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OPTIMAL CAPITAL ACCUMULATION AND STOCK POLLUTION: THE GREENHOUSE EFFECT

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OPTIMAL CAPITAL ACCUMULATION AND STOCK POLLUTION: THE GREENHOUSE EFFECT

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Abstract. In this paper two models are presented to study the relationship between capital accumulation and stock pollution focusing on the greenhouse effect. In the first model we assume a constant population and we analyze pollution control through choice of capital stock level. In that model CO_2 emissions depend on the stock of productive capital and the stock of CO_2 emissions has a negative effect on production through a damage function. The second is a Harrod-neutral technological progress model of optimal growth with increasing population and emission abatement capital. For an economy with constant population the existence of a steady state with stable emissions is guaranteed under the assumption of concavity of the utility and production functions. For an economy with growing population, the saturation of preferences is necessary although not sufficient.

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1. INTRODUCTION

The aim of this paper is to investigate if a path of economic growth compatible with stable emissions and concentration of greenhouse gases exists. Since the early paper of Nordhaus (1977) about the carbon dioxide (CO_2) problem, a growing literature devoted to the economics of the greenhouse effect has emerged. The main issues studied in that literature have been the estimation of costs of CO_2 abatement and of economic damage of climate change. Only a few papers have developed explicit models of economic growth to address the issue of optimal control of CO_2 emissions.

In studies by Nordhaus (1991) and Tahvonen, von Storch and Xu (1992) climatic variables (emissions, concentration of CO_2 and a differential in temperature) are added to a model of an exponentially growing economy. But these additions represent more a juxtaposition than a real integration of climatic elements with economic elements. For example, it is assumed that economic growth is compatible with a stable concentration of CO_2, in order to analyze the optimal level of emissions[1].

Gottinger (1992) presents an interesting way of evaluating the costs of CO_2 abatement using a model of optimal growth and exhaustible resources (stock of fossil fuel energy) with an exogenous rate of technological progress in resource requirements and labor force growth. The author translates the constraint on CO_2 concentration into a constraint on cumulative rate of extraction for a finite time horizon and compares the optimal growth path when the constraint is operative with the unconstrained case. The result is a tendency to postpone capital accumulation.

In the recent special issue on global warming of the journal ***Resource and Energy Economics***[2] Nordhaus has presented a Dynamic

[1] Falk and Mendelsohn (1993) present a model very similar to Tahvonen, von Storch and Xu's model but from a partial equilibrium approach. The same kind of analysis can be found in Harford (1976) for a flow pollutant with adjustment costs in the emission abatement activity.

[2] See Vol. 15, No. 1, March 1993.

Integrated Climate-Economy (DICE) model that has several characteristics in common with an extended version of the model suggested in this paper. He does not however develop the model theoretically, applying it instead to simulate different policy experiments for controlling greenhouse gases[3].

This lack of a complete, consistent neoclassical growth-theoretic approach to the problem is the motivation for our work. We develop a model which integrates a simplified dynamics for CO_2 concentration and a damage function into a model of optimal growth and we study the existence of steady states of the economy compatible with stable emissions of CO_2. Notice that if such long run equilibria exist, they define the optimal level of emissions and concentration of CO_2 from an economic point of view.

Although our theoretical approach represents a departure in the literature on global climate change in the sense that we focus on damage of global warming to productive activities, and only indirectly on consumption, it is related to a previous literature devoted to the study of optimal capital accumulation and control of stock pollutants[4].

The influential paper of Keeler, Spence and Zeckhauser (KSZ) (1972) contains two models of pollution control. The first is a model of optimal capital accumulation with constant population and stock pollution. For this model they assume that a composite commodity can be allocated to consumption, investment and pollution control. Utility depends positively on consumption and negatively on pollution stock. Emissions serve no productive purpose and their flow is assumed to be a fixed proportion by-product of production. In this model there are at most two steady state equilibria. At one of them, no composite commodity is spent on pollution control (they

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[3] In particular he applies the DICE model to evaluate from a pure economic point of view the following scenarios or policy proposals: no controls, optimal policy (maximization of the present value of economic welfare), ten-year delay of optimal policy, twenty percent emissions reductions from 1990 levels and geoengineering.

[4] See the papers of Keeler, Spence and Zeckhauser (1972), Forster (1972a), Brock (1977), Asako (1980), Becker (1982), Heal (1982) and Tahvonen and Kuuluvainen (1993).

called it the Murky Age Equilibrium). It is characterized by an abundance of capital, a high consumption level and an extreme pollution level. The other steady state (the Golden Age Equilibrium) with positive pollution control exhibits lower capital, consumption and pollution levels. In the second model the emissions have a positive marginal product but they do not take into account the dynamics of the capital stock. In this model a fixed supply of labor is allocated to the production of emissions which serve as an intermediate good, or directly to the production of the consumption good[5].

Forster (1972a) has an interesting analysis of pollution in the context of the neoclassical model of economic growth. He introduces the stock of pollution into the production function with a negative marginal product and defines a differential equation for capital depending on capital and pollution stocks assuming a constant rate of savings. On the other hand, he relates pollution to the use of capital and in this way obtains a second differential equation for stock pollution depending on capital and pollution stocks. Then, he studies whether this system of first order differential equations which drives the economy has a steady state. Under the assumptions of concavity and separability for the production function and convexity for the emission function there will exist a unique steady state for the economy, but in general it will not be optimal.

In a similar model by Gifford (1973), the stock of pollution does not have any economic role but firms have to pay an emissions tax. The model assumes a technical change frontier between pollution abatement and labor augmentation. At each time there is an exogenously given amount of technology-improving resources and the firms allocate them to maximize the instantaneous growth rate

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[5] Forster (1972b) points out that the result of the second model of KSZ that a nonzero level of pollution is optimal depends on the assumption of a disutility of the first units of emissions equal to zero. Forster shows that with positive disutility for the first units of emissions no equilibrium at a nonzero level of pollution exists. However, this condition is not sufficient for this result. Luptáčik and Schubert (1982) extend this model defining an emission function which depends on production, consumption and depreciated capital stock, and introducing a more general expression for abatement expenditures.

of the value of output less the emissions tax bill. This plus a constant savings rate reduces the dynamics of the economy to a system of two differential equations, one for the capital-effective labor ratio and another for the ratio between the emission tax bill and output. The main result is that economic growth with a constant level of pollution is possible through the use of a growing tax on emissions, although growth of per capita output is decreased by the control program[6].

Brock (1977) develops the second model of KSZ taking into account the dynamics of the capital stock and introducing emissions directly into the production function. He obtains a polluted Golden Rule for the case of zero discount (Ramsey's approach) and he shows that there is a unique steady state and a unique optimal path which converges monotonically to it.

Asako (1980) examines the optimal program of economic growth for the first model of KSZ adopting the maximin principle. His results show that the constant utility criterion performs fairly satisfactorily within the framework of the model. Becker (1982) analyzes the regular maximin programs in Brock's model. This results in a constant utility path supported by competitive prices with government imposed effluent charges on emission producers and environmental rental charges on consumers. Moreover, he gives sufficient conditions for a regular maximim path to satisfy a Hartwick rule for intergenerational equity.

Heal (1982) reinterprets the model of Ryder and Heal (1973) as a model of optimal capital accumulation and stock pollution. In his model, emissions are generated by consumption. The solution to the model is very sensitive to the specification of the utility function. The issue is whether in a steady state the gain in utility from an increase in consumption is or is not offset by the loss in utility from the associated increase in the stock of pollution. In the first case, multiple steady states may exist. In the second case, there is a unique steady state which can depend in

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[6] Stephens (1976) presents a variation of the model of Gifford taking into account the possibility of capital augmenting technical progress and introducing emissions in the production function. In the last section of the paper, the model with stock pollution and investment in emission abatement is analyzed.

a complex way on the discount rate, the rate of decay of the pollution stock, and the utility function.

More recently, Tahvonen and Kuuluvainen (1993) have solved the model of Brock for a positive discount rate. They have found that at least one steady state exists if the productivity of capital is not bounded when capital tends to zero. Moreover, if the marginal utility of consumption decreases with or is independent of the stock of pollution then the steady state is unique. Additionally if the rate of discount is small enough the equilibrium point is globally stable for bounded solutions. When emissions are uncontrolled the steady state consumption and capital are higher than the case of optimally controlled pollution and the steady state has the saddle point properties[7].

In this paper we present two models to study the relationship between capital accumulation and stock pollution focusing on the greenhouse effect. In the first model we assume a constant population and analyze pollution control through choice of capital stock level[8]. In our mode CO_2 emissions depend on the stock of

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[7] A number of other papers are less directly related to our subject. Maybe the first model of growth and stock pollution is due to D'Arge (1971). In his paper a simple model of the Harrod-Domar type is used to investigate conditions under which it is possible to grow without affecting environmental quality. D'Arge and Kogiku (1973) formulate a model of optimal growth with stock pollution and investment in recycling machinery and equipment, but they assume that the only use of capital is in the recycling process. In Forster (1973b) and (1977), pollution is a stock variable but capital vanishes from the model. Mäler (1974), in the line of the model of D'Arge and Kogiku, presents a model of optimal growth with stock pollution and recycling in which he incorporates two kinds of capital and scarce exhaustible resources. Forster (1975) extends his (1973b) model to examine the effects of introducing a nonconstant exponential pollution decay rate on the steady state equilibrium. On the other hand Forster (1973a), Converse (1974) and Gruver (1976) elaborate different models of economic growth with flow pollution.

[8] The assumption of a constant population is a standard assumption in this kind of literature. See KSZ (1972), Forster (1972a), Brock (1977), Asako (1980), Becker (1982), Tahvonen and Kuuluvainen (1993), Forster (1973a), Forster (1973b) and Gruver (1976). Heal (1982) does not specify clearly his assumption about population. Ryder and Heal (1973) take into account an exogenously

productive capital and the stock of CO_2 emissions (concentration) has a negative effect on production owing to climate change. This negative effect appears in the model through a damage function that depends on CO_2 concentration[9]. As we focus on the greenhouse effect on total production we assume that utility is only a function of per capita consumption. In the second model, we allow population to grow at an exogenously given rate and introduce emission abatement capital which determines the rate of abatement for a given level of productive capital stock.

Our main findings are that there exists a unique steady state for the economy in the first model under the standard assumption of concavity for utility and production functions and that at least one steady state is possible in the model with growing population and investment in emission abatement if there is saturation of preferences. We establish conditions in which increasing per capita consumption with constant emissions and concentration of greenhouse gases is viable.

For the first model, we compare an optimally controlled economy with an uncontrolled one and find that an economy with uncontrolled emissions has higher steady state capital and concentration of CO_2 levels than the optimally controlled economy but that consumption can be higher or lower. Moreover, we analyze stability properties and the comparative statics of steady states.

The paper is organized as follows. In Section 2 the model of optimal capital accumulation and stock pollution with constant population is presented and its properties analyzed. Section 3

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and exponentially growing labor force, but in a model of optimal growth with pollution, this assumption does not work very well. Notice that if emissions depend on per capita consumption the growth of population decreases 'cæteris paribus' the emission of pollutants. The only clear exceptions are Gifford (1973) and Stephens (1976).

[9] We extend here the model of Forster (1972a), in which the stock of pollution enters the production function with a negative marginal product. We introduce this characteristic through a damage function and assume more general conditions for the production function. Moreover we look for the optimal steady state of the economy.

develops a model of optimal economic growth with abatement investment. Concluding comments and some directions for future research are left to Section 4.

2. OPTIMAL CAPITAL ACCUMULATION WITH CONSTANT POPULATION

We will begin with the definition and characterization of variables, parameters and functions of the model.

Variables:

Y = total production
NY = net production
C = consumption
K = capital stock
I = gross investment
Z = stock of pollution or anthropogenic atmospheric concentration of CO_2 equivalent greenhouse gases (GHGs)
E = anthropogenic emissions of CO_2 equivalent GHGs

Parameters:

ρ = social rate of discount
δ = rate of depreciation of capital goods
b = linear adjustment costs of capital (b>1)
α = fraction of CO_2 equivalent emissions that enter the atmosphere
β = rate of removal of CO_2 equivalent from the atmosphere or rate of decay

We assume that the utility function U(C) and production function F(K) are twice continuously differentiable and strictly concave and that the production function satisfies the Inada conditions[10], i.e.:

$$\lim F(K) = 0, \quad \lim F(K) = \infty \tag{1}$$

[10] With fixed labor supply, production can be represented as a function of the capital stock. These condition apply as well for the utility function.

$K \to 0 \qquad K \to \infty$

$$\lim_{K \to 0} F_K(K) = \infty, \qquad \lim_{K \to \infty} F_K(K) = 0 \tag{2}$$

In addition to these two standard functions, we are going to define the greenhouse damage function and the emission function which relate economic variables to emissions and concentration of CO_2 or stock of pollution. The *greenhouse damage function* is defined as a proportion of total production $D(Z)F(K)$ with $D_Z > 0$ and $D_{ZZ} \geq 0$. For this function, there exists a critical value $\bar{Z}$ such that, for that value, the greenhouse damage is maximum $(D(\bar{Z})=1)$. Then, net production will be given by $NY = (1-D(Z))F(K)$ which is defined on a convex subset X of R^2, $X = \{(K,Z): \bar{Z} \geq Z \geq 0 \text{ and } K \geq 0\}$ and exhibits the following properties:

$$\partial NY/\partial K=(1-D)F_K>0, \qquad \partial NY/\partial Z=-D_Z F<0 \tag{3}$$

$$\partial^2 NY/\partial K^2=(1-D)F_{KK}<0, \qquad \partial^2 NY/\partial Z^2=-D_{ZZ}F<0, \qquad \partial^2 NY/\partial K\partial Z=-D_Z F_K<0 \tag{4}$$

$$-(1-D)FD_{ZZ}F_{KK} \geq (D_Z F_K)^2 \tag{5}$$

which determines that that function is concave. Finally the *emission function* relates economic activity and GHGs emissions. As emissions are an output (not planned) from productive process of economy, a joint product of production, we propose the functional relationship $E=E(K)$ with $E_K>0$ and $E_{KK} \geq 0$.

For these functions the dynamics of capital stock and concentration of CO_2 are given by

$$(1-D(Z))F(K) = C + bI \tag{6}$$

$$I = \dot{K} + \delta K \tag{7}$$

$$\dot{Z} = \alpha E(K) - \beta Z \tag{8}$$

using (6) and (7) we obtain

$$\dot{K} = (1/b)[(1-D(Z))F(K) - C] - \delta K \tag{9}$$

Then the optimal path of capital accumulation and control of CO_2 emissions has to maximize

$$\int_0^{\infty} U(C)\ e^{-\rho t} dt \tag{10}$$

with respect to C subject to dynamic restrictions (8) and (9), initial conditions and usual conditions on control and state variables: $(1-D(z))F(K) - C \geq 0$ and $C,\ K,\ Z \geq 0$[11].

In order to solve the above described optimal control problem, we introduce the multipliers associated with constraints and form the Lagrangian

$$\begin{aligned}\mathcal{L}(K,Z,C,\lambda,\eta) = \mathcal{H}(K,Z,C,\lambda) &+ \eta_1((1-D(Z))F(K)-C)\\ &+ \eta_2 K + \eta_3 Z\end{aligned} \tag{11}$$

where

$$\begin{aligned}\mathcal{H}(K,Z,C,\lambda) = U(C) &+ \lambda_1(\frac{1}{b}((1-D(Z))F(K)-C)-\delta K)\\ &+ \lambda 2(\alpha E(K)-\delta Z)\end{aligned} \tag{12}$$

is the usual Hamiltonian.

The corner solutions $Z^*(t)=\bar{Z}$ and $Z^*(t)=0$ are avoided by the properties of the utility function ($\lim_{C\to 0} U_C(C)=\infty$). In this case

[11] These last constraints imply $\bar{Z} - Z \geq 0$ since if $Z > \bar{Z}$ then C must be negative to fulfill $(1-D)F - C \geq 0$ violating the nonnegative constraint.

necessary conditions for an *interior optimum* are the following[12]

$$bU_C(C) = \lambda_1 \tag{13}$$

$$\dot{\lambda}_1 = (\rho+\delta)\lambda - (\lambda_1/b)(1-D(Z))F_K(K) - \lambda_2\alpha E_K(K) \tag{14}$$

$$\dot{\lambda}_2 = (\rho+\beta)\lambda_2 + (\lambda_1/b)F(K)D_Z(Z) \tag{15}$$

PROPOSITION 1. *If assumptions of the model hold then there exists only one steady state (K^∞, Z^∞, C^∞) characterized by the following 'warm' modified golden rule of capital accumulation*

$$(1-D)F_K = b(\rho+\delta) + [1/(\rho+\beta)]FD_Z\alpha E_K \tag{16}$$

Where the left-hand side is the net marginal productivity of capital and $b(\rho+\delta) + [1/(\rho+\beta)]FD_Z\alpha E_K$ is the marginal cost of capital. The second component of marginal cost represents the marginal damage to production owing to an increase in emissions because of an increase in the capital stock.

Proof. From (15) we obtained $\lambda_2^\infty = -[\lambda_1^\infty/b(\rho+\beta)]FD_Z$ letting $\dot{\lambda}_2=0$. Substituting λ_2 and λ_1 for (13) in (14) and letting $\dot{\lambda}_1=0$ yields

$$0 = U_C\{b(\rho+\delta) - (1-D)F_K + [1/(\rho+\beta)]FD_Z\alpha E_K\} \tag{17}$$

so that, if there exists a steady state with $C^\infty>0$, it will have to satisfy

$$(1-D)F_K = b(\rho+\delta) + [1/(\rho+\beta)]FD_Z\alpha E_K$$

[12] If $E_{KK}=0$ or $\lambda_2<0$ the Hamiltonian is *concave* in (K,Z,C) and then the necessary conditions, together with the transversality conditions, are also sufficient (see Seierstad and Sydsaeter (1987, p.385, Th.11)). Notice that concavity of net production guarantee the quasi-concavity of constraint $(1-D(Z))F(K)-C\geq 0$. This implies that if we find a path $(K^*(t),Z^*(t),C^*(t))$ which satisfies necessary conditions we have found an optimal path. On the other hand, the constraint qualification does not apply for an interior optimum since this requires $(1-D(Z^*))F(K^*)-C^*>0$, K^*, Z^*, $C^*>0$.

which is Condition (16). On the other hand, $\dot{Z}=0$ establishes that $Z^{\infty}=(\alpha/\beta)E(K^{\infty})$ and then Condition (16) can be written as a function of K^{∞}. Then a steady state will exist if there is a value for K that fulfills Condition (16) associated to a positive value for consumption.

As $(1-D)F_K$ is a decreasing function with $\lim_{K\to 0}(1-D)F_K = +\infty$ and $\lim_{K\to\bar{K}}(1-D)F_K = 0$ where $\bar{K} = E^{-1}(\beta\bar{Z}/\alpha)$ and $b(\rho+\delta) + [1/(\rho+\beta)]FD_Z\alpha E_K$ is an increasing function in the interval $[0,\bar{K}]$ with initial value $a(\rho+\delta) < +\infty$, then there will exist only one value $K^{\infty} \in (0,\bar{K})$ which satisfies Condition (16). To prove that K^{∞} has associated a $C^{\infty} > 0$, we suppose that $K^{\infty} \geq K'$ where K' is given by $C = 0 = [1-D(\alpha E(K')/\beta)]\ F(K')-b\delta K'$. Subtracting this last expression from Condition (16), we obtain

$$[1-D[\frac{\alpha}{\beta}E(K^{\infty})]]F_K(K^{\infty}) - [1-D[\frac{\alpha}{\beta}E(K')]]\frac{F(K')}{K'} > 0$$

or

$$\frac{1-D[\frac{\alpha}{\beta}E(K^{\infty})]}{1-D[\frac{\alpha}{\beta}E(K')]} > \frac{F(K')/K'}{F_K(K^{\infty})} \tag{18}$$

and as marginal productivity of capital is decreasing we have $D[\alpha E(K^{\infty})/\beta] < D[\alpha E(K')/\beta]$ from which we conclude that $K^{\infty} < K'$ since $D(\cdot)$ is an increasing function, resulting in a contradiction. Then, K^{∞} must be lower than K' and C^{∞} will be positive.□

The graphics of Fig.1 represent a possible steady state solution to the problem. Notice that as the derivative of (1-D)F is equal to $-D_Z(\alpha/\beta)E_KF + (1-D)F_K$, K^{∞} can be higher or lower than K'' which is the stock of capital which maximizes consumption.

FIG. 1

For this model net production (1-D)F is first increasing and then decreasing because the stock of pollution (concentration of CO_2) is increasing with respect to capital stock in the steady state (see (8)).

To develop the stability analysis of the steady state equilibrium let us derive the modified Hamiltonian dynamic system (MHDS) of the system (8), (9), and (13) - (15). Equation (13) defines C as a decreasing function of λ_1. Denote this by $C = C(\lambda_1)$ with $C_{\lambda 1} < 0$. Then the MHDS is

$$\dot{K}=\frac{1}{b}((1-D(Z))F(K)-C(\lambda_1))-\delta K \tag{19.1}$$

$$\dot{Z}=\alpha E(K)-\beta Z \tag{19.2}$$

$$\dot{\lambda}_1=(\rho+\delta)\lambda_1-\frac{\lambda_1}{b}(1-D(Z))F_K(K)-\lambda_2\alpha E_K(K) \tag{19.3}$$

$$\dot{\lambda}_2=(\rho+\beta)\lambda_2+\frac{\lambda_1}{b}F(K)D_Z(Z) \tag{19.4}$$

The Jacobian of (19) evaluated at the steady state is

$$
J=\begin{bmatrix}
\rho+\frac{\alpha}{b(\rho+\beta)}FD_ZE_K & -\frac{D_ZF}{b} & -\frac{C_{\lambda_1}}{b} & 0 \\
\alpha E_K & -\beta & 0 & 0 \\
\theta & \frac{\lambda_1^{\infty}}{b}D_ZF_K & -\frac{\alpha}{b(\rho+\beta)}FD_ZE_K & -\alpha E_K \\
\frac{\lambda_1^{\infty}}{b}F_KD_Z & \frac{\lambda_1^{\infty}}{b}FD_{ZZ} & \frac{FD_Z}{b} & \rho+\beta
\end{bmatrix}
$$

where $\theta=-\frac{\lambda_1^{\infty}}{b}(1-D)F_{KK}-\lambda_2^{\infty}\alpha E_{KK}$ and the characteristic roots μ_i (i=1,...,4) are the solutions of the corresponding characteristic equation

$$
\mu^4-trJ\mu^3+\psi\mu^2-\phi\mu+|J|=0
$$

where ψ and ϕ are, respectively, the sum of all diagonal second and third order minors of J, and $|J|$ is the determinant of J evaluated at the steady state point:

$$
\begin{aligned}
|J|=&-\lambda_1^{\infty}C_{\lambda_1}FD_{ZZ}\left(\frac{\alpha E_K}{b}\right)^2-\frac{\alpha(\rho+2\beta)}{b^2}\lambda_1^{\infty}E_KF_KD_ZC_{\lambda_1} \\
&+\frac{\beta(\rho+\beta)}{b}C_{\lambda_1}\left(\frac{\lambda_1^{\infty}}{b}(1-D)F_{KK}+\alpha_2\lambda_2^{\infty}E_{KK}\right)>0
\end{aligned}
\tag{20}
$$

since $C_{\lambda 1} < 0$ and $\lambda_2^{\infty} < 0$. This allows us to conclude that either all roots are positive or all are negative or there are two positive roots and two negative, since $|J|=\prod_{i=1}^{4}\mu_i$. As trJ= 2ρ all roots cannot be negative and, as the model does not imply any restriction on ψ and ϕ, the following kinds of equilibria are possible: A) If roots are real then the steady state is an unstable <u>improper node</u> if all roots are positive and an unstable <u>saddle point</u> if two roots are positive and the other two roots are negative. B) If roots are complex then the steady state is an <u>unstable spiral point</u> when the two real parts are positive and an <u>asymptotically stable spiral point</u> if the real part of one root is positive and the real root of the other root is negative.[13]

Obviously, these results are not very conclusive but from the Jacobian different conditions can be obtained to satisfy the necessary and sufficient conditions of Th.3 in Dockner (1985, p.96) and to guarantee in this way the saddle point property of the equilibrium.

We now turn to the analysis of comparative statics of the steady state. This analysis is relatively easy since the steady state value of the capital stock is independently determined by Condition (16), which allows us to do the analysis in a sequential way. Differentiating that condition and ordering terms, we obtain

$$\frac{\partial K^{\infty}}{\partial b}=\frac{\rho+\delta}{\sigma}<0 \tag{21}$$

$$\frac{\partial K^{\infty}}{\partial \rho}=\frac{1}{\sigma}\left(b-\frac{\alpha}{(\rho+\beta)^{2}}FD_{Z}E_{K}\right)\begin{Bmatrix} > \\ = \\ < \end{Bmatrix}0 \tag{22}$$

$$\frac{\partial K^{\infty}}{\partial \delta}=\frac{a}{\sigma}<0 \tag{23}$$

where

$$\sigma=-\frac{\alpha}{\beta}D_{Z}E_{K}F_{K}+(1-D)\,F_{KK}-\frac{\alpha}{\rho+\beta}\,(F_{K}D_{Z}E_{K}+\frac{\alpha}{\beta}FD_{ZZ}E_{K}^{2}+FD_{Z}E_{KK})<0\,. \tag{24}$$

[13] Similar results about stability of steady state equilibrium can be found in Ryder and Heal (1973) and Heal (1982).

Then, given the relationship between capital stock and concentration of CO_2 ((8) with $\dot{Z} = 0$), we obtain that

$$\frac{\partial Z^{\infty}}{\partial b}<0,\ \frac{\partial Z^{\infty}}{\partial \rho}\begin{Bmatrix}>\\=\\<\end{Bmatrix}0,\ \frac{\partial Z^{\infty}}{\partial b}<0. \tag{25}$$

On the other hand, from (9) (with $\dot{K}=0$), the following expression is derived when we consider a variation of b:

$$\frac{\partial C^{\infty}}{\partial b}=[(1-D)F_K-\frac{\alpha}{\beta}FD_Z E_K-b\delta]\frac{\partial K^{\infty}}{\partial b}-\delta K^{\infty}. \tag{26}$$

This last equation tells us that the variation of consumption remains undetermined, which was predictable because of the nature of the steady state equilibrium. As is shown in Fig. 1, K^{∞} determines C^{∞} but as K^{∞} can be lower or higher than K'' according to Condition (16), an increase of K^{∞} can increase consumption if $K^{\infty}<K''$ or can decrease it if $K^{\infty}>K''$. This will be true as well for the variations of the other parameters.

The following proposition summarizes all these results:

PROPOSITION 2: *Variations of parameters have the following effects on steady state values of the model:*

a) An increase in the depreciation rate of capital (δ) or in adjustment costs (b) will decrease capital stock and concentration of CO_2.

If $b(\rho+\beta)<(>)\frac{FD_Z\alpha E_K}{\rho+\beta}$ *then an increase in the social rate of discount will decrease (increase) capital stock and* *concentration of CO_2.*

c) The variation of consumption remains undetermined.

To conclude this section we study how an economy without emissions control evolves, i.e., we study the suboptimal growth path of a competitive economy when the agents take the evolution of concentration of CO_2 as if it were exogenously determined.[14]

PROPOSITION 3: *An economy with uncontrolled emissions has higher steady state capital and concentration of CO_2 levels than an economy with optimally controlled emissions.*

Proof. The case where emissions are not optimally controlled can be analyzed by assuming $\lambda_2=0$. Then the steady state value of capital stock will be given by

$$b(\rho+\delta)=(1-D)F_K, \text{ where } Z^\infty=\frac{\alpha}{\beta}E(K^\infty) \text{ again.}$$

Using the same kind of arguments as in Prop. 1 it is shown that that equation has a unique solution $\tilde{K}^\infty$ associated with a positive steady state consumption level ($\tilde{C}^\infty$). To show that $K^\infty < \tilde{K}^\infty$ and $Z^\infty < \tilde{Z}^\infty$ we assume that $K^\infty \geq \tilde{K}^\infty$. Then subtracting the optimal Condition (16) from the above condition we obtain[15]

$$(1-D(\tilde{K}^\infty))F_K(\tilde{K}^\infty)-(1-D(K^\infty))F_K(K^\infty)=-\frac{\alpha}{\rho+\beta}F(K^\infty)D_Z(K^\infty)E_K(K^\infty) \quad (27)$$

from which can be obtained the following relationship:

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[14] In Tahvonen and Kuuluvaineu (1993) the same kind of analysis is done for Brock's (1977) model.

[15] To simplify notation we write damage function directly depending on K.

$$\frac{1-D(K^{\infty})}{1-D(\tilde{K}^{\infty})} > \frac{F_K(\tilde{K}^{\infty})}{F_K(K^{\infty})}. \tag{28}$$

Then as $F_K(K^{\infty}) \leq F_K(\tilde{K}^{\infty})$ since we have assumed that $K^{\infty} \geq \tilde{K}^{\infty}$ and marginal productivity of capital is decreasing, we have that $D(K^{\infty}) < D(\tilde{K}^{\infty})$, that means $\tilde{K}^{\infty} > K^{\infty}$ since $D(\cdot)$ is an increasing function, obtaining a contradiction. For this reason K^{∞} must be lower than $\tilde{K}^{\infty}$ and, given the positive relationship between K and Z, Z^{∞} must be lower than $\tilde{Z}^{\infty}$.□

An interesting corollary from this proposition is that consumption in an uncontrolled emission economy can be higher or lower than in an optimally controlled economy. Again the explanation of this result is given by the characteristics of optimal steady state. If $K^{\infty} > K''$, where K'' is the capital stock which maximizes consumption, an increase in the capital stock will decrease consumption and will increase the concentration of CO_2 (see Fig. 1).

Let us next study the dynamic properties of this kind of steady state, i.e., the stability of the basic growth model with accumulating emissions. In that case the dynamics of the economy are defined by the system of differential equations

$$\dot{K}=\frac{1}{b}[(1-D(Z))F(K)-C(\lambda_1)]-\delta K \tag{29}$$

$$\dot{Z}=\alpha E(K)-\beta Z \tag{30}$$

$$\dot{\lambda}_1=\lambda_1[\rho+\delta-\frac{1-D(Z)}{b}F_K(K)] \tag{31}$$

PROPOSITION 4. *If ß ≤ ρ, the steady state without emission control is a saddle point or an asymptotically stable spiral point.*

Proof. The Jacobian of (29)-(31) evaluated at the steady state is

$$J=\begin{bmatrix} 0 & -\frac{\bar{\lambda}_1^{*}(1-D)}{b}F_{KK} & \frac{\bar{\lambda}_1^{*}}{b}D_Z F_K \\ -\frac{C_{\lambda_1}}{b} & \rho & -\frac{D_Z F}{b} \\ 0 & \alpha E_K & -\beta \end{bmatrix}$$

and the characteristic equation

$$\mu^3 - trJ\mu^2 + \psi\mu - |J| = 0$$

where ψ is the sum of all diagonal second minors of J, and

$$|J| = \frac{\beta}{b^2}\left(C_{\lambda_1}\bar{\lambda}_1^{*}(1-D)F_{KK}\right) - \frac{\alpha}{b^2}C_{\lambda_1}\bar{\lambda}_1^{*}D_Z F_K E_K > 0 \tag{32}$$

since $C_{\lambda_1} < 0$.

On the other hand, we have $trJ=\sum_{i=1}^{3}\mu_i=\beta-\rho$. Then the positiveness of $|J|$ implies that either all roots have positive real parts or two roots have a negative real part and one has a positive real part. If $\beta \leq \rho$ the case of all real parts positive is ruled out and the steady state must be an unstable saddle point or an asymptotically stable spiral point.□

3. OPTIMAL ECONOMIC GROWTH WITH ABATEMENT INVESTMENT

In the previous model we have shown that there exists a steady state for economy which defines the optimal level of emissions (E^{∞}) and concentration of CO_2 (Z^{∞}). In this Section we are going to investigate if these results carry over to an economy with growing population. To do this, we introduce these two variables (E,Z) in the neoclassical model of optimal growth with Harrod-neutral technological progress (for details, see Arrow and Kurz (1970)). As the answer to this question is "no," if the economy does not devote resources to reduce emissions, we shall incorporate a second kind of capital, an emission abatement capital, in the model[16].

We begin with the definition of *efficient labor* and of variables, parameters and functions of the model of optimal growth. Efficient labor is defined as the labor times a factor reflecting technological progress, i.e., $l=e^{\tau t}L$, where L represents labor and τ is a rate of technological progress exogenously determined. As $L=L(0)e^{\pi t}$, where π is the rate of growth of labor, we can write $l=\exp\{(\tau+\pi)t\}L(0)$ and defined all economic variables in efficient labor units[17].

Economic Variables:

y = total production
ny = net production
c = consumption
k_p = stock of productive capital
k_a = stock of emission abatement capital
i_a = gross investment in emission abatement capital

Economic parameters:

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[16] Notice that an economy in a steady state for a model of growth with exogenous technological progress and increasing population is growing. Every variable (production, consumption, capital and investment) is growing at a rate equal to the rate of technological progress plus the rate of growth of population, and the per capita variables are increasing at a rate equal to the rate of technological progress, since the steady state is defined in terms of efficient labor units. This means that, as emissions have been defined as a function of the capital stock, a steady state is not feasible without introduction of emission abatement capital.

[17] We write the variables in small letters to avoid confusion with the absolute value of variables.

ρ = social rate of discount
δ = rate of depreciation of capital goods
b = linear adjustment costs of productive capital (b>1)

e = linear adjustment costs of emission abatement capital (e>1)
τ = rate of technological progress in productive activity
π = rate of growth of labor
σ = elasticity of marginal utility with respect to per capita consumption

We assume that production function is linearly homogeneous. Then the dynamics of the two capital stocks are given by[18]

$$\dot{k}_p = (1/b)[(1-D(Z))f(k_p) - c - ei_a] - \gamma k_p \tag{33}$$

$$\dot{k}_a = i_a - \gamma k_a \tag{34}$$

where $\gamma = \tau + \pi + \delta$.

The utility or welfare of society is determined by the level of per capita consumption. The social utility index function, U(c), is assumed to be homogeneous of degree 1 - σ, σ>0. Then, the optimal growth path of the economy is found by maximizing

$$\int_0^\infty U(c)\,e^{-rt}dt, \text{ where } r = \rho-\pi-(1-\sigma)\tau>0. \tag{35}$$

Finally, we are going to assume a particular specification for emission function

$$E = e^{-\theta t}gk_p, \text{ where } \theta = ak_a, \tag{36}$$

where a>0 is a measure of the efficiency of capital devoted to reducing emissions. In this way we homogenize how technological

[18] Notice that definition of net production allows us to relate Z (concentration of CO_2) to the economic variables defined in efficient labor units in the differential equation for productive capital. We thank Oscar Loureiro for this suggestion.

progress is introduced into the model (remember we are assuming a rate of exogenous technological progress in productive activity (τ)). Using this exponential function we are saying that emission abatement capital determines the rate of abatement for a given level of production. The dependence of θ on k_a means that an increasing stock of capital is required to maintain a constant rate of emission abatement (remember that, although k_a is constant, capital stock K_a is increasing at a rate equal to $\tau+\pi$). So we are assuming that, for a given level of production, emissions will be reduced with the passage of time only if an increasing amount of resources is devoted to this purpose.

On the other hand, we assume that the function is linearly homogeneous with respect to k_p with the aim of being able to write the function in terms of efficient labor units (so it is a technical assumption).

Making the appropriate substitutions, we can rewrite the differential equation for the concentration of CO_2 as

$$\dot{Z} = \alpha\exp\{(\tau+\pi-ak_a)t\}gk_p - \beta Z \ . \tag{37}$$

Then, the proposed model can be stated as the following optimal control problem

$$\max_{\{c,i_a\}} \int_0^\infty U(c)e^{-rt}dt \tag{38}$$

$$\text{s.t. } \dot{k}_p = (1/b)[(1-D(Z))f(k_p) - c - ei_a] - \gamma k_p \tag{39}$$

$$\dot{k}_a = i_a - \gamma k_a \tag{40}$$

$$\dot{Z} = \alpha\exp\{(\tau+\pi-ak_a)t\}gk_p - \beta Z \tag{41}$$

$$k_p(0)=k_{p0}\geq 0,\ k_a(0)=k_{a0}\geq 0,\ T(0)=T_0\geq 0 \tag{42}$$

$$(1-D(Z))f(k_p) - c - ei_a \geq 0 \tag{43}$$

$$c,\ k_p,\ k_a,\ Z \ \geq 0 \tag{44}$$

In this problem the necessary conditions for an interior optimum are:

$$\lambda_1 = bU_c \tag{45}$$

$$\lambda_2 = eU_c \tag{46}$$

$$\dot{\lambda}_1 = \lambda_1(r+\gamma-(1-D)/b\cdot f_{kp})-\lambda_3\alpha\exp\{(\tau+\pi-ak_a)t\}g \tag{47}$$

$$\dot{\lambda}_2 = \lambda_2(r+\gamma)+\lambda_3\alpha at\exp\{(\tau+\pi-ak_a)t\}gk_p \tag{48}$$

$$\dot{\lambda}_3 = \lambda_3(r+\beta)+(1/b)\lambda_1 fD_Z \tag{49}$$

Then if there exists a steady state it must satisfy $\dot{Z}=\dot{k}_p=\dot{\lambda}_1=\dot{\lambda}_2=\dot{\lambda}_3=0$.

PROPOSITION 5. *If there exists a value $\hat{c}$ such that $U_c(\hat{c})=0$ and $b\gamma\leq\varphi'$ where φ' is given by equation system*

$$(1-D)f=\hat{c} + e\gamma(\tau+\pi)/a + \varphi' k_p'$$
$$-D_Z\psi f + (1-D)f_{kp} = \varphi'$$

then there exists at least one steady state for the economy. If $b\gamma=\varphi'$ the steady state is unique. If $b\gamma<\varphi'$ there exist two steady state values for productive capital stock $(k^{\infty}_{p1},k^{\infty}_{p2})$ that satisfy the following inequalities

$$-D_Z\psi f + (1-D)f_{k_p}\begin{Bmatrix} > \\ < \end{Bmatrix} b\gamma \quad for \begin{Bmatrix} k^{\infty}_{p_1} \\ k^{\infty}_{p_2} \end{Bmatrix}.$$

The two values support the same level of consumption and abatement capital stock but different values for the concentration of CO_2, $Z^{\infty}_1<Z^{\infty}_2$.

Proof. Setting $\dot{\lambda}_3=0$, we obtain $\lambda^{\infty}_3=-(1/b(r+\beta))\lambda^{\infty}_1 fD_Z$. Substituting this value in (47) and (48) and using (45) and (46), $\dot{\lambda}_1=\dot{\lambda}_2=0$ gives the equations

$$0=U_c(b(r+\gamma)-(1-D)f_{kp}+(\alpha/(r+\beta))fD_Z\exp\{(\tau+\pi-ak^{\infty}_a)t\}g) \qquad (50)$$

$$0=U_c(e(r+\gamma)-(\alpha/(r+\beta))fD_Z at\exp\{(\tau+\pi-ak^{\infty}_a)t\}gk^{\infty}_p) \qquad (51)$$

On the other hand, from $\dot{Z}=0$, we obtain

$$Z^{\infty}=\psi\exp\{(\tau+\pi-ak^{\infty}_a)t\}k^{\infty}_p, \qquad (52)$$

where $\psi=\alpha g/\beta$. From this last equation it is immediate that a steady state will be possible only if $k^{\infty}_a=(\tau+\pi)/a$, establishing a direct relationship between Z^{∞} and k^{∞}_p in the steady state of economy. This result modifies Conditions (50) and (51), giving

$$0=U_c(b(r+\gamma)-(1-D)f_{kp}+(\alpha/(r+\beta))fD_Z g)$$

$$0=U_c(e(r+\gamma)-(\alpha/(r+\beta))fD_Z atgk^{\infty}_p)$$

but as this last equation is not autonomous with respect to time it will be satisfied only if there exists a value $\hat{c}$ such that $U_c(\hat{c})=0$. However, the existence of a maximum for the utility function is only a necessary condition. Additionally, it is required that the following equation ($\dot{k}_p=0$) has solution

$$(1-D(\psi k^{\infty}_p))f(k^{\infty}_p) = \hat{c} + e\gamma(\tau+\pi)/a + b\gamma k_p \qquad (53)$$

where $\gamma(\tau+\pi)/a=i^{\infty}_a$ and $\hat{c}=c^{\infty}$. But as the left-hand side is a concave function and the right-hand side is a linear function, Equation (53) will have solution only if the slope of the linear function is lower than or equal to a critical value φ' defined by the following equations system

$$\begin{aligned} (1-D)f &= \hat{c} + e\gamma(\tau+\pi)/a + \varphi' k_p' \\ -D_Z\psi f + (1-D)f_{kp} &= \varphi' \end{aligned} \qquad (54)$$

If $b\gamma=\varphi$, equation (53) has a unique solution but, if $b\gamma<\varphi$, the linear function cuts the concave function twice and there exist two values $(k^{\infty}_{p1},k^{\infty}_{p2})$ for the steady state productive capital stock and two values $(Z^{\infty}_1,Z^{\infty}_2)$ for the steady state concentration of CO_2 with the following relationships: $k^{\infty}_{p1}<k^{\infty}_{p2}$ and $Z^{\infty}_1<Z^{\infty}_2$. For the first equilibrium the slope of net production function will be higher

than the slope of the linear function $\hat{c} + e\gamma(\tau+\pi)/a + b\gamma k_p$ while, for the second equilibrium, the reverse will occur. For the other variables, the steady state values do not change $c^{\infty}_1=c^{\infty}_2=\hat{c}$, $i^{\infty}_a=i^{\infty}_{a1}=i^{\infty}_{a2}$ and $k^{\infty}_a=k^{\infty}_{a1}=k^{\infty}_{a2}$.□

In Fig. 2, we illustrate the case for which equation (53) presents two solutions, the role of condition $b\gamma\leq\varphi$ and how the critical value φ' is determined.

FIG. 2

We now turn to a stability analysis of steady states based on the following simplified system of differential equations

$$\dot{k}_p = (1/b)[(1-D(Z))f(k_p) - \tilde{c}] - \gamma k_p \tag{55}$$

$$\dot{Z} = \alpha g k_p - \beta Z \;, \tag{56}$$

where $\tilde{c} = \hat{c} + [e\gamma(\tau+\pi)]/a$.

This simplification is partially justified if we think that once the economy is in a steady state any variation of the productive capital stock is going to affect only the concentration of CO_2, since steady state values of consumption and emission abatement capital and investment do not depend on productive capital or on the concentration of CO_2. Simplification does not remove the essential fact that concentration of CO_2 depends at the steady state on the productive capital stock.

Reducing, when the economy is at the steady state, the dynamics of the model to the differential equation system (55)-(56), we can obtain clear results on stability of the two possible long-run equilibria.

PROPOSITION 6. *$(K^{\infty}_{p1}, Z^{\infty}_1)$ is an unstable <u>saddle point</u> whereas $(K^{\infty}_{p2}, Z^{\infty}_2)$ is an <u>unstable improper node</u> or <u>spiral point</u> if $(1-D)f_{kp}>b(\gamma+\beta)$ and an asymptotically <u>stable improper node</u> or <u>spiral</u>*

point if $(1-D)f_{kp}<b(\gamma+\beta)$.

Proof. For System (55)-(56), the Jacobian matrix is

$$J_E = \begin{vmatrix} \frac{1-D}{b}f_{k_p} - \gamma & -\frac{fD_Z}{b} \\ \alpha g & -\beta \end{vmatrix} \tag{57}$$

and the characteristic equation is

$$\mu^2 + (\beta+\gamma-\frac{1-D}{b}f_{k_p})\mu - \beta(\frac{1-D}{b}f_{k_p}-\gamma) + \frac{\alpha g}{b}fD_Z = 0, \tag{58}$$

where the last term is the Jacobian determinant evaluated at the steady state $|J_E|$. For the first equilibrium $-D_Z\psi f + (1-D)f_{kp}$ is greater than $b\gamma$ which implies that $|J_{E1}|=\mu_1\mu_2 < 0$. The two roots are real and have different sign, defining a saddle point. For the second equilibrium $-D_Z\psi f + (1-D)f_{kp}$ is lower than $b\gamma$ which implies that $|J_{E2}|=\mu_1\mu_2 > 0$. Then the two roots can be real or complex and positive or negative but, as $\mathrm{tr}\ J = (1-D)/b\cdot f_{kp}-\gamma-\beta=\mu_1+\mu_2$, we may conclude that if $(1-D)/b\cdot f_{kp}>\gamma+\beta$ the two roots have to be positive and then the equilibrium point is an unstable improper node (if the roots are real) or an unstable spiral point (if they are complex). In the other case $(1-D)/b\cdot f_{kp}<\gamma+\beta$ the two roots are negative and the equilibrium point is an asymptotically stable improper node (if the roots are real) or an asymptotically stable spiral point (if they are complex[19].□

We conclude this section with a comparative statics analysis of steady state. Using (53) the following signs are obtained

[19] When the equilibrium is unique the Jacobian determinant is zero. Consequently, the two roots are zero and an optimal path to reach the steady state cannot be defined.

$$\frac{\partial k_p^\infty}{\partial \tau} = \frac{\partial k_p^\infty}{\partial \pi} = \frac{(e/a)(\tau + \pi + \gamma) + bk_p}{(1 - D) f_{k_p} - D_Z \psi f - b\gamma} \begin{Bmatrix} >0 \\ <0 \end{Bmatrix} \text{ for } \begin{Bmatrix} k_{p_1}^\infty \\ k_{p_2}^\infty \end{Bmatrix} \tag{59}$$

$$\frac{\partial k_p^\infty}{\partial \delta} = \frac{(e/a)(\tau + \pi) + bk_p}{(1 - D) f_{k_p} - D_Z \psi f - b\gamma} \begin{Bmatrix} >0 \\ <0 \end{Bmatrix} \text{ for } \begin{Bmatrix} k_{p_1}^\infty \\ k_{p_2}^\infty \end{Bmatrix} \tag{60}$$

$$\frac{\partial k_p^\infty}{\partial b} = \frac{\gamma k_p}{(1-D) f_{k_p} - D_Z \psi f - b\gamma} \begin{Bmatrix} >0 \\ <0 \end{Bmatrix} \text{ for } \begin{Bmatrix} k_{p_1}^\infty \\ k_{p_2}^\infty \end{Bmatrix} \tag{61}$$

$$\frac{\partial k_p^\infty}{\partial e} = \frac{(\gamma/a)(\tau + \pi)}{(1-D) f_{k_p} - D_Z \psi f - b\gamma} \begin{Bmatrix} >0 \\ <0 \end{Bmatrix} \text{ for } \begin{Bmatrix} k_{p_1}^\infty \\ k_{p_2}^\infty \end{Bmatrix} \tag{62}$$

$$\frac{\partial k_p^\infty}{\partial a} = \frac{(-e\gamma/a^2)(\tau + \pi)}{(1-D) f_{k_p} - D_Z \psi f - b\gamma} \begin{Bmatrix} <0 \\ >0 \end{Bmatrix} \text{ for } \begin{Bmatrix} k_{p_1}^\infty \\ k_{p_2}^\infty \end{Bmatrix}. \tag{63}$$

The effects of variations of parameters on Z^∞ will have the same signs as these partial derivatives since concentration is positively related to the stock of productive capital ($Z^\infty = \psi k^\infty_p$). On the other hand, as the abatement capital stock only depends on τ, π and a according to the expression $k^\infty_a = (\tau+\pi)/a$, we have

$$\frac{\partial k_a^\infty}{\partial \tau} = \frac{\partial k_a^\infty}{\partial \pi} = \frac{1}{a} > 0, \quad \frac{\partial k_a^\infty}{\partial a} = -\frac{(\tau+\pi)}{a} < 0. \tag{64}$$

The other parameters (δ, b, e) do not have any effect on the steady state value of abatement capital stock. Finally, as the steady state investment in abatement capital is given by $i^{\infty}_{a} = \gamma k^{\infty}_{a}$ the same signs apply for this variable.

The following proposition summarizes these results.

PROPOSITION 7. *Variations of parameters have the following effects on endogenous variables of the model:*

a) For $(K^{\infty}_{p1}, Z^{\infty}_{1})$ all parameters but the rate of efficiency of emission abatement capital are positively related to the productive capital stock and the concentration of CO_2.

b) For $(K^{\infty}_{p2}, Z^{\infty}_{2})$ all parameters but the rate of efficiency of emission abatement capital are negatively related to the productive capital stock and the concentration of CO_2.

c) For the two equilibria, stock of and investment in emission abatement capital increase with the rate of growth of the economy $(\tau+\pi)$, decrease with the rate of efficiency of emission abatement capital (a), and are independent of the other parameters (δ, b and e).

CONCLUSIONS

The two main results of this paper concern the existence of steady states of the economy that are compatible with a stable flow of emissions and concentration of CO_2. For an economy with constant population the existence of a steady state is guaranteed under the standard assumption of concavity of the utility and production functions. For an economy with growing population, there are additional conditions: saturation of preferences and low values for the adjustment costs of productive capital, the rate of growth of the economy and the rate of depreciation of capital goods. In this case there will exist a path of economic growth compatible with constant emissions if society devotes increasing resources to reduce emissions or to improve the efficiency of productive capital in terms of the relation emissions/production. However, if the economy grows very fast, investment in abatement will have to be very high to assure sufficient resources for increasing per capita consumption. Thus, growth may occur only at the price of more emissions and higher concentrations of CO_2 in the atmosphere.

Another interesting result is that in the model of constant population, uncontrolled emissions do not necessarily imply more per capita consumption. With a damage function the model of optimal capital accumulation presents a modified golden rule with a component that reflects the marginal damage caused by an increase in CO_2 concentration. But, as this component depends on the discount rate, the steady state capital stock can be larger or smaller than the capital stock that maximizes per capita consumption. Then, as an uncontrolled economy (the agents take the evolution of concentration of CO_2 as exogenously determined) has a larger capital stock than an optimally controlled economy, the uncontrolled economy can also have lower per capita consumption.

Further research is required in the stability analysis of the models. Mainly for the first model the possibility of an unstable improper node or an unstable spiral point must be carefully checked because the economy cannot reach those kinds of equilibrium points.

One possible extension of the first model can be to incorporate the stock of pollution to the utility function and take into consideration the possibility that resources are allocated to improve the efficiency of productive capital. This extension would complete integration of the greenhouse effect into the models of growth with stock pollution.

Another possible extension would be to take into account different kinds of capital stocks with different structures of adjustment costs in order to capture some of the phenomena involved in the adjustment to changes in GHG concentrations that have not been analyzed here.

Finally, we have assumed that damage caused by concentration of CO_2 through climate change is known with certainty and that it changes in a smooth and continuous way with the size of the CO_2 concentration until it reaches its maximum value. There is, however, great uncertainty about climate change and its effects, and the possibility of sharp discontinuities with disastrous effects should not be neglected. This is another area meriting further research[20].

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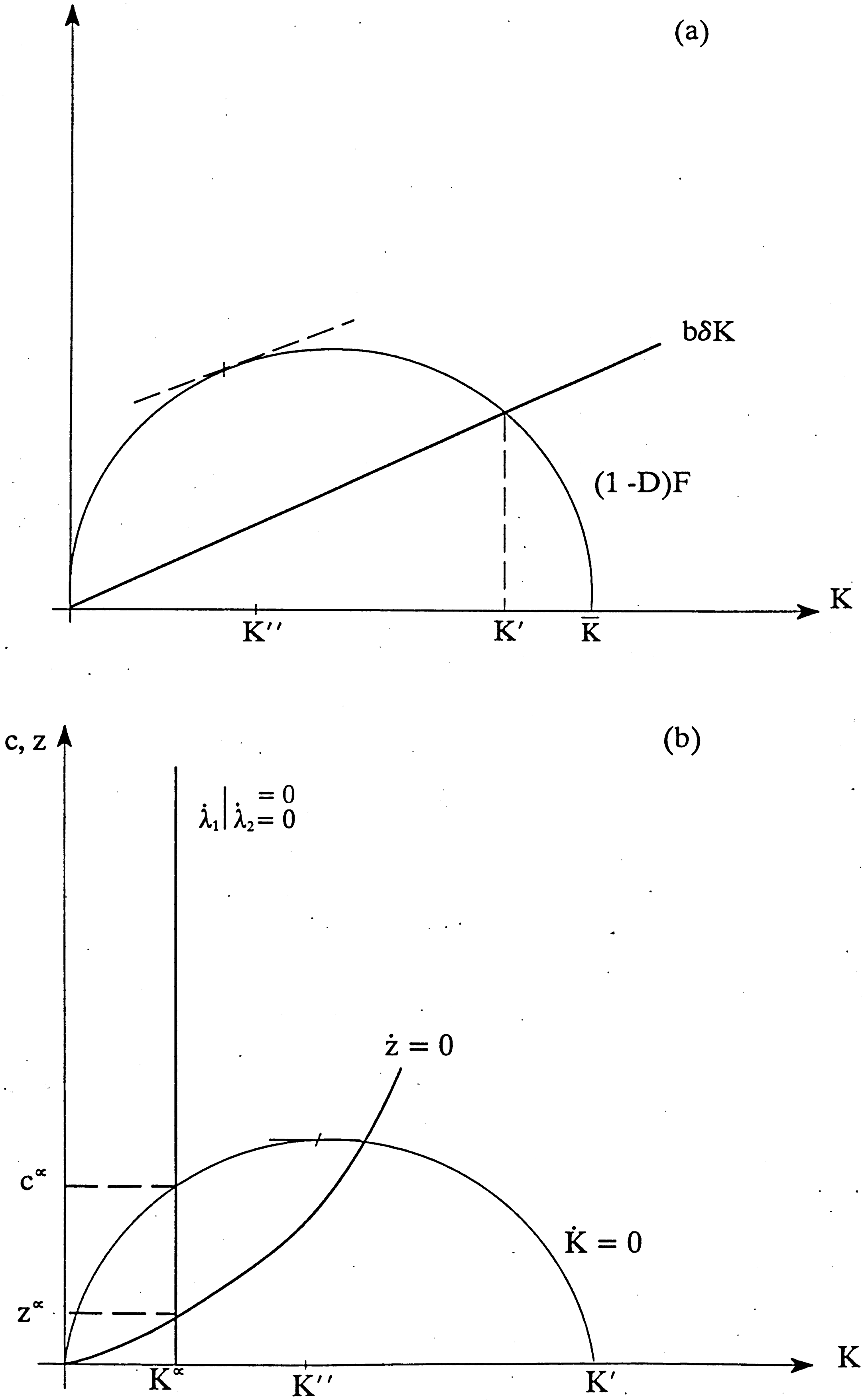

FIG. 1

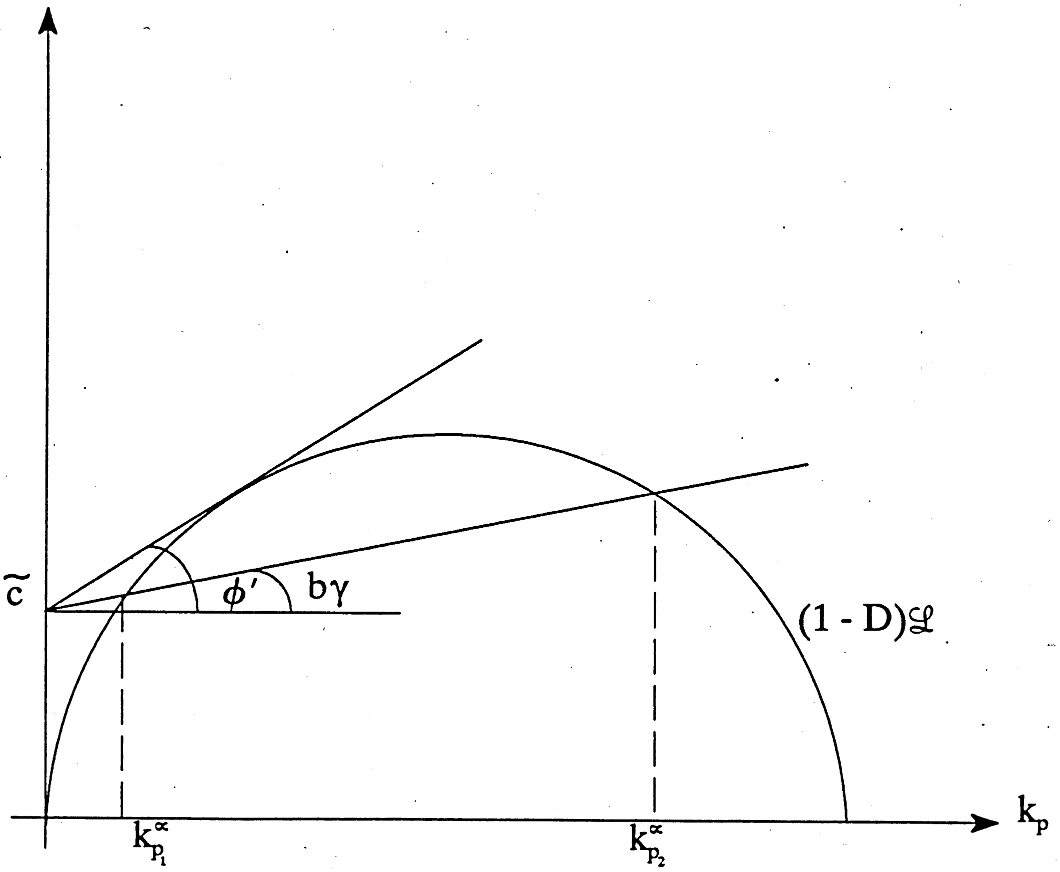

FIG. 2