A comparison with previous models of investment with asymmetric adjustment costs is worth noting here. We define the critical levels in terms of the *variance* of the random variable W. In Tobin's q model, the critical levels are defined in terms of an unobservable q, the effect on the value function (like our J) of an increment of capital. In principle, development of empirical counterparts should be easier for our model.

5.2 Costly reversibility

We turn now to what seems to us the most realistic and therefore the most interesting, case: costly disinvestment. Contrary to what we have assumed thus far, there may be no market for the capital stock accumulated to accommodate water reserves, or in other words, c_s equals zero. Further, and more importantly, there can be a cost of disinvestment, especially when it is recognized that disinvestment implies restoring something like the pre-project environment.

To develop the analysis of this case, we shall need to redefine the *adjustment cost function* of section 2 as

$$AC(I) = [I > 0]c_p I - [I < 0]c_d I. \tag{29}$$

The adjustment cost function has the following interpretation: if $I > 0$, one is investing in infrastructure, at a cost c_p; if $I < 0$, one is investing (disinvesting) in environmental restoration (infrastructure), at a cost c_d.

For this adjustment cost function, social welfare from the management of water reserves and investment in storage capacity can be written as

$$B = U(w) - C(w) - H(K) - [I > 0]c_p I + [I < 0]c_d I$$

and the Bellman equation, equation (8) of section 3, as

$$\begin{aligned} rJ &= \max_{\{w,I\}}[P(w) - H(K) - [I > 0]c_p I + [I < 0]c_d I + \lambda(\alpha K - S) \\ &\quad + (W - w)J_S + (I - \delta K)J_K + a(W,t)J_W + \frac{1}{2}b^2(W,t)J_{WW}] \end{aligned} \tag{30}$$

Then following the development presented in section 3, equation (23) becomes

$$P_{ww}(1/dt)E_t dw = \begin{cases} rP_w + \frac{1}{\alpha}[H_K + (r+\delta)c_p] - \frac{1}{2}P_{www}\left(\frac{\partial w}{\partial W}\right)^2 b^2(W,t) & for\ I > 0 \\ \\ rP_w + \frac{1}{\alpha}[H_K - (r+\delta)c_d] - \frac{1}{2}P_{www}\left(\frac{\partial w}{\partial W}\right)^2 b^2(W,t) & for\ I < 0 \end{cases} \tag{31}$$

and the stochastic steady state defined by $(1/dt)E_t dw = 0$ establishes the optimality condition for the capital stock, analogous to equation (26) of section 4:

$$\begin{aligned} (r+\delta)c_p &= \alpha A(\bar{W}, b^2, K) - H_K(K) \\ -(r+\delta)c_d &= \alpha A(\bar{W}, b^2, K) - H_K(K) \end{aligned} \tag{32}$$

This condition defines a range of inaction for the variance of W, but now the net marginal value must be negative to trigger a disinvestment process. When the net marginal value of capital is negative a decrease in the stock increases the value of capital, so that if the increment in value is greater than the cost of reversal it will be optimal to destroy the infrastructure and recover the natural environment. This effect is shown in Figure 3, and stated in Result 4.

FIG.3

Result 4. *If investment in environment is costly, changes in variance do not affect optimal capital stock in the range defined by the interval* $[(b^2)_m, (b^2)^m]$. *The limits of the interval are calculated for each value of* K *from condition (32)*

$$
\begin{aligned}
(r+\delta)c_p &= \alpha A(\bar{W}, b^2, K) - H_K(K) \;\rightarrow F^m[(b^2)^m, K] = 0 \\
-(r+\delta)c_d &= \alpha A(\bar{W}, b^2, K) - H_K(K) \;\rightarrow F_m[(b^2)_m, K] = 0
\end{aligned}
$$

Again, $(b^2)_m < (b^2)^m$ because the marginal revenue function is increasing with the variance and c_p is always greater than $-c_d$, but now the range of inaction is going to be greater than in the case of asymmetric purchase and sale prices with costless reversibility. This result allows us to conclude that disinvestment can be optimal even when it is necessary to pay to restore the environment, but a range of inaction will appear because of the presence of restoration, or disinvestment, costs. A corollary of this result is that the range of inaction is increasing with the cost of disinvestment. This suggests an interpretation of the earlier literature on project investment with irreversible environmental impacts. Irreversibility of investment in natural resources can be considered an economic phenomenon, related to a sufficiently high cost of reversal[14].

This is clear also when we consider the decision to invest, rather than to disinvest. Suppose disinvestment has occurred, from K^* in Figure 3 to a lower level of reserves, say to the point K^- in the Figure, as a result of a shift in the marginal value curve to $NMV(b^2)'$, which lies below $NMV(b^2)_m$. Now variance b^2 increases, shifting the NMV curve back up. No investment is warranted, however, unless the variance increases all the way to a new critical value, one that implies an intersection of the NMV curve with the $(r + \delta)c_p$ line at a level of reserves above K^-. In particular, a shift of the NMV curve back up to $NMV(b^2)_m$ will not trigger investment in reserves to K^*.

Finally, it is important to note that the linearity assumption for the adjustment cost function is not critical to our results because we know that a convex component of an adjustment cost function will be zero at the long run equilibrium. Suppose we include a convex adjustment cost term in our function: $cI + c_a(I)$, where $c_a'(I) > 0$ and $c_a''(I) > 0$. Then the marginal

[14] A similar result is obtained by Fisher and Hanemann [7].

adjustment cost specified in our optimality condition is $c + c_a'(I)$, and the second term vanishes when $I = 0$. Therefore, neither our optimality condition for the long-run equilibrium capital stock nor our results on the influence of uncertainty will be affected. This point is strengthened if we recall from the theory of investment that adjustment costs do not affect long run equilibria, rather the adjustment (speed) towards the long run equilibria, i.e., they modify the investment process, not the long run value of the capital stock.

6 Conclusions

In this paper we have studied the socially optimal investment in water storage capacity, taking into account that the supply of the resource is uncertain because of the variability of the hydrological cycle. The motivation for the study is the perception that climate change is likely to affect the hydrological cycle, in many regions, by increasing its variability, i.e., increasing uncertainty about future water resources availability.

We model water resources as a stochastic process and focus on the determination of long-run water storage capacity. The model takes into account that to build a certain level of reserves requires investment in public capital stock or infrastructure and that environmental costs are associated with this investment, as the pre-project environment will typically yield some benefit in its natural state. We find that under uncertainty and *convex* marginal benefits there exists an incentive to build a certain level of water reserves (invest in water resources infrastructure) thereby avoiding drastic reductions in consumption that would otherwise be occasioned by drought. Further, we find that long-run water storage capacity is positively related to the level of uncertainty. An increment in the variance of water resources *increases* the long run stochastic equilibrium level of the capital stock or water storage capacity. As an increase in the variance rate means an increase in the probability of occurrence of extreme values of water resources, more reserves will be required to reduce potential future losses.

On the other hand, we show that when adjustment costs are asymmetric there exists a range of inaction for investment in water resources infrastructure. In that case the stability of the long-run capital stock increases with respect to changes in variance. Finally, we study this issue when there is no market for the infrastructure capital stock, and when disinvestment is *costly*. In our model, disinvestment in water resource infrastructure is interpreted to include investment in environmental restoration. We find that reversibility of investment in water resource infrastructure is increasing with the social valuation of environmental assets and decreasing with the cost of reversal, and that here too a range of inaction appears. Moreover, the range of inaction is larger; for cost of reversal sufficiently high, the investment is in effect irreversible.

APPENDIX

Although the concavity of $C(w)$ can cause problems when the second order condition is checked, it is easy to show that it is satisfied for at least one interesting case: the linear one. If we use a quadratic utility function: $U = aw - \frac{b}{2}w^2$, that measures the gross consumer's surplus associated to a linear demand function, and a concave cost function like $C = dw^\beta$ with $d > 0$ and $\beta < 1$, the optimality condition (9) can be written as

$$a - bw - d\beta w^{\beta-1} - J_S(S, K, W) = 0 \tag{33}$$

or

$$\hat{a}(S, K, W) - bw - d\beta w^{\beta-1} = 0 \tag{34}$$

where $\hat{a}(S, W) = a - J_S(S, K, W)$ is assumed strictly positive to avoid the trivial solution $w = 0$.

From (34) we can rewrite the optimality condition for water consumption as net marginal utility equal to marginal cost of withdrawing and distributing water

$$\hat{a}(S, K, W) - bw = d\beta w^{\beta-1} \tag{35}$$

that presents two solutions since the left-hand side is a linear function whereas the right-hand side is an asymptotic decreasing convex function. In this case, the second order condition allows us to select one of the two solutions. This condition requires that

$$-b < \beta(\beta - 1)d(w^{*})^{\beta-2} \tag{36}$$

The slope of marginal net utility has to be less than the slope of marginal cost for the values defined by necessary condition (35). The application of this condition selects the bigger of the two values located by the application of the first order condition.

However, a third condition must be applied to this value before selecting it as the optimal water consumption. Average net utility must exceed average cost in order to have a positive value of the right-hand side of the Bellman equation for the optimal w^{*} (a shutting or closing condition). This third condition requires that the following expression be positive[15]

[15] We do not write the term $\lambda(\alpha K - S)$ because in any case it is zero.

$$RHS = \hat{a}(W,K,S)w^* - \frac{b}{2}(w^*)^2 - d(w^*)^\beta - H(K) - cI + WJ_S$$
$$+(I - \delta K)J_K + a(W,t)J_W + \frac{1}{2}b^2(W,t)J_{WW} \qquad (37)$$

or

$$RHS = \Delta(W,K,S) + \hat{a}(W,S)w^* - \frac{b}{2}(w^*)^2 - d(w^*)^\beta \qquad (38)$$

where $\Delta(W,K,S) = -H(K) - cI + WJ_S + (I - \delta K)J_S + a(W,t)J_W + \frac{1}{2}b^2(W,t)J_{WW}$ is constant with respect to water consumption. Written in this way the right-hand side of Bellman equation can be interpreted as the benefits associated with water consumption and then the third condition would require that these benefits be positive. Figure 4 illustrates a possible solution for the maximization problem in Bellman's equation assuming that $\Delta(W,K,S) > 0$[16].

FIG.4

Notice that in the figure w^* will be a maximum if the value $RHS(w^*)$ is greater than $RHS(0)$ and the restriction $w \leq W + S$ is not operative: $w^* < W + S$.

[16]For $\Delta \leq 0$ we would have a very similar figure with the same kind of solution.

Finally, we can also present some results for the following case: $U = aw^{\alpha}$, with $a > 0$ and $\alpha < 1$, and $C = dw^{\beta}$, with $d > 0$ and $\beta < 1$. If $\alpha < \beta$ and the first order condition is satisfied, the second order condition selects a local maximum that under certain not very restrictive conditions is also a global maximum. However, if $\alpha > \beta$ the local maximum will be a global maximum only under more restrictive conditions than in the previous case. If these condition do not hold the optimal policy consists of consuming all available water in each moment and, therefore, it will not be optimal to store water.

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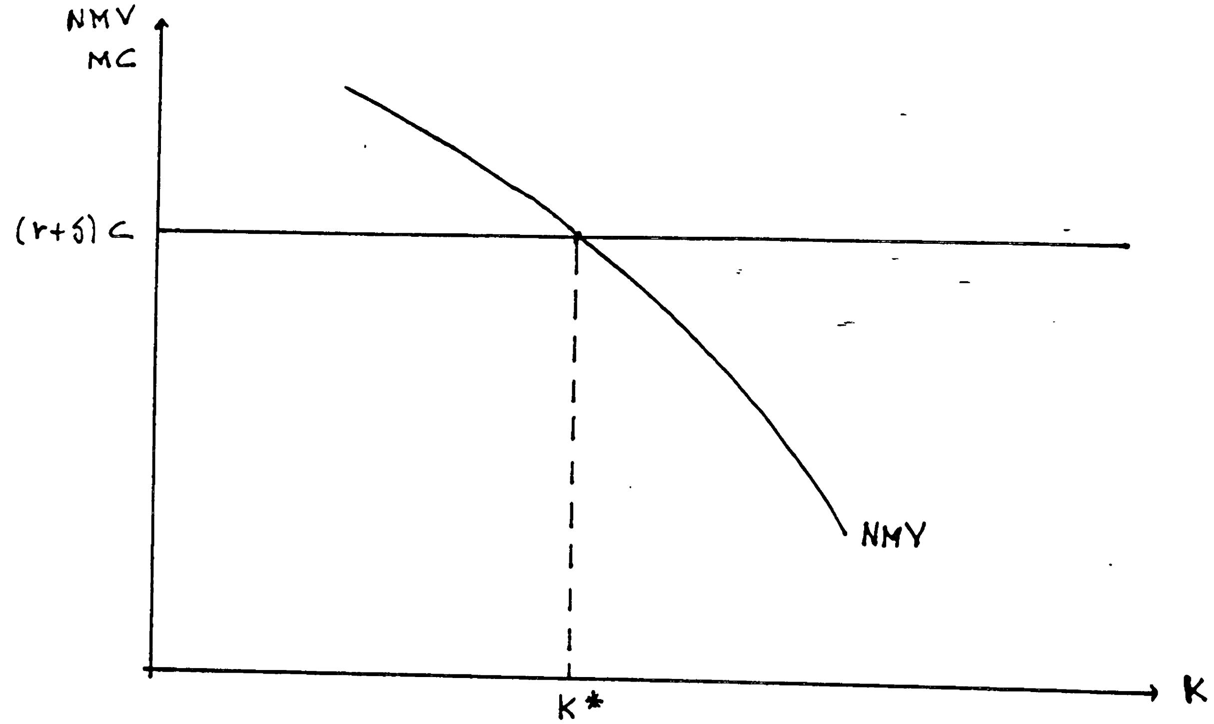
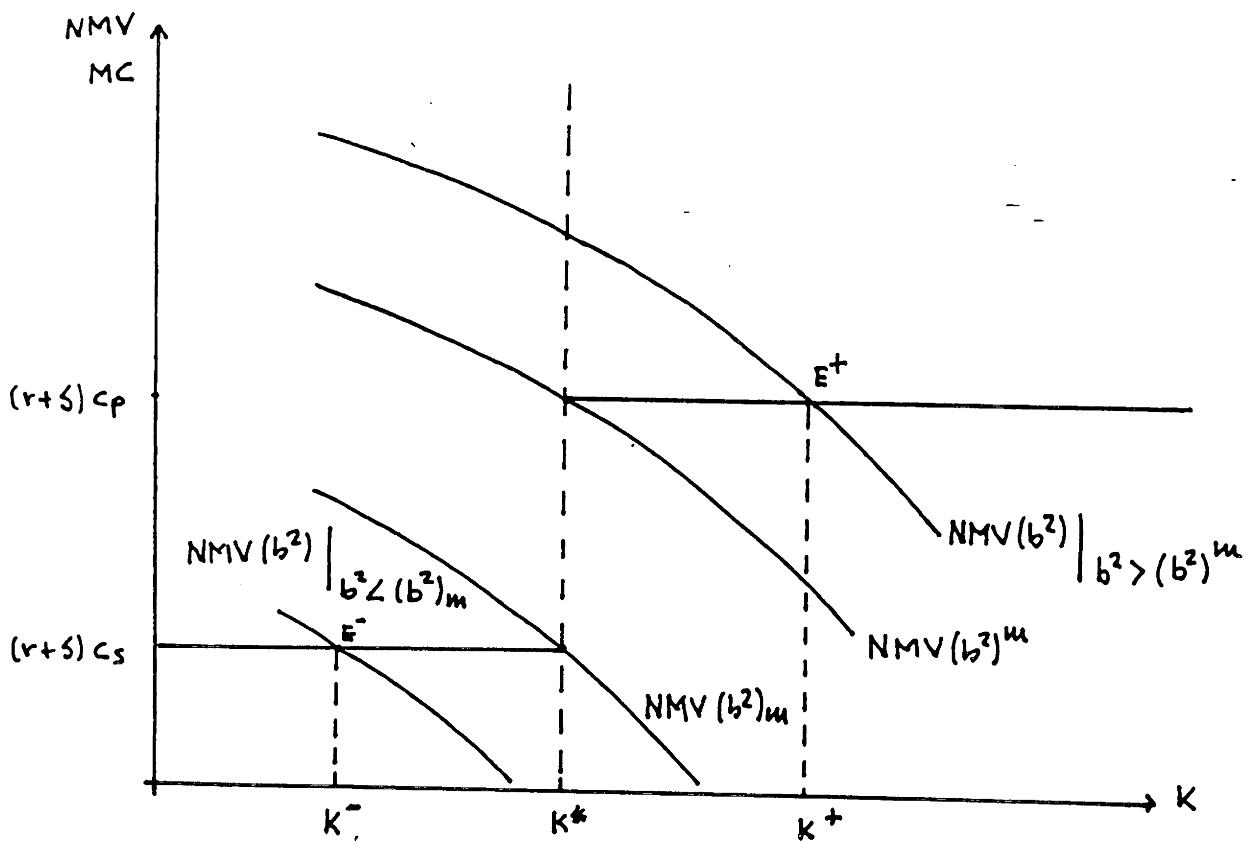


NMV
MC
(r+δ) c
NMY
K*
K

FIG. 1



NMV
MC
(r+δ) cp
(r+δ) cs
E+
E-
NMV (b²) | b² > (b²)^m
NMV (b²)^m
NMV (b²) | b² < (b²)m
NMV (b²)m
K-
K*
K+
K

FIG. 2

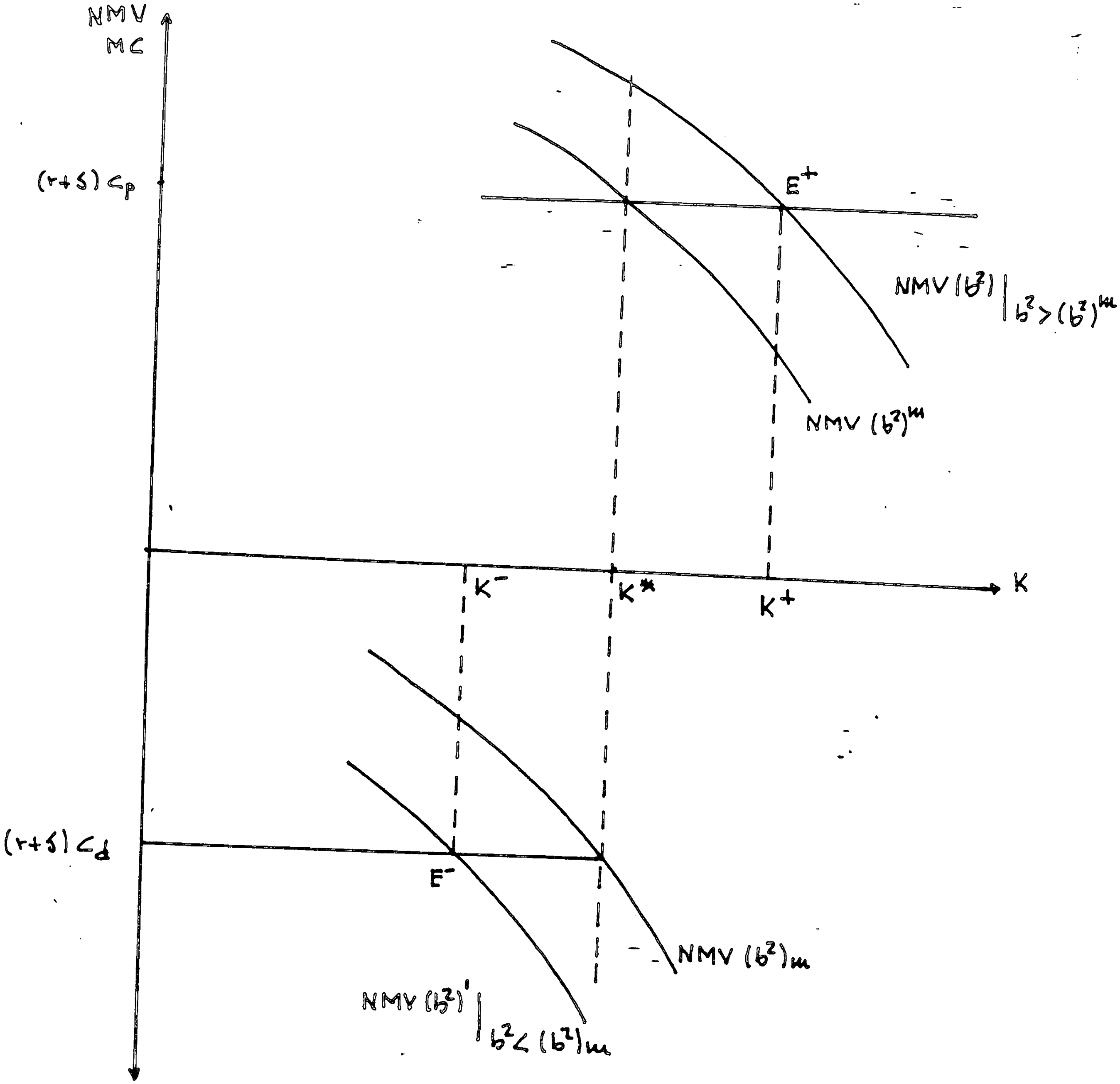

NMV
MC
(r+δ) c_p
E+
NMV (b²) | b² > (b²)^m
NMV (b²)^m
K
K−
K*
K+
(r+δ) c_d
E−
NMV (b²)_m
NMV (b²)' | b² < (b²)_m

FIG. 3

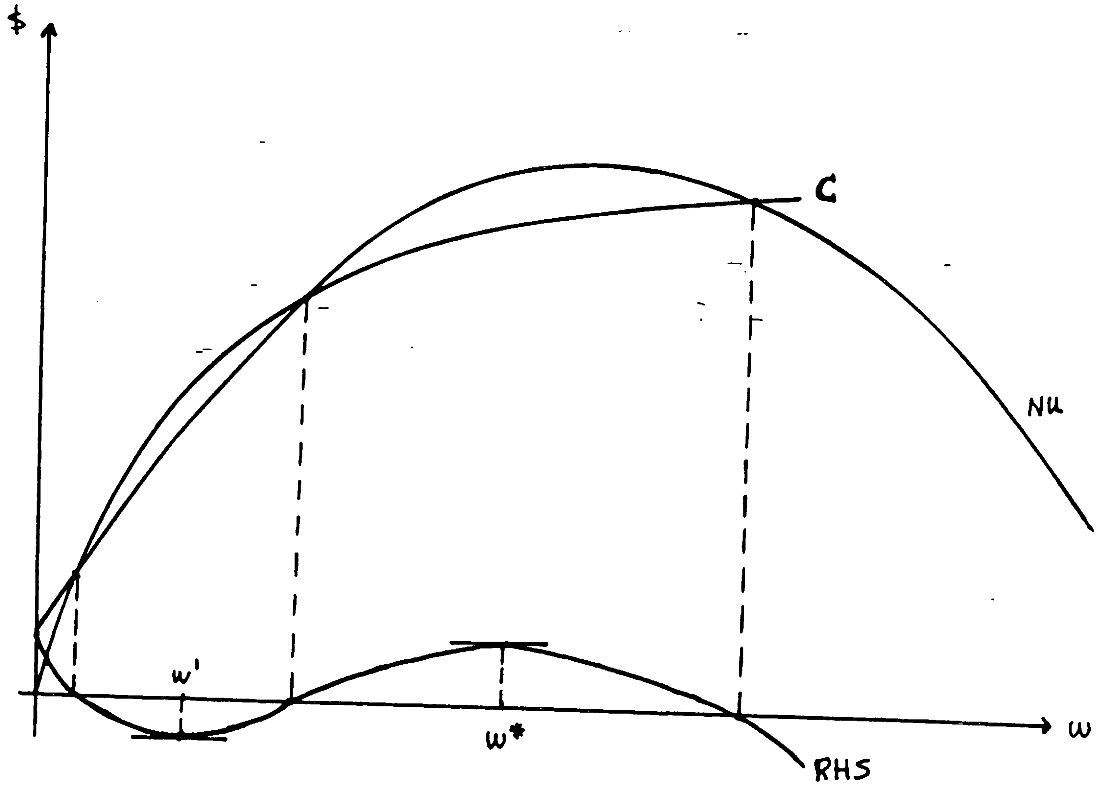


FIG. 4