Working Paper Series

Working Paper No. 467

THE EFFECTS ON PRODUCTION AND PROFITS OF DIFFERENT FORMS OF POLLUTION CONTROL STANDARDS

by

Gloria E. Helfand

DEPARTMENT OF AGRICULTURAL AND RESOURCE ECONOMICS

BERKELEY

CALIFORNIA AGRICULTURAL EXPERIMENT STATION

University of California

The Effects on Production and Profits of Different Forms of Pollution Control Standards

Gloria E. Helfand

Department of Agricultural and Resource Economics
207 Giannini Hall
University of California
Berkeley, California 94720

February, 1988

The Effects on Production and Profits of Different Forms of Pollution Control Standards

Five different specifications of a pollution control restriction are analyzed for their comparative effects on input use, output, and profits. Because of the different effects, the choice of regulatory instrument may reveal the relative power of some interest groups concerned with different kinds of pollutants.

The Effects on Production and Profits of Different Forms of Pollution Control Standards

Environmental restrictions imposed on industries in the United States take a variety of forms. For instance, new electric utility steam generators are required to pollute no more than 520 nanograms of sulfur dioxide per joule of fuel input, a restriction on the amount of pollution it can expel per unit of input (Office of the Federal Register, 1987). In contrast, existing coal-fired power plants are required to install scrubbers, a technology standard (Ackerman and Hassler); and agricultural inputs, such as fertilizers or pesticides, are often restricted directly. These different forms of restrictions will lead to different incentive structures for the regulated industries, and therefore to different levels of input usage, output, and profits.

This paper explores the incentives developed by these and other formulations of environmental regulations and contrasts the characteristics of the industries where each is likely to occur. It will show, for instance, that the form of regulation which maximizes profits for the regulated firm is different from the one that maximizes production in the industry, and therefore that maximizes use of inputs. Because many different interest groups are involved in the development of these environmental regulations, and because they do not all have the same goals for the legislation, consideration of these tradeoffs is likely to have played a role in the choice of policy instruments for the different pollution problems.

The existing studies of alternative specifications of pollution control regulations concentrate on the efficiency of those measures and the aggregate welfare losses associated with them. Holterman concentrates on ways to regulate pollution efficiently if pollution cannot be regulated directly. In her analysis, if a function can be found that links the inputs for the production process to pollution, then taxes can be placed on these inputs that will replicate the optimal approach of taxing pollution directly. Harford and Karp compare the efficiency of different forms of pollution standards against a baseline of no pollution regulation. In their study, which holds output fixed, they find that a standard mandating a fixed ratio of pollution per unit of output is most efficient because it mimics the input mix of no pollution regulation. Thomas looks at the welfare costs of an emissions standard compared to several other forms of regulation for the steel industry: mandating pollution control expenditures, restricting the use of a polluting input, or restricting pollution as a function of the polluting input. He finds that these other policies, except the input restriction, have 30 to 40 percent higher welfare costs than the plain standard, though these welfare costs are only about 1 to 1.4 percent of the value of steel output; however, direct restriction of the fuel input has a welfare cost of about 30 percent of the value of output.

These aggregate welfare measures reflect the efficiency of the regulations, but they do not reflect the political tradeoffs involved in the choice of regulatory instrument. As the literature on the political economy of regulation argues (Stigler, Peltzman, Rausser), politicians may act based not on the efficiency of their actions, but on the interest group support that they derive from them. By looking not at an aggregate welfare measure, but at the disaggregated effects on inputs and output, this analysis attempts to trace some of the political tradeoffs involved in the choice of regulatory instrument.

A Firm's Decisions under Different Forms of Pollution Control Standards

The following analysis will compare the effects of different ways of setting a standard on a firm's input and output decisions. The initial basis for comparison will be the allocation decision that results from the unconstrained profit-maximization problem. Later, comparisons will be made of the differences in impacts of the different forms of standards.

Assume an individual firm is profit-maximizing in a competitive industry with a neoclassical production function $f(x_1, x_2)$, where x_1, x_2 are its inputs. Further assume that $f_1, f_2 > 0$ (where subscripts denote derivatives with respect to input i): that is, both inputs increase production; additionally, $f_{11}, f_{22} < 0$ and the Hessian of the production function is negative definite, indicating that all inputs and the production function itself are subject to diminishing marginal returns. The firm is assumed to exhibit decreasing returns to scale, to give the firm positive profits before regulation.

The firm also produces pollution $A(x_1, x_2)$. Input 1 increases pollution (i.e., $A_1 > 0$), while input

2 decreases pollution ($A_2 < 0$). It is also assumed that $A_{11}, A_{22} > 0$: that is, that pollution increases more than proportionately with increased amounts of the polluting input, while the pollution-abating input exhibits decreasing marginal effectiveness. The formulation of pollution as a joint product, rather than as an input to the production process, is intended to reflect the way that a firm is likely to view pollution: that is, as a completely residual activity, not one that involves a conscious decision about the amount of pollution a firm will "use" in the production process. This formulation permits pollution to be controlled through changes in the input mix as well as changes in total output.

The firm faces an output price p and input prices w_1 and w_2, with the subscripts reflecting to which input the price belongs. They are assumed fixed for the purposes of this analysis.

Before a pollution control constraint is imposed, the firm is assumed to maximize profits. The unconstrained problem becomes

$$\max_{x_1, x_2} \pi = pf(x_1, x_2) - w_1 x_1 - w_2 x_2,$$

which results in a unique solution $x^0 = (x_1^0, x_2^0)$ (the uniqueness arising out of the assumption of the concavity of the production function and decreasing returns to scale).

Graphically, this result can be displayed on an isoquant diagram, such as Figure 1. Here, the polluting input, x_1, is measured along the horizontal axis, while the pollution-abating input, x_2, is measured along the vertical axis. The profit-maximizing combination of these inputs is found at the tangency of the isoquant $f^0 = f(x_1^0, x_2^0)$ with the isocost line whose slope is $-\frac{w_1}{w_2}$.

Because of the uniqueness of this maximum, isoprofit contours can be drawn on the diagram as well. That these contours are convex rings is shown in the following proposition.

Proposition 1: Under the assumptions of concavity of the production function and decreasing returns to scale, the isoprofit contours form convex rings.

Proof:(1) A unique maximum exists, by the assumptions of concavity of the production function and decreasing returns to scale.

(2) The level sets are convex. Define: $x^i = (x_1^i, x_2^i),\ i = a, b, c$.

Let $\pi(x^a) = \pi(x^c) = K$; $x^b = \lambda x^a + (1-\lambda)x^c$. The level sets are convex rings if $\pi(x^b) \geq K$.

By the assumption of concavity of f, $f(x^b) \geq \lambda f(x^a) + (1-\lambda) f(x^c)$. Therefore:

$$\pi(x^b) = pf(x^b) - wx^b$$
$$\geq p[\lambda f(x^a) + (1-\lambda)f(x^c)] - w[\lambda x^a + (1-\lambda)x^c]$$
$$= \lambda[pf(x^a) - wx^a] + (1-\lambda)[pf(x^c) - wx^c] = \lambda K + (1-\lambda)K = K.$$

An isopollution line can be added to this diagram. Totally differentiating the function $A = A(x_1, x_2)$ gives the slope of this line to be $\frac{dx_2}{dx_1} = -\frac{A_1}{A_2} > 0$: that is, as use of the polluting input increases, use of the pollution-abating input must also increase in order to maintain a constant level of pollution. Isopollution contours above and to the left of another contour represent lower levels of pollution: for a given amount of the pollution-abating input, pollution decreases as use of the polluting input decreases. On the diagram in Figure 1, the initial level of pollution is represented by the line $A^0 = A(x_1^0, x_2^0)$. The line A^1, above and to the left of the original contour, represents a lower level of pollution.

Five different pollution control standards will be compared: a fixed level of emissions, a fixed level of emissions per unit of output, a fixed level of emissions per unit of an input, a fixed level of output, and a fixed level of an input. The following subsections will describe each of the standards and will derive the different effects that each has on input use, output, and profits.

Case 1: Standard as a Set Level of Emissions. Let Z_1 be the numerical standard set when pollution is regulated by the amount of total emissions permissible in a certain period of time. It can be represented mathematically as a constraint on the profit-maximization problem with the form $A \leq Z_1$.

One way to analyze the effects of this constraint is to set up the profit-maximization problem, differentiate totally the first-order conditions, and use comparative statics to analyze the effects of this constraint on the levels of input use, outputs, and pollution. These results are reported in Appendix 1. The same results can be derived from a graphical analysis, though, and this approach will be the focus of the remainder of the paper.

Figure 1 describes the effect of this constraint on the regulated firm. The firm still seeks to maximize profits; however, it is constrained to use input combinations along or to the left of the isopollution curve A^1, which represents a lower level of pollution than isopollution curve A^0. Therefore, the new optimal combination of inputs, $x^1 = (x_1^1, x_2^1)$, will be at the tangency of the constraint and the highest attainable isoprofit contour.

In general, as Figure 1 shows, use of the polluting input should decrease, and use of the pollution-abating input should increase. These are the expected results, since the purpose of the constraint is to reduce pollution. The effects on production are ambiguous, since one input is increasing and the other decreasing. Profits are unambiguously lower under this standard, since the firm can always do better in an unconstrained situation than in a constrained one. However, use of the polluting input may actually increase, or use of the abating input may decrease. Graphically, these seemingly perverse results depend on the actual shape of the isoprofit contours; mathematically, these results depend on the sign and magnitude of the f_{12} term, the effect on the marginal product of input 1 [2] as use of input 2 [1] changes. All that is known theoretically about this term is that $f_{11}f_{22} > f_{12}^2$, by the assumption of strict concavity of the production function. This information is not sufficient to determine more about the effects of this constraint on input use and output.

Case 2: Standard as Emissions per Unit of Output. In this case, the pollution control standard is assumed to take the form of a set level of emissions per unit of output. Mathematically, this constraint is represented as $\frac{A}{f} \le Z_2$.

As before, the comparative statics results are reported in Appendix 2, while the primary analysis will focus on the graphical interpretation. Totally differentiating the equation for the constraint gives the slope of the constraint line, $\frac{dx_2}{dx_1} = -\frac{fA_1 - f_1A}{fA_2 - f_2A}$. The denominator is negative according to the assumptions on signs. $fA_1 - f_1A$ is positive if $\frac{A_1}{A/x_1} > \frac{f_1}{f/x_1}$. Since A is assumed to be convex in input 1, the marginal amount of pollution is greater than the average amount at any given point, so $\frac{A_1}{A/x_1}$ is greater than one. On the other hand, f is assumed to be concave in input 1; by analogous reasoning, the marginal product is less than the average product, and it is less than one. Therefore, $fA_1 - f_1A$ is positive, and the slope of this standard is positive.

That the slope of this line is less than the slope of the isopollution lines can be shown by subtraction. $\frac{dx_2}{dx_1}(Z_2) - \frac{dx_2}{dx_1}(Z_1) = \frac{(f_1A_2 - f_2A_1)A}{(fA_2 - f_2A)A_2} < 0$, by the assumed signs of these terms. Therefore, though this line slopes upward, it slopes upward at a lesser angle than does the isopollution line.

This line is illustrated in Figure 1. Once again, the firm seeks the most inward isoprofit contour that meets the standard; the solution is found at the tangency $x^2 = (x_1^2, x_2^2)$. Again, in the "normal" case, use of the polluting input should drop, and use of the abating input should increase. The effects on production remain ambiguous; profits unambiguously decrease. As before, the "perverse" results of use of the polluting input increasing, or use of the pollution-abating input decreasing, are possible, depending on the shapes of the isoprofit contours.

Case 3: Standard as Emissions per Unit of a Specified Input. Here, emissions are regulated per unit of an input, such as the amount of sulfur permissible in a ton of coal used to produce electricity. The mathematical representation of this standard is $\frac{A}{x_i} \leq Z_{3i}$, $i = 1$ *or* 2, where i is the subscript for the input in terms of which pollution is measured.

Two cases are possible here: regulating pollution per unit of the pollution-causing input, as with the example of sulfur dioxide per unit of energy input; or regulating pollution per unit of the abating input, such as regulating biological oxygen demand in water pollution per unit of water used in a production process. Magat *et al.* (p. 35) note concerns raised by the Environmental Protection Agency over use of the latter instrument: the Effluent Guidelines Division was concerned that a regulation of that form, rather than regulation per unit of output (as in Case 2, above), would only increase the use of the "diluting" input without leading to actual cleanup.

Let Case 3a represent the situation where pollution is regulated in terms of the polluting input, and Case 3b represent pollution regulated in terms of the pollution-reducing input. The comparative statics results for this analysis are given in Appendices 3a and 3b.

The slope for $Z_{3a} = \frac{A}{x_1}$ is $-\frac{A_1x_1 - A}{A_2x_1} > 0$; for $Z_{3b} = \frac{A}{x_2}$, the slope is $-\frac{A_1x_2}{A_2x_2 - A} > 0$. $A_1x_1 - A$ is positive if $A_1 > \frac{A}{x_1}$. By the same reasoning used for $fA_1 - f_1A$ in Case 2 above, the marginal pollution at any given level of inputs is greater than the average level by the assumed convexity of A in the polluting inputs, and $A_1x_1 - A$ is indeed positive. $A_2x_2 - A$ is clearly negative by the assumption that A_2 is negative. Thus, both these standards result in upward-sloping constraints, though, through comparisons such as those done for Standard 2, it can be shown that both these lines have slopes less than that of the isopollution lines. However, because similar comparisons also show that they cannot be readily distinguished from each other, or from the slope of the constraint line for Standard 2, the remainder of this analysis will not distinguish among them. These three formulations of the standard will be referred to as the "dilution" standards, since they all involve measuring pollution diluted by either output or input. In Figure 1, the effects of standards 3a and 3b are represented as the same as the effects of standard 2.

Case 4: Standard as a Set Level of Total Output. This method is used in Buchanan and Tullock to examine the distributional effects of taxes versus standards. It does not allow input substitution to reduce pollution while potentially maintaining constant input levels. Mathematically, the constraint in this case can be described as $f \leq Z_4$. If f = cA, where c is a parameter, then this case collapses into Case 1. However, if pollution has another relationship to output or to inputs, then this formulation of a standard is notable for making the firm ignore the pollution problem and concentrate on reducing output.

The comparative statics analysis of this restriction is given in Appendix 4. Graphically, as shown in Figure 1, this formulation of the standard mandates that the firm operate on a given isoquant. The firm will operate with the input mix given by the point of tangency with the highest isoprofit curve. Here, use of both inputs decreases; by definition, output decreases; and profits decline because the problem is constrained away from the optimum.

Case 5: Standard as a Set Amount of a Specified Input. This form of standard may be used in two different ways. A maximum standard can be set on the use of a polluting input -- for instance, a farmer may be prohibited from using more than a specified amount of nitrate fertilizer per acre. Alternatively, imposing a minimum standard on the use of a pollution-reducing input captures the effect of imposing a particular pollution control technology on a firm. Mathematically, this constraint can take two forms: subcase (a) for a pollution-increasing input, and subcase (b) for a pollution-decreasing input. Subcase (a) can be represented as $x_1 \leq Z_{5a}$; subcase (b) as $x_2 \geq Z_{5b}$.

Comparative statics results for these formulations of the standards are given in Appendix 5. Graphically, the limitation on use of the polluting input is a vertical line at the chosen level of input 1; the requirement for a minimum use of the pollution-abating input results in a horizontal line at the chosen level of input 2. These are shown in Figure 2.

For subcase (a), the limit on use of the polluting input obviously causes a reduction in the use of input 1. The effect on input 2 is ambiguous: mathematically, the effect on input 2 depends on the sign of f_{12}. Production will be lower if use of input 2 is not increased much, is unaffected, or decreases. Profits are obviously reduced, as in all previous cases, since the firm would like to use more of this input than is allowed.

Subcase (b) provides some interesting contrasts with subcase (a). Now that a *minimum* amount of input 2 is required, use of input 1 may either increase or decrease, again depending on the sign of f_{12}. Profits nevertheless decrease, since the firm is being forced to use more of an input than it otherwise would desire to use.

Overall, then, most of these different forms of pollution control constraints have their own peculiarities. While Case 5b, the minimum requirement for the pollution-reducing input, increases output, Case 5a, with the constraint on use of the polluting input, and Case 4, the case of a constraint on output, decrease output; the other cases have ambiguous effects on this variable, since they all have use of the polluting input decreasing and use of the pollution-abating input increasing. For all cases except 5b, use of the polluting input drops as the pollution control constraint is tightened; for all cases except 4 and 5a, use of the abating input increases. The following section will describe further comparisons that can be derived from these results.

Comparisons of the Different Standards

Though the results in the previous sections reveal some differences among the different standards, most of the results given there are preliminarily indistinguishable. In this section, more results will be derived through a comparison of the results for the individual standards. Comparison of the impacts of the standards will suggest which interest groups may gain and which may lose by use of the different standards.

The results from the previous section cannot be compared directly, however. Because the standards differ in form -- indeed, even in units of measurement -- then a change of a given magnitude in one standard is not equivalent to a change of the same magnitude in another standard. Therefore, in order to adjust the standards to reflect a common level of change, a baseline of requiring that all of them ultimately achieve the same level of pollution is set.

The comparisons are all made in Figure 1. Their ordering along the isopollution line A^1 is made by comparing the slopes implied by the different standards. For instance, the slope for Standard 2, pollution per unit of output, is known to be positive but with a lesser slope than that of the isopollution line. If it is to be along the same isoprofit contour as is Standard 1, then the tangency will be to the right of the tangency for Standard 1, here called x^1; however, if it is along that isoprofit contour, then it cannot be on the isopollution line, since it lies above that isoprofit contour except at x^1. Therefore, the combination of inputs that maximize profits while achieving Standard 2, x^2, is above and to the right of the combination for Standard 1 along the isopollution line A^1.

Similar comparisons can be made for the other standards. The slope of the standard line for Standard 5b, regulation of the pollution-abating input, is clearly 0, therefore less than the slope for Standard 2, and therefore x^{5b} is to the right and above x^2 along the isopollution line. Standard 5a, regulation of the polluting input, has a vertical slope; therefore, x^{5a} is below and to the left of x^1. Finally, Standard 4, regulation of output itself, is an isoquant with a negative slope, resulting in x^4 being below and to the left of x^{5a} along the isopollution line. Thus is the strict ordering of the inputs determined.

This procedure reveals a strict ordering among the standards for levels of input use, levels of output, and some information on relative profits. The standard that most reduces input use and, therefore, output levels is Standard 4, the restriction on output, followed by Standard 5a, the restriction on the polluting input; this standard also gives the firm higher profits than does Standard 4. Standard 1, the restriction on pollution itself, gives the highest level of profits among any of these standards; however, it has lower levels of input use and therefore output than do the "dilution" standards, pollution per unit of output or input, which in turn have lower levels of input use and output (though higher profits) than mandating a minimum amount of the pollution-abating input.

The relative levels of profits between Standards 4 and 2, Standards 4 and 5b, Standards 5a and 2, and Standards 5a and 5b, cannot be determined from this analysis. On the diagram, Standards 4 and 5b, and Standards 5a and 2 are drawn on the same isoprofit contours only to keep the diagram more simple.

At this point, it is possible to analyze some possible political interests aligned with some of these different standards. For instance, input-related organizations, such as labor unions, are going to look more favorably on formulations of standards that will lead to less decrease, or more increase, in input use; they would be expected to prefer a mandate for the pollution-abating input, or perhaps a restriction on pollution per unit of output or input, over some of the other standards. In contrast, firms themselves would prefer restrictions on the level of pollution, or perhaps on pollution per unit of output or a restriction on the polluting input, since these standards have relatively higher levels of output. If output reduction is considered a benefit rather than a cost, then restrictions on the polluting input or on production will achieve these goals more effectively than the other standards.

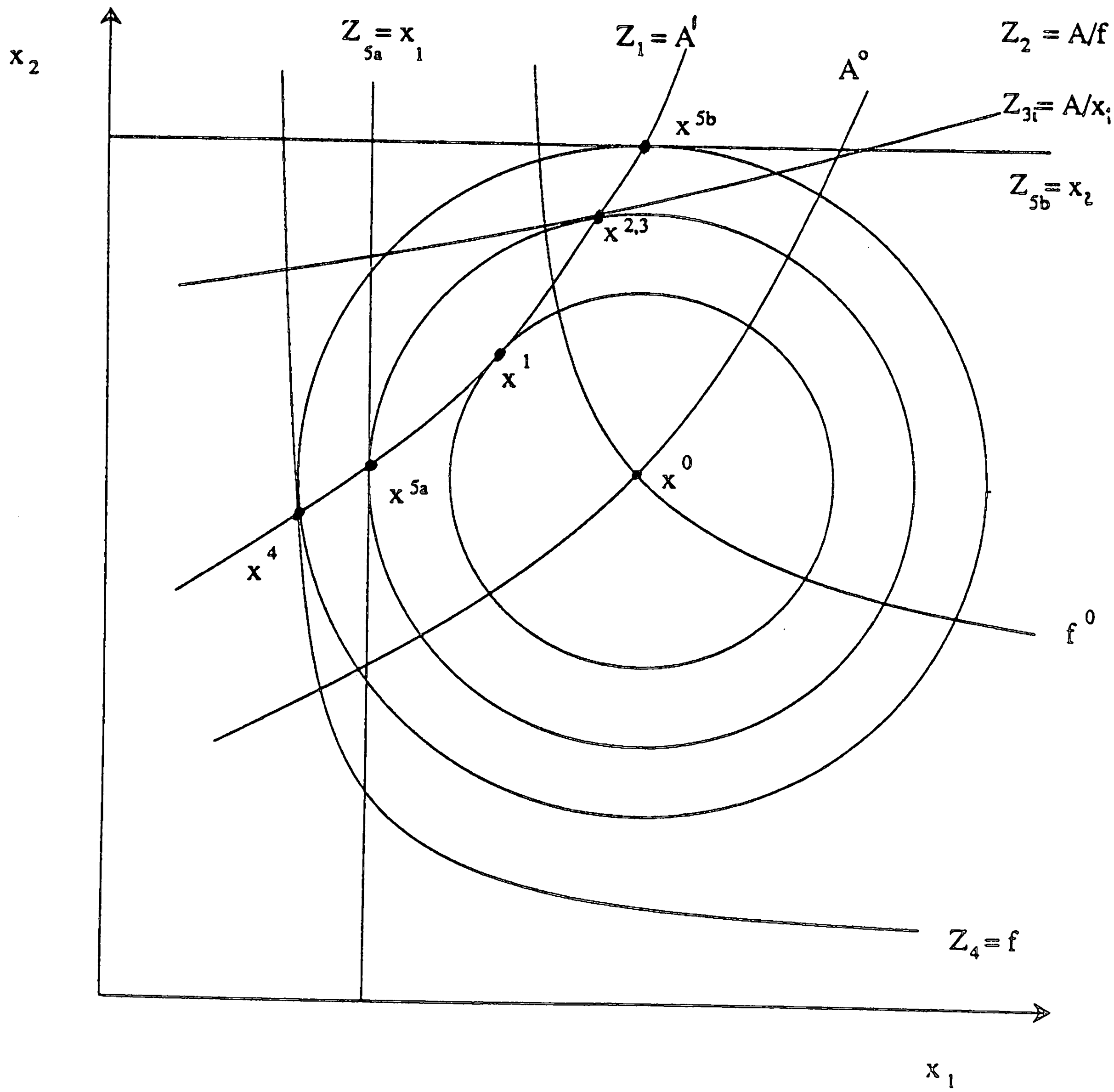
Examples of some environmental policies lend support to these arguments. For instance, many industrial air pollutants from new sources face restrictions on their emissions of pollution per unit of output or input (Office of the Federal Register): in cases such as these, it is likely that industrial workers and the firms themselves were interested in getting regulations that did not unduly restrict production or profits. The use of scrubbers to reduce pollution from coal-fired electricity-generation plants is a more extreme instance: this requirement for the abating input was imposed in large part to prevent achievement of a pollution standard by the substitution of low-sulfur coal for high-sulfur coal. High-sulfur coal comes primarily from the eastern half of the United States, while low-sulfur coal comes primarily from the West. Eastern politicians were reluctant to see eastern coal-mining jobs lost due to this substitution and therefore imposed a form of regulation that had the least adverse effect on these jobs (Ackerman and Hassler). In contrast, some agricultural pollutants are restricted by regulating the polluting input, such as limitations on nitrate fertilizers or on the use of pesticides. In agriculture, because of many federal programs that encourage overproduction, a regulation that leads to less output than another regulation may well be viewed favorably. These examples exhibit the roles that different interest groups are likely to play in the regulatory process, and they reveal how one policy can be manipulated to achieve more than one objective.

Conclusion

This analysis has shown that different formulations of environmental regulations can have different effects on the levels of input use, output, and profits from the production process. From the perspective of the firm, and from the perspective of efficiency, regulating pollution directly is the most desirable form of regulation: it allows the firm the most flexibility in meeting the standard and therefore allows the firm the most profits, and it directly addresses the problem of reducing pollution. However, politicians and regulators often have to face a myriad of interest groups, each pursuing some particular outcome which may not directly relate to the stated goal of the legislation or regulation. These other interest groups may influence the legislation in subtle or not-so-subtle ways in order to achieve their ends. Therefore, the most efficient outcome may not be the outcome selected.

Different pieces of environmental legislation and regulation include a variety of the forms of regulations discussed here. This variety may exist for reasons in addition to those given here: for instance, informational or monitoring problems may contribute to the choice of regulatory instrument chosen. Nevertheless, the kinds of effects shown here are likely to have induced various interest groups to involve themselves in the policy formulation process. Organized labor certainly prefers regulations that do not detract from, or that enhance, the role of labor in the production process; agricultural interests are unlikely to get a sympathetic ear if they complain that a regulation will reduce production of a good for which surpluses exist. Even if the stated goal of a piece of legislation is to reduce pollution, interest groups will seek to achieve other goals as well through that legislation. As shown here, the choice of the regulatory instrument can reveal some of those other goals.

Figure 1: Effects of Different Pollution Control Specifications



Appendix 1: Comparative Statics Analysis for Restriction of Pollution

See text for definitions of variables. Note that all derivatives are evaluated at the point where the constraint is just barely binding -- that is, where λ is zero.

$$\frac{dx_1}{dZ_1} = \frac{f_{12}A_2 - f_{22}A_1}{2f_{12}A_1A_2 - f_{22}A_1^2 - f_{11}A_2^2} \tag{1.1}$$

$$\frac{dx_2}{dZ_1} = \frac{f_{12}A_1 - f_{11}A_2}{2f_{12}A_1A_2 - f_{22}A_1^2 - f_{11}A_2^2} \tag{1.2}$$

$$\frac{d\lambda}{dZ_1} = \frac{p[(f_{12})^2 - f_{11}f_{22}]}{2f_{12}A_1A_2 - f_{22}A_1^2 - f_{11}A_2^2} \tag{1.3}$$

$$\frac{df}{dZ_1} = \frac{f_1(f_{12}A_2 - f_{22}A_1) + f_2(f_{12}A_1 - f_{11}A_2)}{2f_{12}A_1A_2 - f_{22}A_1^2 - f_{11}A_2^2} \tag{1.4}$$

$$\frac{dA}{dZ_1} = 1 \tag{1.5}$$

Appendix 2: Comparative Statics Analysis for Restriction on Pollution per Unit of Output

See text for definitions of variables. Note that all derivatives are evaluated at the point where the constraint is just barely binding -- that is, where λ is zero.

$$\frac{dx_1}{dZ_2} = \frac{f^2[f_{22}(f_1A - fA_1) - f_{12}(f_2A - fA_2)]}{2f_{12}(f_1A - fA_1)(f_2A - fA_2) - f_{22}(f_1A - fA_1)^2 - f_{11}(f_2A - fA_2)^2} \tag{2.1}$$

$$\frac{dx_2}{dZ_2} = \frac{f^2[f_{11}(f_2A - fA_2) - f_{12}(f_1A - fA_1)]}{2f_{12}(f_1A - fA_1)(f_2A - fA_2) - f_{22}(f_1A - fA_1)^2 - f_{11}(f_2A - fA_2)^2} \tag{2.2}$$

$$\frac{d\lambda}{dZ_2} = \frac{pf^4[(f_{12})^2 - f_{11}f_{22}]}{2f_{12}(f_1A - fA_1)(f_2A - fA_2) - f_{22}(f_1A - fA_1)^2 - f_{11}(f_2A - fA_2)^2} \tag{2.3}$$

$$\frac{df}{dZ_2} = \frac{f^2[(f_1A - fA_1)(f_1f_{22} - f_2f_{12}) + (f_2A - fA_2)(f_2f_{11} - f_1f_{12})]}{2f_{12}(f_1A - fA_1)(f_2A - fA_2) - f_{22}(f_1A - fA_1)^2 - f_{11}(f_2A - fA_2)^2} \tag{2.4}$$

$$\frac{dA}{dZ_2} = \frac{f^2[(f_{22}A_1 - f_{12}A_2)(f_1A - fA_1) + (f_{11}A_2 - f_{12}A_1)(f_2A - fA_2)]}{2f_{12}(f_1A - fA_1)(f_2A - fA_2) - f_{22}(f_1A - fA_1)^2 - f_{11}(f_2A - fA_2)^2} \tag{2.5}$$

Appendix 3a: Comparative Statics Analysis for Restriction on Pollution per Unit of the Polluting Input

See text for definitions of variables. Note that all derivatives are evaluated at the point where the constraint is just barely binding -- that is, where λ is zero.

$$\frac{dx_1}{dZ_3} = \frac{f_{12}A_2x_1^3 - f_{22}(A_1x_1 - A)x_1^2}{2f_{12}(A_1x_1 - A)A_2x_1 - f_{22}(A_1x_1 - A)^2 - f_{11}A_2^2x_1^2} \tag{3a.1}$$

$$\frac{dx_2}{dZ_3} = \frac{f_{12}(A_1x_1 - A)x_1^2 - f_{11}A_2x_1^3}{2f_{12}(A_1x_1 - A)A_2x_1 - f_{22}(A_1x_1 - A)^2 - f_{11}A_2^2x_1^2} \tag{3a.2}$$

$$\frac{d\lambda}{dZ_3} = \frac{px_1^4[(f_{12})^2 - f_{11}f_{22}]}{2f_{12}(A_1x_1 - A)A_2x_1 - f_{22}(A_1x_1 - A)^2 - f_{11}A_2^2x_1^2} \tag{3a.3}$$

$$\frac{df}{dZ_3} = \frac{(f_1f_{12} - f_2f_{11})A_2x_1^3 + (f_2f_{12} - f_1f_{22})(A_1x_1 - A)x_1^2}{2f_{12}(A_1x_1 - A)A_2x_1 - f_{22}(A_1x_1 - A)^2 - f_{11}A_2^2x_1^2} \tag{3a.4}$$

$$\frac{dA}{dZ_3} = \frac{(f_{12}A_1 - f_{11}A_2)A_2x_1^3 + (f_{12}A_2 - f_{22}A_1)(A_1x_1 - A)x_1^2}{2f_{12}(A_1x_1 - A)A_2x_1 - f_{22}(A_1x_1 - A)^2 - f_{11}A_2^2x_1^2} \tag{3a.5}$$

Appendix 3b: Comparative Statics Analysis for Restriction on Pollution per Unit of the Pollution-Abating Input

See text for definitions of variables. Note that all derivatives are evaluated at the point where the constraint is just barely binding -- that is, where λ is zero.

$$\frac{dx_1}{dZ_3} = \frac{f_{12}(A_2x_2 - A)x_2^2 - f_{22}A_1x_2^3}{2f_{12}(A_2x_2 - A)A_1x_2 - f_{11}(A_2x_2 - A)^2 - f_{22}A_1^2x_2^2} \tag{3b.1}$$

$$\frac{dx_2}{dZ_3} = \frac{f_{12}A_1x_2^3 - f_{11}(A_2x_2 - A)x_2^2}{2f_{12}(A_2x_2 - A)A_1x_2 - f_{11}(A_2x_2 - A)^2 - f_{22}A_1^2x_2^2} \tag{3b.2}$$

$$\frac{d\lambda}{dZ_3} = \frac{px_2^4[(f_{12})^2 - f_{11}f_{22}]}{2f_{12}(A_2x_2 - A)A_1x_2 - f_{11}(A_2x_2 - A)^2 - f_{22}A_1^2x_2^2} \tag{3b.3}$$

$$\frac{df}{dZ_3} = \frac{(f_1f_{12} - f_2f_{11})(A_2x_2 - A)x_2^2 + (f_2f_{12} - f_1f_{22})A_1x_2^3}{2f_{12}(A_2x_2 - A)A_1x_2 - f_{11}(A_2x_2 - A)^2 - f_{22}A_1^2x_2^2} \tag{3b.4}$$

$$\frac{dA}{dZ_3} = \frac{(f_{12}A_1 - f_{11}A_2)(A_2x_2 - A)x_2^2 + (f_{12}A_2 - f_{22}A_1)A_1x_2^3}{2f_{12}(A_2x_2 - A)A_1x_2 - f_{11}(A_2x_2 - A)^2 - f_{22}A_1^2x_2^2} \tag{3b.5}$$

Appendix 4: Comparative Statics Analysis for Restriction on Output

See text for definitions of variables. Note that all derivatives are evaluated at the point where the constraint is just barely binding -- that is, where λ is zero.

$$\frac{dx_1}{dZ_4} = \frac{f_2f_{12} - f_1f_{22}}{2f_1f_2f_{12} - f_1^2f_{22} - f_2^2f_{11}} \tag{4.1}$$

$$\frac{dx_2}{dZ_4} = \frac{f_1f_{12} - f_2f_{11}}{2f_1f_2f_{12} - f_1^2f_{22} - f_2^2f_{11}} \tag{4.2}$$

$$\frac{d\lambda}{dZ_4} = \frac{p[(f_{12})^2 - f_{11}f_{22}]}{2f_1f_2f_{12} - f_1^2f_{22} - f_2^2f_{11}} \tag{4.3}$$

$$\frac{df}{dZ_4} = 1 \tag{4.4}$$

$$\frac{dA}{dZ_4} = \frac{(f_2f_{12} - f_1f_{22})A_1 + (f_1f_{12} - f_2f_{11})A_2}{2f_1f_2f_{12} - f_1^2f_{22} - f_2^2f_{11}} \tag{4.5}$$

Appendix 5a: Comparative Statics Analysis for Restriction on the Polluting Input

See text for definitions of variables. Note that all derivatives are evaluated at the point where the constraint is just barely binding -- that is, where λ is zero.

$$\frac{dx_1}{dZ_5} = 1 \tag{5a.1}$$

$$\frac{dx_2}{dZ_5} = -\frac{f_{12}}{f_{22}} \tag{5a.2}$$

$$\frac{d\lambda}{dZ_5} = \frac{p[(f_{12})^2 - f_{11}f_{22}]}{-f_{22}} \tag{5a.3}$$

$$\frac{df}{dZ_5} = \frac{f_1 f_{22} - f_2 f_{12}}{f_{22}} \tag{5a.4}$$

$$\frac{dA}{dZ_5} = \frac{f_{22}A_1 - f_{12}A_2}{f_{22}} . \tag{5a.5}$$

Appendix 5b: Comparative Statics Analysis for Restriction on the Pollution-Abating Input

See text for definitions of variables. Note that all derivatives are evaluated at the point where the constraint is just barely binding -- that is, where λ is zero.

$$\frac{dx_1}{dZ_5} = -\frac{f_{12}}{f_{11}} \tag{5b.1}$$

$$\frac{dx_2}{dZ_5} = 1 \tag{5b.2}$$

$$\frac{d\lambda}{dZ_5} = \frac{p[(f_{12})^2 - f_{11} f_{22}]}{f_{11}} \tag{5b.3}$$

$$\frac{df}{dZ_5} = \frac{f_2 f_{11} - f_1 f_{12}}{f_{11}} \tag{5b.4}$$

$$\frac{dA}{dZ_5} = \frac{f_{11}A_2 - f_{12}A_1}{f_{11}} \tag{5b.5}$$

References

Ackerman, Bruce A., and William T. Hassler. *Clean Coal/Dirty Air.* New Haven, CT: Yale University Press, 1981.

Buchanan, James M., and Gordon Tullock."Polluters' Profits and Political Response: Direct Controls versus Taxes." *American Economic Review* 65 (March 1975): 139-147.

Harford, J., and Gordon Karp. "The Effects and Efficiencies of Different Pollution Standards." *Eastern Economics Journal* 9 (1983): 79-89.

Holterman, Sally. "Alternative Tax Systems to Correct for Externalities, and the Efficiency of Paying Compensation." *Economica* 43 (February 1976): 1-16.

Magat, Wesley A., Alan J. Krupnick, and Winston Harrington. *Rules in the Making: A Statistical Analysis of Regulatory Agency Behavior.* Washington, D.C.: Resources for the Future, 1986.

Office of the Federal Register, National Archives and Records Administration. *Code of Federal Regulations,* Volume 40, Part 60. Washington, D.C.: U. S. Government Printing Office, 1987.

Peltzman, Sam. "Toward a More General Theory of Regulation." *Journal of Law and Economics* 19 (1976): 211-240.

Rausser, Gordon C. "Political Economic Markets: PERTs and PESTs in Food and Agriculture." *American Journal of Agricultural Economics* 64 (1982): 821-833.

Stigler, George J. "The Theory of Economic Regulation." *Bell Journal of Economics and Management Science* 2 (Spring 1971): 3-21.

Thomas, Vinod. "Welfare Cost of Pollution Control." *Journal of Environmental Economics and Management* 7 (1980): 90-102.