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SOVEREIGN DEBT AS INTERTEMPORAL BARTER

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Abstract

Borrowing and lending between sovereign parties is modelled as intertemporal barter that smooths the consumption of a risk-averse party subject to endowment shocks. The surplus anticipated in the relationship offers sufficient incentive for cooperation by all parties, including any other competitive lenders who may be potential entrants. The sole punishments consist of renegotiation-proof changes in the paths of future payments. This implicit long-term relationship may be fulfilled as the continual renegotiation of a simple, incomplete short-term debt contract with associated "debt overhang." The analysis suggests that the crucial role of the explicit contract is the identification of the parties to the relationship.

1 Introduction

Respect for sovereign immunity has long been recognized as a crucial constraint facing lenders to sovereign states (for example, Keynes [1924]). Intertemporal exchange is restricted by the absence of a supranational legal authority to enforce the terms of agreements across national borders. The history of lending to sovereigns shows the consequences of lenders' inabilities to enforce repayments specified in loan contracts. Overall payments on sovereign loans during the 19th and 20th century have not come close to discharging the original contractual obligations in an overwhelming number of cases, and there have been many defaults as identified by historians.[1]

Although debt service has fallen far short of formal contractual obligations, the lack of collateral has not meant that lenders did not recover their principal on average. In fact, economic historians have shown that lending to sovereign nations has been very profitable overall with average returns comparing favorably to those on contemporaneous domestic government debt issued in lender nations.[2] Even loans in default were frequently profitable ex post.[3]

When payment deviations or defaults occurred, it has been widely noted that there was generally no abrupt termination of the borrower-lender relationship as often seen in domestic bankruptcy. Instead, most have been ongoing relationships that continue through renegotiation under the guises of rescheduling, partial payment, new loans, debt repurchase and so on. Settlement has been achieved in almost all instances on a case by case basis through bilateral negotiations.[4] Indeed, all parties may view a default as "excusable," an equilibrium phenomenon in appropriate states of the underlying international financial relationship.[5]

The subject of our paper is the equilibrium intertemporal exchange relationship that underlies a formal contract for loans between sovereigns. The desire for consumption-smoothing with an uncertain endowment stream generates gains from intertemporal trade in our model. This incentive has figured prominently in the literature on debt dating from Eaton and Gersovitz [1981].[6]

In our model, the sole motivation for any payments made by any of the parties is the surplus anticipated from continuation of the exchange relationship. Our analysis contrasts with the existing literature in two respects. The first is that every action taken by an agent is voluntary; payments are made only if doing so raises the surplus to the agent looking forward in the relationship, taking into account the previously observed actions of others. In the model, there are no external agents to provide exogenous enforcement of any actions taken by market participants, so that commitment opportunities do not exist. In effect, sovereign immunity applies symmetrically to any agent, whether called a lender or a borrower. Punishments that encourage participants to engage in intertemporal trade consist solely of changes in the division of the surplus generated within the relationship. The second distinction is that agents can always renegotiate the terms of the relationship, notably any punishments used, to their mutual benefit.

We study a model of intertemporal barter. In equilibrium, market participants make unilateral payments to each other on different dates: all exchange is intertemporal. We show that an equilibrium path that is Pareto superior to permanent autarky can be achieved by self-interested agents in a long-term self-enforcing relationship. In our model, punishment threats that enforce cooperation in intertemporal exchange must be immune to the possibility that an agent can abandon one consumption-smoothing relationship to begin another with an entrant. We show that intertemporal exchange can be sustained when there are many competitive agents and no third party enforcement whatsoever, that is, under the anarchy that characterizes international relations.[7]

Commitment in the presence of government enforcement is important in domestic credit markets, where loans are frequently collateralized. In a simple loan, a lender completes her obligation by making an initial payment to the borrower in trade for contingent rights to claim collateral. The borrower commits to make payments that are less than the value of the collateral if the government is able and willing to reallocate collateral across agents. Long-term relationships need not be an intrinsic feature of lending in this case.

The wide array of models of sovereign debt can similarly be characterized as dependent on positive or negative awards administered by a third party whose credibility is assumed.[8] For example, in the bargaining model of Bulow and Rogoff [1989a], it is implicit that a third party exists to protect the exporting country from interference in its trade. By taking a "loan", the exporter sells this protection service to the "lender". In this case, the exporter and the lender Nash bargain each period over the amount paid as "protection money" to keep the lender from interfering with the country's trade. Essentially, the lender buys a monopoly franchise to the country's exports by making the initial payment, and "repayment" are the equilibrium surpluses going to the monopsonist each period. If the lender held this right initially, there would be no "loan" and subsequent transactions would be the same, but it would be more obvious that the relationship is one of repeated contemporaneous bilateral trade.

Empirical examples of contemporaneous trade of goods for sanctions are found by Bulow and Rogoff [1989a] in the history of sovereign borrowing. However, evidence of a marked reluctance on the part of lenders or their governments to interfere with a non-performing debtor's trade is found by Eichengreen and Portes [1989b] in the historical record and by Sachs [1989] in the recent experience of Brazil, Ecuador and Peru.

Another motive for repayment, introduced to formal models by Eaton and Gersovitz [1981], is the possibility of interference with a country's intertemporal trade through an embargo on further loans for smoothing its consumption over a fluctuating income stream. In equilibrium, the agents play standard trigger strategies: any deviation from the equilibrium path of intertemporal trade triggers reversion to permanent autarky. These punishments, however, create losses for both the lender and the borrower in that they could be better off returning to a new equilibrium foresaking punishment by mutual agreement.

When there are at least two potential parties to smooth the borrower's consumption, all of them must participate in punishment of the borrower to maintain credibility of the threat. If intertemporal exchange can be sustained, this requires that other lenders forego a share of the gains from trade to punish a recalcitrant borrower. Bulow and Rogoff [1989b] argue that reputation alone cannot work when there are other potential lenders. The borrower can simply abandon her relationship with one lender when she is required to make a repayment and start up another achieving more surplus with a new entrant. They argue that third party enforcement of lender seniority rights is necessary. In doing so, they assume a commitment opportunity for the new lender: she can commit to make a future payment to the borrower that when that event is realized, she would prefer not to make looking forward in her relationship with the borrower.[9]

We model intertemporal exchange when no agent can force a payment from another, either directly or by appeal to a third party, using an infinitely-repeated game with a finite number of agents. In the absence of commitment opportunities, we impose two criteria on the equilibria of this game. The first of these is that any equilibrium must be subgame-perfect, as in most sovereign debt models. Each party is free to choose her best strategy after any history of play. As in many repeated games, there can be a lot of subgame-perfect equilibria for this model, and many pairs of these provide equilibrium payoffs that are Pareto-ranked. In some of these, intertemporal exchange is enforced by punishment threats, such as permanent autarky, that give all agents lower payoffs that they realize on the equilibrium path itself. Without commitment, the possibility arises that the agents can collectively decide to abandon one subgame-perfect equilibrium for another after someone deviates.

In the literature on repeated games, the recognition of this possibility has lead to notion of renegotiation-proofness.[10] The idea behind renegotiation-proofness is that players decide to follow a particular self-enforcing equilibrium by negotiating at the outset. Since they can negotiate at the beginning, it seems plausible that they can renegotiate later after any history. The issue that the various definitions in the literature tackle is that if a punishment is not credible when renegotiation is possible then any equilibrium path that must be enforced by it becomes unviable.

We adopt a particularly strict notion of renegotiation-proofness to show that efficient subgame perfect equilibrium outcomes can be sustained when any mutually beneficial renegotiation is allowed. This is strong subgame perfection, defined by Rubinstein [1980]. There are three reasons for our choice. The first is that because we are addressing the claim that credible reputational punishments do not exist, we want to impose the most stringent requirement for credibility that is available. The second is that there are many competing notions of renegotiation-proofness in the game theory literature, but strong subgame perfection satisfies all of them so our claim that intertemporal barter is possible holds for any existing concept of renegotiation-proofness for this model. The third is that strong subgame perfection is defined for n-person games, so our equilibrium is also proof to renegotiation by a coalition formed by a subset of the agents. Lastly, we show that a strong subgame perfect equilibrium exists for our game; one of the reasons for subsequent definitions of renegotiation-proofness appearing is that in many games none exists.[11]

We demonstrate that an equilibrium exists for the model of intertemporal barter such that the equilibrium path and punishments are all efficient (subject to subgame-perfection). These paths differ in their allocation of the surplus from the consumption-smoothing relationship. Our punishments of the participants in a smoothing relationship have a simple and appealing interpretation as moratoria on resource transfers to the deviating agent until she cooperates in a new efficient equilibrium path that yields all of the surplus to her counterpart. In the presence of free entry by many potential lenders, all of the initial surplus is captured by the borrower. Our punishments of a potential lender who does not observe punishment of the borrower should she deviate consist of symmetric treatment: other lenders induce her to deviate from the equilibrium established with the new lender. These punishments can be complex but have a simple intuitive interpretation. Other agents are expected to cheat a cheater; they will do so simply because it makes them better off. This type of punishment has indeed been observed in trading relations, for example, by Grief [1993] for the case of the Maghribi traders of the late Medieval period.[12]

The next section of the paper presents the model and notation. The discussion of equilibrium for the model is then divided into two parts. Section 3 demonstrates that a strong perfect equilibrium exists and gives some of the properties of the renegotiation-proof punishments for the two-agent case with a single lender. Section 4 discusses the dynamics of payments and the division of surplus along the equilibrium path for the two-agent model. To simplify things, we set up the model so that the equilibrium dynamics can be taken directly from an existing model of implicit wage contracts. In Section 5, we describe the punishment dynamics, which are quite different from the trigger strategies assumed in previous models. In Section 6, we then construct punishments for a strong perfect equilibrium that support bilateral intertemporal exchange when there are many potential competitive lenders with no external means of enforcement.

Section 7 discusses the implementation of the long-term equilibrium relationship using short-term contracts. Without commitment, short-term contracts suffice for the same reasons as in Rey and Salanie [1990] and Fudenberg, Holmstrom and Milgrom [1990]. With complete information and common knowledge in the model, every agent knows the equilibrium being followed so that contracts are unnecessary. However, the model suggests the hypothesis that in the absence of exogenous enforcement mechanisms, a contract between two parties may serve as a tombstone advertisement, informing other agents of the equilibrium strategies chosen by them, thereby enabling third parties to recognize default.

2 Model

We use a simple model of an infinite-horizon economy in discrete time in which there are gains from intertemporal exchange. There are two types of infinite-lived agents and a single non-storable good. Each agent receives an endowment of the good in each period. For simplicity, we assume that one agent is risk-averse and has a stochastic endowment stream. There are $J \geq 1$ risk-neutral agents, each of whom receives a constant endowment stream. For convenience, the risk-averse agent is called the borrower, and each risk-neutral agent is called a lender. The endowment received by every agent in any given period, as well as all past and present actions, are common knowledge.

An agent can give part or all of her endowment to others, but no other agent can force her to make such a transfer either through her own action or by appeal to an external authority. There is no third party to enforce agreements between agents, so that neither the borrower nor any lender can commit to make a future transfer from her endowment to any other agent. Each agent can always choose to consume her entire endowment.

The borrower's preferences over consumption streams are assumed to be represented by the following function:

$$U^0 = E\sum_{t=1}^{\infty}\beta^t u(c_t^0)\ , \tag{1}$$

where $u(c_t^0)$ is increasing, strictly concave and continuously differentiable, and $0 < \beta < 1$. The expectation is taken with respect to the distribution of consumption plans, $(c_1^0,\ c_2^0,\ \ldots)$, conditional on information available on date 1. The borrower's endowment is observed at the beginning of each period, before any transfers or consumption take place. The preferences for each lender can be represented by:

$$U^j = E\sum_{t=1}^{\infty}\beta^t c_t^j\ , \tag{2}$$

where c_t^j equals consumption in period t by lender j and the expectation is again taken with respect to date 1 information. For simplicity, we have assumed that the discount factor, β, is the same for all agents.

We assume that the borrower's endowment each period is a random variable with a stationary Markov distribution. This can depend on past realizations of the endowment through the most recent one, but is independent of the past actions of any agent. For our analysis, we require that the borrower's endowment is always risky over the infinite horizon so that she always has a desire to smooth her future consumption. To simplify the model, we assume that the support of the distribution of the endowment is fixed and finite consisting of $N > 1$ values, labelled in increasing order, $y^1 < y^2 < ... < y^N$. The endowment of the borrower at date t is y_t. The distribution of y_t^n is given by $p^n(y_{t-1})$, for $n = 1, 2, ..., N$ and$t = 1, 2, ...$ We call the history of endowment realizations through date t, $\omega_t \equiv (y_1, y_2, ..., y_t)$, the history of nature or the event at date t. We assume that the endowment for each lender equals y^N in every period.[13]

By assumption any transfer of part of an agent's endowment is voluntary, so that at any time she can choose to consume her endowment in every period forevermore. We define the surplus for an agent under a given consumption plan as the difference between the utility she realizes from the plan and the utility achieved under permanent autarky. At time t, the borrower realizes the surplus

$$V_t^0 = [u(c_t^0) - u(y_t)] + E_t \sum_{i=1}^{\infty} \beta^i [u(c_{t+i}^0) - u(y_{t+i})] \tag{3}$$

from the consumption plan, $(c_t, c_{t+1}, ...)$, and each risk-neutral agent realizes the surplus

$$V_t^j = \tau_t^j + E_t \sum_{i=1}^{\infty} \beta^i \tau_{t+i}^j \, , \tag{4}$$

where $\tau_t^j = (c_t^j - y^N)$ is the net transfer received by agent j in period t.

The assumption that there are no commitment opportunities implies that $V_t^i \geq 0$, for each i, $0 \leq i \leq J$, and $t \geq 1$.

2.1 Game Representation of the Model

This model can be represented as a repeated game with $J + 1$ players. We label the risk-averse borrower as player 0 and the risk-neutral lenders as players $j = 1, 2, ..., J$. At each date (stage), the borrower chooses an action which is the vector of length J of (gross) transfers that she makes to each lender. Each transfer must be non-negative and the sum cannot exceed y_t. The transfer made by the borrower to lender j is denoted a^{0j}, and the action for the borrower is denoted $a^0 \equiv (a^{01}, ..., a^{0J})$. The borrower's set of feasible actions in the stage game played on date t is the simplex

$$A^0(y_t) = \{a^0 \in \mathbf{R}^J : \sum_{j=1}^{J} a^{0j} \leq y_t, a^{0j} \geq 0, \quad for \quad j = 1, 2, ..., J \}.$$

For the case of a single potential lender, this is just the interval, $[0,\ y_t]$.

An action for each lender is also a J-vector of tranfers to the other agents, each non-negative and summing to y^N. A transfer from lender j to agent $i \neq j$ is denoted by a^{ji}. The action space is analogous to that for the borrower with y_t replaced by y^N. For the case with one lender, it is the interval $[0, y^N]$.

The stage-game payoff to the borrower is equal to the single-period surplus she receives which is

$$\pi^0(a_t) = u(y_t - \sum_{j=1}^{J} a^{0j} + \sum_{j=1}^{J} a^{j0}) - u(y_t) \ ,$$

and the stage-game payoff to each lender is equal to

$$\pi^j(a_t) = \sum_{i \neq j} a^{ij} - \sum_{i \neq j} a^{ji} \ , \quad for \quad j = 1, ..., J,$$

where a_t is the entire vector of actions for all the players, $(a_t^0,\ ...,\ a_t^J)$.

The single-shot game has a unique Nash equilibrium in which no player makes a transfer to any other.

The infinite-horizon game resembles a repeated game, but, strictly speaking, it is not a repeated game unless the borrower's endowment is iid. At any stage in an infinitely-repeated game, the remainder of the game is identical to the original game at date 1. In this model, the payoffs and feasible actions for all the players in any continuation of the game after date t depend on the current state, y_t. However, the analysis of infinitely-repeated games can be applied to this extended repeated game because the principles of dynamic programming apply. For simplicity, we will simply refer to this as a repeated game.

We introduce some notation for the repeated game. For a profile of actions taken in a feasible event, ω_t, at date t, we use the notation $a_t = a(\omega_t) = (a^0(\omega_t),\ ...,\ a^J(\omega_t))$ to indicate that the action taken by each agent at date t is contingent on the event ω_t. Even though the set of feasible actions for each agent depends only on the current endowment, we allow for the possibility that the action taken at time t may be conditioned on past states of nature as well. A path, s, is a sequence of event-contingent action profiles, one for each possible event at each date $t : s \equiv \{a(\omega_t)\}_{t=1}^{\infty}$.

The history of actions for the game after t periods is given by $h_t \equiv (a_1, a_2, ..., a_t)$. The set of feasible histories of actions, $H(\omega_t)$, depends on the event at date t. A strategy for either player determines the action that she takes in each event, ω_t, as a function of the histories of both actions and nature before date t, (h_{t-1}, ω_{t-1}).[14] σ^i denotes a strategy for agent i. A strategy profile, σ, is the vector of strategies for all the agents, $\sigma \equiv (\sigma^0, ..., \sigma^J)$. A strategy profile is the set of rules that determine the actions taken by each agent in every possible contingency.

A given strategy profile generates a path, s, that the agents begin to follow on date 1. This path is followed until at least one of them deviates by taking an action in some event different from that specified by s. If and when someone deviates, the profile prescribes a new path for all the agents to follow beginning in the next period. If an agent deviates from this new path, then the profile specifies yet another path for the agents to follow from then on. Following convention, any path initiated after deviation by at least one agent from an ongoing path is called a punishment. Initial paths will be labelled using s and punishments using q. In general, punishments depend on who deviated and on the history of actions and event when the deviation took place. The strategy profile depends on the history of actions because it determines what path is being followed on a given date.

Our description is completed by defining payoffs in the repeated game. The payoff for player j at date t is the surplus over permanent autarky she realizes from her consumption plan under the path generated by the strategy profile, σ, given the history (h_{t-1}, ω_{t-1}) in the state of nature, y_t. Noting that the surplus, V_t^j, is a function of a path, s, and event ω_t, we have that

$$\begin{aligned} V_t^i &= V^i(s, \omega_t) = \pi^i(a(\omega_t), y_t) + E_t\left[\sum_{t'=1}^{\infty} \beta^{t'-t} \pi^i(a(\omega_{t'}), y_{t'}) | \omega_t\right] \\ &= \pi^i(a(\omega_t), y_t) + \beta\, E\, V^i(s, \omega_{t+1}) \,, \end{aligned}$$

and use the new notation

$$\Pi^i(h_{t-1}, \omega_{t-1}, y_t; \sigma) = V^i(s, \omega_t)$$

to define the payoff in the repeated game, where s is the path generated by the profile σ in the history, (h_{t-1}, ω_{t-1}).

3 Equilibrium with Two Agents

Before discussing the bilateral intertemporal exchange relationship, we define the equilibrium concept used to model the outcomes of repeated negotiations in the general case. Because none of the agents can commit to future courses of action, strategies are first restricted to form subgame-perfect equilibria. In a subgame-perfect equilibrium, each agent i chooses the strategy that maximizes her payoff, Π^i, in the game remaining after any feasible history taking the strategies of all other agents as given. Hereafter, we shorten subgame-perfect to perfect for convenience.

One perfect equilibrium for this game is permanent repetition of the unique Nash equilibrium for the stage-game: nobody ever gives anybody else anything. In general, there will be many other perfect equilibria. There are Pareto gains from risk sharing in this economy, so that feasible actions, contingent on the state of nature, exist that give each agent a higher expected payoff on every date than they would realize under permanent autarky. The well-known folk theorems for infinitely-repeated games imply that the model possesses perfect equilibria sustaining any feasible payoffs preferred by all the agents to permanent repetition of the Nash equilibrium if the discount factor is a large enough number less than one.

In many of the perfect equilibria of this model, intertemporal exchange is enforced by punishment threats, such as permanent autarky, that give all agents lower payoffs than they achieve in the path itself. Without commitment, the possibility arises that a group of some or all of the agents could collectively decide to abandon one perfect equilibrium for another after somebody deviates from the equilibrium path. Recognizing that the traders in our model can renegotiate the equilibrium at any time to their mutual benefit, just as they are able to negotiate the initial equilibrium, we restrict the equilibria to those that are renegotiation-proof, using strong perfection as defined by Rubinstein [1980].

Under this notion of renegotiation-proofness, any perfect equilibrium path is a candidate for renegotiation away from an ongoing path or punishment. Subsequent notions of renegotiation-proofness are not as strict in that they allow as candidates for renegotiation only paths that are themselves renegotiation-proof by the same criteria. When a strong perfect equilibrium exists, it satisfies every other definition of renegotiation-proofness in the current literature. Therefore, by showing that at least one strong perfect equilibrium exists, we demonstrate that intertemporal exchange is possible for any definition of renegotiation-proofness. We use strong perfection for refinement because the definition is straightforward and imposing a strict renegotiation requirement strengthens the argumemt that reputation without commitment can sustain intertemporal exchange in a model without information imperfections.

We turn to formalities and define a strong perfect equilibrium for our model. First, we define a strong perfect equilibrium for the case of only two agents, a borrower and a single lender. A strategy profile is a strong perfect equilibrium if after every history, there is no alternative perfect equilibrium that sustains a Pareto superior pair of payoffs for the two agents. This implies that the payoffs achieved in any punishment associated with a strong perfect equilibrium are on the Pareto frontier of the set of all payoffs sustained by all the perfect equilibria for the continuation of the game.

For the case of more than two players, a strong perfect equilibrium is a perfect equilibrium such that after every possible history no coalition of players can raise the payoff for each of its members by choosing another perfect equilibrium strategy profile that keeps the strategies of all the players outside the coalition fixed. Formally, a strategy profile σ is a strong perfect equilibrium if for all non-empty coalitions, C, formed from the set of agents, $\{0, 1, ..., J\}$,

$$\Pi^i(h_{t-1}, \omega_{t-1}, y_t; \sigma) \geq \Pi^i(h_{t-1}, \omega_{t-1}, y_t; \sigma')$$

for each $i \in C$ and every strategy profile σ' satisfying the restriction that $\sigma'_j = \sigma_j$ for all $j \notin C$.

To find a strong perfect equilibrium for the bilateral case, we begin by characterizing the sets of all perfect equilibrium paths and payoffs for the two-agent version of the model. The results of Abreu [1988] provide a simple way to determine these. Abreu proves that any (pure-strategy) perfect equilibrium path can be generated by a strategy profile that punishes agent i for deviating by switching to an equilibrium path that gives agent i her worst perfect equilibrium payoff under assumptions satisfied by this model.[15] The lowest possible equilibrium payoff for any agent is zero in this repeated game in any state of nature. Therefore, the set of all perfect equilibrium paths, including the renegotiation-proof punishments we seek, can be found using as punishments any perfect equilibrium paths that give a deviant agent zero surplus.

In any perfect equilibrium strategy profile, the equilibrium path, $s = \{a(\omega_t)\}_{t=1}^{\infty}$, and punishments, denoted q_{t+1}^i, must satisfy the following inequality for every agent i in every possible event, ω_t, for all $t \geq 1$:

$$\pi^i(a(\omega_t), y_t) + \beta\ E\ V^i(s, \omega_{t+1}) \geq \max_{a^i \in A^i(y_t)} \pi^i(a^i, a^{-i}(\omega_t), y_t) + \beta\ E\ V^i(q_{t+1}^i, \omega_{t+1}) \ ,$$

where $a^{-i}(\omega_t)$ denotes the vector of actions taken in $a(\omega_t)$ by all agents other than i and q_{t+1}^i is the perfect equilibrium path that starts at time $t+1$ if agent i deviates at time t. This just says that q_{t+1}^i is a sufficient threat to keep agent i from deviating from the initial path s in event ω_t. The q_{t+1}^i are paths of event-contingent action profiles that begin the period after agent i deviates, and, in general, can be very complicated. Similar inequalities must hold whenever the ongoing path is a punishment due to an earlier deviation.

When all punishments give zero surplus to a deviating agent, this inequality simplifies to

$$\max_{a^i \in A^i(y_t)} \pi^i(a^i, a^{-i}(\omega_t), y_t) \leq \pi^i(a(\omega_t), y_t) + \beta\ E\ V^i(s, \omega_{t+1}) \ . \tag{5}$$

The left-hand side of (5) is maximized by choosing $a^i = 0$, that is, by making no payments to any other agent. The right-hand side of inequality (5), which is $V^i(s, \omega_t)$, must be non-negative for any equilibrium path s. When there is a single lender, this implies that inequalities (6) and (7) must hold for the borrower and the lender, respectively, for every perfect equilibrium path in each ω_t.

$$u(y_t + a^1(\omega_t)) - u(y_t) \leq u(y_t + a^1(\omega_t) - a^0(\omega_t)) - u(y_t) + \beta\ E\ V^0(s, \omega_{t+1}) \tag{6}$$

$$a^0(\omega_t) \leq a^0(\omega_t) - a^1(\omega_t) + \beta\ E\ V^1(s, \omega_{t+1}) \ . \tag{7}$$

[16]

Since $V^i(s, \omega_{t+1}) \geq 0$ for each agent i in every equilibrium path s, the inequality for agent $i's$ payoff will be satisfied whenever $a^i(\omega_t) = 0$. The punishment binds only if $V^i(s, \omega_t)$ equals zero which requires that $a^i(\omega_t) > 0$. There can be many equilibria in which simultaneous positive transfers are made by the two agents. Inequalities (6) and (7) imply that any such equilibrium path can be replaced by another not involving simultaneous positive payments but using one-way positive payments equal to the net payment achieved by any pair of simultaneous transfers. All of the consumption paths and payoffs possible under subgame perfection can be supported by equilibria using only unilateral transfers. We adopt this restriction to simplify our exposition and to emphasize that intertemporal exchange is the essence of any perfect equilibrium path other than permanent autarky. From here on, all transfers are implicitly unilateral unless otherwise noted.

The following result is straightforward to prove (see Appendix):

Proposition 1 *(a) For each state of nature, y^n, the set of all payoffs sustained by some perfect equilibrium, W^n, is non-empty, compact and convex. (b) Its Pareto frontier,$V^0 = F(V^1, y^n)$, is decreasing in V^1 and contains as its endpoints, two points given by $(\bar{V}^0(y^n), 0)$ and $(0, \bar{V}^1(y^n))$, where $\bar{V}^0(y^n)$ is the maximum of V^0 over W^n and $\bar{V}^1(y^n)$ is the maximum of V^1 over W^n.*

At any date, the set of all perfect equilibria for the remainder of the game depends only on the state of nature since y_t is a Markov random variable. The folk theorems for repeated games imply that there exists a value $\hat{\beta} < 1$ such that whenever the discount factor β is larger than $\hat{\beta}$, W^n includes pairs of payoffs that are positive for both agents. Figure 1 portrays the set of all perfect equilibrium payoffs in a state of nature y^n for any n, $1 \le n \le N$, for $\beta > \hat{\beta}$. The optimality principal of dynamic programming implies that any perfect equilibrium path generating payoffs on the frontier of W^n at date t also generates payoffs on the frontier of W^k at date $t+1$, for each state $k = 1, ..., N$.

Existence of a strong perfect equilibrium follows from Proposition 1. Because the Pareto frontier of the set W^n contains the corner payoffs, $(\bar{V}^0(y^n), 0)$ and $(0, \bar{V}^1(y^n))$, there are (different) efficient perfect equilibrium paths that give either one of the agents zero surplus, the payoff she would receive under permanent autarky. Any perfect equilibrium path on the Pareto frontier can be supported using paths that sustain these payoff pairs as punishments in place of permanent autarky.[17]

Let $\hat{q}_t^0$ denote a perfect equilibrium path sustaining the payoff pair $(0,\ \bar{V}^1(y_t))$ at date t and $\hat{q}_t^1$ denote the equilibrium path sustaining $(\bar{V}^0(y_t),\ 0)$ at date t, for$y_t = y^1, ..., y^N$. $\hat{q}_t^i$ is an efficient perfect equilibrium path for the continuation of the repeated game beginning at date t that gives agent i zero surplus at date t in every possible state, y_t. Therefore, the actions taken by both agents under $\hat{q}_t^i$ are contingent on the history of nature from date t onwards but do not depend on the history of nature before date t, so that $\hat{q}_{t+1}^i$ is just $\hat{q}_t^i$ shifted forward one period.[18]

The punishments $\hat{q}_t^0$ and $\hat{q}_t^1$ are efficient perfect equilibrium paths that give the borrower and lender, respectively, all of the surplus from the bilateral consumption-smoothing relationship. These punishments differ from permanent autarky whenever $\beta > \hat{\beta}$, since $\bar{V}^0(y_t)$ and $\bar{V}^1(y_t)$ are greater than zero in that case.

Either party's surplus must be non-negative in every event at every date. This implies that whenever an agent receives a net payment, her surplus on that date must be positive. To assure, for example, that $\hat{q}_t^0$ gives the borrower zero surplus at date t, the borrower must not receive any payment from the lender until after she has made some payment to the lender. The borrower must make a payment to offset the present value of her future positive surplus. For the lender's surplus, $\bar{V}^1(y_t)$, to be positive under $\hat{q}_t^0$, the borrower must make a positive payment to the lender in some event after date t before the lender makes any new transfers to the borrower.

Define $\hat{\sigma}$ to be any strategy profile of the following form:

(i) beginning at $t = 1$, any efficient perfect equilibrium path s is initiated,

(ii) if agent i unilaterally deviates at any time $t > 1$ from the ongoing path, then switch to $\hat{q}_{t+1}^i$,

(iii) if both agents deviate simultaneously at $t > 1$, then continue on the present path.

Note, that if agent i deviates from the punishment $\hat{q}_{t'}^i$ at time $t > t'$, then $\hat{q}_{t+1}^i$ begins at $t+1$– the punishment restarts.

This strategy profile has an appealing interpretation. For a sufficiently patient lender and borrower pair ($\beta > \hat{\beta}$), the borrower and lender make positive payments to each other in different events following the initial equilibrium path to at least partially smooth the borrower's consumption. Either party to the relationship makes a positive payment only because she anticipates receiving return payments in the future that compensate for her lower current consumption in present value. If the borrower, for example, deviates at some date t by not making an equilibrium payment, then the lender will not make any payments to the borrower starting on date $t + 1$ until after the borrower pays the lender the amount required under a punishment $\hat{q}_v^0$ for some $v \geq t + 1$.

This is because the punishment keeps being reinitiated until the borrower cooperates in her own punishment by making the first payment to the lender. Punishments of the form $\hat{q}_v^0$ imply that the lender imposes a moratorium on payments to the borrower lasting until the borrower makes the (state-contingent) payment to the lender that gives the lender all the surplus from a new intertemporal exchange relationship. The borrower does not gain in equilibrium by deviating from $\hat{q}_{t+1}^0$ so that a moratorium should be short-lived as she cooperates in the new consumption-smoothing relationship starting in date $t+1$. The path $\hat{q}_{t+1}^0$ is a credible punishment in the sense that no alternative perfect equilibrium path gives the lender a higher payoff. The lender can only make herself worse off, reducing her current consumption as well as her present value surplus, if she refrains from punishing the borrower. Punishment of the lender operates analogously.

From the results of Abreu [1988], this strategy profile is a subgame perfect equilibrium for our model.[19] As constructed, the payoff pair for every feasible history of actions, including play off the equilibrium path, is Pareto optimal within the set of subgame perfect equilibrium payoffs for the repeated game. This establishes the following result:

Proposition 2 *The strategy profile $\hat{\sigma}$ is a strong perfect equilibrium for the infinite-horizon repeated game that exists for all β, $0 < \beta < 1$.*

In the profile $\hat{\sigma}$, the participants cannot negotiate after any history of nature or actions to switch to another perfect equilibrium without making one of them worse off. In this model, the set of strong perfect equilibria coincides with the set of strongly renegotiation-proof equilibria as defined by Farrell and Maskin [1989]. But there are many other renegotiation-proof equilibria. In fact, the entire set of perfect equilibrium payoffs can be achieved using perfect equilibria that are weakly renegotiation-proof (by the definition of Farrell and Maskin) or consistent bargaining equilibria (as defined by Abreu, Pearce and Stacchetti [1991]).[20]

The strong perfect equilibrium is a bargaining equilibrium for this repeated intertemporal exchange economy. It is an equilibrium for continual recontracting in a sequential exchange relationship where any renegotiation must benefit each participant in equilibrium.[21] It is not the Nash bargaining equilibrium for the extensive form game used by Rubinstein [1982] to model the division of surplus in simultaneous exchange. Here, a transfer of something of value is made in only one direction on any date. It is exchanged for a future payment offered without commitment. In constrast, the strategic Nash bargaining equilibrium models (sequential) negotiation that ends with the simultaneous exchange of something of value from each party. The example of the next two sections illustrates an essential difference between sequential exchange and simultaneous exchange: the division of the surplus in an equilibrium of the form $\hat{\sigma}$ varies with the event and history of actions.

Before proceeding, we note that punishments giving a deviating agent her lowest equilibrium payoff may not be necessary to support all perfect equilibrium paths, although in general these will be needed to support efficient ones. A special case arises when an unconstrained Pareto optimum (complete smoothing of the borrower's consumption over states of nature) can be supported in a perfect equilibrium. This is possible when the common discount factor, β, satisfies $1 > \beta > \bar{\beta}$, where $\bar{\beta}$ is the smallest β that satisfies inequalities (8) and (9) for some perfectly-smoothed consumption level c and each y^n.

$$(u(c) - u(y^n)) + E\left[\sum_{t=1}^{\infty} \beta^t (u(c) - u(y_t)) | y_0 = y^n\right] \geq 0 \tag{8}$$

$$(y^n - c) + E\left[\sum_{t=1}^{\infty} \beta^t (y_t - c) | y_0 = y^n\right] \geq 0 \tag{9}$$

It is easy to check that both (8) and (9) can be satisfied with strict inequality if y_t is iid for large enough $\beta < 1$. In that case, some Pareto-optimal equilibrium can be supported by punishments that give positive surpluses to both agents. Note that if we define $\hat{\beta}$ to be the lowest β for which outcomes other than permanent autarky can be sustained by some perfect equilibrium, then $\bar{\beta} > \hat{\beta}$.[22]

4 Dynamics of the Equilibrium Path with Two Parties

We next portray the dynamics of equilibrium paths for our renegotiation-proof equilibria for the two-agent economy using an example. This will help us to characterize the equilibrium punishments in the next section and explain self-enforcement with many potential lenders in Section 6. Under our assumptions, an efficient equilibrium path can be derived as the solution to a dynamic programming problem. In the case that the endowment of the risk-averse agent is iid, this programming problem has been solved by Thomas and Worrall [1988] in their analysis of self-enforcing wage contracts. In their model, a risk-averse worker's opportunity spot wage is stochastic and an implicit wage contract serves to smooth the worker's consumption over an infinite horizon. In their model, Thomas and Worrall assume trigger strategy punishments are used by the firm and worker to enforce the contract. In this section, we summarize the derivation of efficient perfect equilibrium paths for our model and use the results of Thomas and Worrall to explain how things work in an iid example.

Our proof of Proposition 1 implies that the set of all perfect equilibrium consumption paths for the borrower is convex and the efficient frontier of the set W^n is strictly concave, for y_t generated by a stationary Markov process. Furthermore, the efficient frontier of payoffs is continuously differentiable on the interior of its domain, and the consumption plan (and, consequently, the path of net payments) sustaining any particular payoff pair on the frontier in perfect equilibrium is unique for each state of nature.[23] As a result of these facts, the dynamics of efficient equilibrium paths of payments can be found by solving the following dynamic program for all $t \geq 1$:

$$F(V^1(\omega_t), y_{t+1}) = \max\{[u(c(\omega_t) - u(y_t)] + \beta\ E[F(V^1(\omega_{t+1}), y_{t+1})|y_t]\} \tag{10}$$

subject to

$$[y_t - c(\omega_t)] + \beta\ E[V^1(\omega_{t+1})|y_t] \geq V^1(\omega_t) \tag{11a}$$

$$F(V^1(\omega_{t+1}), y_t) \geq 0\ , \quad for\ y_{t+1} = y^1, ..., y^N \tag{11b}$$

$$V^1(\omega_{t+1}) \geq 0\ , \quad for\ y_{t+1} = y^1, ..., y^N \tag{11c}$$

The maximum is taken with respect to $c(\omega_t)$ and $\{V^1(\omega_{t+1})|y_{t+1} = y^1, ..., y^N\}$, and the initial surplus for the lender must satisfy $V^1(y_1) \in [0, \bar{V}^1(y_1)]$. The net transfer from the borrower to the lender is just the difference, $y_t - c(\omega_t)$.

For any feasible initial division of surplus, $V^1(y_1)$, this problem solves for a unique consumption path for the borrower.[24] The first-order and envelope conditions for an interior solution to this program yield the following N Euler conditions, one for each value of $y_{t+1} \in \{y^1, ..., y^N\}$:

$$u'(c(\omega_t)) = u'(c(\omega_t, y_{t+1}))(1 + \varphi(\omega_t, y_{t+1})) - \psi(\omega_t, y_{t+1}), \tag{12}$$

where $\beta\, p^n(y_t)\varphi(\omega_t, y^n) \geq 0$ for each $n = 1, ..., N$ are the multipliers associated with each of the N constraints (11b) and $\beta\, p^n(y_t)\psi(\omega_t, y^n) \geq 0$ for each $n = 1, ..., N$ are the multipliers associated with N constraints (11c). Note, that the notation $\omega_{t+1} \equiv (\omega_t, y_t)$ is used.

This implies that consumption is smoothed between dates, across states of nature, unless one of the perfection constraints (11b or 11c) is binding for the next period. If one of the constraints is binding, then consumption is as close to equal across dates as possible. The Euler equation implies that consumption in period $t+1$, $c(\omega_{t+1})$, depends on consumption in period t, $c(\omega_t)$ and on the distribution of y_{t+1} (hence on y_t for a Markov distribution) as well as on the realization of y_{t+1}. Therefore, in general the entire history of nature, ω_{t+1}, determines equilibrium consumption in period $t+1$ given the initial division of surplus between the two agents. As Thomas and Worrall show, when y is iid, period $t+1$ consumption depends on both consumption in period t and y_{t+1} if at least one of the self-enforcing constraints is binding in some state. When none is binding, consumption is fully smoothed and, so, independent of the state of nature at every date.

Associated with each of the endpoints of the efficient frontier of W^n, $\bar{V}^0(y^n)$ and $\bar{V}^1(y^n)$, are consumption levels for the borrower, $\bar{c}(y^n)$ and $\underline{c}(y^n)$, respectively, for each $n = 1, ..., N$. These depend only on the state of nature y^n. An efficient equilibrium path starting in state y^n with the borrower consuming $\bar{c}(y^n)$ gives the payoff pair $(\bar{V}^0(y^n), 0)$, and an efficient equilibrium path starting in state y^n with the borrower consuming $\underline{c}(y^n)$ sustains the payoff pair $(0, \bar{V}^1(y^n))$. If $y_t = y^n$, for some n, then $\underline{c}(y^n) \leq c(\omega_t) \leq \bar{c}(y^n)$ if $c(\omega_t)$ is part of a solution for the dynamic program (10). This follows from Proposition 1, uniqueness of consumption path and the assumption that the stochastic process is Markov. The proof is a straightforward extension of the proof given by Thomas and Worrall for their model with an iid spot wage.

By letting y_t be iid, we can adopt all of the results proved by Thomas and Worrall. They prove that the upper and lower bounds on the risk-averse agent's consumption satisfy the following conditions:

$$\underline{c}(y^1) < \underline{c}(y^2) < ... < \underline{c}(y^N)\ , \qquad \bar{c}(y^1) < \bar{c}(y^2) < ... < \bar{c}(y^N)\ ,$$

$$\underline{c}(y_1) = y^1\ , \quad \bar{c}(y^N) = y^N\ , \quad and \quad \underline{c}(y^n) \leq y^n \leq \bar{c}(y^n)\ , \quad for\ each\ n.$$

When some consumption-smoothing is possible, the lower and upper bounds for the borrower's consumption are not equal and y^n is stictly between them for all states other than y^1 and y^N. That is, in any of the middle states, either party can make some positive payment to the other and still realize positive surplus over permanent autarky. The proofs given in Thomas and Worrall [1988] can be extended without difficulty to the case of a Markov endowment process displaying first-order stochastic dominance.[25]

The Euler conditions imply dynamics for the borrower's consumption in an efficient equilibrium path as follows. If state y^n occurs in period $t+1$ and $c(\omega_t)$ satisfies $\underline{c}(y^n) \leq c(\omega_t) \leq \bar{c}(y^n)$ (whatever the value of y_t), then $c(\omega_{t+1}) = c(\omega_t)$, where $\omega_{t+1} = (\omega_t, y^n)$. If $c(\omega_t) < \underline{c}(y^n)$, then $c(\omega_{t+1}) = \underline{c}(y^n)$, and if $c(\omega_t) > \bar{c}(y^n)$, then $c(\omega_{t+1}) = \bar{c}(y^n)$. These dynamics imply that the borrower's surplus at date t rises if the efficient equilibrium path is changed to one giving her higher date t consumption. For the same change, the lender's surplus at date t falls.

Thomas and Worrall also demonstrate the folk theorem for their model by showing that there is a $\bar{\beta} < 1$ such that for all $\beta \geq \bar{\beta}$, $\underline{c}(y^N) \leq \bar{c}(y^1)$. That is, using the Euler equations, full smoothing of the risk averter's consumption is possible in some efficient equilibrium if both agents are sufficiently patient. They also prove that there is a $\hat{\beta} > 0$ for this type of model such that if $\beta \leq \hat{\beta}$, then the only equilibrium is permanent autarky.

Figure 2a illustrates an equilibrium path for an example economy with iid borrower endowment and three states of nature. The vertical bars portray the range for the borrower's consumption under all efficient perfect equilibrium paths for each state. The distance from c_t to the 45° line equals the payment made by or to the borrower. The arrow paths show how consumption evolves. In this example, the equilibrium path starts in state y^1 with all of the surplus going to the borrower. State y^2 occurs next, followed in sequence by y^3 and y^2 again. This illustrates that consumption is Markovian even if endowments are iid when at least one perfection constraint is binding. That is, when the inability of either party to commit is binding, there is uninsurable risk and consumption smoothing is incomplete. That consumption is smoothed as much as possible from one period to the next is a direct consequence of Bellman's principle for dynamic programming.

Efficient paths in the example of Figure 2a converge to a unique stationary state (with probability one in finite time) consisting of the four points, $(y^1, \bar{c}(y^1))$, $(y^2, \bar{c}(y^1))$, $(y^2, \underline{c}(y^3))$ and $(y^3, \underline{c}(y^3))$.

The second point is reached when $y_t = y^2$ if $y_{t-1} = y^1$ or $y_{t-1} = y^2$ and $c_{t-1} = \bar{c}(y^1)$, while the third occurs when $y_t = y^2$ if $y_{t-1} = y^3$ or $y_{t-1} = y^2$ and $c_{t-1} = \underline{c}(y^3)$.

The example also shows how the borrower's and lender's share of the surplus for the continuation of the repeated game vary over time. The borrower realizes all of the surplus from the efficient perfect equilibrium path at the start. That is, all of the surplus available from the intertemporal exchange relationship. At date 3, the lender receives all of the surplus for the continuation of the game. The division of the surplus in any event, ω_t, depends on both ω_t and the initial division of the surplus for an efficient perfect equilibrium path.

The example of Figure 2a assumes a common discount factor β between $\hat{\beta}$ and $\bar{\beta}$. If we let β rise, the bars drawn will lengthen. Once β reaches $\bar{\beta}$, $\bar{c}(y^1)$ will equal $\underline{c}(y^N)$ and the stationary state for any equilibrium path will be completely smoothed. Figure 2b illustrates a sample equilibrium path under our profile $\hat{\sigma}$ for $\beta > \bar{\beta}$. The path begins at date 1 in the highest state, y^3, with all of the surplus realized by the borrower: $V^1_{t=0} = 0$. The borrower's first-period consumption is y^3, and neither party makes a payment at date 1. The borrower's consumption cannot be smoothed at this level because doing so would imply that lender never receives payment, so that along the equilibrium path the borrower's consumption falls over time until state y^1 is realized and $c^0_t = \bar{c}(y^1)$ thereafter. While the borrower's consumption is fully smoothed in the stationary state, it is not completely smoothed beginning at date 1 in this equilibrium. The borrower cannot pay some of her endowment in the first period in state y^3 in exchange for larger payments from the lender in other states because the lender's commitment constraint is binding in state y^1. Any reduction in the borrower's first-period consumption would reduce her utility among efficient perfect equilibria. Full consumption smoothing is not always possible no matter how close to one β is because both parties lack the ability to commitment their future actions. A full Pareto optimum is possible if the surplus in the relationship is divided differently at date 1, for example, if all the surplus goes to the lender in state y^3 at date 1, so that the borrower's date 1 consumption is $\underline{c}(y^3)$. The folk theorems establish the existence of some perfect equilibria generating Pareto optima, but do not imply that all efficient perfect equilibria are Pareto optimal as β approaches one.

5 Dynamics of Punishments in the Two-Agent Economy.

Our next step is to characterize the renegotiation-proof punishments for the strong perfect equilibria of the two-country version of the model. The punishment $\hat{q}^i_{t+1}$ is an efficient equilibrium path that gives the agent who deviated on date t zero surplus for the continuation game at date $t+1$.

An example with iid endowments provides intuition for our argument that these punishments are sensible candidates for credible threats in the application to sovereign debt. Suppose that the borrower deviated from an equilibrium path at date t. For her surplus to be zero at date $t+1$, the borrower's consumption at date $t+1$ must equal $\underline{c}(y_{t+1})$ for every possible realization of y_{t+1} if $\hat{q}^0_{t+1}$ is to be efficient. Since $\underline{c}(y^n)$ is less than or equal to y^n for each n, this implies that the borrower must make a payment at date $t+1$ in all but the lowest state if $\beta > \hat{\beta}$. If $y_{t+1} = y^1$, then the borrower does not make or receive a payment.

Figure 3a depicts a sample path for the punishment $\hat{q}^0_{t+1}$ in the example of Figure 2a. The sequence of realizations for periods $t+1$ through $t+4$ is $\{y^2, y^1, y^3, y^2\}$. The punishment is the efficient equilibrium path starting in date $t+1$ at the point $(y^2, \underline{c}(y^2))$ in Figure 3a. Now, suppose the borrower deviates again, this time from the punishment, by not paying the difference between y^2 and $\underline{c}(y^2)$ at date $t+1$ to the lender. Under our equilibrium profile of the form $\hat{\sigma}$, the punishment restarts as $\hat{q}^0_{t+2}$ giving the borrower zero surplus in the subgame beginning at date $t+2$. Therefore, the best that the borrower can do if she deviates at $t+1$ in state y^2 is to make no payment and consume her endowment, y^2, at $t+1$. The result is that the lender can hold the borrower to zero surplus at $t+1$, so that the borrower has no incentive to deviate from the punishment $\hat{q}^0_{t+1}$.

Figure 3a portrays that the borrower only receives transfers from the lender after making a payment to the lender. It also shows what happens if the borrower repeatedly deviates. She consumes her endowment until she makes a payment that yields all of the surplus to the lender from the initiation of an efficient equilibrium. This gives our interpretation that punishments consist of moratoria on payments to the deviant that last until the deviant cooperates in the efficient punishment. If the borrower follows $\hat{q}^0_{t+1}$ in the example of Figure 3a, the moratorium lasts at most one period. That is, there is a single-period moratorium in $t+1$, if under the equilibrium path in force at time t, the lender would have made a positive payment to the borrower in state y^2 at $t+1$.

If the borrower deviates at $t+2$ as well, the observed moratorium under the sample path would last for periods $t+1$, $t+2$ and $t+3$.

Using the example of Figures 2a and 3a, suppose that the borrower and lender are following an equilibrium path in the stationary state as described already. Note that the punishment $\hat{q}_t^0$ and the equilibrium path coincide if $y_t = y^3$, the highest state. Since the borrower's surplus is zero in this event along the equilibrium path, she could deviate at date t and suffer no loss to her payoff. All of the social loss from deviation from the efficient equilibrium path is borne by the lender. The lender has no incentive to give up any of her surplus at date t, since when it comes time to punish the borrower (if the borrower does deviate) the lender maximizes her surplus and current consumption by imposing the moratorium.

The lender's punishment for any deviations from an ongoing path is analogous. Under $\hat{q}_{t+1}^1$, the lender pays the borrower the amount $(\bar{c}(y_{t+1}) - y_{t+1})$ at $t+1$. If she deviates from this punishment at date $t+1$, then $\hat{q}_{t+2}^1$ starts. The interpretation is that the borrower puts the lender under a payments moratorium until she receives a payment from the lender that gives up all the surplus from that date forward to the borrower.

The example of Figure 3a illustrates our earlier point that worst strong perfect equilibrium punishments are needed if the discount factor is too low for the borrower's consumption to be completely smoothed in any perfect equilibrium of the repeated game. Figure 3b illustrates our earlier argument that renegotiation-proof punishments do not need to leave the deviant with zero surplus for $\beta > \bar{\beta}$ using the example of Figure 2b . Suppose that the initial path for the strong perfect equilibrium gives all of the surplus to the borrower at date 1 as in Figure 2b. One efficient perfect equilibrium punishment is to provide the borrower with a constant consumption level equal to c^* as shown.

6 Self-enforcement with Multiple Potential Entrants

So far, our focus on the two-party model restricts the concept of renegotiation to agreements between those parties. In practice, markets such as that for international lending have more than two participants. In this section, we extend our argument that a strong perfect equilibrium exists for intertemporal exchange under anarchy to the case of multiple potential lenders. We need to show that the punishment of the borrower can be enforced when it is possible for another lender to start up a new intertemporal exchange relationship with the borrower. The strong perfect equilibria we describe will be coalition-proof.[26]

To motivate our strategy profile, suppose that there are two lenders and the borrower is following an efficient perfect equilibrium path with lender 1. Assume the borrower deviates at date t. Adapting the punishment $\hat{q}^0$ to the three player game, lender 1 imposes an embargo on payments to the borrower until she pays an amount that gives up all of the surplus in a new efficient equilibrium path. Under this embargo, one can imagine that lender 2 offers to start a consumption-smoothing path with the borrower that gives the borrower positive surplus in every state by asking her to pay less on the first date. If lender 1 stays with the embargo, then the borrower and lender 2 can play one of the $\hat{\sigma}$ equilibria in the induced two-player game. If this behavior is formalized in a strategy profile for the three-player game, a consumption-smoothing path could never get started because the first lender would never be repaid for any transfer she makes.

In extending the equilibrium to the $J+1$ agent case, we need to specify punishments of the interfering lender as well as punishments of the borrower. Our proposed strategy profile will include punishments of each potential lender if she deviates from an ongoing punishment of the borrower. For a strong perfect equilibrium, our punishments are more complicated to describe with multiple lenders than with one. We sketch a strategy profile in which the punishment used depends on how and in what history an infraction occurred.[27] Our construction begins with the perfect equilibrium path s in which the borrower and lender 1 follow an efficient intertemporal exchange path and lenders $j = 2, \ldots, J$ make and receive no payments any period. If the borrower deviates at time t, the punishment $\hat{q}^0_{t+1}$ for the $J+1$ agent equilibrium is imposed. This is the efficient path in which lenders $j \geq 2$ make and receive no payments but the payments by the borrower and lender 1 to each other are the same as in $\hat{q}^0_{t+1}$ for the two-agent version. If the borrower alone deviates from her punishment, it starts again as before.

A punishment of any lender $j \geq 2$ who deviates from $\hat{q}^0_{t+1}$ is derived as follows. A deviation occurs only if lender j makes a payment to the borrower. Lender j will do this in equilibrium only if she anticipates receiving a payment in return in some future event. When such an event occurs, it is time to punish agent j. Let this be time t'. Suppose that the first lender offers the borrower a new efficient path s' beginning at date t' with a smaller positive payment from the borrower to lender 1 than she owes lender j, giving the borrower more surplus. The borrower cannot commit to refuse this offer once made, and the first lender will make it because it raises her surplus. In $q^j_{t'}$, the punishment of agent j, the only payments after date t' are made between lender 1 and the borrower. The strategy profile for the subgame reached at date t' has the same form as the one for the repeated game at date 1 but generates the equilibrium path s'.

In a strategy profile formalizing these actions, no lender other than the first can ever be assured of payment from the borrower. As a consequence, none of them will ever make a payment to the borrower, so that in turn the borrower will never make a payment to another potential lender in a subgame reached by her deviation. If lender 1 adopts the strategy described, then the only perfect equilibrium of the induced repeated game for any coalition of agents excluding lender 1 is permanent autarky. For any coalition that includes the borrower and lender 1, an efficient perfect equilibrium is possible.

We offer a proof of the following proposition in the appendix:

Proposition 3 *There exists a strong perfect equilibrium for the repeated game with $J+1$ agents, for $J \geq 1$.*

Lender 1 can be punished for deviating from an ongoing path using an analogous punishment to $\hat{q}^1_{t+1}$ or by allowing any other lender to start a new efficient equilibrium path generated by the strong perfect equilibrium profile, in the period after lender 1 deviates, that gives all the surplus to the borrower. The equilibrium paths described thus far are unique up to the initial division of the surplus in the relationship. With multiple potential lenders, we assume that the initial path gives zero surplus on date 1 to the first lender, consistent with free entry.

We can interpret this equilibrium as follows. At date 1, some lender makes a payment to the borrower that exhausts the lender's surplus from intertemporal exchange. In doing so, she obtains a monopoly franchise on future intertemporal exchange with the borrower. This is self-enforcing even though there are other potential lenders because no agent can ever commit to make a particular payment in the future.

Proposition 3 demonstrates existence of an efficient coalition-proof equilibrium, not uniqueness. For example, an alternative could use similar forms of punishment but allow that whenever an event is reached such that the lender's equilibrium surplus is zero, a new lender takes over.

Our result contrasts with the claim of Bulow and Rogoff [1989b] that the threat of non-cooperation alone cannot support lending and repayment in an infinitely repeated game of smoothing a sovereign's consumption. In their proof, they assume that lenders can commit, so that they can offer contracts that obligate them to make future payments in exchange for a current payment from the borrower.[28] Under this assumption, a defecting lender in our model could become an insurer who takes payments from the risk-averse party (the borrower in our model) in return for a credible promise to make indemnity payments that, at the time of payment, give her negative surplus from the relationship. This defection is profitable for the lender, and it would cause the initial lender to realize negative surplus from any initial loan. In a model with asymmetric commitment opportunities, intertemporal trade to smooth the risk-averter's consumption is possible, but it can be initiated only by a payment from the party that cannot commit to the party that can.[29]

Our model of intertemporal barter under anarchy assumes that both sides of the market have symmetric lack of capacity for commitment, so we directly affirm that reputation alone can sustain intertemporal exchange including cases where the initial payment flows to the party whose consumption is smoothed, as observed in lending to sovereign states. Note also that reputation refers purely to past actions in this model; under common knowledge there is no need for an agent to signal her type via actions to sustain intertemporal barter. Incomplete information, introduced for example by assuming a borrower type that values honesty for its own sake (as assumed by Cole and Kehoe [1992]) is not needed to construct a reputational model of sovereign borrowing.[30]

7 Implementation using Short-term Contracts

The equilibrium paths of transfers between the borrower and a lender can be interpreted as the equilibrium outcome of lending and repayment using simple debt contracts subject to renegotiation and sovereign risk. Simple debt contracts specify an initial loan and subsequent repayments (principal plus interest). For international loans, the explicit terms of repayment are not generally followed in equilibrium, in contrast with the case for many domestic lending relationships.

The designation of collateral is typical for domestic lending in developed market economies. In a simple contract, the lender's obligation is discharged at the initiation of the loan. If a borrower who defaults loses collateral of greater value than the repayment, adherence to the explicit conditions of the loan is subgame perfect; renegotiation is not an issue. This requires the existence, capability and commitment of a third party to allocate collateral contingent on debtor performance. If these conditions are fulfilled, lender commitment is moot, and borrower commitment can be induced by third party enforcement.

International loans are different because sovereign immunity limits third party enforcement of explicit contracts via the international allocation of collateral. Our model captures lending and repayment between sovereigns by assuming that no party can commit to make payments that leave her negative surplus looking forward at any date along an equilibrium path.

A popular tactic in the literature has been to reverse the balance of commitment in the collateralized loan contract by assuming that the lender always commits to fulfill any contractual obligations (as would be plausible if the lender's obligations were collateralized). In two-party models, such collateralization make the threat of reverting to autarky credible as a punishment of a deviant borrower. Such commitments can support financial relationships of the type discussed by Grossman and van Huyck [1988] and interpreted with explicit dynamics by Worrall [1990].[31] Atkeson [1991] extends this analysis to an infinitely repeated game of repeated moral hazard in which a lender cannot observe actions taken by the borrower.

In models with more parties, the possibility of lender commitment can break the incentive for the borrower to reciprocate by rendering the threat of punishment incredible as in Bulow and Rogoff [1989b]. Even if lending cannot occur, the possibility of international insurance remains with the "borrower" paying in advance as in a conventional insurance contract. In all of these papers, insurance is part of the contract offered by a lender because she can commit to make payments in future events. Expected profit may be non-negative at the outset, but in some future event, her surplus can become negative (looking forward from that event).[32]

Grossman and van Huyck [1989] argue that the renegotiation of simple debt contracts may be part of an implicit state-contingent contract. They suggest that debt contracts need only specify the largest repayment made in equilibrium over possible states of nature when the borrower cannot commit. When lender commitment is feasible, by definition, there must be some third party to enforce lender obligations. This requires an explicit contract to inform third parties of the commitments undertaken by lenders. In our model, there is no exogenous party to enforce any commitments, so that with a single lender and borrower, no explicit contract is needed; explicitness, after all, is for third parties. Because any mutually beneficial renegotiation is possible, constrained by subgame perfection, our punishments are not grim reversions to autarky.

With free entry by multiple potential lenders and no exogenous enforcement of commitments, an explicit simple debt contract could play the modest role of identifying the lender who makes the initial transfer accepted by the borrower and of disclosing terms of repayment. A perfect equilibrium for an extensive-form game is commonly interpreted as an implicit contract negotiated at the outset between all players. In the strong perfect equilibria constructed for this model, all lenders but the first simply need to know that an efficient bilateral relationship was formed so that they know that there is nothing to be gained by making payments to the borrower. The other lenders do not need to participate in the negotiation of the implicit contract: they just need to be informed of the bilateral relationship so that they cooperate in equilibrium. A simple debt contract might serve to publicize the relationship between the borrower and her lender as a "tombstone" advertisement. This publicity facilitates cooperation by the other potential lenders by making seniority common knowledge. In common parlance, such cooperation is called "respect for seniority," meaning that other lenders will not deal with the borrower until her obligation to the initial lender has been discharged.

In contrast, much of the earlier literature assumes exogenous enforcement of lender seniority by the lenders' governments: any payments to a junior lender by a borrower in default are reallocated by force of law to the senior lender. Under the asymmetric commitment opportunities assumed in Bulow and Rogoff [1989b], seniority enforcement must extend to other financial transactions (in particular, the cash-in-advance insurance transactions) if international lending is to occur.

The equilibrium path of net payments for intertemporal barter in our model can be replicated using one-period loan contracts with state-contingent repayments. In the absence of commitment, long-term relationships can be supported in this model using a sequence of one-period contracts, as in the model of long-term agency relationships analyzed by Fudenberg, Holmstrom and Milgrom [1990]. Longer maturity contracts are unenforceable when they specify behavior that cannot be supported by a sequence of one-period contracts.

The one-period contracts consist of a loan, ℓ_t, and repayment schedule, $R_{t+1}(y_{t+1})$, for each $y_{t+1} = y^1, ..., y^N$. ℓ_t is given by

$$\ell_t(y_t) = c(\omega_{t-1}, y_t) - y_t - R_t(y_t) \ .$$

The lender's surplus at date t is given by

$$V^1(\omega_{t-1}, y_t) = R_t(y_t) + E[\sum_{i=t+1}^{\infty} \beta^{i-t}(-\ell_i + \beta \ R_{i+1}(y_{i+1}))|y_t] \ .$$

Along the equilibrium path, any potential lender earns zero expected profit, so that

$$-\ell_i + \beta \ R_{i+1}(y_{i+1}) = 0 \ , \quad \textit{for all } i \geq t \ , \textit{ and}$$

$$V^1(\omega_{t-1}, y_t) = R_t(y_t) \ , \quad \textit{for all } t \geq 1 \ .$$

The equilibrium repayments are always non-negative, since lender surplus is non-negative in every event. In general, further insurance will be desirable, so that allowing lenders to commit will lead to negative repayments in equilibrium.

The state-contingent repayments may be interpreted as the outcomes of renegotiations of a simpler contract specifying ℓ_t and $R_t^* = \max\{R_t(y_t)\}$. The assumption of lender non-commitment is the same as the assumption that lenders are only willing to negotiate repayments down to zero and make net resource transfers that earn non-negative expected profits. Since lender surplus is the sum of the current net transfer from the borrower and the discounted surplus in the continuation, R_t^* will exceed the net resource transfer made by the borrower in any event for date t in all but exceptional cases. The renegotiation of R^* may appear in accounting schemes as debt write-downs, reschedulings, or new loans (not to be confused with net resource transfers) without changing the equilibrium path of net transfers in any way.[33]

8 Conclusion

Borrowing and lending between sovereigns can be modelled as intertemporal trade without exogenous enforcement of commitments. The surplus in a consumption-smoothing relationship by itself provides sufficient incentive for cooperation by all participants and potential entrants along the equilibrium path and in punishments if an agent has deviated. The explicit or implicit assumption in other models of sovereign debt that a third party is available to enforce commitments, including respect for lender seniority or monopoly rights in commodity trade, is not essential to sustain lending to sovereigns.

Renegotiation is captured in two senses by our equilibrium. The first is that used in repeated games: any equilibrium can be renegotiated to choose another. The second is the renegotiation of an incomplete formal contract (for example, a one-period debt contract) to fulfull an implicit long-term contract. The equilibrium path specifies the net transfer of resources in each event of nature that could result from the renegotiation of sequences of formal short-term contracts. In the application to sovereign debt, the equilibrium provides the observable net payments between countries and derives punishments that prevent defection in the absence of exogenous enforcement, even in the presence of multiple competitive lenders.

By modelling renegotiation-proof intertemporal exchange, this paper captures the essence of credit transactions without collateral. In contrast, the "constant recontracting model" of Bulow and Rogoff [1989a] portrays repeated simultaneous commodity trade in a bilateral monopoly. In that model, a "loan" is the one-time payment for a monopoly franchise to the purchase of a country's exports, and a "repayment" is the monopsonist surplus gained in simultaneous exchange each period. In our model, a unilateral transfer from one party to another is made each period, and the lender and borrower make payments at different dates throughout the relationship. This relationship is permanent even though it may be guided by formal short-term contracts that often appear violated.

Our results show that intertemporal barter under anarchy is feasible without appealing to the threat of exogenous force (Hirshleifer [1995], *p.* 28). The punishments we proposed to ensure that potential entrants do not interfere with the borrower-lender relationship incorporate the ethic, "cheat the cheater." That is, if the borrower deviates and makes a payment to a new entrant, then the new lender is punished only if she reciprocates. Any new lender maximizes her utility in equilibrium by cheating the deviant borrower if she gets the chance. A prominent example of this type of ethic is observed by Grief [1989] in the records of Maghribi traders in the Mediterranean in the 11th century.

This model could be extended to a number of credit situations in which third party enforcement is lacking or incredible and there are many borrowers and lenders. Examples might include informal credit markets in poor rural economies, medieval trade and interenterprise credits in the ex-Soviet-sphere republics.

Appendix

Proof of Proposition 1:

(a) The set W^n is non-empty because it always includes the origin. Compactness can be proven by application of Theorem 4 of Abreu, Pearce and Stacchetti [1990]. Their proof, based on the notion of self-generation, are written under the assumption that the action space is finite at each stage. Careful inspection of their proof reveals that the Theorem is valid for a discounted game of perfect information when the action space for each player is a compact interval and the stage-game payoffs are continuous. Compactness of the set W^n for each n follows from Theorem 4.[34] An alternative proof of compactness follows from application of Tychonoff's Theorem.

Convexity could also be proved by application of their Theorem 5, although it is simpler to show that the set of perfect equilibrium paths for any initial state y^n is convex. Suppose that s and s' are two equilibrium paths. The convex combination of s and s' is given by $s^\lambda \equiv \{a^\lambda(\omega_t)\}_{t=1}^{\infty}$ where $a^\lambda(\omega_t) = \lambda a(\omega_t) + (1-\lambda)a'(\omega_t)$ for every ω_t and $0 \le \lambda \le 1$. Since $u(c)$ is concave, $V^0(s^\lambda, \omega_t) \ge \lambda\, V^0(s,\omega_t) + (1-\lambda)V^0(s',\omega_t) \ge 0$. Also, $V^1(s^\lambda,\omega_t) = \lambda\, V^1(s,\omega_t) + (1-\lambda)V^1(s',\omega_t) \ge 0$. Therefore, s^λ is a perfect equilibrium path. Since $u(c)$ is concave, convexity of W^n follows.

(b) That the Pareto frontier of W^n is decreasing is straightforward (simply reduce $a^0(y^n) - a^1(y^n)$ in any efficient path providing positive surplus to each agent). Together with compactness and non-emptiness, this assures that there are points in W^n, $(\bar{V}^0(y^n), 0)$ and $(0, \bar{V}^1(y^n))$, such that $\bar{V}^i(y^n)$ is the maximum of $V^i(y^n)$ over W^n.

Proof of Proposition 2:

For the $J+1$ person repeated game, the set of payoffs sustainable using subgame perfect equilibria given initial state y^s is given by

$$\begin{aligned}\{(V^0, V^1, ..., V^J) | V^0 &= \nu^0, \sum_{j=1}^{J} V^j = \nu^1,\ and\ V^j \ge 0,\ for\ all\ j = 1,\ ...,\ J, \\ for\ (\nu^0, \nu^1) &\in W^n\},\end{aligned}$$

where W^n is the set of all perfect equilibrium payoffs for the two-person repeated game, defined in Section 3. To demonstrate the proposition, we construct a strong subgame perfect equilibrium that sustains the efficient perfect equilibrium payoff $(\bar{V}^0,\ 0,\ 0,\ ...,\ 0)$. The initial path is given by $s = \{a(\omega_t)\}_{t=1}^{\infty}$ such that the $J+1$ -tuple$a(\omega_t) = (a^0(\omega_t), a^1(\omega_t), 0, ..., 0)$ where $(a^0(\omega_t),\ a^1(\omega_t))$ is the efficient path of unilateral transfers for the single lender and borrower economy that sustains the initial surplus pair, $\bar{V}^0$ and 0. The punishments $\hat{q}^0_{t+1}$ and $\hat{q}^1_{t+1}$ are also extended to the $J > 1$ case by setting all payments made by or to lenders $j \neq 1$ equal to zero in every event.

For the punishment $\hat{q}^0_{t+1}$ to be an equilibrium path for a subgame reached by a deviation from an initial efficient perfect equilibrium path, it is necessary that the borrower's surplus can be held to zero in this subgame. Take a given perfect equilibrium strategy profile σ for the $J+1$ players. Suppose that under σ a subgame can be reached (at date $t+1$) in some history (h_t, ω_t), for $t > 1$, such that the equilibrium action for the borrower is to make a positive payment to some lender $j \neq 1$ (or collection of lenders excluding lender 1) in period $t+1$ and the equilibrium (infinite-horizon) payoff for the borrower is positive. This is the type of subgame that needs to be handled.

Suppose lender 1 adopts the following change of her strategy, $\hat{\sigma}^1$. She will follow her part of an efficient perfect equilibrium path that begins on date $t+1$, $\{a^1(\omega_{t+i})\}$ for $i \geq 1$, as long as the borrower follows the same path of actions, $\{a^0(\omega_{t+i})\}$. The chosen path begins with a non-negative payment by the borrower to lender 1 that leaves the borrower with higher payoff than she would receive under σ in equilibrium for this subgame. The existence of such a path follows from Proposition 1. If the borrower deviates for any $i \geq 1$, then lender 1 follows her path of actions under $\hat{q}^0_{t+i+1}$. Lender 1 reinitiates this punishment each period that the borrower deviates from it until any subgame is reached that given $\sigma_{-1} \equiv \{\sigma^0, \sigma^2, ..., \sigma^J\}$, the strategy profile for all players other than lender 1, the borrower pays another lender in the equilibrium path for the subgame. In such a subgame the lender follows a strategy analogous to that described thus far. The payment by the borrower to lender 1 and particular efficient perfect equilibrium path to be followed by lender 1 thereafter depend on σ_{-1}. In histories such lender 1 deviates, her strategy is to take the actions required of her under $\hat{q}^1_{t+i+1}$.

The equilibrium response of the borrower in the subgame (h_t, ω_t) taking $\{\sigma^2, ..., \sigma^J\}$ as given is to deviate from the equilibrium path under σ by making the payment $a^0(\omega_{t+1})$ to lender 1 and no payments to lenders 2 through J. With lender 1's strategy chosen as $\hat{\sigma}^1$, the borrower will never make a positive payment to some other lender. Given $\hat{\sigma}^1$, the other lenders, $j = 2, ..., J$, will choose strategies such that in this subgame, they never make positive payments to the borrower. The only perfect equilibrium in the induced continuation subgame for the borrower and lenders 2 through J given lender 1's strategy as $\hat{\sigma}^1$ is permanent autarky. None of these J players will ever make an equilibrium payment to another. The equilibrium actions for the borrower in the subgame are given by the path $\{a^0(\omega_{t+i})\}$ for $i \geq 1$. This strategy for lender 1 assures that in equilibrium for every possible subgame for $t > 1$, no other lender will ever make a positive to the borrower, assuring efficient punishments that give the borrower zero surplus if she deviates are coalition-proof. This assures that the borrower and lender 1 can start an efficient equilibrium path at date 1. In any possible subgame under a profile $\hat{\sigma}$, the equilibrium path is an efficient perfect equilibrium path for the continuation game. The profile is not unique. Notably, whenever lender 1's surplus is zero is equilibrium a new lender could take over.

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Notes

[1]Lindert and Morton [1989] examined 1552 external bonds of ten borrowing governments (approximately the top ten borrowers over the past thirty years) outstanding in 1850 or floated between then and 1970, following all through to settlement or the end of 1983. Defaults were not only common but widespread in their sample; most of the countries had some defaults in each of the periods 1820-1929 and the 1930s (*p.* 61). A detailed summary of experience by country is presented in their Table 2.8.

[2]Eichengreen and Portes [1989b] examined 125 London overseas issues and a sample of 250 United States foreign issues floated in the 1920's. (Nearly half of latter, by value, lapsed into default (*p.* 233)). In their samples, British bonds had an overall internal rate of return of 5 percent, higher than domestic investments (Eichengreen and Portes [1989a, *p.* 77], while United States loans to national governments had an internal rate of return of 4.6 percent, compared to the 4.1 percent yield on United States treasury bonds over the 1920s (pp. 35, 38). These yields were, however, substantially below those offered ex ante, which were generally between 7 and 8 percent (*p.* 27). Overall, the bonds in the Lindert and Morton [1989] sample proved profitable; the average 2 percent ex ante premium over domestic government bonds became a 0.42 percent permium ex post (*p.* 77). Further, they find (*p.* 59), that "there is no clear evidence of a systematic difference in realized returns" between the onds of their ten borrower governments and United States domestic coporate bonds.

[3]Eichengreen and Portes [1989b, *p.* 234] report that, in their 1920s samples, "The typical default reduced the internal rate of return by 4.3 percent for dollar loans, but 1.4 to 2.3 percent for sterling loans." They note, for example, that all sterling loans to Brazil in that period went into default, but they yielded positive internal rates of return between 1.1 and 2.3 percent.

[4]See Eichengreen and Lindert [1989].

[5]For an early expression of this view, see Wallich [1943]. The term "excusable default" is due to Grossman and van Huyck [1988]. The idea that defaults might not always violate the underlying equilibrium relationship helps explain the findings of Lindert and Morton [1989] and Eichengreen [1989] that defaulters have not generally suffered subsequent discrimination in credit terms, and also the finding of Ozler [1988] for loans from 1968-81 that the average penalty for past defaults was only a small fraction of interest spreads.

[6]Examples of other models that adopt a consumption-smoothing motive for international financial flows include Atkeson [1991], Craig [1991], Grossman and van Huyck [1988], Kehoe and Levine [1993], Kletzer [1989 and 1994] and Worrall [1990]. Cole and English [1992] study expropriation of equity investment in a consumption-smoothing model, and Kletzer, Newbery and Wright [1992] study loan, futures and options contracts for international smoothing in the presence of sovereign risk.

[7]There are many models of sovereign borrowing that study the possibility a reputation for repayment can sustain international lending. These include models of infinite-horizon relationships with complete information, such as Eaton and Gersovitz [1981], Kletzer [1984, 1989], Eaton, Gersovitz and Stiglitz [1986], Grossman and van Huyck [1988], Worrall [1990], Atkeson [1991] and Kehoe and Levine [1993] that all assume some form of exogenous enforcement. Other authors model reputations using games of incomplete information. Examples include Cole and Kehoe [1992], Cole, Dow and English [1994] and Gale and Hellwig [1989]. Our model of anarchy in international relations is related to Hirshleifer [1995]. In particular, see p. 27. In contrast to Hirshleifer's generic model, we assume implicitly that fighting is ineffective for appropriating international resources, as is true if Hirshleifer's "decisiveness parameter" is zero.

[8]These include the enforcement of trade sanctions and of creditor seniority privileges either explicitly or implicitly assumed.

[9]Both Eaton [1990] and Chari and Kehoe [1993] make the point that the sovereign borrower can appeal to an external authority to enforce her loans to the party identified as the lender in Bulow and Rogoff [1989b] while the lender cannot enforce loans to borrower.

[10]A number of authors have proposed definitions of renegotiation-proofness in infinitely and finitely repeated games. Important definitions and results on renegotiation-proofness in infinitely-repeated games for our analysis are given by Farrell and Maskin [1989], Abreu, Pearce and Stacchetti [1991], Evans and Maskin [1989] and Bernheim and Ray [1989]. Farrell [1984] introduces the concept of renegotiation-proofness developed by Farrell and Maskin [1989] and Pearce [1987] introduces the competing approach taken by Abreu, Pearce and Stacchetti [1991]. A brief survey of the literature is given in Fudenberg and Tirole [1991], section 5.4.

[11]Another reason for alternative definitions is the argument that this is too strong a definition of renegotiation-proofness, ruling out equilibria that might survive the possibility of renegotiation to other credible (that is, renegotiation-proof) equilibria.

[12]Our approach to modelling credible punishment of sovereigns differs from the analysis of sanctions of Eaton and Engers [1992] in two essential ways. The first is that they model a bilateral relationship and so are not concerned with our main issue — the problem that new entrants might benefit by not cooperating in a punishment. The second is that they study Markov perfect equilibria of a game in which the power to sanction is exogenous to borrowing and lending, as in Bulow and Rogoff [1989a], rather than subgame perfect equilibria of a game in which the incentives to cooperate derive from the surplus internal to the intertemporal smoothing relationship.

[13]By assuming that the endowment of each lender equals the upper bound of the borrower's endowment, we assure that there are enough resources each period for any single lender to fully smooth the borrower's consumption over time. This is merely a simplifying assumption that allows us to concentrate on the role of the inability of agents to

commit for incomplete consumption smoothing without the obvious effects of global resource constraints.

[14]As stated, this defines pure strategies: an element of the vector σ^i identifies a feasible action at date t for agent i with each possible history for date t, (h_{t-1}, ω_t). Mixed strategies could be formed, but our notation and exposition anticipate that equilibria for this model will exist in pure strategies.

[15]Specifically, the following conditions are satisfied by the stage game: the (finite-dimensional) action space for each agent is compact, the payoffs for each agent are continuous and an equilibrium in pure strategies exists.

[16]In the two-agent case, the actions for each are a scalar.

[17]More generally, these punishments can be used to support an equilibrium providing any payoff pair in the interior of the set in Figure 1.

[18]That is, the action profile, a_{t+v}, taken at date $t+v$ under $\hat{q}_t^i$ depends on the realization of $(y_t, ..., y_{t+v})$, so that a_{t+v} is the same function of a $v+1$ -length vector for every $t \geq 1$.

[19]In Abreu's terminology, this is a simple stategy profile. Part (iii) is a convenient and simple choice that could be replaced in a number of ways. The essential result of Abreu [1988] is that any perfect equilibrium path can be generated using a profile that gives the worst perfect equilibrium payoff (minmax payoff) to any deviating player.

[20]It can be verified that any perfect equilibrium path can be generated by a simple strategy profile that is weakly renegotiation-proof by using punishments that give a deviating player zero continuation surplus. The geometry of the set W^s assures that such punishments exist that neither Pareto dominate or are Pareto-dominated by the chosen perfect equilibrium path in any feasible event. The same profiles also satisfy the Abreu, Pearce and Stacchetti definition of a consistent bargaining equilibrium.

[21]Abreu, Pearce and Stacchetti [1991] and Abreu and Pearce [1991] argue that equal bargaining power results from the adoption of a Paretian criterion in bargaining situations, as is the case in this model. Abreu, Pearce and Stacchetti [1991] use the term bargaining equilibrium for a renegotiation-proof equilibrium of a repeated game.

[22]Theorem 6 of Abreu, Pearce and Stacchetti [1990] implies that for $0 < \beta_1 < \beta_2 < 1$, W^n for β_1 is a subset of W^n for β_2 (with the modification of their proofs to this model). It could be proved that the correspondence associating each $\beta \in (0, 1)$ with each set W^n is continuous under the convexity and compactness assumptions of this model. Since the set W^n consists only of the origin as β goes to 0 and includes Pareto optimal allocations for β sufficiently close to one, there must be such values $\hat{\beta}$ and $\bar{\beta}$.

[23]These results can be proved in a number of ways, for example, by extending the proof of Lemma 1 of Thomas and Worrall [1988] to the case of a Markov chain.

[24]The borrower's consumption at time t depends on the initial division of surplus, as well as on ω_t. To avoid cumbersome notation, we suppress this dependence and write $c_t^0 = c(\omega_t)$ to denote a consumption path.

[25]First-order stochastic dominance means that $\sum_{k=m}^{N} p^k(y^n)$ for each given $m = 1, ..., N$ is non-decreasing in y^n, for $n = 1, ..., N$.

[26]Bernheim, Peleg and Whinston [1987] propose an alternative, weaker definition of coalition-proofness. That definition is made recursively, so that it applies to finitely repeated games. See Fudenberg and Tirole [1991] for a discussion. We use the stricter concept of a strong perfect equilibrium since at least one exists for our game and it gives a tougher test for the equilibrium to survive.

[27]The strategy profile proposed in the $J > 1$ case is not a simple strategy profile, as defined by Abreu [1988].

[28]In their paper, Bulow and Rogoff note that they make this assumption but write that it is unnecessary for their claim that reputational equilibria alone will not work. See Bulow and Rogoff [1989b], page 45, lines 12-16. Cohen [1991, page 94] makes a similar claim in a consumption-smoothing model. He imposes the constraint that the borrower will just be indifferent in period $t+1$ between autarky and repayment if she repays in period t. Therefore, repaying in period t can only make her worse off. There is a problem: the continuation values are fixed rather than derived from equilibria for the subgames reached, so that his argument does not address whether lending and repayment can be self-enforcing.

[29]Worrall [1990] solves for the efficient smoothing path in a two-party model under the one-sided commitment assumption made by Bulow and Rogoff [1989b] when y_t is iid. Since he does not allow for free entry by other potential lender-insurers, enforcement is not an issue. Kletzer, Newbery and Wright [1992] show how option contracts can be used in combination with one-period loans to approximate Worrall's efficient solution.

[30]Cole and Kehoe [1992] pursue the possibility, adumbrated in Bulow and Rogoff [1989b], of a reputational equilibrium in which the borrower is concerned about the implications for her other market relationships of the reputation (the conditional probability that she is "honest") that she establishes in the loan market. Other models of incomplete information include Eaton [1990], Cole, Dow and English [1991] and Thomas [1992].

[31]Grossman and van Huyck [1989] do not solve for the equilibrium of their model, but rather assert that consumption is iid if borrower income is iid. Worrall [1990] shows that consumption is not iid outside a steady state that is reached in finite time with probability one.

[32]Worrall [1990] and Atkeson [1991] explicitly state this assumption. Grossman and van Huyck [1988] are a bit unclear about the assumptions being made, but the description of the model and their analysis are only consistent with the assumption that the lender commits. It is straightforward to show that along the equilibrium paths of the Atkeson and Worrall models the lender's surplus is negative in some event for the general case.

[33]Fernandez and Rosenthal [1990] model the negotiation of a repayment in an extensive-form game model with exogenous default penalty. They model the one-time termination of a debt relationship, rather than renegotiation in a long-term relationship with endogenous incentives. Gale and Hellwig [1989] present a similar model with incomplete information about the borrower's type.

[34]To apply the results of Abreu, Pearce and Stacchetti [1990], the payoffs are first renormalized by multiplying by β. Define the payoff $\beta V^i(a, \nu)$ as $\beta(\pi_i(a, y) + E\nu^n)$. Let W denote the N-tuple of the sets W^n, $\{W^1, ..., W^N\}$, and ν denote the N-tuple of the continuation payoffs, $\{\nu^1, ..., \nu^N\}$. The pair (a, ν) is admissible with respect to W if $\nu^n \in W^n$ and $V^i(a, \nu)$ is maximal with respect to the action $a_i \in A_i(y^n)$, for each agent $i = 0, 1$. The set $B(W)$ defined by Abreu, Pearce and Stacchetti is given by $\{\beta V(a, \nu) | (a, \nu) \text{ is admissible } w.r.t.\ W\}$.