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RISK ATTITUDES OVER WEALTH UNDER DISCRETE STATUS LEVELS

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David E. Buschena and David Zilberman*

This paper gives a rigorous development of Friedman and Savage's explanation for individuals' simultaneous gambling and insurance purchases. Preferences for risky wealth distributions are considered in light of a stochastic and discrete factor, where the probability of this discrete factor depends on the wealth level.

RISK ATTITUDES OVER WEALTH UNDER DISCRETE STATUS LEVELS

Introduction

This paper analyzes representations for behavior under risk where preferences depend on continuous wealth and also on a discrete variable referred to as status. This discrete variable may correspond with being a) solvent or foreclosed upon, b) self-employed or a wage earner, c)renting or owning a productive resource such as land, etc. The model further views behavior where the probability of reaching a desired status level depends on wealth. Preferences over income and status levels are taken to satisfy assumptions allowing representation by a multivariate expected utility (EU) function. Furthermore, preferences over marginal distributions over wealth alone are shown under the axioms to also be represented by a univariate "marginal" EU function under these status levels. This univariate utility representation is not necessarily concave; under reasonable assumptions, it is S-shaped a la Friedman and Savage. It may also lead to behavior that is approximated by safety rules a la Roy. When consumer utility is affected by multiple status variables, marginal risk preferences for changes in wealth may drastically change with the level of wealth, and consumers may switch from risk aversion to risk-loving behavior as these wealth levels change.

The paper proceeds as follows. Section I develops the model and sets forth an EU representation of preferences for joint probability distributions over discrete status levels and wealth. Section II defines another EU representation for preferences on marginal distributions over wealth alone, where the probabilities of the status levels depend on wealth. Section III offers a discussion of the model, its applications, a general graphical analysis of the behavior of this representation of preferences over marginal wealth distributions and its relationship to alternative models of behavior that reflect shifts in well-being near critical income levels.

I. The Status Model

* We are grateful to Jim Chalfant, Brian Wright and Eddie Dekel for their insights.

Preferences over probability distributions $k \in P$ from the n+1 dimensional compact space K are taken to depend on wealth and on an n-dimensional vector of discrete "status" levels, $[s_i]$, for $i=\{1,2,...n\}$. Status levels affect preferences over these joint distributions and have probability distributions that in turn depend on wealth. The two status level case corresponds directly with scenarios of solvency/bankruptcy, life/death, health/illness, and other binary cases where either a favorable or an unfavorable outcome will be realized. The model with greater than two status levels describes the case of a lumpy or a vertically differentiated consumption goods such as housing, or non-physical variables such as occupation levels.

Let the marginal probability distribution[1] over wealth be given by $g(y)$ and let the conditional probability of achieving a particular status level (s_i) for a wealth level be denoted (for all status levels) by $p(s_i/y) = H_i(y)$. There are many situations where these conditional probabilities will be non-deterministic in wealth as the probability of occurrence for status levels would depend on other variables in addition to income. For example, a) the wealth level required to avoid foreclosure proceedings depends on variables that are lender-specific and on variables that reflect the general economy of the region or the nation, b) the ability to purchase status goods with future income depends on wealth and on these goods' ex-ante unknown prices, and c) the costs of obtaining political benefits (such as right-to-work or citizenship papers, political office, or operating licenses) may be influenced by wealth, but would also depend on numerous political and economic factors.

Assumptions on the conditional probability vector for the status' levels occurrence are, for all i:

A1a $\sum_{i=1}^{n} H_i(y) = 1,$

A1b. $H_i(y) \geq 0$

A1c. $\partial H_i / \partial y = h_i(y) > (<) 0$, for i > (<) j, with j the current status level.

A1d. $\sum_{i=1}^{n} h_i(y) = 0$

A1e $\frac{\partial h_i(y)}{\partial y} = h_i'(y)$

[1]For expository ease, we assume that there are probability distributions for the status level; in reality, no objectively known distribution would be available.

The assumptions require that these conditional probabilities are well behaved; they sum to one and are bounded above by 1 and below by 0 for all wealth levels. Increases in wealth increase the probability of the desired (higher) status level; marginal changes in the probabilities of the status levels from a change in wealth are offsetting. The elements of the vector of conditional probabilities for the status levels are also taken to be twice continuously differentiable in wealth.

Expected Utility Representation for Preferences over Joint Probability Distributions

An EU representation of the preferences for the joint distributions over both income and the status levels exists under certain assumptions on preferences. The existence of this representation is a result of a direct application of the multivariate utility representation under general probability measures developed by Fishburn (1982).

Property 1: Under the fulfillment of the preference axioms (E0 to E5) given in the Appendix, there exists a real valued function $U([s_i],y)$ on the wealth/status space that represents preferences, $\succ$, for the joint probability distribution vectors over wealth[1] and the status level vector for the distributional alternatives $k^j([s_i],y)=g^j(y)[H^j(y)]$, given by[3]

$$(1) k^1([s_i],y) \succ k^2([s_i],y) \Leftrightarrow$$

[1]Preferences are taken to be defined here over final wealth levels, rather than over changes in income as in Kahneman and Tversky.

[3] All integration is over Rieman-Stieljies integrals.

$$\int_y [\sum_{i=1}^{n} U(s_i,z)H_i^1(z)]g^1(z)dz \geq \int_y [\sum_{i=1}^{n} U(s_i,z)H_i^2(z)]g^2(z)dz$$

Further, this representation is unique up to an affine transformation, so that $U([s_i],y)$ and $a+bU([s_i],y)$ represent the same preferences.

See Fishburn for the proof of this Property.

Define $U(s_i,y)$ ar the value of $U[s_i],y)$ when status level si is realized. Denote the first and second derivatives of the utility representation with respect to wealth for a particular status level s_i are denoted by $U_y(s_i)$ and $U_{yy}(s_i)$. The representation $U(s_i,y)$ is taken throughout the paper to be a monotonically increasing preference representation for both wealth and the level of the discrete status variable for all wealth and status levels:

A2a. $U_y(s_i) \geq 0$

A2b. $U(s_j,y) \leq U(s_i,y)$ for all i>j.

II. A Representation for Preferences Over Wealth.

In this section we define a marginal expected utility (MEU) representation for preferences over marginal distributions over wealth, where the conditional probability distributions of the status levels depend on the wealth level. That is, the MEU representation is defined for changes over wealth, where the wealth levels also affect the probabilities of the status levels. Under such preferences, this MEU representation is the true model for representing preferences over wealth shifts, but it may differ considerably from the representation of preferences commonly assumed to be concave in wealth . After showing that this MEU exists (in Property 1) under the assumptions needed for the existence of the EU representation over joint distributions $U(s_i,y)$, we investigate the curvature properties of this MEU representation, particularly those of the second derivative with respect to wealth, in Proposition 2.

<u>Proposition 1</u>:

Agents' preferences $\succ^*$ over marginal distributions over wealth, g(y), correspond in a conditional manner with the preference ordering $\succ$ over the joint distributions $k(s_i,y)$ over wealth and the status levels. These preferences can be represented by the function:

$$V(y) = E_{si}[U(s_i, y)] = \sum_{i=1}^{n}[U(s_i,y)H_i(y)]$$

under Assumptions E0-E5 for multivariate utility. That is, preferences over distributions $g^j(y)$ over wealth can be represented by the representation $V(y)$ in the following sense;

$$g^1(y) \succ^* g^2(y) \Leftrightarrow \int_y V(z)g^1(z)dz \geq \int_y V(z)g^2(z)dz.$$ Further, this representation is unique up to an affine transformation, so that V(y) and a+bV(y) represent the same preferences.

Proof: The preference ordering $\succ$ for the joint distributions over wealth and status $k(s_i,y)$ is also well defined over the marginal distributions, g(y), over wealth under E0, so the result in Property 1 holds under the assumptions E0-E5. Further, given $U(s_i,y)$ and from Fubini's theorem (Apostol), under Lebesque integration, the value of the integral over the range of y using $V(y)$ is equal to the value of the integral using $U(s_i, y)$ over the same range of y given by the marginal distribution g(y); thus, preferences $\succ^*$ over $g(y)$ correspond to $\succ$ for marginal densities of wealth obtained from $k([s_i]y)$. Since $a+bU([s_i],y)$ represents the same preferences as $U([s_i],y)$ and again from properties of Lebesque integration, V(y) also represents the same preferences as a+bV(y).

The true preferences over wealth alone can be represented by a well defined function V(y) unique up to an affine transformation under the assumptions E0-E5; this MU representation depends both on the joint utility representation, $U(s_i, y)$, and the vector of conditional status level probabilities, $H(y)$.

Properties of the Marginal Preference Representation.

Most empirical and theoretical evaluation of preferences over wealth ignores the influence of discrete factors (status) on preferences, and the effect of wealth on these status level probabilities. We consider agent's first and second degree risk attitudes under the MEU function for wealth spreads, focusing on the implications that differ from those obtained from analysis when such effects of wealth on the occurrence of status levels is ignored.

The first derivative of $V(y)$ is non-negative under A1a-A1e and has the form:

$$(2) \quad V_y(y) = \sum_{i=1}^{n}[U_y(s_i)H_i(y) + h_i(y)U(s_i,y)] = E_{si}[U_y(s_i)] + \sum_{i=1}^{n} h_i(y)U(s_i,y)$$

The marginal effect of changes in wealth on the MEU preference representation is decomposed into two effects: a) $E_{si}(U_y(s_i)$: the expected marginal change in the joint

EU functions given no change in the status level probabilities, and b) $\sum_{i=1}^{n} h_i(y)U(s_i, y)$: the marginal change in the conditional probability of the status levels weighted by the joint wealth/status level pair. The sign of $V_y(y)$ is non-negative since the marginal utility of wealth is non-negative for all status levels under (A2a), since $\sum_{i=1}^{n} h_i(y)U(s_i, y) \geq 0$ from A1b, and from A1c.

The second derivative with respect to wealth of the MEU representation, $V_{yy}(y)$ reflects first and second order effects of wealth on the joint status-level/wealth utility representation $U(s_i, y)$ and upon the probabilities of reaching these levels, given by $H_i(y)$:

$$V_{yy}(y) = \sum_{i=1}^{n} [U_{yy}(s_i)\, H_i(y) + 2h_i(y)U_y(s_i, y) + h_i'(y)\, U(s_i, y)] \tag{4}$$

We are primarily interested in the sign of $V_{yy}(y)$ over the range of wealth.

> Proposition 3: The second order effect of wealth on the preference representation $V_{yy}(y)$ may quite likely be positive for agents who otherwise show risk aversion for wealth under a given level of status, $U_{yy}(s_i,y)<0$. When the status level depends stochastically on wealth, agents' willingness to take risks over wealth distributions depends on the first and second order effects of both the joint wealth/status EU representations and the distributions for the status levels.

Discussion of Proposition 3

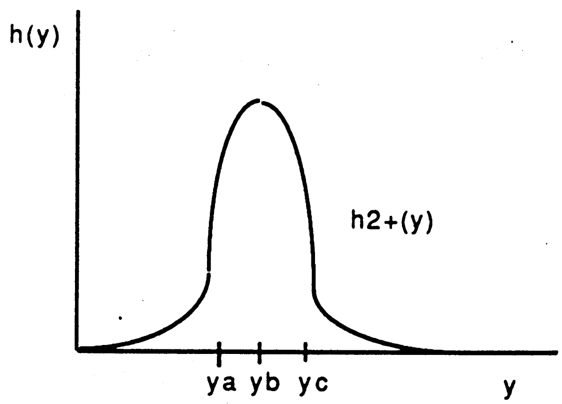
The first n terms in $V_{yy}(y)$ can be denoted by $E_{si}U_{yy}(s_i) = \sum_{i=1}^{n} U_{yy}(s_i)H_i(y)$, an expectation corresponding with the second degree of curvature in the standard EU model over wealth alone; this term is negative when $U(s_i,y)$ is concave in wealth.

The sum of the cross terms, $\sum_{i=1}^{n} 2h_i(y)U_y(s_i)$, could have either a positive or a negative sign and reflects the effects from a first order change in the probabilities of the status levels from an increase in wealth, weighted by the marginal utility for each status level. The sign of the final collection of terms, $\sum_{i=1}^{n} h_i'(y)U(s_i,y)$, may be positive when $h'_{j+1}(y)>0$ for s_j representing the currently realized status levels, where s_j^* is such that: $s_1<s_j^*<s_n$. A positive value for this final collection of terms is likely when the second order gain in probability, weighted by the utility representations over status levels, shows increases in the probabilities of favorable status levels, with the subsequent offsetting decreases in less favorable levels under A1b. Explicit statement of conditions yielding the sign of $V_{yy}(y)$ for specific wealth levels, such as in Myles, is not possible since the realized utility maximizing status level is not completely dependent on wealth nor chosen by the agent; the sign of $V_{yy}(y)$ for various wealth levels must be empirically determined.

Risk attitudes in wealth for the case of two status levels depends greatly on the behavior of the conditional function giving the probability of the status levels. The first derivative of this function gives the probability that a particular level of wealth is required to achieve a status level (a density function); this function is illustrated in Figure 1 for a unimodal symmetric distribution $h_2^+(y)$. The area under $h_2^+(y)$ over the range $[0,y]$ gives the probability of achieving status level 2 given a particular wealth level y.

Near the mode of the distribution (y_b), the second derivative of the MEU function will depend much on the cross-terms $2h_1(y)U_y(s_1,y)+2h_2(y)U_y(s_2,y)$ when $U_y(s_1,y)$ is large enough. Also, the second order effect of wealth on the probability of the second status level, $h_2'^+(y)$, may well be positive if $y \le y_b$ and negative if $y \ge y_b$, depending on the nature of $U(s_1,y)$ and $U(s_2,y)$ with respect to wealth changes. When $y \le y_1$, increases in the probability of the desired status level, $h_2'^+(y)$, is low and does not change greatly with increased wealth, agents would make choices as if they were restricted to the lower status level s_1 while $V_{yy}(y)$ will likely be near zero or negative. When $y_a \le y \le y_b$, the probability of reaching the desirable status level is increasing at an increasing rate (large value of $h_2^+(y)$ and a positive $h_2'^+(y)$), likely giving a positive sign on $V_{yy}(y)$. For wealth levels between y_b and y_c, $h_2^+(y)$ is large but $h_2'^+(y)$ is negative; the sign on $V_{yy}(y)$ is likely to be positive initially near y_b and to become negative as wealth moves closer to y_c, depending on the magnitudes of the utility functions $U(s_i,y)$ and their corresponding first derivative functions with respect to wealth. Beyond y_c, the status level s_2 is highly probable with $h_2^+(y)$ and $h_2'^+(y)$ small, so agents would likely treat the status level as effectively fixed at s_2 and be risk averse.

Figure 1



III. Implications of the Model

The risk attitudes represented by the function $V(y)$ is illustrated for two status levels in Figure 2(a, b, c). In 2a, the joint status level/wealth utility representation, $U(s_i, y)$, increases as the status level increases from s_1 to s_2, while the probability of achieving the second status level is given alternatively by one of the probability functions $H_2^+(y)$ or $H_2^*(y)$ in Figure 2b. The smooth function $H_2^+(y)$ indicates some degree of status-level uncertainty for a large range of wealth levels; this gradual increase in the probability of achieving the second status level may reflect the influence of factors beyond only wealth on this probability. In contrast, the $H_2^*(y)$ curve's rapid increase at y^* stems from a deterministic rule where the status levels depend only on wealth. With $H_2^+(y)$, the preferences over wealth are given by the smooth curve $V^+(y)$, while $H_2^*(y)$ gives $V^*(y)$, reflecting the abrupt shift in status level at y^*.

Figure 2a

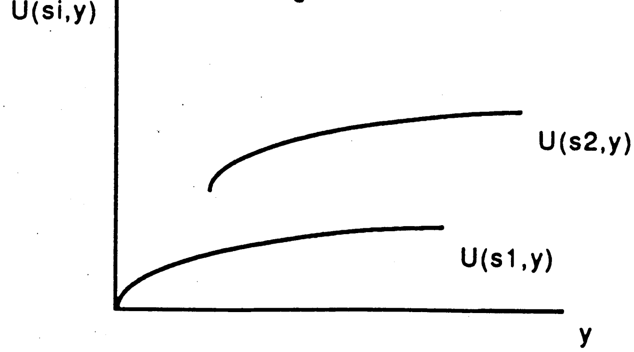


Figure 2b

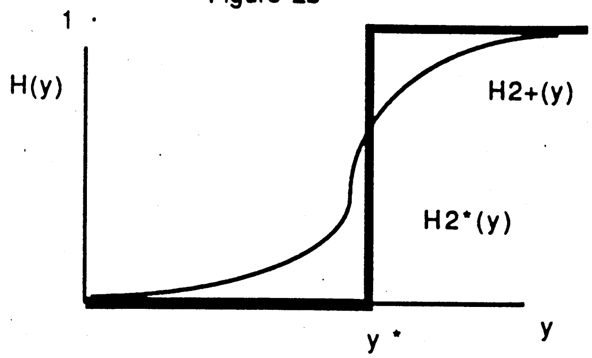
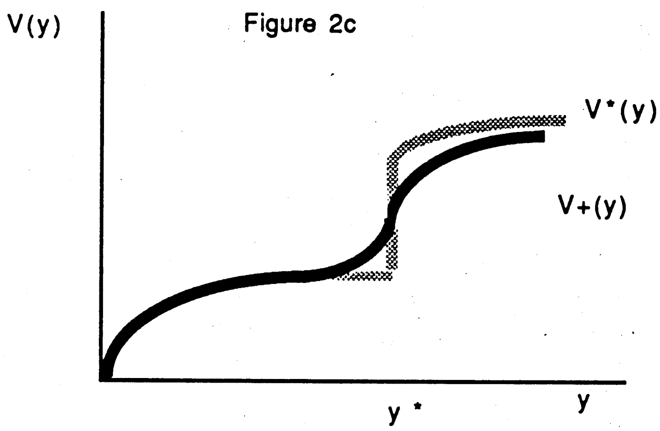
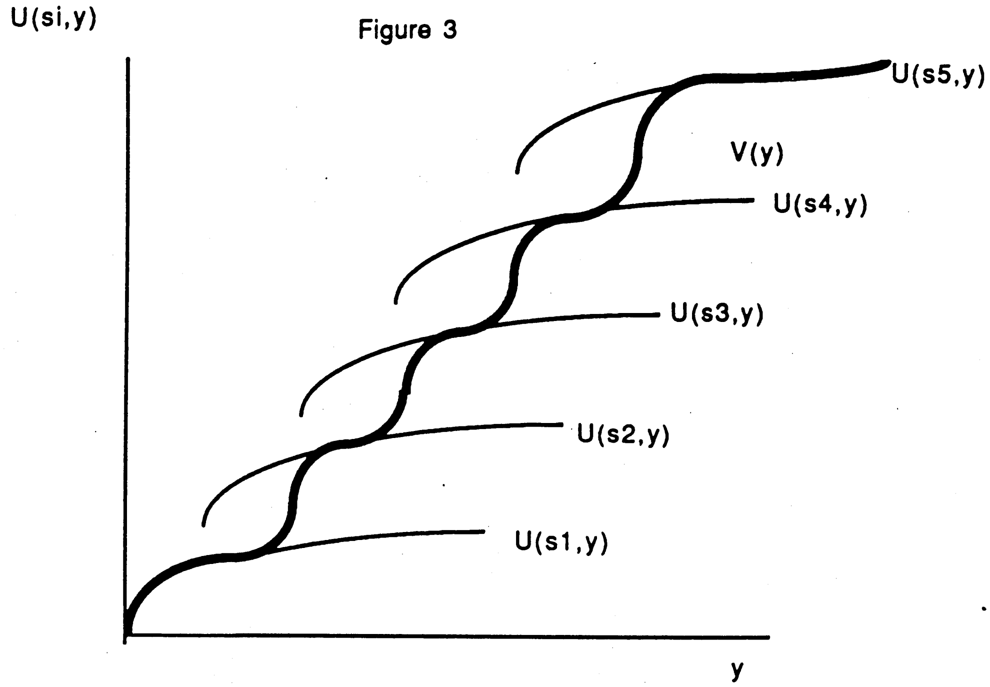


Figure 2c



The nature of the anticipated model misspecification from the lack of consideration for discrete status levels becomes clear; the true preference representation over wealth, $V(y)$, may well be convex for some wealth levels, as illustrated in Figure 2 and also in Figure 3 for five status levels. Important areas of convexity of V(y) in wealth may show that decision makers are risk averse when there is a small probability of status level change, risk preferring with a large probability of status improvement, and extremely risk averse for gambles where there is some chance of a reduction in status. Because the curvature properties of $V(y)$ are affected by both wealth and the status levels, summary measures of behavior that depend on the curvature properties of the preference representation, such as the Arrow-Pratt absolute and relative risk-aversion coefficients and the risk premium, may have unanticipated signs or magnitudes at some levels of wealth.



If the probabilities of each status level become degenerate as the result of well-known rigid rules such as set threshold levels of wealth governing lender activities; behavior for wealth levels around y^* in Figure 2c may move from the gradually shifting gambling behavior associated with $V^+(y)$ to the extreme switching between risk aversion and preference associated with $V^*(y)$. Decision makers under the probability situation described by $H_2^*(y)$ with potential wealth either only above or only below y^* will be more risk averse for wealth spreads than they would be in the case of wealth levels both above and below y^* that carry potential status-level changes. Individuals with potential wealth spreads on either side of y^*, who may previously have gambled to some degree, may exhibit behavior which is more complex; they may be willing to pay a considerable amount to avoid risk, or they may pay a considerable sum to gamble if the certain level lies on the lower $U(s_0, y)$ curve.

IV. Synthesis of Alternative Models of Behavior

In the safety-rule models, decision makers are held to either minimize the probability of the occurrence for outcomes below the threshold (Roy), or maximize some objective function such as profit subject to a limited probability of occurrence for outcomes below the threshold level (Telser(1953) and Katoka(1963)). Roy's safety rule model requires agents to minimize the probability $P(y \leq d^*)$ that y is less than some critical level of wealth d^* and can be incorporated directly into the two-level status model; the utility over wealth for status level s_1, $U(s_1, y)$, is extremely low (in a cardinal sense) relative to the utility over wealth for status level s_2 for all levels of wealth, i.e., the gap between $U(s_1, y)$ and $U(s_2, y)$ is large and $H_2(y) = 0$ for wealth below a threshold level as for $H_2^*(y)$ in Figure 2b. Telser's and Katoka's models can be approximated by the status formulation.

In their 1948 article, Friedman and Savage suggest an extension of EU theory for preferences over wealth to allow for agents' simultaneous purchase of insurance for small wealth spreads (risk aversion) and lottery tickets for larger spreads in wealth (risk preference). They postulate the shape of the EU curve to have concave portions for both low and very high wealth levels and a convex portion for medium to high wealth levels, as for $V^+(y)$ in Figure 1c. Friedman and Savage suggest an interpretation for their utility function as "regarding the . . . (concave) segments as corresponding to qualitatively different socio-economic levels, and the (convex) segment to the transition between the two levels." Their interpretation calls for diminishing marginal utility for wealth changes that do not shift agents out of a socioeconomic class and also for increasing marginal utility for wealth levels that move toward a class shift. The status model under a gradual change in the probability of a status level offers a rigorous framework for behavior modeled by Friedman and Savage for multiple status levels (class shifts) and incorporates the bankruptcy/solvency and vertical product differentiation scenarios by treating variables, such as lender's behavior and nondivisible status good prices, as stochastic.

Conclusions and Extensions of the Status Model

The common assumption of concavity in wealth for utility representing preferences in wealth is far from innocuous. This assumption gives global risk aversion, positive risk-premia and has been used in a considerable number of economic applications. Concave utility in wealth, however, is only an approximation of true agent behavior; this approximation may be quite good for some instances, but may be quite far from true behavior when discrete factors such as financial standing, lumpy levels of production technology or durable goods, or subsistence levels of consumption are of major concern.

APPENDIX

The following axioms are necessary and sufficient to give a real valued function representing preferences for joint distributions defined over the status levels and wealth. The joint distributions are over the (n+1)-tuple of probability measures over status levels and wealth, $k^i, k^j \in P = P^y x P^s$, for $P^s = P^{s^1} x P^{s^2} x \ldots P^{s^n}$. This convex set of probability measures is defined over a Boolean algebra of subsets of the outcome (wealth/status) space YxS, $S = S^1 x S^2 x \ldots S^n$.

Definition: The probability measures are taken to be finitely additive, so that, for any finite subset defined on the outcome space through the indicator j:

$$j=\{1,\ldots M\},\ P(\bigcup_{j=1}^{M}(Y_j,S_j)) = \sum_{j=1}^{M} P(Y_j,S_j).$$

Further, as in Fishburn (82), define A^y, A^s, $A^s = A^{s^1} x A^{s^2} x \ldots A^{s^n}$ as a Boolean algebra over Y and S, respectively, that contain the singleton subsets {y} and $(s^i\}$ for all points $y \in Y$ and $s^i \in S^i$; therefore, $YxS \in \bigcup_i A^i$ The marginal probability measures (P^y, P^s) are well defined on (A^y, A^s), given the defintion of P on the Boolean altgebra of subsets of the outcome space YxS. The preference ordering $\succ(\succ,\sim)$ on $P = P^y x P^s$ also extends to these marginal probability measures. Further, the probabilities $p^* \in P^y$ and $p^{si*} \in P^{si}$ are defined to give $y \in Y$ and $s^i \in S^i$ with probability one (certainty); the preferences $\succ$ are defined so that $(y',[s']) \succ (y'',[s''])$ is equivalent to $(p^{y'*}, p^{s'*}) \succ (p^{y''*}, p^{s''*})$; this property allows for the extension of preferences over distributions to those over outcomes.

The following rigourously defines the nature of preferences on marginal distributions. Conditional preference intervals are taken to exist as in Fishburn (1982). Let $P^{(j)} = P^s$ if $j=y$ and $P^{(j)} = P^y x P^{(si)}$ if $j=s_i$, with $P^{(si)} = P^{s1} x P^{s2} x \ldots P^{si-1} x P^{si+1} X \ldots P^{sn}$ if $i=s^i$. Then the preference ordering $\succ$ also exteneds to $p^{(j)} \in P^{(j)}$ and is denoted by $(\succ, p^{(j)})$; that is, $p^j(\succ, p^{(j)}) q^j$ if and only if $(p^j, p^{(j)}) \succ (q^j, p^{(j)})$. A <u>preference initerval conditional</u> is defined to be a subset $B^j \in \{Y,S\}$ on $p^{(j)} \in P^{(j)}$ if $d_j \in B^j$ whenever $c_j, e_j \in B^j$, $(c_j^*, p^{(j)}) \succ (d_j^*, p^{(i)})$ and $(c_j^*, p^{(j)}) \succ (e_j^*, p^{(i)})$. A conditional preference interval B^j exists if there is a $p^{(j)} \in P^{(j)}$ for which B^j is preference interval conditional on $p^{(i)} \in P^{(i)}$

The following axiom defines the closure of the probability measures on $A = A^y x A^s$ for finite convex combinations and conditional measures.

Definition: Closure under Finite Convex Comibinations satisfied if, for $0<\lambda<1$:

$\lambda p^y+(1-\lambda)q^y \in P^y$ and $\lambda p^y+(1-\lambda)q^y \in P^y$,
$\lambda p^{si}+(1-\lambda)q^{si} \in P^{si}$ and $\lambda p^{si}+(1-\lambda)q^{si} \in P^{si}$,
whenever $p^y, q^y \in P^y$ and $p^{si}, q^{si} \in P^{si}$.

E0. A^y and A^s are taken to contain every conditional preference interval in Y and S, respectively, and P^y, P^s are closed under finite convex combinations and under the formulation of conditional measures:

The following three axioms are generalizations of those used to define the existence of a funtional representation of preferences in the univariate case. The axioms call for a well-defined ordering of preferences (E1), a multivariate version of indepedence of preferences (E2) and an Archimedian property (E3).

E1. The preference ordering on P is an asyemmetric weak order.

Definition: For probability measures $x,z \in P$, $x \equiv_i z$ indicates that $x_j = z_j$ for all $j \neq i$.

E2. For all joint probability measures over the wealth/status space, and for $i,j \in \{y,s^1,s^2,...s^n\}$ for any $0<\lambda<1$, if $b \equiv_i d$, $c \equiv_j a$, $b \succ c$ and $a \sim d$, then:
$\lambda b+(1-\lambda)d \succ \lambda c+(1-\lambda)a$, where:

E3. For all $a,b,c \in P$ and for all $i \in \{y,s^1,s^2,...s^n\}$, if $a \equiv_i c$, $a \succ b$ and $b \succ c$, then there exist $\alpha, \beta \in (0,1)$ such that $\alpha a+(1-\alpha)c \succ b$ and $b \succ \beta a+(1-\beta)c$.

The next axiom defines a form of dominace for the conditional probability measures.

E4. If $i,j \in \{y,s^1,s^2,...s^n\}$, $j \neq i$, p^i, $q^i \in P^i$, $a^i \in A^i$, then
$(p^i,p^{(i)}) \succ (q^i,p^{(i)})$ if $(p^{ai*},p^{(i)}) \succ (p^i p^{(i)})$ for all $a^i \in A^i$ and
$(q^i,p^{(i)}) \succ (p^i,p^{(i)})$ if $(q^{ai*},p^{(i)}) \succ (p^{ai*},p^{(i)})$ for all $a^i \in A^i$.

Axiom E4 says that, if $p^* a^i(\succ,p^{(i)})q^i$ for all $a^i \in A^i$, then $p^i \succ q^i$. This axiom uses the definition of p^i as a set of probability measures on A^i for.satisfaction of dominance by the preference ordering.

A final axiom is needed in the case of finite additivity of the probability measures for potentially unbounded function u and uses the following definitions:

$$\langle p^{(i)};x^i) = \{w^i \in X^i : (p^{x^{i*}}, p^{(i)}) \succ (p^{w^{i*}}, p^{(i)})\}$$

$$\langle p^{(i)};x^i] = \{w^i \in X^i : (p^{x^{i*}}, p^{(i)}) \succ (p^{w^{i*}}, p^{(i)})\}$$

$$(x^i;p^{(i)}\rangle = \{w^i \in X^i : (p^{x^{i*}}, p^{(i)}) \succ (p^{w^{i*}}, p^{(i)})\}$$

$$[x^i;p^{(i)}\rangle = \{w^i \in X^i : (p^{x^{i*}}, p^{(i)}) \succ (p^{w^{i*}}, p^{(i)})\}$$

Define $v^i \in \{Y, S^i\}$, where $v^0 \in Y$ $v^i \in S^i$.
In the above defintions, $x^i, zi \in \{y, s^1, s^2, \ldots s^n\}$ Condtional probability measures $P^i{}_T(W) = P^i(T \cap W)/P^i(T)$, for $W, T \in A^i$

E5. a. If $i \in \{y,s\}$, $p_j \in P^j$, such that:

$P^i([x^i;p^{(i)}\rangle) = 1$ for some x^i

$P^i((x^i;p^{(i)}\rangle) > 0$ for every x_i

$P_i(\langle p^{(i)};x^i) > 0$ for some x_i

Then there exists a z^i that defines the set $C=(z^i;p^{(i)}\rangle \in A^i$ such that, for v^i and $p^{i1*}, p^{i0*} \in P^i$ with $(p^{i1}, p^{(i)}) \succ (p^{i0}, p^{(i)})$:

$$p^i(C)(p^{w^{i*}};p^{(i)}) + [1 - p^i(C)](p^{i1*}, p^{(i)}) \succ p^i(C)(p^{v^{i*}};p^{(i)}) + [1 - p^i(C)](p^{i0*}, p^{(i)})$$

b. If $i \in \{y,s\}$, $p^{(i)} \in P^{(i)}$, p^{i0*}, $p^{i1*} \in P^i$ such that:

$P^i(\langle p^i, x^i]) = 1$ for some x^i

$P^i(\langle p^i, x^i)) > 0$ for every x^i

$P^i([x^i, p^j\rangle) > 0$ for some x^i

Then there exists a $u^i \in \{y, s^1, \ldots s^n\}$ that defines the set $U=(p_j;u_i\rangle$ such that, for u^i:

$$p^i(U)(p^{iv*};p^{(i)}) + [1 - p^i(U)](p^{il*}, p^{(i)}) \succ p^i(U)(p^{ui*};p^{(i)}) + [1 - p^i(U)](p^{i0*}, p^{(i)})$$

Fishburn (82) proves the existence of a utility representation $U(s^i,y)$ if and only if E0-E5 hold on the preferences.

References

Becker, Gary S. "A Theory of the Allocation of Time." *Econ J.* 75(1965):493-517.

Boudon, Raymond. The Logic of Relative Frustration. in Rational Choice, ed. by Jon Elster. New York: NY University Press, 1986.

Fafchamps, Marcel. *Sequential Decisions Under Uncertainty and Labor Market Failure: A Model of Household Behavior in the African Semi-Arid Tropics.* Ph. D. Dissertation, U.C. Berkeley, 1989.

Finkelsthain, Israel and James Chalfant. U.C. Berkeley. (1991).

Fishburn, Peter C. *The Foundations of Expected Utility.* Dordrecht, Holland: Reidel, 1982.

_______. "Expected Utility: An Anniversary and a New Era." *J. Risk and Uncertainty* 1(1988):267-83.

Friedman, Milton and Leornard J. Savage. " The Utility Analysis of Choices Involving Risk." *Journal of Political Economy* 56(1948):279-304.

Katoka, Shinji. "A Stochastic Programming Model." *Econometrica* 31(1963): 181-196.

Myles, Gareth D. "Vertical Product Differentiation Can Imply the Friedman-Savage Utility Function." *Economic Letters* 33 (1990):5-9.

Quiggin, John. "On the Optimal Design of Lotteries." *Economica* 58(1991):1-16.

Roy, A.D. "Safety First and the Holding of Assets." *Econometrica* 20(1952):431-449.

Telser, Lester G. "Safety First and Hedging." *Review of Economic Studies.* 23(1955):1-16.

Whitney, Marilyn D. "Why Do the Poor Gamble? Imperfectly Divisible Goods and Choice Under Risk." Working Paper, University of California, Davis, 1990.