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## Optimal conversion of European beech – models and preliminary results

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#### Abstract

In this study we analyse economic aspects of conversion of even-aged beech (Fagus sylvatica L.) stands to near-natural, uneven-aged forests. Growth is modelled using a matrix approach based on a new distance-independent individual tree growth model developed for beech in Denmark. The analyses include characteristics of optimal conversion strategies and their consequences for cash flow, liquidation value, steady-state diameter distributions and long-term harvest policies. We also examine effects of factors such as discount rate, cost structure, prices, site quality, recruitment, and initial state of the stand on optimal conversion strategy and long-term development of the forest. Here we present the applied models and a few preliminary results.

*Key words:* continuous cover, economic optimisation, Fagus sylvatica L., matrix model, near-natural forestry

#### Introduction

In recent years, near-natural, continuous-cover management systems have attracted increasing attention. In part this is due to the ecological advantages of only harvesting smaller patches of trees, or even single trees, at a time; in part it is a consequence of the potential economic advantages of avoiding large investments in regeneration and tending. A range of studies have focused on the silvicultural aspects of the conversion process, e.g. Hanewinkel & Pretz (2000), Kenk & Guehne (2001), and Sterba & Zingg (2001). Gradually, economic aspects have also atracted attention, e.g. Tarp et al. (2000), Buongiorno (2001), Hanewinkel (2001), Price (2003), Tarp et al. (2005), and Price & Price (2006).

Among harvest regimes mentioned as possible paths towards uneven-aged stand structures the most common is undoubtedly 'target diameter harvesting', i.e. a range of harvest regimes characterised by putting emphasis on the largest trees of the stand. Normally the concept includes, however, also thinning of medium-sized or even small trees that have reached 'their target' due to other characteristics than size alone. It is assumed to depend on species and site conditions how harvesting of the largest trees should be combined with thinning and how the overall harvest strategy should depend on initial state of the stand, ecological goals, prices, etc.

The ultimate goal of this study is to thoroughly examine the economics of conversion to uneven-aged beech stands by: (i) identifying optimal conversion strategies; (ii) examining how such strategies depend on initial state of the forest, price functions, discount rate, fixed costs, growth and recruitment; (iii) comparing results with continued even-aged management. The presentation here is limited to description of the applied models and some preliminary results.

#### Models and methods

Describing the growth of even-aged stands and future uneven-aged forest properly within the same model framework is demanding. Using an individual-tree model provides maximum flexibility with respect to simulation of thinning practices and patterns, but in a study that emphasises optimal conversion and associated harvest policies this approach is unnecessarily time consuming and requires complicated thinning rules. Consequently this study applies a

diameter-class approach where an individual-tree growth model is used to create, and dynamically update, the transformation matrices needed to forecast future states of the stand. The basic growth model is the diameter growth model of Nord-Larsen (2006):

$$\Delta d(d_{i,t}, \alpha_0, G_t, G_{L,t}) = \exp(\alpha_0 + \alpha_1 \ln(d_{i,t} + \alpha_5) + \alpha_2 d_{i,t} + \alpha_3 G_t + \alpha_4 G_{L,t} / \ln(d_{i,t} + \alpha_6))$$

where  $\Delta d$  is diameter increment from time t to time t+1,  $d_{i,t}$  is the breast height diameter of tree i at time t,  $G_t$  is the basal area of the stand,  $G_{L,t}$  is the basal area of trees with breast height diameters exceeding that of tree i,  $h_{i,t}$  is the height of tree i, and  $\alpha_0 \dots \alpha_6$  are model parameters (Table 1). The parameter  $\alpha_0$  is site specific. Here it was estimated using  $\alpha_0 = \beta_0 + \beta_1 \ln(H_{50})$ , where  $H_{50}$  is a site index expressed as dominant height at age 50;  $\beta_0$  and  $\beta_1$  are model parameters (Table 1).

Mortality was described using Nord-Larsen's (2006) mortality model:

$$M_{i,t} = M(d_{i,t}, G_{L,t}) = (1 + \exp[\eta_0 + \eta_1/d_{i,t} + \eta_2 G_{L,t}])^{-1},$$

where M is the probability that the subject tree dies within the next year and  $\eta_0 \dots \eta_2$  are model parameters.

Height was estimated using the following regression of height on diameter:

$$h_{i,t} = 1.3 + \frac{H_{50}}{22} \times 41 \left( d_{i,t} / (d_{i,t} + 6.46) \right)^3$$

The parameters of this model were estimated by Emborg et al. (1996) for beech in Suserup Forest, a semi-natural forest on the island of Zealand. However, the coefficient  $H_{50}/22$  is not part of the original model and was introduced here to enable using the model at sites with site indices ( $H_{50}$ ) differing from 22 m. Total and commercial volume were estimated using the volume functions developed by Madsen (1987).

Recruitment was estimated using a model type inspired by Morsing (2001, p. 55-56). The model is expressed as:

$$R_t = R(V_t) = \delta_0 - \delta_1 V_t$$

In Morsing's work the model was used to express the number of trees in the first diameter class but here  $R_t$  denotes the number of trees entering the first diameter class within a five-year period,  $V_t$  is the total standing volume per hectare and  $\delta_0$  and  $\delta_1$  are model parameters. In practice, recruitment varies a lot between sites and over time so, while the general pattern of decreasing recruitment with increasing standing volume is likely to be correct, at least for a wide range of volumes, the precision of predictions are likely to be low, even for locally estimated parameters.

At a given point in time (t) the state of the stand is described as a vector  $N_t = [n_{1,t}...n_{m,t}]^T$  of stem numbers in each of j=1...m fixed-width (w) diameter classes. The harvest at time t,  $F_t = [f_{1,t}...f_{m,t}]^T$  is calculated using a policy vector  $Q_t = [q_{1,t}...q_{m,t}]^T$ . All elements of  $Q_t$ ,  $0 \le q_{j,t} \le 1$ , and  $f_{j,t} = n_{j,t} \cdot q_{j,t}$ . The state of the stand at time t is projected to time  $t+\Delta t$  using:

$$= \begin{bmatrix} n_{1,t+\Delta t} \\ n_{2,t+\Delta t} \\ n_{3,t+\Delta t} \\ \vdots \\ n_{m,t+\Delta t} \end{bmatrix} = \begin{bmatrix} a_1 & 0 & 0 & \cdots & 0 \\ b_1 & a_2 & 0 & \cdots & 0 \\ 0 & b_2 & a_3 & \cdots & 0 \\ 0 & 0 & b_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & b_{m-1} & a_m \end{bmatrix} \begin{bmatrix} n_{1,t}(1-q_{1,t}) \\ n_{2,t}(1-q_{2,t}) \\ n_{3,t}(1-q_{3,t}) \\ n_{4,t}(1-q_{4,t}) \\ \vdots \\ n_{m,t}(1-q_{m,t}) \end{bmatrix} + \begin{bmatrix} R_t \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

where  $R_t$  is the recruitment, A is the transition probability matrix,  $a_1...a_m$  are probabilities that a tree will remain in the diameter class and  $b_1...b_{m-1}$  are probabilities of moving to the next

higher class within the period  $\Delta t$ . At each time step the probabilities  $a_i$  and  $b_i$  are estimated

$$\begin{split} b_j &= \left( \Delta t \, \Delta d(w[j-0.5], H_{s_0}, G_t, G_{L,jt}) \big/ w \right) \times \left( 1 - M(w[j-0.5], G_{L,jt}) \right)^{\Delta t} \\ a_j &= \begin{cases} \left( 1 - \Delta t \, \Delta d(w[j-0.5], H_{s_0}, G_t, G_{L,jt}) \big/ w \right) \\ &\times \left( 1 - M(w[j-0.5], G_{L,jt}) \right)^{\Delta t} & for \ j < m \\ \left( 1 - M(w[j-0.5], G_{L,jt}) \right)^{\Delta t} & for \ j = m \end{cases} \end{split}$$

Growth and mortality within each class are assumed to correspond to those of a tree with diameter equal to the mid diameter of the class,  $d_i = w[j - 0.5]$ . In addition, mortality is assumed to be homogeneously distributed within the class.

The stumpage value of the harvest  $(S_t)$  is calculated by application of the volume function of Madsen (1987) for merchantable volume >10 cm, v(), and a stumpage price function P():

$$S_{t} = \sum_{j=1}^{m} n_{j,t} q_{j,t} \left[ P(d_{j,t}) v(d_{j,t}, h(d_{j,t})) - C_{a} \right],$$

where  $C_a$  is a fixed access cost and the price function P() is the polynomial:

$$P(d_{j,t}) = \phi_0 + \phi_1 d_{j,t} + \phi_2 d_{j,t}^2 + \phi_3 d_{j,t}^3 + \phi_4 d_{j,t}^4 + \phi_5 d_{j,t}^5$$

with parameters  $\phi_0 \dots \phi_5$ .  $C_a$  is mainly of interest for small trees with zero merchantable volume and is introduced here to make sure that the option of cutting lots of non-commercial trees does not become unrealistically attractive.

At the initial stage (t = 0) we assume that an even-aged stand exists. In practice we describe this stand by its diameter corresponding to mean basal area,  $D_g$ , standard deviation of diameter, s(d), skewness of the diameter distribution,  $\gamma_1(d)$ , and stand basal area, G. Based on these the parameters of a three-parameter Weibull distribution were computed. Finally, this distribution was used to predict the number of trees in each diameter class at t = 0.

Table 1 Applied parameters of models describing growth, site quality, mortality, recruitment and stumpage price. The parameters of the stumpage price function are valid for prices as of 2003.

	Model				
Par. No.	$\Delta d()^{\dagger}$	$a_0(H_{50})$	$M()^{\dagger}$	R()	$P()^{\ddagger}$
0	-	-3.4642	7.4157	230.8	2.3021
1	0.7281	0.7938	-36.2820	0.255	423.61
2	-0.0021	-	-0.0875	-	-2991.5
3	-0.0164	-	-	-	9987.5
4	-0.2109	-	-	-	-13907
5	22.5321	-	-	-	6844.4
6	1.0000	-	-	-	-

<sup>&</sup>lt;sup>†</sup> Diameter and diameter increment in millimetres, G and  $G_L$  in m<sup>2</sup>ha<sup>-1</sup>. <sup>‡</sup> Price in  $\in$  m<sup>-3</sup>, diameter in metres.

Assuming that the initial state of the stand is given, our goal is to maximise the discounted value of the future net cash flow. Consequently, the objective function to be maximised is the expectation value:

$$W^* = \max_{Q} \left\{ \sum_{t=0}^{\infty} S_t (1+r)^{-t} \right\}$$
 where *r* is the discount rate.

Since it is impractical to operate with an infinite sequence of policies ( $Q = Q_0, Q_1, ...$ ), since we are focusing on conversion and are therefore mainly interested in the first roughly 100 years, and as we assume that conversion leads to a long-term equilibrium forest that will be treated in the same way indefinitely, we restricted the number of different policies to 7. Each

of the first six policies was applied three times and the seventh policy was applied for the rest of the future. Moreover, as in practice the time horizon of simulations must be limited the objective function was approximated as:

$$W^* = \max_{Q_0 \dots Q_6} \left\{ \sum_{\tau=0}^{\rm T} S_{\tau \Delta t} \; (1+r)^{-\tau \, \Delta t} + \frac{(1+r)^{-({\rm T}+1) \, \Delta t}}{3 \; \Delta t \; r} \; \sum_{\tau={\rm T}-2}^{\rm T} S_{\tau \Delta t} \right\} \; . \label{eq:W*}$$

Obviously, the use of an annuity based on the last three periods ( $\tau = T - 2...T$ ) reduces precision but for commonly used discount rates of 0.01-0.04 and a time horizon of 500 years (T = 100;  $\Delta t = 5$  years) it emerges that the proportion of the total expectation value contributed by the discounted value of this annuity is in the order of  $10^{-3}$ - $10^{-9}$ . We therefore consider the approximation acceptable.

Due to the fact that future stand states are, in part, determined by the present harvest policy the optimisation problem is characterised by strong inseparability of policy vectors. Consequently, the objective function has a large number of local maxima and it is necessary to use a robust global optimisation procedure to search for the optimal harvest policy. We used a simulated annealing algorithm (Cerny 1985; Kirkpatrick et al. 1983) and tuned its parameters to provide stable results for discount rates from 0.01 to 0.04. With the applied price function it emerged that optimal solutions always implied that all trees were felled before 60 cm and we therefore used m=12 diameter classes with a width of w=5 cm. Consequently the total number of policy variables to be optimised was 84. In most cases the optimisation algorithm managed to do this within 200,000 iterations. To further improve the results a greedy coordinate search was used to identify the maximum objective value of the basin of attraction pointed out by the simulated annealing algorithm. Finally, elements of the harvest policies in the optimal solution that had no effect on the objective value were zeroed and it was checked that, for all policy elements, the partial derivatives with respect to the objective function were negative, i.e.  $\forall j,t: \partial W/\partial q_{j,t} < 0$ .

#### Results

Most optimisation runs were done for a stand with the following initial characteristics:  $D_g = 25$  cm, G = 30 m<sup>2</sup>ha<sup>-1</sup>, s(d) = 5 cm,  $\gamma_1(d) = 0.05$ ,  $H_{50} = 22$  m. The corresponding parameters of the Weibull distribution function are: location: 13.31; scale: 12.37; shape: 2.172. With respect to stumpage prices we used functions expressing the relationship between diameter and price in 2003 (parameters in Table 1). These were based on data from Dansk Skovforening et al. (2003). Real prices were calculated using a consumer price index (Statistics Denmark 2005) and the exchange rate was 7.4601 DKK/ $\epsilon$  (Anon. 2005). An access cost of  $C_a = \epsilon$  0.1 was applied for each tree harvested.

Replicated optimisation runs showed that the results were remarkably stable and policy elements that were either 0 or 1 generally did not vary between runs. For policy elements that were somewhere between 0 and 1, standard deviations between individual runs remained small for common discount rates of 0.02-0.04. However, for discount rates of 0.06-0.08 solutions ended up in 2-3 slightly different basins of attraction. Expectation values of the resulting policies were very stable and for discount rates of 0.02-0.08 between-run coefficients of variation were in the order of 10<sup>-4</sup> or less.

Optimisation was done for discount rates ranging from 10<sup>-6</sup> to 10<sup>-1</sup>. The dynamics characterising the optimal solutions are shown in Figure 1 for discount rates from 0.01 to 0.07. The optimal path is illustrated by the development of remaining volume and liquidation value after cutting and the optimal policy is illustrated by the volume and value of the harvest. The graphs clearly show that the first roughly 100 years are characterised by very strong dynamics. This is the period where the harvest policy changes every 15 years and the period where the remaining part of the original stand is harvested. When the seventh harvest

policy is introduced the stand has been put on the track and is allowed to converge towards its long-term steady state. Since stand basal area and the maximum diameter of trees left in the forest decrease with increasing discount rate, high discount rates lead to fast growth and short expected life-time of trees. Consequently, for high discount rates the time needed for the forest to converge towards the final state is relatively short and the fluctuations are dampened very fast compared to the dynamics observed for lower discount rates. An important consequence of the lower maximum diameter resulting for high discount rates is that, although the annual harvest does not vary much between discount rates and is in fact greatest for a discount rate of 6%, the value of the harvest decreases considerably with increasing discount rate.

The way that the long-term equilibrium diameter distribution is approached in the optimal solution depends on initial state of the stand, discount rate and access cost. Figure 2 shows an example for a discount rate of 2%. The initial state of the stand corresponds to the base case described above. The graphs show that initially (t = 0) the stand is harvested from below. Later, at t = 15, when the next policy is activated thinning is discontinued for some time but from t = 30 harvest from above (45-50 cm) is started and at t = 60 the harvest diameter is 50-55 cm. Finally, at t = 120 and t = 500 the last harvest policy is active and trees are harvested in the diameter class 45-50 cm. Closer examination of the patterns reveals that from year 45 to 85 trees are left to grow beyond the size found at the long-term equilibrium where the harvest diameter is 45-50 cm, and from year 0 to 60 the number of trees (after harvest) in the greater-size diameter classes exceed the number of trees found in those classes in the steady-state forest.

The steady-state harvest policy (at t=500) and the distribution of the steady-state harvest are illustrated in Figure 3 for discount rates from 0.0001% to 10%. With respect to the harvest policies it is noted that, as expected, the size at which all remaining trees are removed ( $q_{j,t}=1$ ) is decreasing with increasing discount rate. More interestingly, it is observed that for discount rates of 1-2% felling takes place in the largest diameter classes only but for other discount rates at least one class of smaller-diameter trees is thinned. This can be thought of as a combined tending and commercial felling operation. For high discount rates the two size ranges tend to merge and tending and commercial felling can no longer be distinguished. By running the optimisation with different assumptions regarding access cost and recruitment it was observed that, in agreement with common sense, the use of tending depends on both. If lots of trees are recruited more need to be removed and to the extent that natural mortality leaves too many, implying that diameter growth becomes relatively slow, thinning is introduced. Similarly, if access cost is low it becomes an attractive solution to thin trees before they have reached commercial size.

Steady-state diameter distributions are shown in Figure 4. In agreement with the observations mentioned above the maximum diameter generally decreases with increasing discount rate. In addition, due to the decreasing standing volume and basal area with increasing discount rate (cf. Figure 1) the recruitment increases and the mortality in the smaller diameter classes decreases. Consequently, for discount rates from 0.0001% to 5% the remaining stem number after harvest in the diameter class 0-5 cm is observed to increase with increasing discount rate. For greater discount rates it appears that two alternative strategies are competing: (a) thinning many pole-sized trees and leaving the remaining trees to grow to 35-40 cm, and (b) thinning fewer pole-sized trees and reducing the final target diameter. Strategy (a) tends to win for discount rates of 0.07-0.08 and since it is characterised by slightly greater standing volumes than (b) the stem numbers in the diameter class 0-5 cm are lower than for discount rates of 0.06 and 0.09-0.10.

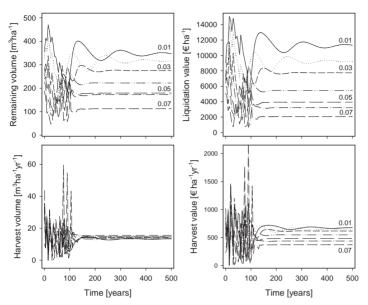


Figure 1. Dynamics of remaining volume, liquidation value after harvest, harvest volume and harvest value. Results for discount rates from 0.01 to 0.07.

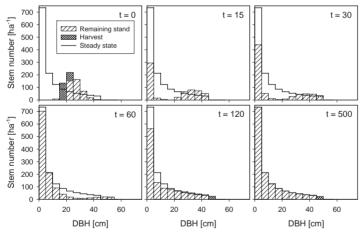


Figure 2.Diameter distributions at  $t=0,\,15,\,30,\,60,\,120$  and 500 years. The long-term diameter distribution (remaining stand after harvest) is shown with thick black lines. A discount rate of 2% and an access cost  $\in$  0.1 per tree are assumed.

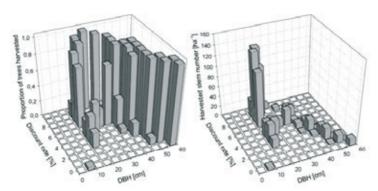


Figure 3.Steady-state harvest policy (left) and harvested stem number (right) in diameter classes 0-5...55-60 cm for discount rates from 0.0001 to 10 per cent. Empty classes are shown in white.

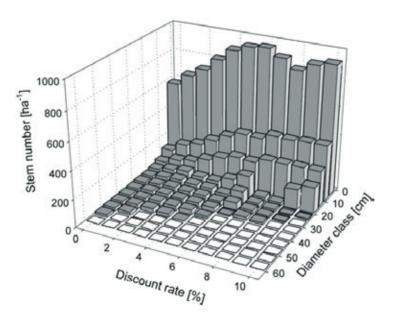


Figure 4. Steady-state diameter distributions after harvest for discount rates from 0.0001 to 10 per cent. Empty classes are shown in white.

#### Discussion

A major advantage of the applied matrix approach is that, although the model is initialised for, and continues to operate within, a stand covering a certain area the computational effort is not influenced by this area and the associated number of stems. In comparison, an individual-tree approach based on a tree list does not allow operating with very large stands (lists), implying that optimality results would be characterised by random variation caused by the discreteness of the approach.

The applied individual-tree growth model was developed on the basis of a large dataset from even-aged stands. Consequently, the model can be expected to better describe competition within the stand existing at the beginning of the simulations than the unevenaged stand that gradually develops. However, at present no better model components are available for Danish conditions.

The effect of stand density on recruitment is only supported by data from a single stand (Morsing 2001) but, in reality, recruitment may vary considerably from site to site and over time. In the simulations it is assumed that recruitment can be described as a linear function of volume. This may indeed be true within the range of conditions typically found in forest stands but for very high and, particularly, very low stand densities it is not likely to be an adequate description of reality.

Obviously, the use of a fixed price function constitutes a large simplification of reality. However, in the present context use of the same price function for the initial, evenaged population of trees and for future generations of uneven-aged stands seems even more disputable. Quite likely the initial, even-aged stand is the outcome of an intensive (and costly) regeneration process and, that being the case, the density and quality of the stand may be everywhere nearly the same. However, since future regeneration is assumed to happen spontaneously and will usually be most successful in medium-sized gaps, a more heterogeneous stand can be expected. The average quality of the stand may be better or worse than that of the existing stand but the extent to which this depends on the chosen harvest strategy is not accounted for by the applied model. Fundamentally, it can be expected that the quality of trees left in the stand after active thinning (i.e. target diameter harvesting in small to medium size classes) is higher than that of trees left as a result of natural selection. It is not likely that the natural selection process will favour survival of high quality trees and, therefore, it seems safe to assume that stumpage prices will be lower if no thinning is done than if a quality-oriented thinning programme is introduced. This assumption will be incorporated in future analyses.

The results indicated that always harvesting the largest trees may not in all cases be the most profitable practice. At the start of the conversion the diameter of the largest trees may still be so low and their expected future price increase so great that harvesting these largest trees is not justified from an economic point of view. Instead, to reduce basal area and increase diameter growth in such cases thinning from below may be preferable in the beginning. Similarly, at steady state the recruitment may be so high and the natural mortality so relatively low that it becomes profitable to supplement harvest of the largest trees with thinning among pole-sized trees. As a consequence the overall harvest will include both large and relatively small trees, making the harvest resemble the pattern observed by Sterba & Zingg (2001, Fig. 3).

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