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## Working Paper Series

## WORKING PAPER NO. 534

MEASURING INCOME MOBILITY WITH CENSUS DATA by

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DEPARTMENT OF AGRICULTURAL AND RESOURCE ECONOMICS

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# DEPARTMENT OF AGRICULTURAL AND RESOURCE ECONOMICS DIVISION OF AGRICULTURE AND NATURAL RESOURCES UNIVERSITY OF CALIFORNIA 

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MEASURING INCOME MOBILITY WITH CENSUS DATA


#### Abstract

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*We are indebted to George Judge for his helpful comments.

In recent years an increasing amount of attention has been focused on what happens to the distribution of income and to the incomes of key groups such as the poor during the development process. Our knowledge about the links between growth and distribution are based on comparisons of snapshots of income distributions at different points in time. ${ }^{1}$

There are two problems with the snapshot approach to the distribution of income. First, since in actual economies, the number of individuals in the population is growing, everyone's relative position in the income pyramid will be affected by where the new entrants come into the labor force. Thus one cannot track incomes over time for groups such as the rich or the poor by comparing their average incomes at different points in time because one is not dealing with the same group of individuals (Morley 1981). The second problem is that there may be upward mobility for some individuals and downward mobility for others, so that the average earnings over time of the group as a whole may be completely unrepresentative of what happened to the individuals in the group over that time period.

These observational problems stem from the fact that we do not have panel surveys which would enable us to track the incomes of particular individuals over time. In this paper we will describe a method by which such inferences can be made using successive cross section household censuses. The method permits us to estimate mobility over time between different parts of the income pyramid, as well as to determine the influence of economic growth on upward mobility. Obviously the advantage of the method we are suggesting is that panel data are rare in LDC's and very costly to produce whereas censuses are
now available for a number of years and a number of countries. In parts one and two we describe the method and the data problems we encountered in using it, in part three we apply the model to Brazil and in part four we draw some conclusions Erom the exercise.

## PART I. THE ESTIMATION PROBLEM

In thinking about the distriburion of income or earnings, one must, at the outset, distinguish between two different notions-one is the distribution of income across all the occupations in the society and the other is the distribution across the individuals in the society. Obviously at any point in time those two measures are the same. But they are not the same over time because individuals change jobs. Thus the structure of earnings could well remain constant over time while the earnings of individuals did not. That is pareicularly true in an economy where the population is growing. This is not to deny the smportance of the snapshot approach, but rather to complement it with a measure which comes closer to telling what happened to individuals in the labor force over time because we think that is a key element in the evaluation of income inequality.

One can think of the distribution of income at any point in time as the result of a first order Markov process in which the probability that any individual will be in income class $j$ ar rime $t+1$ will depend on which income class he or she was in ar eime $\tau$. Formally our interest in what happens to the income of particular groups over time could then be solved by estimating the transition matrix of the Markov process. 2 What we have from the reported censuses are the row and column sums of transition matrices, whose ijth elements are the number
of people in income class $j$ at time $t$ and income class $i$ at time $t+1$. We are looking for some way of estimating these cell entries, given our observation of the row and column sums. Since there are only 2 n data points and $n^{2}$ unknowns, we need some additional data or restrictions to make progress.

Telser (1963) addressed this problem in the context of market shares for cigarettes using a time series approach. If one takes a sufficient number of observations of the distribution (in his case of cigarette smokers across brands) and if one assumes these distributions are generated by a first order Markov process, Telser showed how to derive an unbiased regression estimator of the unknown elements of the transition matrix. The method gives the transition matrix which minimizes the difference between the actual distribution at time $t+1$ and the distribution predicted by applying the transition matrix to the distribution at time t. Lee, Judge and Zellner (1970) propose alternative Bayesian and non-Bayesian approaches to the estimation of transi. tion probabilities from time series data on marginal totals and examine the properties of these estimates.

Unfortunately the time series approach is not practical for the income distribution problem in LDCs because we do not have a sufficient number of censuses. But we can use regional data from the censuses themselves as an alterative. If we have regional data, and can assume either that the same first order Markov mechanism operates in each region, or that it differs across regions in a predictable way, we can proceed, as Telser did, to use regression analysis to find the transition matrix which minimizes the difference between the observed and the
predicted regional distributions at time $t+1$, given the observed distribution at time $\varepsilon$.

A similar problem has been addressed in sociology and political science. In 1953 Goodman proposed a simple regression to estimate the incerior elements in a Eour way rable of individual characteristics when only the regional row and column sums of the two characteristics are known. His technique made the assumption that the interior conditional probabilities were constant across regions. Crewe and Payne (1976) applied the same general rechnique to get an estimate of the percentage of different occuparional groups voting for the two British political parties. They extended Goodman's technique by assuming that the conditional probabilities were a function of exogenous factors that vary across regions. They derived a best linear unbiased estimator which simultaneously produced an estimate of the transition matrix and of the effect of the exogenous variables on that eransition matrix. Their model was applied to a two by two case-.two parties and two broad occupational classes. Our model is a simple extension of Crewe and Payne to the $n$-dimension case, where the $n$ dimensions are income classes and where we are trying to find the proportion of those in income class $f$ in time $t$ who move to class $i$ at eime $t+1$.

Let $P$ be an nxn eransition matrix whose ijth element, $P_{i j}$, is the proportion of those in income class $j$ at time $t$ who move to class $i$ at time $t+1$. Let $X_{i}$ and $Y_{i}$ be the observed fraction of the toral popularion in income class $i$ ar time $t$ and $t+1$ respectively. $N$ is the number of mutually exclusive income classes. By definition, in matrix notation,
(1)

$$
\begin{aligned}
Y & =P * X \\
\text { or } Y_{i} & =\sum_{j} P_{i j} X_{j} \quad(i-1, \ldots, n)
\end{aligned}
$$

Equation one looks like a regression model where we observe the $X$ 's and the $Y$ 's and estimate the unknown transition parameters $P_{i j}$. Clearly only $n-1$ of these equations are independent. However, rather than dropping one of the equations, we make the equivalent restriction that the sum of each column of $P_{i j}$ 's be equal to one. We further require that each estimated $P_{i j}$ falls between zero and one. The problem with equation (1) is that we do not have enough data to estimate the $P_{i j}$. In our case we have 5 income classes so we are trying to estimate 25 elements of the transition matrix, but we have only five observations of the marginal totals $X_{i}$ and $Y_{1}$.

We proceed by using regional observations. If the Markov process could be assumed to be the same across regions, we could increase the number of observations by taking regional observed values of the distribution. However it is probably unreasonable to assume that mobility is the same across regions. Instead, one would expect it to vary positively with many variables like income growth and labor force structure that vary across regions. Surely one's chances of moving up the distribution ladder are higher in fast growing or highly industrialized regions. Following Crewe and Payne (1976) it is straightforward to modify equation (1) to take account of regional variations in the transition matrix.

We hypothesize that transition probabilities are a function of observable characteristics $Z$ that differ across regions. Thus in the simplest form with only one $Z$ variable:

$$
\begin{equation*}
P_{i j}=a_{i j}+b_{i j} Z \tag{2}
\end{equation*}
$$

In (2) $Z$ is a variable with region-specific values. In our case, the growth rate of income was used. More complex Eormulations, in which the $P_{i j}$ depend on more variables, are possible, but were precluded in our estimation by the small number of degrees of Ereedom we had.

If we now substitute equation (2) into equation (1) we get:

$$
\begin{equation*}
Y_{i}=\sum_{j}\left(a_{i j}+b_{i j} Z\right) X_{j} \quad(1 \propto 1, \ldots n) \tag{3}
\end{equation*}
$$

This is the equation system we will estimate under the two restric. tions:

$$
\begin{align*}
& 0 \leq P_{i j} \leq 1 \quad \text { for all } 1, j  \tag{4}\\
& \sum_{i} P_{i j}=1 \text { for } j=1 \ldots n
\end{align*}
$$

Unfortunately, available statistical packages cannot incorporate both restrictions. Packages which allow for estimation of systems of equations will incorporate the cross-equation constraint (5) but not the within-equation inequality constraint (4). Bayesian packages, which can incorporate the inequality constraine, do not allow for estimation of systems of equations and thus prohibit incorporation of cross-equation constraints.

To circumvent this problem, one can estimate a system of equations explicity incorporating restriction (5), and perform a non-linear eransformation on the coefficients (transition probabilities) that restrices their values to between 0 and 1 , thus incorporating restric.
tion (4). For estimation without a $Z$ variable, such a transformation could take the form:

$$
Y_{i}=\sum_{j} e^{-a^{2}} i j X_{j}
$$

Here $P_{i j}=e^{-a_{1 j}^{2}}$ and thus must fall between 0 and 1 for all values of $a_{i j}$. This method is relatively straightforward for the simple case, where $a \quad Z$ variable is excluded, but proves intractable with the inclusion of such a variable.

The alternative we used was treating the problem as a non-linear programming problem representing the ordinary least squares approach. The objective function minimized is the sum of squared errors and the constraints are given by equations (6), (7) and (8) below. This yields a non-linear programming problem with non-linear inequality constraints.

A representative equation of the constraint set is given by:

$$
\begin{equation*}
Y_{i}^{r}-\sum\left(a_{i j}+b_{i j} Z^{r}\right) X_{j}^{r}+\epsilon^{r} \tag{6}
\end{equation*}
$$

where the $r$ superscript indicates regional observations and $\epsilon$ is the statistical error term.

In our estimation, we required that the inequality constraint hold for all values of $Z$ in the sample and that the cross-equation constraint hold for the mean value of $Z$ in the sample. That is:

$$
\begin{align*}
& 0 \leq P_{i j}^{r} \leq 1 \quad \text { for all } i, j \text { and } r  \tag{7}\\
& \sum_{i} \overline{P_{i j}^{r}}-1 \quad \text { for } j=1, \ldots, n
\end{align*}
$$

Where $P_{i j}^{r}=a_{i j}+b_{i j} Z^{r}$ and $P_{i j}^{r}$ is the sample mean of $P_{i j}^{r}$.
The above procedure yields untiased estimates of the parameters under the usual assumptions that the distribution of the error term is iid.

To estimate standard errors of the estimated coefficient we used the "delete-one jackknife technique" (i.e., subsample size = sample size - 1) by subsampling with replacement from the data and estimating the coefficients for each subsample. Such a technique was required because standard calculations do not incorporate the information contained in the restrictions and so give misleading estimates of the standard errors. Following Efron (1982) the jackknife estimate of the standard error of a paramenter estimate $\hat{\theta}_{i j}$ is calculated according to:

$$
\text { s.e. } \hat{\theta}_{i j}=\sqrt{\frac{n-1}{n} \sum_{k=1}^{n}\left(\hat{\theta}_{i j}^{k}-\overline{\hat{\theta}}_{i j}\right)^{2}}
$$

Where $\hat{\theta}_{i j}^{k}$ is the kth subsample estimate of the parameter, and $\overline{\hat{\theta}}_{i j}$ is the sample mean of the subsample estimares of the paramerer.

PART II. GENERATING THE NECESSARY DATA

Central to the procedure we are using here are sets of regional observations of the distribution of income by age and sex. We have chosen Brazil for this purpose, partly because this regional data is available for 1970 and 1980, and partly because of the interest inherent in Brazil's experience as a country which gained notoriety for its rapid but inequitable growth path over the period 1960-80. Our
estimate will permit us to make what we think is a crucial distinction between what happened to the distribution of income and what happened to the income of those who were in the distribution at a point in time. We hope that by describing in some detail both the econometric procedure and how the necessary data can be obtained from census tapes, we will encourage others to apply the same technique in other countries.

The goal, in the data preparation about to be described, is to obtain an estimate of the income distribution in 1980 of those in the distribution who survived from 1970-a population we label "survivors". That means that we have to remove new entrants from the observed 1980 group in those cohorts where the 1980 labor force is larger than the 1970; and we have to remove those who rerire from the 1970 group in those cohorts where the 1970 labor force is larger than in 1980. The problem is that we have no way of knowing which of the 1980 workers are new entrants in expanding cohorts, or which will retire in those age groups which shrank during the $1970^{\circ} \mathrm{s}$. We can, however, determine many of the characteristics of new entrants and retirees. For each age cohore we first disaggregate by sex education and region with each combination defining a cell. We then subtract the 1970 from the 1980 totals in each cell. If the difference is positive we know there were new entrants between 1970 and 1980 , with the particular characteristics of the cell. We assumed that the new entrants had the same distribution of income as the total observed for that cell in 1980. That permits us to estimate the income distribution of survivors in each cell by subtracting, element by element, the vector of new entrants from the 1980 cell totals.

If we then aggregate across the 120 cells ( 2 sex, 5 for education and 12 regions) in each age cohort, we obtain an estimate of the 1980 income distribution of survivors. It is a vector giving the observed value of the $X_{i}{ }^{\prime}$ s which we will compare with the $Y_{i}{ }^{\prime}$ s obtained from the observed 1970 distribution.

For older age cohorts which had retirements instead of new entrants over the $1970^{\circ}$ s, we use a procedure similar to that described above to estimate the 1970 income distribution of those who would retire during the next decade. In any cell if the 1980 number is smaller than the 1970 number we know there were net retirements. Here we assumed that the retirees had the same income profile in 1970 as the rest of the members of the cell, and we subtract element by element the vector of retirees from the total distribution of the population in the cell in 1970. We then aggregate across the ten cells as before to get a survivors ${ }^{\circ}$ distribution for 1970. This distribution gives the vector of $Y_{i}$ 's which we will use in our regressions along with the $X_{i}$ 's obtained from the observed 1980 distribution.

The other complication encountered in adapring census data for income-mobility estimates is regional migrarion. Clearly, in a country like Brazil, there is a substantial amount of interregional migration. Since we assume that mobility differs across region we have the choice of either excluding migrants, placing them in their destination populations or in their originating regions. We chose the last of the three options because it allows us to use the observed regional distributions without making a correction for migrants similar to the one made for retirements. The disadvantage with our procedure is that for regions with shbrantial outmigration the income grayth that we use
is not equal to that of the originating region since some part comes from migrants to faster growing areas. An interesting question which we can fairly easily explore in an extension to this work is the effect of migration on mobility. How much do those who migrate contribute to the observed mobility patterns? Did migrants do better than those they left behind? We can get a good answer to both of these questions by either comparing the transition matrices of migrants and non-migrants or by putting migrants into the destination population.

The Brazilian public use census tapes upon which this work is based, are a $1 \%$ sample of the demographic censuses of 1970 and 1980 . They contain data on earnings, age, sex, occupation, education, current and previous residence, time in present residence, and many other variables. We aggregated the data into the twelve regions shown in the appendix. We treated as migrants all those who had resided in their current region for less than ten years. We then reassigned migrants to the region they reported as their previous residence.

We divided the 1970 population into six age groups: 15-19, 20-29, 30-39, 40-49, $50-59$ and over. We created five education groups, classifying individuals according to the last grade passed. They are: no education: elementary ( 5 years or less) : middle school ( $6-8$ years): high school (9-12 years); and university.

Income is reported in current cruzeiros. We converted the 1970 data to 1980 cruzeiros using the Rio de Janeiro cost of living index. We then created the following five real income classes:

$$
\text { No income, 0-3599, 3600-4999, 5000-11999, > } 12000
$$

We set the upper limit of class one at 3599 CRS because the 1970 minimum wage was 3600 measured in 1980 cruzeiros. Over the subsequent
decade, the minimum wage rose in real rerms reaching 4149 CRS or $\$ 79$ in 1980. That means, of course, that any worker holding a less than minimum wage job in 1970 would move from class one to class two by finding a minimum wage paying job in 1980. Note that the income variable that we used ostensibly includes earned income from all sources, but there is a substantial degree of underreporting, particularly of income from capital.

PART III. ESTIMATES OF MOBILITY IN BRAZIL

Before the oil shocks, Brazil was often held up as the quintes. sential example of inequitable growth. Between 1960 and 1980 it enjoyed one of the worlds highest growth rates with per capita income rising by $3.9 \%$ per year. But the benefits of this prodigious boom do not appear to have been distributed at all equally across the working population. The Gini coefficient rose from . 50 to . 59 and the average income of the top 20 g grew half again as fast as that of the bottom 60\%. During the $1970^{\circ}$ s, the period we will concentrate on here, the income share of the bottom $60 \%$ shrank from $21.2 \%$ to $19.7 \%$ while the top 20\% rose from 61.7\% to 63.3\%. ${ }^{3}$ There is a long literature suggesting reasons for this unfortunate pattern. (See Bacha and Taylor (1978), Fishlow. (1972), Langoni (1973), Morley and Williamson (1975), Morley (1982), Fields (1977), Pferrerman and Webb (1979), Denslow and Tyler (1983), Hoffman and Kageyama (1986)).

We do not wish to add to this literature here. Instead we wish to look behind the aggregate numbers to get a measure of mobility during the $1970^{\circ}$ s. Granted that the average income of the worst jobs rose more slowly than that of the best. Granted that wage differentials widened. What happened to the average incomes of those who held these worst jobs in 1970 over the ensuing decade? How did they fare relative
to those who were further up the income pyramid in 1970? These are some of the main questions we will attempt to answer here.

One cannot answer these questions with published data because it does not distinguish survivors from new entrants. Since new entrants tend to occupy lower paying jobs, their presence biases downward any comparison based on all respondents. To see this we have displayed three separate distributions in table one: that of the entire observed labor force over 15 years of age, that of survivors, defined as those who were present in both 1970 and 1980 and that of new entrants.

TABLE 1 about here

It is obvious that by any measure there was a lot of upward mobility in Brazil during the $1970^{\circ}$ s. $44 \%$ of the male labor force earned less that the 1970 minimum wage of 3600 cruzeiros. Ten years later only $25 \%$ earned that litele. But the full extent of income growth for those at the botrom of the income pyramid is hidden by the presence of new entrants, $31 \%$ of whom earned less than 3600 Cr\$ in 1980. When we look just at male survivors we find that the $46 \%$ of the labor force earning less than the minimum wage dropped to 19 g ten years later. The difference between the survivors and the new entrants distribution is not so great for females, but is still significant: For survivors as a group, income grew by $9 \%$ per year over the decade. That compares to reported growth of 48 for the labor force as a whole. Clearly, while Brazilian style growth led to a widening of income differentials, it generated a substantial amount of upward mobility as well.

We now look more closely at the det?iled evidence on mobility by using our model to estimate transition matrices for different. sex and age groups. We estimared separacely for males and females and for four age groups: $15-19,20-39,40-59$ and 60 years and older. Each of the middle sroups represents aggregation of two of the previonsly defined age groups. This aggregation increases the degrees of freedom in the estimation process. The aggregation is limited to these groups in order to be consistent with a priori assumptions on mobility patterns.

Table two shows the constrained equation estimates and Table three the transition matrices derived from the coefficients in Table two. To derive Table three we set the regional growth variable at its average level for the relevant age group. Thus, for example, to get the entry ( $P_{01}$ ) for males in age group one rake the coefficient on $X 1$ and subtract from it .008 times the average growth rate of income for this age group (19.73\%).

$$
\left(P_{01}-.192-.008(19.73)-.033\right)
$$

Consider now the mobility patcerns implied by Table three. For males first, there is very litele downard mobility, except for the oldest age group. The upper off diagonal of all the matrices is either zero or a small number. ${ }^{* *}$ IE you were lucky enough to be in the rop Income group in 1970 the chances were better than $85 \%$ that you would stay there. IE you were in $X_{3}$ and were less than 40 in 1970 (age group 0-2), the chances were better than $95 \%$ that you would either stay where you were or move up to the top group. ( $84 \%$ of the $20-39$ year group moved up).

[^0]What about those at the botrom of the distribution in 1970? They also shared in the Eavorable mobility patterns. If you had zero income as a male teenager in 1970, you had a 933 chance of moving up at least one class; and a 498 chance of moving up at least two classes--implying a move up to a job earning more than the minimum wage. If you were in the 20-39 group and earning less than the minimum wage in 1970 (in $X_{1}$ ), you had a $44 \%$ chance of moving up at least one income class. Note here that the minimum wage itself increased in real terms from 3600 to 4149 CRS so part of this mobility is an expansion in the number of jobs covered by minimum wage legislation. Consistent with this interpretation, it appears that upward mobility was greater for those who started further up the labor pyramid. For the $20-39$ age group, compare the very high probabilities that those who started in $X_{2}$ would move up to $Y_{3}$ or $Y_{4}$ or that those who started in $X_{3}$ would move to $Y_{4}$ with the much less favorable prospects for those starting in $X_{1}$. The growth process appears to have helped everyone. But it was most favorable for those placed high enough in the income distribution to take advantage of the rapid expansion in jobs with relatively high educational requirements and wages.

The reader may object that this differential pattern is not found in the $X_{0}$ column where there is also large upward mobility. But $X_{0}$ represents those earning zero income and is a somewhat special case. Most of this group are teenagers many of whom undboubtedly worked on farms or in family business while attending school and who entered the formal labor market during the decade. When they did, many were able to get good jobs. Note also that this group represents less than 33 of the non-teen age labor force.

The picture for female workers is a good deal less favorable than it is for males. There is far more downward mobility and less upward mobility. Whereas about $89 \%$ of males who started in $X_{2}$ or $X_{3}$ in the 20 39 age group moved up at lnast one income class, only about $62 \%$ of similarly placed women did. In the same age group $4 / 18$ of Xl males moved up, compared to only $24 \%$ of the females. For males the zero income class appears to be transitory, with relatively few remaining over the decade. Except for women over 60 the situation is entirely different for females, where the probability is quite high that, if one started in a zero income job, one would still be there ten years later. Thus the data tell us that when measuring mobility one must distinguish by sex.

Consider next the age income profiles underlying the transition matrices. In Table four we show the average annual real income growth rates of different age cohorts of survivors. For this purpose we use a more disaggregated breakdown than the one used for our regression analysis. Table four makes clear the very steep income gradient during the early working years. Over the 1970's decade young workers gained relative to other survivors. Since those young workers tended to start at the bottom of the income pyramid ${ }^{4}$, much of the upward mobility we have documented in Tables 2-3 must, in fact, have been young workers moving up and out of their low paying entry level jobs, which were taken by the next generation of new entrants.

TABLE 4 about here

In trying to interpret and understand what this mohility evidence implies, it is important to go back to the social significanon of
inequality. We have argued that the snapshot approach gives a misleading picture because it does not allow us to track individuals over time. The mobility data confirm that the published aggregate data hides a substantial amount of upward mobility for those who were at the bottom of the 1970 income pyramid. That is comforting to the defenders of trickle down growth. However, if following Paglin [1975], one argues that it is cohort inequality that matters--that is, one's prospects relative to the other members of the same cohort-then Brazil's growth process does not look so appealing. For, as we pointed out, those who started out in the better positions in 1970, also tended to fare better over the ensuing decade. Thus growth seems to have widened the divergences within each age cohort, particularly among male workers, even while it made possible upward mobility for almost everyone.

Consider now the role of the growth variable itself in the regressions. Recall that we hypothesized that the transition probabilities should be related to the overall growth rate of regional income. Upward mobility should be higher and downward mobility lower in the faster growing regions. We should therefore expect the coefficients on the growth variable to be negative in the upper off diagonal of Table 2 , where we are estimating downward mobility, and positive in the lower off diagonal. Table five shows the sign and significance of the growth coefficients in the Table two regressions. We included diagonal elements ( $P_{i i}$ ) together with the downward mobility group and performed a one-tailed t-test at the $95 \%$ confidence level to evaluate the significance of the coefficients. ${ }^{5}$

## TABIE 5 about here

As the reader can see, our hypothesis of a positive growth effect on mobility is confirmed by the data. The bulk of the downward mobility coefficients are negative and the upward positive, and all but one of the seven significant coefficients have the right sign. Another way to see the growth effect on mobility is to compare the $P_{i j}{ }^{\prime} s$ in Table three with the coefficients on the $X_{j}{ }^{\prime} s$ in Table two. The former are calculated at the average growth rate of the relevant age class, while the latter would be the estimated transition probability if income growth were zero. Thus, for example, we see that for the 20-39 age males, if you were in $X_{3}$ in 1970, with zero growth you had a $19 \%$ chance of falling to $Y_{2}$, a $30 \%$ chance of staying where you started and a $45 \%$ chance of moving up to the top class. Rapid growth dramatically changes those probabilities. At the average growth of $9.9 \%$, the probability of Ealling to $X_{2}$ Ealls to $5 \%$ while the probability of moving to the top class rises from $45 \%$ to $84 \%$. One can see the same large growth impact in other cells of the matrix.

One should however enter a cautionary note here. While the growth effect appears to be large, it does not appear to be very significant in a statistical sense. As the reader can see from Table five only 6 of the 73 coefficients are significant and have the right sign. Since the overall fit of these sets of regressions is quite good, that has to mean that the initial position (the $X^{\prime} s$ ) must play a big explanatory role. Indeed, if nus looks at Tahl. $\quad \because$ morn rinscly, that appears to be true particularly along the diagonal. $X_{1}$ is a powerfil predictor of Y , in both the male and female $20-59$ age groups. The same is true for
$X_{3}$ and $Y_{3}$ or $X_{4}$ and $Y_{4}$ in the male regressions and for $X_{2}$ and $X_{3}$ in the 40-59 group of females and for the $X_{4}$ and $Y_{4}$ in the $20-39$ females.

So much for the implications of our estimates. How good are the estimates? How well does our procedure work? There are several ways to address that question. Judging by overall goodness of fit, the estimates, particularly for non-teenagers look very good, explaining more than $90 \%$ of the variance in the regional 1980 distributions given the 1970 distribution. When one looks at the individual equations it is clear that one of the reasons is the very high significance level of $X_{1}$ in the $Y_{1}$ regression in most sex and age classes. The same can be said of $X_{4}$ in the $Y_{4}$ regression for $40-59$ year old males and $20-39$ aged females: Overall, 24 of the 144 coefficients in Table 2 for males and 24 of the 156 for females are significant at the $5 \%$ level.

As an alternative test of goodness of fit, we applied our estimated transition matrix to the all-Brazil data for each age-sex group. ${ }^{6}$ To do this we first calculated weighted averages across states of (1) the per capita GNP growth rate, (2) shares of the population in the given income classes in $1980\left(Y_{j}{ }^{\prime}\right.$ s) and (3) shares of the popula. tion in the income classes in $1970\left(\mathrm{X}_{\mathrm{j}}\right.$ 's). For each age-sex group the weights used were the number of workers in each state corresponding to the age-sex group. We then calculated forecast values for each group by applying the growth-rate-dependent transition probabilities (using the weighted growth rate for the group) to the weighted $X_{j}$ 's to produce predicted values for the weighted $Y_{j}$ 's. These results are presented in Table six along with the actual weighted values.

In order to quantitatively evaluate the fit of our estimates to the actual values, we regressed the actual all-Brazil weighted popularion shares for each income class on the predicted all-Brazil popula. tion shares in a linear regression with no constant. Our observations are the different age-sex groups in each income class. We ran these regressions as an iterated SUR system.

Our syotem is:

$$
\begin{aligned}
& Y_{0}^{W}=\beta_{0} \hat{Y}_{0}^{W}+\epsilon_{0} \\
& Y_{1}^{W}=\beta_{1} \hat{Y}_{1}^{W}+\epsilon_{1} \\
& Y_{2}^{W}=\beta_{2} \hat{Y}_{2}^{W}+\epsilon_{2} \\
& Y_{3}^{W}=\beta_{3} \hat{Y}_{3}^{W}+\epsilon_{3} \\
& Y_{4}^{W}=\beta_{4} \hat{Y}_{4}^{W}+\epsilon_{4}
\end{aligned}
$$

Where the $X_{j}^{W} \rho_{s}$ are the actual weighted population shares for income class $j$ and the $\hat{Y}_{j}^{W}$ 's are the predicted population shares calculated Erom our estimated transition probabilities.

If our estimates are "good" we should see unitary coefficients on the predicted popularion shares (unbiasedness) and high $R^{2}$ values. The results presented in Table seven confirm both expectations. Prediction errors are small and all coefficients are very close to one.

TABLE 7
RESULTS OF MODEL TEST

|  | $\beta_{0}$ | $\beta_{1}$ | $\beta_{2}$ | $\beta_{3}$ | $\beta_{4}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Coefficients | .9930 | .9144 | 1.0188 | 1.0553 | 1.0387 |
| Standard Error | .0296 | .0125 | .0209 | .0117 | .0086 |

We tested the collective proximity of our estimated coefficients to unity through a likelihood ratio test based on comparing the value of the likelihood function for the unrestricted estimation with the value of the likelihood function when all the $\beta^{\prime}$ s were restricted to equal one. This statistic is distributed as a chi-square with five degrees of freedom (five linear restrictions: $\beta_{i} \approx 1, i \Rightarrow 0, \ldots 4$ ). From our results, we cannot reject the null hypothesis that all the $\beta^{\prime} s$ are equal to one. That is, our estimates of the $Y_{j}{ }^{\circ}$ s for allBrazil are unbiased. Our estimates also have very high $R^{2}$ and very low mean square forecast error. We thus do remarkably well.

## CONCLUSION

This paper develops a method for estimating Markov income-mobility matrices using census data. It allows for the first time to use nonpanel data to make estimates of intracohort income mobility over time. The method uses regional observations as the basis for a non-linear programming approximation to a SUR regression with additional interval constraints on the estimated parameters.

The method was successfully applied to Brazil where it generates a very accurate and unbiased estimate to the predicted 1980 distribution of the labor force across income classes, given the observed 1970 distribution. Our results confirm previous assertions that the
observed distribution, which became more unequal over the 1970s, also hides a great deal of upward mobility. We show that the probability of upward mobility for survivors was high in every age cohort, and was also positively related to regional income growth. All income classes benefitted from growth, but it also appears that those who started further up the distribution in 1970 tended to fare better over the ensuing decade. Thus while growth was beneficial to all groups, it also widened income divergences within each age cohort.

TARIEE 1

> DISTRIBUTION OF LABOR FORCE, SHRVIVORS NND IFW FHTRANTS BY INCONE CLASS $1970-1980$
> (in percent)

## MALES

| Income Class | A11 over 15 |  | Survivors |  | New Entrants$\qquad$$1980$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1970 | 1988 | 1970 | 1980 |  |
| 0 | $6.7 \%$ | $5.3 \%$ | $7.5 \%$ | 1. $7 \%$ | $1.0 .6 \%$ |
| 0-3599 | 37.6 | 18.4 | 37.2 | 17.0 | 20.5 |
| 3600-4999 | 23.1 | 13.6 | 22.8 | 12.2 | 15.6 |
| 5000-1.2000 | 18.9 | 34.9 | 18.6 | 36.8 | 32.0 |
| > 2000 | 13.7 | 27.8 | 13.8 | 32.3 | 21.1 |

## FEMALES

| 0 | $9.0 \%$ | $7.9 \%$ | $9.0 \%$ | $6.8 \%$ | $8.8 \%$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $0-3599$ | 50.4 | 32.7 | 49.7 | 31.6 | 33.6 |
| $3600-4999$ | 18.0 | 15.3 | 1.8 .4 | 1.4 .8 | 1.5 .8 |
| $5000-12000$ | 14.9 | 28.9 | 15.4 | 30.4 | 27.7 |
| $>12000$ | 7.7 | 15.1 | 7.5 | 16.4 | 14.1 |

Source: Census Tapes



Tade 2 (continued)
R-8qu880 0.873
Females 40.58

R-squarodes.463
Incomo Category in 1970
32

Note: Figures in italics are jackknife estimates of standard errors.



（panu！̣uoo）$\varepsilon$ өgre」
Table 3 (continued)
Females $\quad 40.59$

|  |  | 10 | $\times 1$ | $\begin{gathered} \text { © Caleg } \\ \quad ¥ 2 \\ \hline \end{gathered}$ | 73 | \% 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Income <br> Category <br> in 198 | $\forall 0$ | 0.518 | 0.090 | 0.000 | 0.000 | 0 |
|  |  | 0.206 | 0.019 | 0.034 | 0.019 | 0 |
|  | Y 1 | 0.230 | 0.768 | 0 | 0 | 0 |
|  |  | 0.163 | 0.029 | 0.000 | 0.000 | 0.000 |
|  | $y 2$ | 0.118 | 0.072 | 0.379 | 0.009 | 0.163 |
|  |  | 0.115 | 0.014 | 0.849 | 0.106 | 0.158 |
|  | $y 3$ | 0.110 | 0.061 | 0.481 | 0.650 | 0.163 |
|  |  | 0.125 | 0.010 | 0.168 | 0.138 | 0.120 |
|  | $y 8$ | 0.029 | 0.013 | 0.180 | 0.340 | 0.678 |
|  |  | 0.077 | 0.010 | 0.120 | 0.177 | 0.201 |

Females 60 and over
Category in 1970
R.squared $=.873$
R-squared $=.463$

$n$
$n$
$n$
0
0

TABIE I

ANNUAL GROWTH RATES OF REAL INCOME BY COHORT

| AGE IN 1970 | MALE | FEMALE |
| :---: | :---: | :---: |
| $15-19$ | 19.48 | $15.9 \%$ |
| $20-29$ | 11.5 | 7.9 |
| $30-39$ | 7.8 | 4.8 |
| $40-49$ | 6.5 | 3.9 |
| $50-59$ | 5.5 | 3.5 |
| +60 | 7.9 | 3.2 |
| Overall Brazil | 10.6 | 7.9 |

Source: Computed from census tapes.

SIGN AND SIGNIFICANCE OF GROWTH COEFFICIENTS

| Downward | Upward |
| :---: | :---: |
| Positive Negative Positive Negative |  |


| Males |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| $15-19$ | 0 | 3 | $4(2)$ | $5(1)$ |
| $20-39$ | 0 | 4 | 8 | 2 |
| $40-59$ | 2 | 1 | 7 | 2 |
| Total | 2 | 8 | $19(2)$ | $9(1)$ |
| Females |  |  |  |  |
| $15-19$ | 0 | $4(2)$ | $6(1)$ | 0 |
| $20-39$ | 0 | 4 | $6(1)$ | 4 |
| $40-59$ | 0 | $11(2)$ | $16(2)$ | 8 |
| Total | 0 |  |  | 4 |

(number of significant coefficents in parentheses)

Table 6. Forecase Resulss


|  | Welghted Population Shares for Males 60 and Over |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| y0 | $y 1$ | $y 2$ | $y 3$ | $y 4$ |
| .0492 | 0.2722 | 0.1693 | 0.3296 | 0.2098 |
|  | Predicted Population Shares |  |  |  |
| .0231 | 0.2819 | 0.1707 | 0.3258 | 0.2018 |

$y 0$
.0774
.0751

| Weighted Population Shares for Females | 15.19 |  |  |
| :---: | :---: | :---: | :---: |
| y 4 | $y 2$ | y | y |
| .3446 | .1595 | .3239 | .0976 |
| Predicted Populallon Shares |  |  |  |
| .4002 | .1469 | .3499 | .0829 |

```
    yo
```

.0623
.0663
yo .0770
.0800
90
.1004
.0862

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${ }^{1}$ The paper by Adelman 1989, in which the dependent variables are the rates of change of the income share between 1960 and 1970 and 1970 and 1980, is an exception.
${ }^{2}$ This view of income mobility was introduced by Prais (1955), and adopted hy Shorrocks (1976), Adelman and Whittle (1980) and Adelman, Subbarao and Vashistra (1985).
$3^{3}$ Bone11i and Sedlacek (1988), p. 14.
${ }^{4}$ The average 1979 income of workers aged 15-29 in 1970 was 4, 289 CRS compared to an average of 6,159 for the population as a whole.
${ }^{5}$ Degrees of freedom for the regressions are calculated according to:
df - \# of equations $X$ \# of observations - \# of coefficients +\# of cross equation constraints $+\#$ of binding inequality constraints

Yielding:

## Regression

Males $\quad 15-19$
20-39
40-59
60 and over
15-19
20-39
4.0-59

60 and over

## Degrees of Freedom

$$
\begin{array}{ccc}
60-50+5+17 & =32 & \\
120-50+5+12 & =87 & \\
120-50+5+16 & =91 & \\
60-50+5+16 & & 31
\end{array}
$$

$$
60-50+5+18=33
$$

$$
120-50+5+14=89
$$

$$
120-50+5+12-87
$$

$$
60-5()+5+23=38
$$

${ }^{6}$ The all-Brazil data was omitted from the regional census data on which the regression estimates of the $P_{i j}$ are based.

## DATA APCFINDK

The appendix gives a brief description of the census tapes we used and outlines the steps to calculate the income distribution of workers in 1970 who will stay in the labor force until at least 1980 and the income distribution of workers in 1080 who already worked in 1970. Hence we consider only workers who neither retire nor enter the work force during the period considered. If a worker migrated during the decade we determined her region of origin.

For 1970 we used public use data available in the one-percent census sample file. This public use data tape records information about 910,808 individuals living in 193,889 households of various cypes. We excluded the record of every individual younger chan 14 years old or who is not in the labor force; 274,529 records of individuals remained. For 1980 the public use tape has 197,413 households and 893,278 individuals. We extracted the records of 202,453 workers over 25 years of age for our mobility estimations. Each of the data sets is a weighted sample of the entire Brazilian population.

We are interested in a particular set of characteristics to
describe the individual. The characteristics are: region (12 outcomes), sex (2 outcomes), age group ( 6 outcomes), level of education ( 5 outcomes), income group (5 outcomes) and economic sector, which was only used in an intermediare step. The variable "region" describes where the worker was living in 1970. We divided Brazil into twelve regions whose boundaries are in many cases identical with the state boundaries. The reported income was
 to 1980 levels. Where respondents did not report inecure, $\cdots$....isisned the averige income of worliers with the samm wharareristire.

For 1980 we used two different versions of the publice use tane because the official version did not contain information about the individual's previous residence. Fortunately we had a second version with this information as well as time of residence. This information was used to create the variable 1970 region for each 1980 respondent. We then constructed a $12 \times 12$ migration matrix for each possible age-education-sex combination. The elements of this matri\% contain the number of workers who lived in the ith region in 1970 and in the $j$ th region in 1980. We converted these entries to percentages of the 1980 population and applied these percentages to the official public use tape. We used the same percentages for each income class in the appropriate characteristics cell. This procedure redistributes the 1980 labor force into the regions they inhabited in 1970 and is thus compatible with the 1970 data.

If we now subtract the number of workers reported in the 1970 census for each particular set of characteristics from the number in 1980, we get a positive or negative number, indicating either net new entrants or retirements. Where new entrants are positive we calculate the number of surviving workers by subtracting the number of workers who have entered the labor force from the number of workers in 1980 with the same set of characteristics. Where new entrants are negative, we subtract our estimate of retirements from the 1970 total to get an estimate of those in the 1970 labor force who survived to 1980. Assuming that the proportion of new entrants or retirements are the same across income classes within each cell, we can then calculate the number of survivors in each income class in 1970 and 1980. This is the basic input to all the distribution data and regressions reported in the paper.


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