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WORKING PAPER NO. 636

### ESTIMATING INCOME MOBILITY FROM CENSUS DATA

by

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## I. Introduction

[In this paper we propose a methodology for estimating income mobility from census data collected at two points in time. Income mobility information is useful in assessing the degree of economic and social stratification in a given country. It is also useful in analyzing the extent of equality of opportunity, since it provides information on how the starting position of individuals affects their subsequent income prospects. Finally, income mobility information is useful in assessing how economic growth affects the lifetime income prospects of different cohorts.]

Income mobility is an important supplement to income distribution data for assessing social equity. One can have two countries with identical income distributions, but very different mobility histories. One country can be characterized by Horatio Alger mobility, whereas another can be characterized by rigid immobility where the poor are perpetually doomed to the same station in life. Clearly, the more mobile society is more equitable.

Income mobility can be most directly estimated from panel data collected from the same cohort over time. Unfortunately, such data are virtually nonexistent for developing countries. By contrast, census data are collected decennially in most developing countries. Hence the utility of an approach to estimating income mobility which relies only on census data.

In the next section we describe the general methodology we are proposing. Treating the distribution of income at any point in time as the result of a first-order Markov process, we develop an approach for estimating individual income mobility from successive cross-section income data. Section III describes how the census data is prepared for use in estimation. Section IV contains an econometric application of the methodology to an important and much-analyzed period in recent Brazilian economic history--the decade from 1970 to 1980. The results are validated using tests

of goodness of fit in section V. We illustrate the usefulness of these estimates in section VI, where we apply the results of section IV to calculate the expected lifetime income stream of a male entering the work force in 1970. We conclude with a brief discussion of the results.

## II. The Estimation Problem

One can think of the distribution of income at any point in time as the result of a first order Markov process, in which the probability that any individual will be in income class  $j$  at time  $t+1$  depends on which income class he was in at time  $t$ . To see what happens to the income of particular groups over time then requires estimating the transition matrix of the Markov process. The  $ij$ -th element in this transition matrix is the number of people who have moved into income class  $i$  at time  $t+1$  from income class  $j$  at time  $t$ . The censuses report the row and column sums of the transition matrices. We need some way of estimating the  $ij$  cell entries of the transition matrix from observations on the row and column sums. Since there are only  $2n$  data points and  $n^2$  unknowns, we need either additional data or additional restrictions to make progress.

Telser (1963) addressed this problem in the context of market shares for cigarettes using a time-series approach. If one takes a sufficient number of observations of the distribution (in his case, the distribution of smokers across brands) and if one assumes these distributions are generated by the same first-order Markov process, Telser showed how to derive an unbiased regression estimator of the unknown elements of the transition matrix. The method gives the transition matrix which minimizes the difference between the actual distribution at time  $t+1$  and the distribution predicted by applying the transition matrix at time  $t$ . Lee, Judge, and Zellner (1970) proposed alternative Bayesian and non-Bayesian approaches to

the estimation of transition probabilities from time series data on marginal totals and examined the properties of these estimates.

Unfortunately, the time-series approach is not practical for the income mobility problem in LDCs because we do not have a sufficient number of censuses. But we can use regional data from the censuses themselves as an alternative. If we have regional data, and can assume either that the same first-order Markov mechanism operates in each region, or that the process differs across regions in a predictable way, we can proceed, as Telser did, to use regression analysis to find the transition matrix which minimizes the difference between the observed and the predicted regional distribution at time  $t+1$ , given the observed distribution at time  $t$ .

A similar problem has been addressed in sociology and political science. In 1953 Goodman proposed a simple regression to estimate the interior elements in a four-way table of individual characteristics when only the regional row and column sums of the two characteristics are known. His technique made the assumption that the interior conditional probabilities were constant across regions. Crewe and Payne (1976) applied the same general technique to derive an estimate of the percentage of different occupational groups voting for the two British political parties. They extended Goodman's technique by assuming that the conditional probabilities were a function of exogenous factors that vary across regions. They derived a best linear unbiased estimator which simultaneously produced an estimate of the transition matrix and of the effect of the exogenous variables on that transition matrix. Their model was applied to a two-by-two case -- two parties and two broad occupational classes. Our model is a simple extension of Crewe and Payne to the  $n$ -dimension case, where the  $n$  dimensions are income classes and where we are trying to find the proportions of those in income class  $j$  at time  $t$  who move to class  $i$  at time  $t+1$ .

Let  $P$  be an  $n \times n$  transition matrix whose  $ij$ -th element,  $P_{ij}$ , is the proportion of those in income class  $j$  at time  $t$  who move to class  $i$  at time  $t+1$ . Let  $X_j$  and  $Y_i$  be the

observed fraction of the total population in income classes  $j$  and  $i$  at times  $t$  and  $t+1$  respectively. The number of mutually exclusive income classes is  $n$ . By definition, in matrix notation,

$$(1) \quad Y = P * X$$

$$\text{or} \quad Y_i = \sum_j P_{ij} X_j \quad (i, j = 1, \dots, n)$$

Equation one looks like a regression model where we observe the  $X$ 's and the  $Y$ 's and estimate the unknown transition parameters  $P_{ij}$ . Clearly only  $n-1$  of these equations are independent. However, rather than dropping one of the equations, we make the equivalent restriction that the sum of each column of  $P_{ij}$ 's be equal to one. We further require that each estimated  $P_{ij}$  falls between zero and one. The problem with equation (1) is that we do not have enough data to estimate the  $P_{ij}$ . In our case we have 5 income classes so we are trying to estimate 25 elements of the transition matrix, but we have only five observations of the marginal totals  $X_j$  and  $Y_i$ .

We proceed by using regional observations. If the Markov process could be assumed to be fundamentally the same across regions except for the influence of specific exogenous variables that vary across regions, we could increase the number of observations by taking regional observed values of the distribution. One would expect mobility to be higher in fast growing or highly-industrialized regions. Following Crewe and Payne (1976) it is straightforward to modify equation (1) to take account of regional variations in the transition matrix induced by these variables.

We hypothesize that the transition probabilities are functions of observable characteristics  $Z$  that differ across regions. Thus, in simplest form with only one  $Z$  variable with region-specific values, we have

$$(2) \quad P_{ij} = a_{ij} + b_{ij}Z$$

In our case,  $Z$  was the growth rate of income. More complex formulations, in which the  $P_{ij}$  depend on more variables, are possible, but were precluded in our estimation by the small number of degrees of freedom we had.

If we now substitute equation (2) into equation (1) we get:

$$(3) \quad Y_i = \sum_j (a_{ij} + b_{ij}Z)X_j \quad (i, j = 1, \dots, n)$$

This is the equation system we will estimate under the two restrictions

$$(4) \quad 0 \leq P_{ij} \leq 1 \quad \text{for all } i, j$$

$$(5) \quad \sum_i P_{ij} = 1 \quad \text{for } j = 1, \dots, n$$

Unfortunately, available statistical packages cannot incorporate both restrictions. Packages which allow for the estimation of systems of equations will incorporate the cross-equation constraint (5) but not the within-equation inequality constraint (4). Bayesian packages, which can incorporate the inequality constraint, do not allow for the estimation of systems of equations and thus prohibit incorporation of cross-equation constraints.

To circumvent this problem, one can estimate a system of equations explicitly incorporating restriction (5), and perform a non-linear transformation on the coefficients (transition probabilities) that restricts their values to between 0 and 1, thus incorporating restriction (4). For estimation without a  $Z$  variable, such a transformation could take the form

$$Y_i = \sum_j e^{-a_{ij}^2} X_j$$

Here  $P_{ij} = e^{-a_{ij}^2}$  and thus must fall between 0 and 1 for all values of  $a_{ij}$ . This method is relatively straightforward for the simple case, where a Z variable is excluded, but proves intractable with the inclusion of such a variable.

The alternative we used was to write the problem as a non-linear programming problem with non-linear inequality constraints. The objective function minimized is the sum of squared errors and the constraints are given by equations (6), (7) and (8) below. This yields an OLS estimate for a system of equations subject to both cross-equation and inequality constraints.

A representative equation of the constraint set is given by

$$(6) \quad Y_i^r = \sum_j (a_{ij} + b_{ij}Z^r)X_j^r + \epsilon^r$$

where the r superscript indicates regional observations and  $\epsilon^r$  is the statistical error term.

In our estimation, we required that the inequality constraint hold for all values of Z in the sample and that the cross-equation constraint hold for the mean value of Z in the sample. That is

$$(7) \quad 0 \leq P_{ij}^r \leq 1 \quad \text{for all } i, j, \text{ and } r$$

$$(8) \quad \sum_i \bar{P}_{ij}^r = 1 \quad \text{for } j = 1, \dots, n, \bar{P}_{ij}^r = a_{ij} + b_{ij}\bar{Z}^r$$

where  $P_{ij}^r = a_{ij} + b_{ij}Z^r$ ,  $\bar{P}_{ij}^r = a_{ij} + b_{ij}\bar{Z}^r$ , and  $\bar{Z}^r$  is the sample mean for Z.

the sample by sex, education, and region. Each age-sex-education-region combination defined a data cell. We then subtracted the 1970 population from the 1980 population in the corresponding cell. (In establishing correspondence between cells, age in 1980 equals age in 1970 plus ten years.) If the result of the subtraction was positive, there must, on a net basis, have been new entrants into that cell; if negative, there must have been net retirements. In the case of new entrants, we simply set the 1980 population equal to the 1970 population for that cell. We assumed that since we have matched the cells by sex, age, education, and region, on the average, the new entrants into each cell since 1970 have the same incomes as the other members of that cell in 1980. We could therefore assign the frequency distribution of income of the unadjusted 1980 cell to the adjusted population in that cell. By aggregating across the 120 cells (2 sex, 5 education, and 12 regions) in each age cohort, we obtained an estimate of the 1980 size distribution of income of those who were represented in the 1970 sample. The result is an estimated vector of  $Y_i$  for 1980 which contains only sample members present in 1970.

The procedure used to adjust cells which had net retirements over the 1970s was similar. Here we assumed that the retirees had the same income profile in 1970 as the rest of the members of the cell. We then set the number of people in a given cell in 1970 equal to the number in the corresponding age-sex-education-region cell in 1980. Aggregating across cells as before yields a vector of estimated  $X_i$  for 1970 to use in our regressions along with the previously-estimated  $Y_i$  in 1980.

#### IV. Estimates of Mobility in Brazil

Before the oil shocks, Brazil was often held up as the quintessential example of inequitable growth. Between 1960 and 1980 it enjoyed one of the world's highest growth rates with income per capita rising by 3.9% per year. But the benefits of this

prodigious boom were not at all equally distributed across the working population. The Gini coefficient rose from .50 to .59 and the average income of the top 20% grew 50% faster than that of the bottom 60%. During the 1970's the income share of the bottom 60% shrank from 21.2% to 19.7% while that of the top 20% rose from 61.7% to 63.3%. There is a long literature suggesting reasons for this pattern. (See Barros *et al.* (1992), Bacha and Taylor (1978), Fishlow (1972), Langoni (1973), Morley and Williamson (1975), Morley (1982), Fields (1977), Pferrerman and Webb (1979), Denslow and Tyler (1983), and Hoffman and Kageyama (1986)). We do not wish to add to this literature here.

Rather, we focus on the differential mobility patterns at different income levels. We ask, *inter alia* : How likely was it that someone who began the decade in the lower income classes would be better off in 1980? How did the mobility of the worse off classes compare to the mobility of those who were further up the income pyramid in 1970?

One cannot answer these questions with published data because the published data does not distinguish those who were included in both 1970 and 1980 from new entrants. Since new entrants tend to occupy lower-paying jobs, their presence biases downward any comparison based on all respondents. To see this we have displayed three separate distributions in table 1: the distribution of the entire observed labor force over 15 years of age, the distribution of those who were present in both 1970 and 1980 (labelled "survivors" in the table), and the distribution of new entrants.

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Table 1. Distribution of Labor Force, Survivors, and New Entrants by Income Class in 1970 and in 1980.

Income Class	<u>MALES</u>				
	All over 15		Survivors		New Entrants
	<u>1970</u>	<u>1980</u>	<u>1970</u>	<u>1980</u>	<u>1980</u>
0	6.7%	5.3%	7.6%	1.7%	10.6%
0-3599	37.6	18.4	37.2	17.0	20.5
3600-4999	23.1	13.6	22.8	12.2	15.6
5000-12000	18.9	34.9	18.6	36.8	32.0
>12000	13.7	27.8	13.8	32.3	21.1

  

Income Class	<u>FEMALES</u>				
	All over 15		Survivors		New Entrants
	<u>1970</u>	<u>1980</u>	<u>1970</u>	<u>1980</u>	<u>1980</u>
0	9.0	7.9	9.0	6.8	8.8
0-3599	50.4	32.7	49.7	31.6	33.6
3600-4999	18.0	15.3	18.4	14.8	15.8
5000-12000	14.9	28.9	15.4	30.4	27.7
>12000	7.7	15.1	7.5	16.4	14.1

Source: Census Tapes

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While the distribution of the income of new entrants is more evenly spread across categories than that of survivors, it is also clearly biased towards the lower income categories. For both males and females the percentage of new entrants in each of the three lower income categories is higher than for the survivors, and the new entrants representation in the higher categories is correspondingly lower. The table also shows that, in comparison to the population as a whole, survivors are substantially more likely to find themselves in the upper income classes.

Let us now turn to the results of our estimation procedure. Table A-1 of the appendix presents estimated coefficients of the mobility matrices corresponding to equation (3). The base coefficient corresponds to  $a_{ij}$  and the growth rate coefficient

corresponds to  $b_{ij}$ . The R-squared values and jackknife standard errors appear with the estimates.

To obtain all-Brazil estimates of mobility, we re-estimated the matrices fixing the value of the growth rate ("Z") variable at its population-weighted mean value for each age group. These results appear in Table A-2 with R-squared values and estimated standard errors. For visual clarity, we also present the results in Table 2 in the text without the statistical measures.

Consider now the mobility patterns implied by Table 2. Our estimates suggest that there is very little downward mobility for males: with few exceptions the upper off-diagonal figures of all the matrices are either zero or a small number. If a working-age Brazilian male was lucky enough to be in the top income group in 1970 the chances were better than 85% that he would stay there. If he was in income class three and was less than 40 years old in 1970, the chances were better than 95% that he would either stay where he was or move up to the top group.

What about those at the bottom of the distribution in 1970? From our estimates it appears that they also shared in the favorable mobility patterns. A male teenager with zero income in 1970 had a 93% chance of moving up at least one class and a 49% chance of moving up at least two classes -- implying a move up to a job earning more than the minimum wage. A male in the 20-39 age group earning less than the minimum wage in 1970 (in  $X_1$ ), had a 44% chance of moving up at least one income class. Part of this mobility is explained by an increase in the minimum wage itself, from 3600 to 4149 CR\$ in real terms, and part by an expansion in the number of jobs covered by minimum wage legislation.

Nevertheless, upward mobility was greater for those who started further up the labor pyramid. For example, compare the very high probabilities that those aged 20-39 who started in  $X_2$  would move up to  $Y_3$  or  $Y_4$ , or that those who started in  $X_3$  would move to  $Y_4$ , with the much less favorable prospects for those starting in  $X_1$ .

Table 2. Estimated Mobility Matrices, Brazil 1970-1980.

Males 15-19

		Income Category in 1970				
		x0	x1	x2	x3	x4
Income Category in 1980	y0	.072	.031	0	0	0
	y1	.438	.246	0	0	0
	y2	.051	.244	0	0	0
	y3	.343	.342	.559	.438	0
	y4	.095	.137	.441	.562	1.000

Males 20-39

		Income Category in 1970				
		x0	x1	x2	x3	x4
Income Category in 1980	y0	.169	.035	0	0	0
	y1	.023	.525	0	0	0
	y2	.097	.197	.058	.053	.041
	y3	.309	.220	.933	.107	.087
	y4	.402	.024	.008	.839	.871

Males 40-59

		Income Category in 1970				
		x0	x1	x2	x3	x4
Income Category in 1980	y0	.160	.044	0	0	0
	y1	.011	.633	0	0	0
	y2	0	.168	.167	.143	.029
	y3	.004	.137	.765	.501	0
	y4	.825	.018	.068	.356	.971

Males 60 and over

		Income Category in 1970				
		x0	x1	x2	x3	x4
Income Category in 1980	y0	0	.034	0	0	.026
	y1	.010	.516	0	0	0
	y2	.481	.235	.004	.067	.247
	y3	.510	.194	.863	.314	0
	y4	0	.021	.134	.618	.728

Table 2 (continued)

## Females 15-19

		Income Category in 1970				
		x0	x1	x2	x3	x4
Income Category in 1980	y0	.309	.037	0	0	0
	y1	.492	.496	0	0	0
	y2	.133	.153	.118	.077	.502
	y3	.066	.230	.882	.502	0
	y4	0	.084	0	.421	.498

## Females 20-39

		Income Category in 1970				
		x0	x1	x2	x3	x4
Income Category in 1980	y0	.505	.066	0	0	0
	y1	.048	.696	0	0	0
	y2	.181	.089	.186	.241	.033
	y3	.266	.096	.700	.341	.369
	y4	0	.053	.115	.418	.598

## Females 40-59

		Income Category in 1970				
		x0	x1	x2	x3	x4
Income Category in 1980	y0	.514	.090	0	0	0
	y1	.230	.764	0	0	0
	y2	.118	.072	.379	.009	.163
	y3	.110	.061	.441	.650	.163
	y4	.029	.013	.180	.340	.674

## Females 60 and over

		Income Category in 1970				
		x0	x1	x2	x3	x4
Income Category in 1980	y0	0	.114	0	0	0
	y1	.168	.687	.193	0	0
	y2	0	.130	0	.470	.509
	y3	.176	.055	.807	.530	.491
	y4	.657	.015	0	0	0

Note: These results are presented in Appendix A with corresponding standard errors and R-squared values

Thus, the growth process appears to have benefited most those placed high enough in the income pyramid to take advantage of the rapid expansion in jobs with relatively high educational requirements and wages.

The reader may object that this differential pattern is not found in the  $X_0$  category representing those earning zero income. But the  $X_0$  category is a somewhat special case. The overwhelming majority of this group are teenagers (less than 3% of the non-teenage labor force falls in this group), many of whom undoubtedly worked on farms or in family businesses while attending school and then entered the formal labor market some time during the 1970's. Many found good jobs when they entered the formal labor market. For the relatively small number of cases in  $X_0$  who are older, the zero-income starting point probably represented transitory unemployment at the time of the 1970 census.

The mobility picture for female workers is a good deal less favorable than for males. We find that there is far more downward mobility and considerably less upward mobility. Whereas about 89% of males aged 20-39 who started in  $X_2$  or  $X_3$  moved up at least one income class, only about 62% of similarly-placed women did. In the same age group, 44% of  $X_1$  males moved up, compared to only 24% of females. For males the zero income class appears to be transitory, with relatively few remaining there over the decade. The situation is entirely different for females less than 60 years old. If a working-agewoman started in a zero income job, the probability is quite high that she would still be there ten years later. It is likely that an important contributor to this pattern is the lack of monetary valuation of domestic work.

Consider next the age-income profiles underlying the transition matrices. In Table 3 we show the average annual real income growth rates of different age cohorts of "survivors". For this purpose we use a more disaggregated breakdown than the one used for our regression analysis. Table 3 makes clear the very steep

income gradient during the early working years. Over the 1970's young workers gained relative to other survivors. Since those young workers tended to start at the bottom of the income pyramid, much of the upward mobility we have documented in the mobility matrices must, in fact have been young workers moving up and out of their low-paying, entry-level jobs, which were then taken by the next generation of new entrants.

-----  
Table 3. Annual Growth Rates of Real Income by Cohort of Survivors

Age in 1970	Male	Female
15-19	19.4%	15.9%
20-29	11.5	7.9
30-39	7.8	4.8
40-49	6.5	3.9
50-59	5.5	3.5
60+	7.9	3.2
Overall Brazil	10.6	7.9

Source: Computed from census tapes.

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In trying to understand what this mobility evidence implies, it is important to go back to the social significance of inequality. The mobility data confirm that the published aggregate data hides a substantial amount of upward mobility for those who were at the bottom of the 1970 income pyramid. This may seem comforting to the defenders of trickle-down growth. However if, following Paglin [1975], one argues that it is cohort inequality that matters - that is, one's prospects relative to other members of the same cohort - then Brazil's growth process does not look so appealing. For, as we pointed out, those who started out in better positions in 1970,

also tended to fare better over the ensuing decade. Thus growth seems to have widened the divergences within each age cohort, particularly among male workers, even while it made upward mobility possible for most.

## V. Validation of Estimates

How good are the mobility estimates? How well does our procedure work? There are several ways to address that question. Judging by the overall goodness of fit, the estimates in Table A-1, particularly for non-teenagers, look very good indeed, explaining more than 90% of the variance in the regional 1980 distributions given the 1970 distribution.

But how do our estimates perform in forecasting? As an alternative, more stringent, test of goodness of fit, we applied our estimated transition matrix to the all-Brazil data for each age-sex group.<sup>1</sup> Since the all-Brazil data was omitted from the regional census data on which regression estimates of the  $P_{ij}$  are based, this test is analogous to an "out of sample" forecast. To perform the test we first calculated weighted averages across states of (1) their per capita GNPs, (2) their population share in the given income classes in 1980 ( $Y_j$ 's) and (3) their population shares in the income classes in 1970 ( $X_j$ 's). For each age-sex group, the weights used in deriving the weighted averages were the number of workers in each state corresponding to that age-sex group. We then calculated forecast values for each group by applying the growth-rate-dependent transition probabilities (using the weighted growth rate for the group) to the weighted  $X_j$ 's to produce predicted values for the weighted  $Y_j$ 's. The results are presented in Table 4 along with the actual weighted values from the data.

Table 4. Forecast Results, All Brazil

Actual Weighted Population Shares for Males 15-19				
Y0	Y1	Y2	Y3	Y4
.0323	0.1812	0.1362	0.4172	0.2331
Predicted Population Shares				
.0361	0.2471	0.1392	0.3804	0.2063
Actual Weighted Population Shares for Males 20-39				
Y0	Y1	Y2	Y3	Y4
.0136	0.1494	0.1111	0.3691	0.3568
Predicted Population Shares				
.0196	0.1802	0.1039	0.3575	0.3382
Actual Weighted Population Shares for Males 40-59				
Y0	Y1	Y2	Y3	Y4
.0151	0.2106	0.1349	0.3314	0.3079
Predicted Population Shares				
.0150	0.2208	0.1335	0.3267	0.3007
Actual Weighted Population Shares for Males 60 and Over				
Y0	Y1	Y2	Y3	Y4
.0192	0.2722	0.1693	0.3296	0.2098
Predicted Population Shares				
.0231	0.2819	0.1707	0.3258	0.2018
Actual Weighted Population Shares for Females 15-19				
Y0	Y1	Y2	Y3	Y4
.0774	.3416	.1595	.3239	.0976
Predicted Population Shares				
.0751	.4002	.1469	.3499	.0829
Actual Weighted Population Shares for Females 20-39				
Y0	Y1	Y2	Y3	Y4
.0623	.2835	.1485	.3181	.1876
Predicted Population Shares				
.0663	.3138	.1398	.3019	.1764
Actual Weighted Population Shares for Females 40-59				
Y0	Y1	Y2	Y3	Y4
.0770	.3861	.1358	.2453	.1558
Predicted Population Shares				
.0800	.4021	.1272	.2312	.1583
Actual Weighted Population Shares for Females 60 and Over				
Y0	Y1	Y2	Y3	Y4
.1004	.4751	.1509	.2507	.0230
Predicted Population Shares				
.0862	.4874	.1626	.2223	.0390

To evaluate the goodness of fit of our estimates, we regressed the actual all-Brazil weighted population shares for each income class  $j$ ,  $Y_j^w$ , on the predicted all-Brazil population shares,  $\hat{Y}_j^w$ , calculated from our estimated transition probabilities, in a linear regression with no constant. We ran these regressions as an iterated SUR system consisting of

$$Y_j^w = \beta_j \hat{Y}_j^w + \varepsilon_j \quad j=0,\dots,4$$

If our estimates are "good" we should see unitary regression coefficients on the predicted population shares (unbiasedness) and high R-squared values. The results presented in Table 5 confirm both expectations. Prediction errors are small and all coefficients are very close to one.

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Table 5. Results of Model Test

	$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$
Coefficients	.9930	.9144	1.0188	1.0553	1.0387
Standard Error	.0296	.0125	.0209	.0117	.0086

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We tested the collective proximity of our estimated coefficients to unity through a likelihood ratio test based on comparing the value of the likelihood function for the unrestricted estimation with the value of the likelihood function when all the  $\beta$ 's were restricted to equal one. This statistic is distributed as a chi-square with five degrees of freedom (five linear restrictions:  $\beta_j = 1, j = 0,\dots,4$ ). From our results, we cannot reject the null hypothesis that all the  $\beta$ 's are equal to one. That is, one cannot reject the hypothesis that our estimates of the  $Y_j$ 's for all-Brazil are unbiased.

Our estimates also have very high R-squares and very low mean square forecast error. We thus do remarkably well.

## VI. Expected Lifetime Income Under Different Growth Scenarios

In this section we explore one application our estimates, calculating lifetime income prospects. If the observed mobility patterns remained constant over the lifetime of an individual, what would be the individual's expected lifetime income? How would these prospects differ under alternative growth scenarios?

We model the expected lifetime income stream of a 17-year-old Brazilian male entering the labor force in 1970. We can track his income trajectory using the successive mobility matrices we have estimated. By multiplying the appropriate matrices we can calculate his probability of being in any income class at the end of each ten-year interval through out his working life. For example if we would like to know in which income class we are likely to find our worker after 40 years, and we represent our estimated matrix for males in age group A by  $M_A$ , then this 40-year mobility matrix is calculated according to

$$M_{\text{after 40}} = M_{40-59} * M_{20-39} * M_{20-39} * M_{15-19}.$$

We calculate these mobility matrices for each ten-year interval from entrance into the labor market to retirement. In order to examine the income stream of a 17-year-old Brazilian male, however, we need to convert the probabilities in the mobility matrices into expected income. To do this we assume that each worker in an income class receives the mean income of that class. Thus at the end of each time interval we calculate his expected income at that time, contingent on his starting income class, by multiplying the probability that he is in a given class by the

mean income of that class and then summing across classes. We divide the ten-year intervals into two periods and assign the expected income at the start of the period to the first five years and the expected income at the end of the period to the second five years.

In Table 6 we present the lifetime income stream of a male worker calculated as described above. The mean starting income of each income class appears in the first row of the table. The present value of the income stream appears in the right-hand side of this table.<sup>1</sup> This table indicates that mobility makes the annual incomes 25 years hence almost independent of the starting point of the individual.<sup>2</sup> Over time, income prospects converge. The average annual income of the poorest group with a job (X1) is 12% of that of the top group in the observed data; in 25 years, it becomes 85% of the top group. Nevertheless, expected lifetime incomes, especially in present value terms, still clearly show the impact of the starting point of the individual. The ratio of the lowest to the highest present value of expected lifetime income is only 29%. It is therefore clear that, despite the substantial upward mobility we have documented, a Brazilian teenager's lifetime income prospects are very strongly influenced by his starting point in the income pyramid.

In Table 7 we present the results of similarly calculated present values of expected lifetime incomes for females alongside those for males. The picture for females is quite different from that for males. The present value of lifetime incomes is considerably lower for females, especially women starting in the X0 category, whose lifetime income prospects are only about 40% of those for males. For higher income categories, the prospects of women average to about 70% of comparably situated males. The distribution of lifetime incomes is considerably more unequal than that for males, since, as we observed already, mobility is

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<sup>1</sup>The discount factor was set at 8.5%, the 1970-1980 growth rate of the Brazilian economy calculated from World Bank figures. In long-term equilibrium, the growth rate and the interest rate must be equal.

<sup>2</sup>This is, of course, a property of ergodic Markov chains.

Table 6. Expected Lifetime Income Stream for 17-year-old Male Entering the Labor Force in 1970

Undiscounted Income Stream for Individual Starting in Indicate Income Class					Discount Rate=.085		Present Value of Income Stream					Expected Lifetime Income	
Age	X0	X1	X2	X3	X4	Discount Factor	Age	X0	X1	X2	X3	X4	
17	0	3000	4300	8500	25000	1	17	0	3000	4300	8500	25000	
18	0	3000	4300	8500	25000	.92	18	0	2765	3963	7834	23041	
19	0	3000	4300	8500	25000	.85	19	0	2548	3653	7220	21236	
20	0	3000	4300	8500	25000	.78	20	0	2349	3367	6655	19573	
21	0	3000	4300	8500	25000	.72	21	0	2165	3103	6133	18039	
22	6873	8763	15825	18605	24495	.67	22	4571	5495	10524	12373	16290	
23	6873	8763	15825	18605	24495	.61	23	4213	5065	9700	11404	15014	
24	6873	8763	15825	18605	24495	.56	24	3883	4668	8940	10511	13838	
25	6873	8763	15825	18605	24495	.52	25	3578	4302	8240	9687	12754	
26	6873	8763	15825	18605	24495	.48	26	3298	3965	7594	8928	11755	
27	6873	8763	15825	18605	24495	.44	27	3040	3655	6999	8229	10834	
28	6873	8763	15825	18605	24495	.41	28	2802	3368	6451	7584	9985	
29	6873	8763	15825	18605	24495	.38	29	2582	3104	5946	6990	9203	
30	6873	8763	15825	18605	24495	.35	30	2380	2861	5480	6442	8482	
31	6873	8763	15825	18605	24495	.32	31	2193	2637	5050	5938	7817	
32	13581	14209	22322	22418	22339	.29	32	3995	4180	6566	6594	6571	
33	13581	14209	22322	22418	22339	.27	33	3682	3852	6051	6077	6056	
34	13581	14209	22322	22418	22339	.25	34	3393	3550	5577	5601	5562	
35	13581	14209	22322	22418	22339	.23	35	3127	3272	5140	5162	5144	
36	13581	14209	22322	22418	22339	.21	36	2882	3016	4738	4758	4741	
37	13581	14209	22322	22418	22339	.20	37	2657	2780	4367	4385	4370	
38	13581	14209	22322	22418	22339	.18	38	2448	2562	4024	4042	4028	
39	13581	14209	22322	22418	22339	.17	39	2257	2361	3709	3725	3712	
40	13581	14209	22322	22418	22339	.15	40	2080	2176	3419	3433	3421	
41	13581	14209	22322	22418	22339	.14	41	1917	2006	3151	3164	3153	
42	16797	18648	21926	21959	22020	.13	42	2185	2426	2852	2857	2865	
43	16797	18648	21926	21959	22020	.12	43	2014	2236	2629	2633	2640	
44	16797	18648	21926	21959	22020	.11	44	1856	2061	2423	2427	2433	
45	16797	18648	21926	21959	22020	.10	45	1711	1899	2233	2237	2243	
46	16797	18648	21926	21959	22020	.09	46	1577	1751	2068	2061	2067	
47	16797	18648	21926	21959	22020	.09	47	1453	1613	1897	1900	1905	
48	16797	18648	21926	21959	22020	.08	48	1339	1487	1748	1751	1756	
49	16797	18648	21926	21959	22020	.07	49	1234	1370	1611	1614	1618	
50	16797	18648	21926	21959	22020	.07	50	1138	1263	1485	1487	1492	
51	16797	18648	21926	21959	22020	.06	51	1049	1164	1369	1371	1375	
52	18481	19862	22312	22332	22369	.06	52	1063	1143	1284	1285	1287	
53	18481	19862	22312	22332	22369	.05	53	980	1053	1183	1184	1186	
54	18481	19862	22312	22332	22369	.05	54	903	971	1091	1092	1093	
55	18481	19862	22312	22332	22369	.05	55	833	895	1005	1006	1008	
56	18481	19862	22312	22332	22369	.04	56	767	825	926	927	929	
57	18481	19862	22312	22332	22369	.04	57	707	760	854	855	856	
58	18481	19862	22312	22332	22369	.04	58	652	700	787	788	789	
59	18481	19862	22312	22332	22369	.03	59	601	646	725	726	727	
60	18481	19862	22312	22332	22369	.03	60	554	595	668	669	670	
61	18481	19862	22312	22332	22369	.03	61	510	548	616	617	618	
62	19715	20729	22522	22536	22561	.03	62	502	528	573	574	574	
63	19715	20729	22522	22536	22561	.02	63	462	486	528	529	529	
64	19715	20729	22522	22536	22561	.02	64	426	448	487	487	488	
65	19715	20729	22522	22536	22561	.02	65	393	413	449	449	449	
Expected Lifetime Income	636174	707741	935438	985788	1127480			85887	106985	171534	202896	301238	

Table 7. Present Value of Expected Lifetime Income Under Alternative Assumptions on the Mean Growth Rate of Income.

	Starting Income Group				
	X0	X1	X2	X3	X4
I. Males					
A. Growth Rate 20% Below Observed Value	68485	112277	166584	192390	292255
B. Observed Growth Rate	85887	108985	171534	202896	301238
C. Growth Rate 20% Above Observed Value	126564	107565	176916	237918	308465
II. Females					
A. Growth Rate 20% Below Observed Value	31394	72935	100686	159336	224414
B. Observed Growth Rate	36374	76455	106072	162636	226754
C. Growth Rate 20% Above Observed Value	41239	80380	112755	166952	229774

substantially less for females. The highest paid women have lifetime income prospects 6.23 times higher than those with the lowest income prospects. The comparable ratio for males is only 3.5.

How might these lifetime prospects differ under alternative growth regimes? Is a higher growth rate likely to increase or decrease the convergence of expected income? To address this question, we can use Table A-1 to calculate the mobility matrices under different assumptions on the average rate of growth of income (remember, Table A-1 has both a base component and a growth-rate dependent component of each transition probability). We chose to simulate lifetime expected income with 1) a rate of growth 20% higher than the observed growth rate and 2) a rate of growth 20% lower than the observed growth rate. These two scenarios give us values for the rate of income growth that fall well within our regional observations. The results are presented in Table 7.

Table 7 portrays the effects of economic growth on lifetime income prospects. Growth benefits most groups and both sexes. But its effect is strongest on the those with no income in 1970 (X0), where for males a 20% increase in growth rate increases the present value of expected life cycle income by 48%. Minimum-wage male job holders in X1, however, lose from faster growth: their lifetime income prospects are 3.7% higher at the lower growth rate. That the poor have difficulty in adjusting to faster growth has been found to be the case in the 19th century as well (Morris and Adelman, 1988). During the early phase of the Industrial Revolution, poverty increased faster in Germany, a rapidly growing nation, than in France, a slow growing country. The resources of the working poor are too limited to cope with change, even if the environment becomes more favorable. Slower change gives them a better chance to adapt, through migration, training, job change, family limitation, etc.<sup>3</sup> Economic growth substantially lowers overall lifetime inequality

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<sup>3</sup>The working poor in low paying jobs in the United States did not benefit from fast growth in the Reagan

for workers of both sexes, but the effect of growth on female income prospects is much smaller than for males. A twenty percent increase in growth rate lowers the ratio of the top lifetime income to the bottom lifetime income by 31% for males as compared to only 11% for females.

## VII. Conclusion

This paper develops a method for estimating Markov income-mobility matrices using census data. It allows, for the first time, the use of non-panel data for deriving estimates of intracohort income mobility over time. The method uses regional observations as the basis for a non-linear programming approximation to a SUR regression system with additional interval constraints on the estimated parameters.

The method was successfully applied to Brazil where it generates a very accurate and unbiased estimate of the predicted 1980 distribution of the labor force across income classes, given the observed 1970 distribution. Our results confirm previous assertions that the observed distribution, which became more unequal over the 1970s, also hides a great deal of upward mobility. We show that the probability of upward mobility for survivors was high in every age cohort. All income classes benefited from growth, but the benefits from growth were distributed unequally: those who started out further up the distribution in 1970 tended to fare better over the ensuing decade. Thus while growth was beneficial to all groups, it also widened annual income differences within each age cohort.

Our technique allowed us to compute life cycle incomes and life cycle income distributions. We show that, due to mobility, life cycle incomes were distributed considerably more equally than starting incomes for workers of both sexes. We also

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years either.

showed that those with no income in 1970 (X0 class) are most sensitive to growth and that faster growth benefits all except male minimum-wage earners, who actually lose, in absolute terms, from faster growth.

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Table A-1. Regression results, Brazil 1970-1980.

Males Ages 15-19

R-squared=.633

		Income Category in 1970									
		X0		X1		X2		X3		X4	
		Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef
Income Category in 1980	Y0	0.084	-0.0006088	0.189	-0.008	0	0	0	0	0	0.000
		0.425	0.017	0.245	0.010	0	0	0	0	0	0.000
	Y1	3.137	-0.138	-0.886	0.058	0	0	0	0	0	0.000
		0.743	0.033	0.471	0.023	0.000	0.000	0.000	0.000	0.000	0.000
	Y2	-0.193	0.013	0.250	0.000	0	0.000	0.060	-0.003	0.261	-0.012
		0.113	0.007	0.375	0.020	0.000	-0.000	0.657	0.030	2.876	0.126
	Y3	-1.306	0.085	0.491	-0.008	1.051	-0.025	2.689	-0.118	0	0.000
		0.899	0.046	0.521	0.027	1.691	0.093	2.929	0.129	0.000	0.000
	Y4	-0.362	0.023	-0.166	0.016	0.761	-0.016	2.585	-0.102	1.130	-0.008
		0.657	0.040	0.421	0.023	2.749	0.143	2.066	0.129	1.426	0.093

Males Ages 20-39

R-squared=.918

		Income Category in 1970									
		X0		X1		X2		X3		X4	
		Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef
Income Category in 1980	Y0	0.587	-0.042	0.074	-0.004	0	0	0	0	0	0.000
		0.657	0.053	0.048	0.004	0	0	0	0	0	0.000
	Y1	0.082	-0.006	0.797	-0.027	0	0	0	0	0	0.000
		0.739	0.480	0.139	0.014	0.000	0.000	0.000	0.000	0.000	0.000
	Y2	-0.111	0.021	0.221	-0.002	-0.095	0.015	0.193	-0.014	0.150	-0.011
		0.705	0.062	0.110	0.010	0.259	0.029	0.326	0.029	0.350	0.034
	Y3	0.185	0.012	0.042	0.018	0.845	0.009	0.389	-0.028	0.317	-0.023
		1.189	0.115	0.192	0.019	0.566	0.053	0.758	0.058	0.518	0.038
	Y4	-0.428	0.083	-0.007	0.003	-0.015	0.002	0.445	0.039	0.848	0.002
		0.734	0.067	0.072	0.010	0.278	0.043	0.619	0.077	0.451	0.043

Males Ages 40-59

R-squared=.942

		Income Category in 1970									
		X0		X1		X2		X3		X4	
		Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef
Income Category in 1980	Y0	0.525	-0.062	0.059	-0.003	0	0	0	0	0	0.000
		1.012	0.120	0.038	0.005	0.001486708	0.00017265	0.000	0.000	0.000	0.000
	Y1	0.037	-0.004	0.831	-0.034	0	0	0	0	0	0.000
		2.240	0.264	0.072	0.014	0.000	0.000	0.000	0.000	0.000	0.000
	Y2	0	0.000	0.155	0.002	0.265	-0.017	0.023	0.020	-0.036	0.011
		0.000	0.000	0.077	0.014	0.355	0.062	0.456	0.086	0.091	0.029
	Y3	0.012	-0.001	0.097	0.007	0.227	0.091	1.298	-0.135	0	0.000
		2.134	0.249	0.086	0.014	0.259	0.029	0.317	0.048	0.000	0.000
	Y4	0.425	0.068	-0.022	0.007	-0.080	0.025	0.112	0.041	1.035	-0.011
		2.379	0.278	0.019	0.005	0.110	0.034	0.225	0.058	0.101	0.029

Males Ages 60 and over

R-squared=.898

		Income Category in 1970									
		X0		X1		X2		X3		X4	
		Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef
Income Category in 1980	Y0	0.000	0.000	0.046	-0.001	0.000	0	0	0.000	-0.010	0.004
		0.000	0.000	0.033	0.003	0.017	0.001094486	0	0.000	0.050	0.007
	Y1	-0.004	0.002	0.647	-0.016	0	0.000	0	0.000	0	0.000
		0.166	0.073	0.080	0.010	0.000	0.000	0.000	0.000	0.000	0.000
	Y2	-0.183	0.083	0.197	0.005	-0.001	0.001	0.026	0.005	0.561	-0.039
		0.262	0.106	0.076	0.013	0.056	0.027	0.232	0.027	0.206	0.013
	Y3	1.161	-0.081	0.111	0.010	1.052	-0.024	0.716	-0.050	0	0.000
		1.416	0.176	0.129	0.020	0.126	0.036	0.355	0.030	0.000	0.000
	Y4	0	0.000	-0.008	0.004	-0.051	0.023	0.438	0.022	0.380	0.043
		0.000	0.000	0.013	0.007	0.289	0.040	0.786	0.070	0.441	0.043

Table A-1(continued)

## Females 15-19

R-squared=.630

		Income Category in 1970									
		X0		X1		X2		X3		X4	
		Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef
Income Category in 1980	Y0	-0.282	0.035	0.172	-0.008	0	0	0	0	0	0.000
		0.919	0.050	0.116	0.007	0	0	0	0	0	0.000
	Y1	2.386	-0.113	0.096	0.024	0	0	0	0	0	0.000
		1.552	0.080	0.537	0.033	0.000	0.000	0.000	0.000	0.000	0.000
	Y2	-0.382	0.031	0.246	-0.006	0.571	-0.027	0.371	-0.018	2.434	-0.115
		0.415	0.033	0.537	0.030	1.144	0.070	1.728	0.086	2.335	0.116
	Y3	-0.190	0.015	0.061	0.010	1.025	-0.009	2.434	-0.115	0.000	0.000
		1.270	0.070	0.242	0.013	1.008	0.056	0.471	0.036	2.186	0.106
	Y4	0.000	0.000	0.024	0.004	0.000	0.000	2.041	-0.097	2.412	-0.114
		0.106	0.007	0.129	0.007	0.000	0.000	1.386	0.076	3.774	0.222

## Females 20-39

R-squared=.906

		Income Category in 1970									
		X0		X1		X2		X3		X4	
		Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef
Income Category in 1980	Y0	0.498	0.001	0.126	-0.009	0	0	0	0	0	0.000
		0.321	0.038	0.043	0.005	0	0	0	0	0	0.000
	Y1	-0.038	0.012	0.851	-0.022	0	0	0	0	0	0.000
		0.086	0.029	0.072	0.010	0.000	0.000	0.000	0.000	0.000	0.000
	Y2	0.221	-0.006	0.034	0.008	-0.146	0.047	0.722	-0.068	0.098	-0.009
		0.259	0.029	0.043	0.005	0.168	0.014	0.283	0.034	0.364	0.034
	Y3	0.729	-0.063	-0.010	0.015	0.309	0.055	0.163	0.025	1.108	-0.105
		0.523	0.067	0.072	0.010	0.710	0.086	1.084	0.139	0.652	0.067
	Y4	0	0.000	0.018	0.005	-0.090	0.029	0.288	0.018	1.084	-0.069
		0.001	0.000	0.038	0.005	0.101	0.029	0.297	0.058	0.350	0.062

## Females 40-59

R-squared=.873

		Income Category in 1970									
		X0		X1		X2		X3		X4	
		Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef
Income Category in 1980	Y0	0.345	0.035	0.130	-0.008	0.000	0	0	0	0	0.000
		0.623	0.086	0.062	0.010	0.014	0.010	0.005	0.039	0.000	0.000
	Y1	0.400	-0.036	0.752	0.002	0	0	0	0	0	0
		0.317	0.038	0.067	0.010	0.000	0.000	0.000	0.000	0.000	0.000
	Y2	0.018	0.021	0.021	0.011	0.547	-0.035	0.016	-0.001	0.284	-0.025
		0.058	0.019	0.048	0.010	0.245	0.043	0.182	0.014	0.278	0.029
	Y3	0.016	0.020	0.009	0.011	0.451	-0.002	0.948	-0.063	0.283	-0.025
		0.197	0.043	0.034	0.005	0.129	0.034	0.168	0.024	0.360	0.077
	Y4	0.004	0.005	0.016	-0.001	0.262	-0.017	0.131	0.044	0.433	0.051
		0.058	0.010	0.038	0.010	0.312	0.067	0.341	0.086	0.388	0.053

## Females 60 and over

R-squared=.463

		Income Category in 1970									
		X0		X1		X2		X3		X4	
		Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef	Base Coef	Growth Coef
Income Category in 1980	Y0	0	0.000	0.111	0.000	0	0.000	0	0.000	0	0.000
		0.000	0.000	0.047	0.006	0.000	0.000	0.000	0.000	0.000	0.000
	Y1	0.285	-0.026	0.614	0.016	0.063	0.028	0.000	0.000	0.000	0.000
		0.914	0.104	0.225	0.041	0.259	0.092	0.000	0.000	0.000	0.000
	Y2	0	0.000	0.119	0.002	0.000	0.000	0.798	-0.072	0.840	-0.072
		0.000	0.000	0.057	0.006	0.386	0.035	0.515	0.063	0.199	0.070
	Y3	0.057	0.026	0.063	-0.002	0.937	-0.028	0.847	-0.069	0.834	-0.075
		2.593	0.073	0.171	0.028	0.275	0.092	0.316	0.070	1.078	0.123
	Y4	0.416	0.053	0.025	-0.002	0.000	0.000	0.000	0.000	0.000	0.000
		0.718	0.092	0.038	0.006	0.060	0.013	0.136	0.002	0.095	0.041

Note: Figures in italics are jackknife estimates of standard errors.

Table A-2. Estimated Mobility Matrices, Brazil 1970-1980.

## Males 15-19

R-squared=.633

		X0	X1	X2	X3	X4
Income Category in 1980	Y0	0.072 0.066	0.031 0.033	0 0	0 0	0 0
	Y1	0.438 0.153	0.246 0.076	0 0.000	0 0.000	0 0.000
	Y2	0.051 0.033	0.244 0.017	0 0.000	0.000 0.090	0.000 0.385
	Y3	0.343 0.176	0.342 0.080	0.559 0.305	0.438 0.458	0 0.000
	Y4	0.095 0.149	0.137 0.053	0.441 0.305	0.562 0.484	1.000 0.385

## Males 20-39

R-squared=.918

		Income Category in 1970				
		X0	X1	X2	X3	X4
Income Category in 1980	Y0	0.169 0.149	0.035 0.010	0 0	0 0	0 0
	Y1	0.023 0.206	0.525 0.034	0 0.000	0 0.000	0 0.000
	Y2	0.097 0.144	0.197 0.014	0.058 0.077	0.053 0.096	0.041 0.139
	Y3	0.309 0.153	0.220 0.024	0.933 0.110	0.107 0.197	0.087 0.139
	Y4	0.402 0.182	0.024 0.019	0.008 0.149	0.839 0.211	0.871 0.134

## Males 40-59

R-squared=.942

		Income Category in 1970				
		X0	X1	X2	X3	X4
Income Category in 1980	Y0	0.160 0.307	0.044 0.010	0 0.000455604	0 0.000	0 0.000
	Y1	0.011 0.681	0.633 0.024	0 0.000	0 0.000	0 0.000
	Y2	0 0.000	0.168 0.014	0.167 0.101	0.143 0.144	0.029 0.077
	Y3	0.004 0.652	0.137 0.014	0.765 0.086	0.501 0.086	0 0.000
	Y4	0.825 0.724	0.018 0.014	0.068 0.091	0.356 0.139	0.971 0.077

## Males 60 and over

R-squared=.901

		Income Category in 1970				
		X0	X1	X2	X3	X4
Income Category in 1980	Y0	0 0.000	0.034 0.013	0.000 0.007	0 0	0.026 0.040
	Y1	0.010 0.411	0.516 0.033	0 0.000	0 0.000	0 0.000
	Y2	0.481 0.607	0.235 0.040	0.004 0.139	0.067 0.153	0.247 0.149
	Y3	0.510 0.574	0.194 0.056	0.863 0.179	0.314 0.153	0 0.000
	Y4	0 0.000	0.021 0.033	0.134 0.146	0.618 0.216	0.728 0.156

Table A-2 (continued)

## Females 15-19

R-squared = .680

		Income Category in 1970				
		X0	X1	X2	X3	X4
Income Category in 1980	Y0	0.309	0.037	0	0	0
		0.103	0.020	0.000	0.000	0
	Y1	0.492	0.496	0	0	0
		0.322	0.050	0.000	0.000	0.000
	Y2	0.133	0.153	0.118	0.077	0.502
		0.126	0.036	0.172	0.289	0.468
	Y3	0.066	0.230	0.882	0.502	0.000
		0.136	0.023	0.172	0.129	0.050
	Y4	0.000	0.084	0	0.421	0.498
		0.033	0.010	0.000	0.222	0.239

## Females 20-39

R-squared = .906

		Income Category in 1970				
		X0	X1	X2	X3	X4
Income Category in 1980	Y0	0.505	0.066	0	0	0
		0.072	0.010	0	0	0
	Y1	0.048	0.696	0	0	0
		0.106	0.029	0.000	0.000	0.000
	Y2	0.181	0.089	0.186	0.241	0.033
		0.072	0.010	0.082	0.082	0.120
	Y3	0.266	0.096	0.700	0.341	0.369
		0.091	0.019	0.144	0.235	0.216
	Y4	0	0.053	0.115	0.418	0.598
		0.000431145	0.010	0.106	0.173	0.153

## Females 40-59

R-squared = .873

		Income Category in 1970				
		X0	X1	X2	X3	X4
Income Category in 1980	Y0	0.514	0.090	0.000	0.000	0
		0.206	0.019	0.034	0.019	0
	Y1	0.230	0.764	0	0	0
		0.163	0.029	0.000	0.000	0.000
	Y2	0.118	0.072	0.379	0.009	0.163
		0.115	0.014	0.149	0.106	0.158
	Y3	0.110	0.061	0.441	0.650	0.163
		0.125	0.010	0.168	0.134	0.120
	Y4	0.029	0.013	0.180	0.340	0.674
		0.077	0.010	0.120	0.177	0.201

## Females 60 and over

R-squared = .463

		Income Category in 1970				
		X0	X1	X2	X3	X4
Income Category in 1980	Y0	0	0.114	0	0	0
		0.000	0.027	0.000	0	0
	Y1	0.168	0.687	0.193	0	0
		0.531	0.093	0.507	0.000	0.000
	Y2	0	0.130	0.000	0.470	0.509
		0.000	0.036	0.229	0.335	0.531
	Y3	0.176	0.055	0.807	0.530	0.491
		0.474	0.083	0.494	0.352	0.564
	Y4	0.657	0.015	0.000	0.000	0.000
		0.730	0.033	0.066	0.153	0.295

Note: Figures in italics are jackknife estimates of standard errors.