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SAMPLING PERFORMANCE OF SOME JOINT ONE-SIDED
PRELIMINARY TEST ESTIMATORS UNDER SQUARED ERROR LOSS

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Abstract

In this paper we evaluate under a squared error loss measure the risk characteristics of preliminary test estimators that evolve when an estimation decision is taken as a result of a particular inequality hypothesis test based on the data at hand. The sampling performances of the pretest estimators that result from two one-sided multivariate hypothesis tests, H_o: $A\boldsymbol{\theta} = \mathbf{0}$ versus H_a: $A\boldsymbol{\theta} \geq \mathbf{0}$ and H_o: $A\boldsymbol{\theta} \geq \mathbf{0}$ versus H_a: $A\boldsymbol{\theta} \ngeq \mathbf{0}$, are evaluated and compared, and the unsatisfactory sampling (risk) performances of these inequality pretest estimators, over part of the parameter space, are established. Over much of the hypothesis specification error part of the parameter space, the pretest estimator resulting from the test mechanism H_o: $A\boldsymbol{\theta} = \mathbf{0}$ is risk inferior to the pretest estimator resulting from H_o: $A\boldsymbol{\theta} \geq \mathbf{0}$. Power functions for the two one-sided tests are presented and the difficulty of using the power criteria in practice is noted.

Key Words: Squared Error Loss, Inequality Estimators, Inequality Hypothesis Tests, Likelihood Ratio Tests, Preliminary Test Estimators, Risk Functions, Power Function.

SAMPLING PERFORMANCE OF SOME JOINT ONE-SIDED PRELIMINARY TEST ESTIMATORS UNDER SQUARED ERROR LOSS[1]

1. INTRODUCTION

In much of the work concerned with measurement in the sciences there is uncertainty as to the agreement between the stochastic sampling model underlying the data generation process and the statistical model that is employed for estimation and inference purposes. As a consequence, investigators begin with an initial specification and sometimes modify their models by testing the statistical significance of some or all of a class of hypotheses. This process makes the model and the estimation procedure dependent on the outcome of the tests of hypotheses and leads to what has been termed in the literature preliminary test or sequential estimators. For traditional equality hypotheses, this class of statistical procedures has been studied starting with Bancroft (1944) and the results of some of the contributions in this area are summarized in Judge and Bock (1978).

Recently, within the context of the general linear statistical model, multivariate analogues of one-sided (inequality hypotheses) tests have been explored by Bartholomew (1959); Kudo (1963); Osterhoff (1969); Barlow, et al. (1972); Yancey, Judge and Bock (1981); Yancey, Bohrer and Judge (1982); Gourieroux, Holly and Monfort (GHM) (1982);

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and Wolak (1985). In general the likelihood ratio framework has been used in developing a test statistic and determining acceptance and rejection regions. In order to facilitate the interpretation of these test results in econometric practice, in this paper we evaluate, under a squared error loss measure, the risk characteristics of the preliminary test estimators that evolve when an estimation decision is taken as a result of an inequality hypothesis test outcome based on the data at hand. Power results are also given for the two one-sided tests.

2. STATISTICAL MODEL, ESTIMATORS AND GENERAL LINEAR HYPOTHESES

Let the K-dimensional random vector $\underset{\sim}{b}$ have a multivariate normal distribution with mean $\underset{\sim}{\beta}$ and covariance matrix Σ. The location vector is unknown, and the objective is to estimate the K-dimensional location vector $\underset{\sim}{\beta}$ using an estimator $\delta(\underset{\sim}{b})$ under a squared loss measure

$$L(\underset{\sim}{\beta}, \delta(\underset{\sim}{b})) = (\delta(\underset{\sim}{b})-\underset{\sim}{\beta})'(\delta(\underset{\sim}{b})-\underset{\sim}{\beta}) \tag{2.1}$$

where the sampling performance of the estimator $\delta(\underset{\sim}{b})$ will be evaluated by its risk function

$$\rho(\underset{\sim}{\beta},\delta(\underset{\sim}{b})) = E[L(\underset{\sim}{\beta},\delta(\underset{\sim}{b}))]. \tag{2.2}$$

One common problem that gives rise to the above statistical model involves estimating the location vector for the normal linear statistical model

$$\underset{\sim}{y} = X\underset{\sim}{\beta} + \underset{\sim}{e} \tag{2.3}$$

where $\delta_o(\underset{\sim}{b}) = \underset{\sim}{b} = (X'X)^{-1}X'\underset{\sim}{y}$ is the maximum likelihood (ML) estimator, $\Sigma = \sigma^2(X'X)^{-1}$, X is a (T x K) design matrix, $\underset{\sim}{e} \sim N(\underset{\sim}{0}, \sigma^2 I_T)$ and σ^2 is an unknown scale parameter.

In addition to the sample information $\underset{\sim}{y}$, consider nonsample information or general linear inequality hypotheses of the form

$$R\underset{\sim}{\beta} \geq \underset{\sim}{r} \tag{2.4}$$

where R is a (J x K) known matrix of full row rank $J \leq K$ and $\underset{\sim}{r}$ is a J-dimensional known vector. Following Judge and Bock (1978, p. 84) and GHM (1982), one can, by an appropriate invertible affine transformation, reparameterize the statistical model and the inequality hypotheses to the following form:

$$\underset{\sim}{y} = XS^{-1/2}Q'QS^{1/2}\underset{\sim}{\beta} + \underset{\sim}{e} = Z\underset{\sim}{\theta} + \underset{\sim}{e}$$

$$\text{or} \qquad Z'\underset{\sim}{y} = \underset{\sim}{\theta} + Z'\underset{\sim}{e} \text{ or } \underset{\sim}{g} = \underset{\sim}{\theta} + \underset{\sim}{v} \tag{2.5}$$

$$\text{and} \qquad R\underset{\sim}{\beta} = RS^{-1/2}Q'QS^{1/2}\underset{\sim}{\beta} = A\underset{\sim}{\theta} \geq \underset{\sim}{0} \tag{2.6}$$

where $\underset{\sim}{\theta} = QS^{1/2}\underset{\sim}{\beta}$, $Z'Z = QS^{-1/2}X'XS^{-1/2}Q' = QQ' = I_K$, $\underset{\sim}{v} \sim N(\underset{\sim}{0}, \sigma^2 I_K)$ and A is a positive definite triangular matrix. The ML estimator of $\underset{\sim}{\theta}$ is $\underset{\sim}{g} = Z'\underset{\sim}{y} \sim N(\underset{\sim}{\theta}, \sigma^2 I_K)$.

For expository purposes, let us assume that the scale parameter σ^2 is known and equal to one and that K = 2. The nature of the results remains virtually unchanged for higher dimensions and when σ^2 is replaced by a suitable estimator s^2, where $(T-K)s^2/\sigma^2 \sim \chi^2_{(T-K)}$. Under this specification, consider the following inequality hypothesis,

$$A\underset{\sim}{\theta} = \begin{bmatrix} 1 & 0 \\ -a & 1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \geq 0 \text{ or } \theta_1 \geq 0 \text{ and } \theta_2 - a\theta_1 \geq 0 \tag{2.7}$$

Following GHM (1982), and using $a > 0$, this inequality hypothesis specification is depicted in Figure 1. The cone A in Figure 1 is a set of (θ_1, θ_2) such that $\theta_1 \geq 0$ and $\theta_2 - a\theta_1 \geq 0$. In order to obtain the least squares estimator for $\underset{\sim}{\theta}$, under restrictions (2.7), one has to replace all observations of $\underset{\sim}{g}$ by their orthogonal projections on the cone A. Thus the inequality restricted estimator $\underset{\sim}{g}^* = (g_1^*, g_2^*)'$, consistent with the general linear inequality (2.7), is

$$\underset{\sim}{g}^* = \begin{bmatrix} g_1^* \\ g_2^* \end{bmatrix} = I(g_1 \geq 0) I(g_2 - ag_1 \geq 0) \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \qquad \text{(Region A)}$$

$$+ I(g_1 \leq 0) I(g_2 \geq 0) \begin{bmatrix} 0 \\ g_2 \end{bmatrix} \qquad \text{(Region B)}$$

$$\tag{2.8}$$

$$+ I(g_2 \leq 0) I(g_2 + (1/a) g_1 \leq 0) \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \text{(Region C)}$$

$$+ I(g_2 + (1/a) g_1 \geq 0) I(g_2 - ag_1 \leq 0) \left[\frac{ag_2 + g_1}{1 + a^2} \right] \begin{bmatrix} 1 \\ a \end{bmatrix} \text{ (Region D)}$$

where $I(\cdot)$ is a zero-one indicator function. Under a squared error loss measure $L(\underset{\sim}{\theta},\underset{\sim}{g}^*) = \|\underset{\sim}{\theta}-\underset{\sim}{g}^*\|^2$, the frequentist risk characteristics of $\underset{\sim}{g}^*$ have been evaluated by Judge and Yancey (1986).

3. INEQUALITY HYPOTHESIS STRUCTURES AND TEST STATISTICS

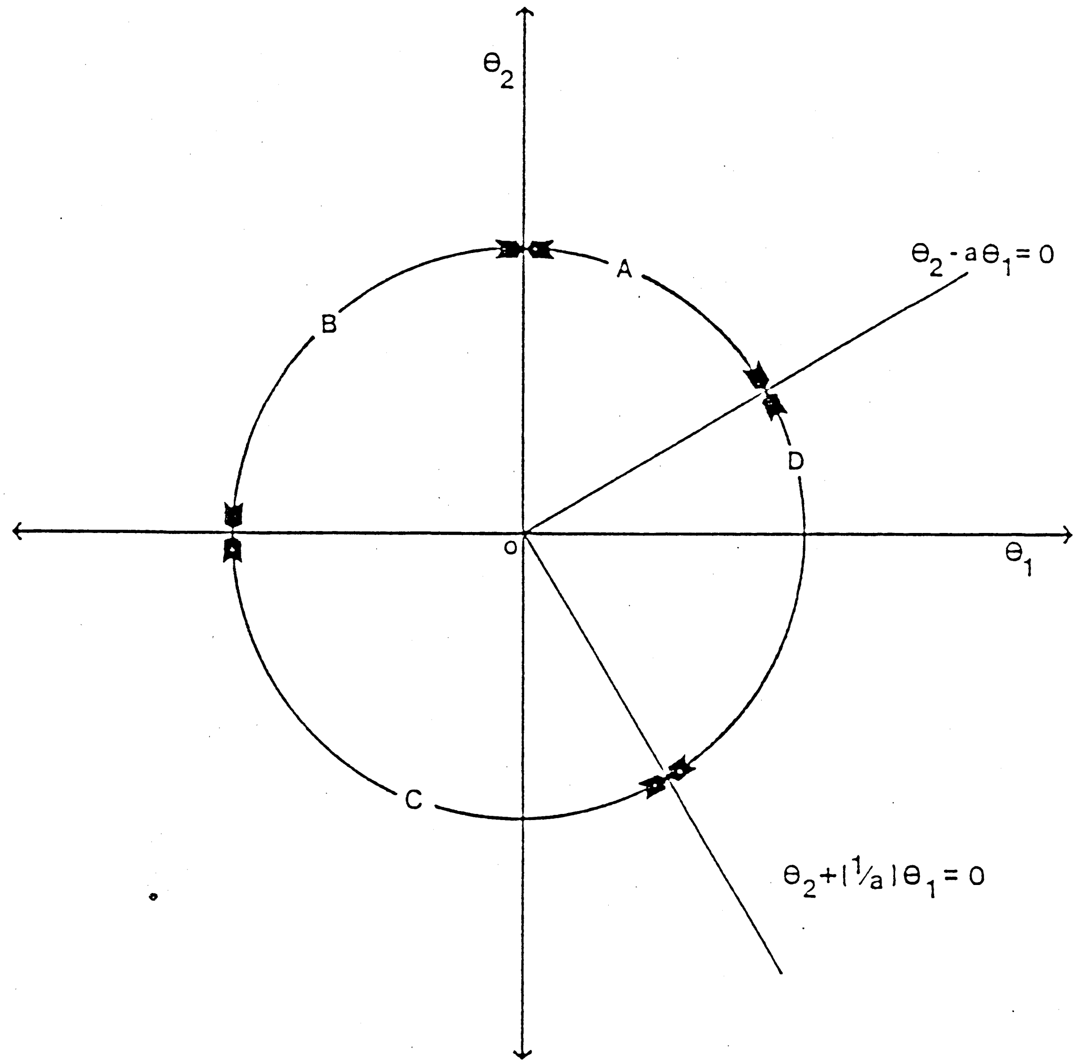
In many areas of science, when drawing conclusions concerning a set of phenomena, individual or linear combinations of parameters are assumed to be nonnegative, nonpositive, or to lie between upper and lower bounds. For example, in economics in the theory of the firm, marginal productivities are nonnegative and economies of scale parameters may be increasing, constant or decreasing. In the theory of the household, the substitution matrix in consumer demand theory requires that all latent roots of the substitution matrix be non-positive. Likewise, many functions are assumed to be monotonic, convex, or quasiconvex. In all areas of science in particular situations, questions naturally arise as to the correctness of the assumptions and, within the context of statistical inference, how these assumptions may be tested by making use of the data at hand to determine the appropriate statistical model and the corresponding method of estimation.

3.1 A One-Sided Multivariate Hypothesis Test

Following the multivariate one-sided hypothesis testing literature reflected in the work of Barthelomew (1959), Kudo (1963), Osterhoff (1969), GHM (1982), and Hillier (1985), consider the testing framework

$$H_o: \; A\underset{\sim}{\theta} = \underset{\sim}{0} \text{ and } H_a: \; A\underset{\sim}{\theta} \geq \underset{\sim}{0}, \tag{3.1}$$

Figure 1. The $\underset{\sim}{A}\underset{\sim}{\theta} \geq \underset{\sim}{0}$ Restriction Set



which implies, when A is of full row rank under H_o, that $\underset{\sim}{\theta} = \underset{\sim}{0}$, against H_a where at least one strict inequality holds in the alternative hypothesis. In the two-parameter case, the alternative hypothesis consists of $[\theta_1>0,\ \theta_2-a\theta_1>0]$, or $[\theta_1>0,\ \theta_2-a\theta_1=0]$, or $[\theta_1 = 0, \theta_2 > 0]$. The likelihood ratio (LR) test LR $= -2 \log (\tilde{L}/\hat{L})$ corresponding to H_o: $A\underset{\sim}{\theta} = \underset{\sim}{0}$ versus H_a: $A\underset{\sim}{\theta} \geq \underset{\sim}{0}$ is

$$u_1 = \|\underset{\sim}{g}-\underset{\sim}{g}_o\|^2 - \|\underset{\sim}{g}-\underset{\sim}{g}^*\|^2 = \|\underset{\sim}{g}^*\|^2 \tag{3.2}$$

where $\underset{\sim}{g}_o = \underset{\sim}{0}$ and $\underset{\sim}{g}^*$ are the ML estimators of $\underset{\sim}{\theta}$ under H_o and the alternative hypothesis, respectively.

Under this test statistic, H_o is rejected if $\|\underset{\sim}{g}^*\|^2 \geq c_1^2$ or if one of the following is met:

$$\text{(i)}\ g_2^2 \geq c_1^2 \text{ and } g_1 < 0,\ g_2 \geq 0 \qquad \text{(Region B)}$$

$$\text{(ii)}\ g_1^2 + g_2^2 \geq c_1^2 \text{ and } g_1 \geq 0,\ g_2 - ag_1 \geq 0 \qquad \text{(Region A)}$$

$$\text{(iii)}\ (g_1^*)^2 + (g_2^*)^2 \geq c_1^2 \tag{3.3}$$

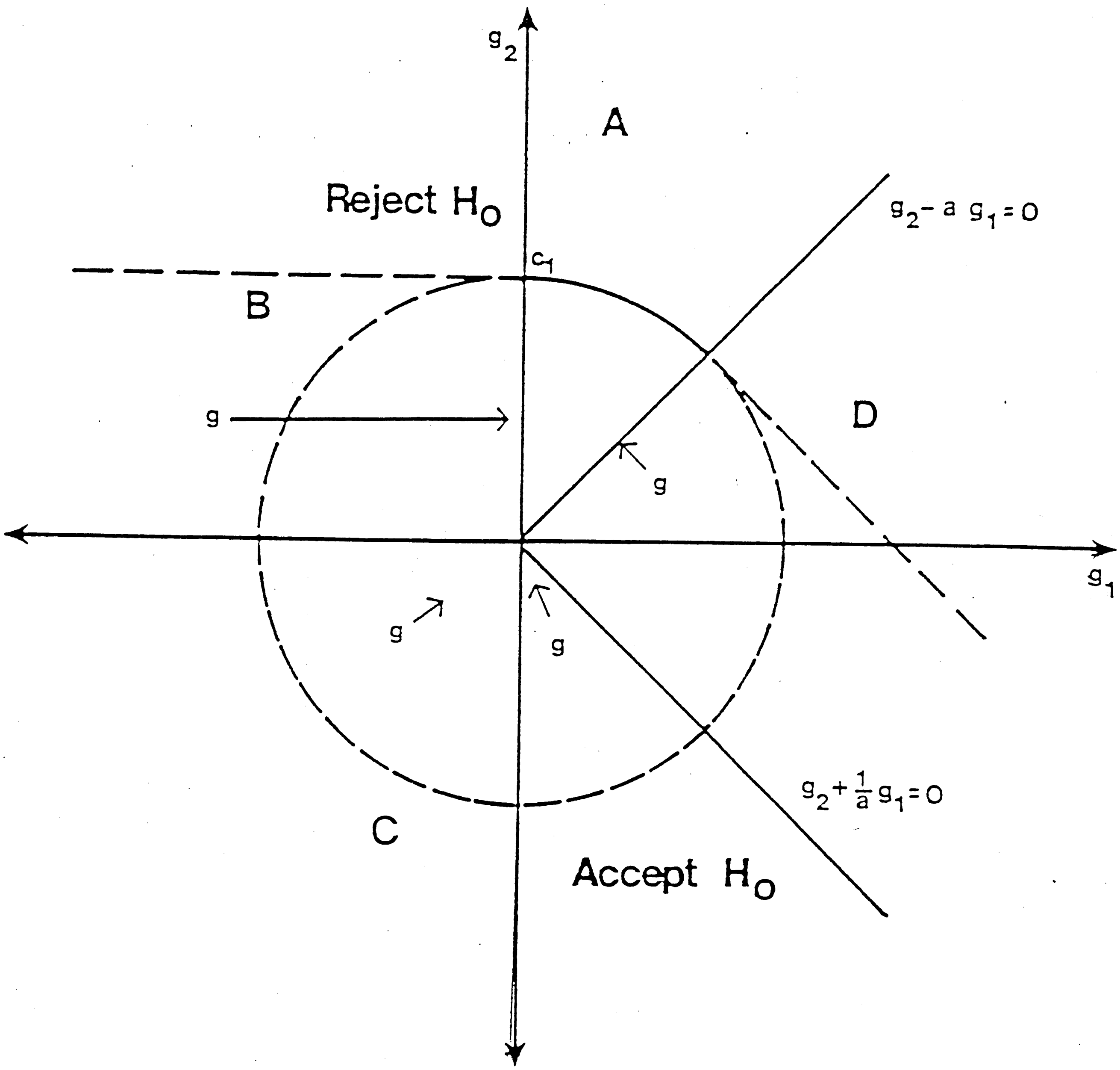
$$\text{and } g_2 - ag_1 < 0,\ g_2 + \frac{1}{a} g_1 \geq 0 \qquad \text{(Region D)}$$

where c_1 is the critical value of the test and the acceptance and rejection regions are depicted in Figure 2.

The distribution of u_1 is that of a weighted sum of Chi-square distributions and, given α, the critical value c_1 is determined by

$$\alpha = P(g_1<0,\ g_2\geq 0)P(g_2^2 \geq c_1^2\ |g_1, g_2 \text{ belongs to Region B})$$

Figure 2. Acceptance and Rejection Regions for the One-Sided Multivariate Hypothesis Test H_o: $\underset{\sim}{A\theta} = \underset{\sim}{0}$ Versus H_a: $\underset{\sim}{A\theta} \geq \underset{\sim}{0}$.



$$+ P(g_1 \geq 0,\ g_2 - ag_1 \geq 0)P(g_1^2 + g_2^2 \geq c_1^2\ |g_1, g_2 \text{ belongs to Region A})$$

$$+ P(g_2 - ag_1 < 0,\ g_2 + \frac{1}{a} g_1 \geq 0)P(g_1^{*2} + g_2^{*2} > c_1^2\ |g_1, g_2 \text{ belongs to Region D}) \tag{3.4}$$

$$= (1/4)P(\chi^2_{(1)} > c_1^2) + qP(\chi^2_{(2)} \geq c_1^2) + (1/4)P(\chi^2_{(1)} \geq c_1^2)$$

where q is the fraction of 2π radians included in the angle between the lines $g_2 = 0$ and $g_2 - ag_1 = 0$.

3.2 A One-Sided Inequality Hypothesis Test

In economics and other areas of science there may be instances where one may want to test the validity of the inequality constraints versus a restricted alternative. Looking at the test mechanism within a decision theoretic context, there may be times when the data and the inequality hypotheses are in conflict and thus one might want to abandon the inequality restriction and go with the data, that is, the unrestricted ML estimator. In this case one might want to consider, in the notation of (2.7), the hypothesis test structure proposed by Judge and Yancey (1986), Wolak (1985), Yancey, Judge and Bock (1981), and Perlman (1969) which can be expressed as

$$H_o:\ A\underset{\sim}{\theta} \underset{\sim}{\geq} \underset{\sim}{0} \text{ and } H_a:\ \underset{\sim}{\theta}\ \varepsilon\ R^2 \tag{3.5}$$

where H_a implies that $\underset{\sim}{\theta}$ is unrestricted under the alternative hypothesis. This procedure tests the compatibility of the data and the inequality constraints. This is in contrast to problem (3.1), where equality constraints are tested under the assumption that the parameters satisfy the inequality constraints.

The likelihood ratio test statistic (Judge and Yancey 1986) corresponding to (3.5) is, in reference to Figure 3,

$$u_2 = \|\underset{\sim}{g}-\underset{\sim}{g}^*\|^2 \tag{3.6}$$

This outcome of the test statistic for this hypothesis structure leads to the rejection of H_o if

$$\begin{array}{lll} \text{(i)}\ g_1 < 0,\ g_2 \geq 0 \text{ and } g_1^2 \geq c_2^2 & \text{(Region B)} & \\ \text{(ii)}\ g_2 < 0,\ g_2 + (1/a)g_1 < 0 \text{ and } g_1^2 + g_2^2 \geq c_2^2 & \text{(Region C)} & (3.7) \\ \text{(iii)}\ g_2 - ag_1 < -c_2(1+a^2)^{1/2},\ g_2 + (1/a)g_1 \geq 0. & \text{(Region D)} & \end{array}$$

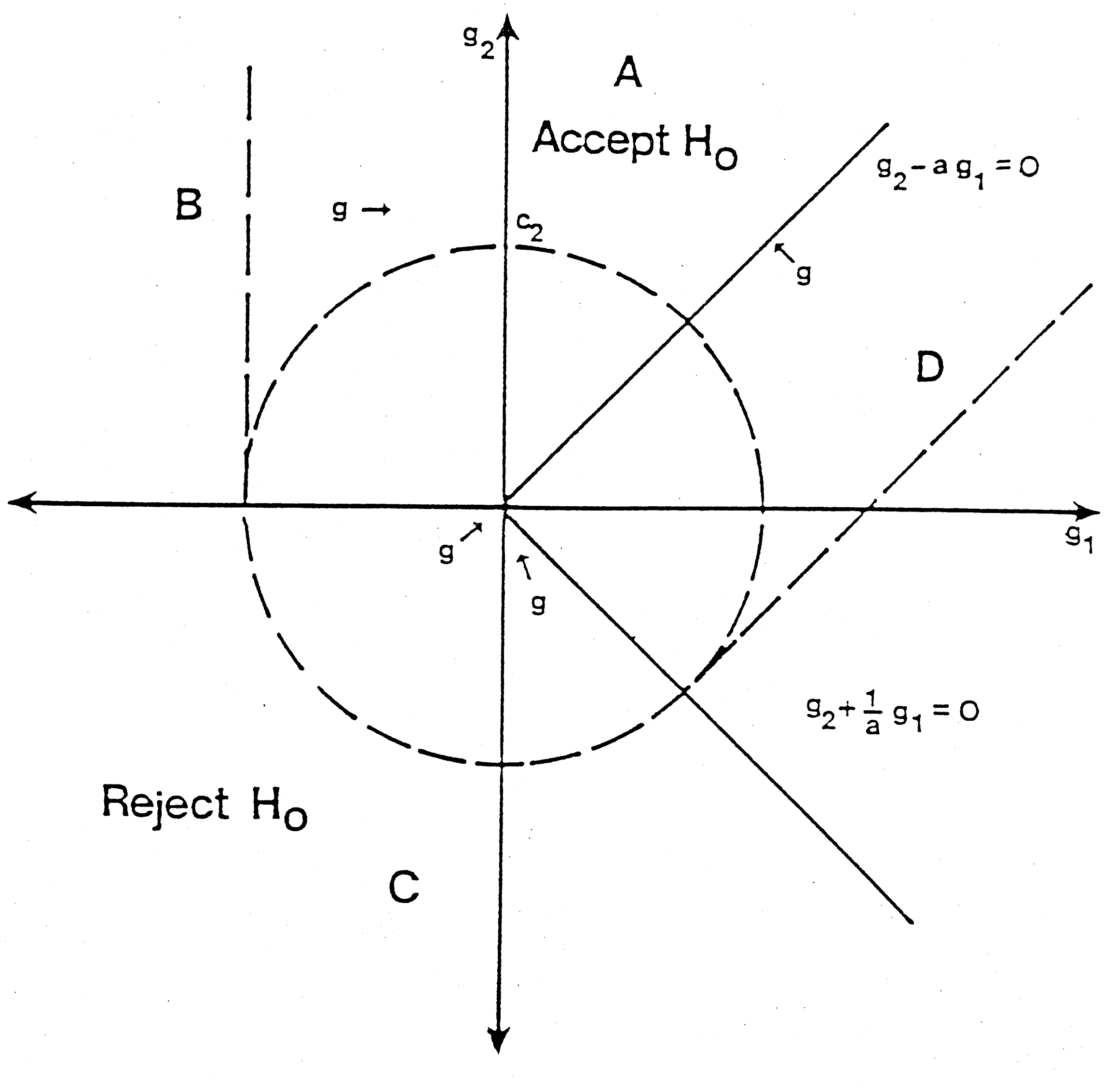
The distribution of u_2 is calculated with $\underset{\sim}{\theta} = \underset{\sim}{0}$ since this is the <u>least favorable</u> value of $\underset{\sim}{\theta}$ under the null hypothesis. In Wolak (1985) and Judge and Yancey (1986), the distribution of u_2 is shown to be a weighted sum of Chi-squared distributions and, given α the size of the test, the critical value of c_2^2 is determined as follows:

$$\begin{aligned} \alpha = {} & P(g_1<0,\ g_2\geq 0)P(g_2^2\geq c_2^2\ |g_1,g_2 \text{ belong to Region B}) \\ & + P(g_2<0,\ g_2+(1/a)g_1<0)\ P(g_1^2+g_2^2\geq c_2^2\ |g_1,g_2 \text{ belong to Region C}) \\ & + P(g_2-ag_1<-c_2(1+a^2)^{1/2},\ g_2+(1/a)g_1\geq 0) \\ & \qquad P((g_1^*)^2+(g_2^*)^2\geq c_2^2\ |g_1,g_2 \text{ belong to Region D}) \end{aligned}$$

$$= 1/4\ P(\chi^2_{(1)}\geq c_2^2) + ((1/2)-q)P(\chi^2_{(2)}\geq c_2^2) + 1/4\ P(\chi^2_{(1)}\geq c_2^2) \tag{3.8}$$

where q has been previously defined in conjunction with (3.4).

Figure 3. Acceptance and Rejection Regions for the One-Sided Inequality Hypothesis Test H_o: $A\underset{\sim}{\theta} \geq \underset{\sim}{0}$ Versus $\underset{\sim}{\theta} \in R^2$



4. INEQUALITY TYPE PRETEST ESTIMATORS

As is apparent from Section 3, the alternative test structures definitely have an impact on the appropriate critical values and the relevant acceptance and rejection regions. One way to trace out the statistical implications of the two test structures and to reach a decision concerning the use of the tests in practice, is to look at the results for each test structure within the context of a squared error loss measure and the implied inequality pretest estimator.

4.1. The One-Sided Multivariate Pretest Estimator, H_o: $A\theta = 0$

When the hypothesis test structure is H_o: $A\theta = 0$ and H_a: $A\theta \geq 0$, the pretest estimator, using the test statistic (3.2) is

$$\begin{aligned}\delta_1(g) &= (I(c_1 \leq u_1 < \infty)g^* + I(0 \leq u_1 \leq c_1^2)0 \\ &= g^* - I(0 \leq u_1 \leq c_1^2)g^*\end{aligned} \tag{4.1}$$

where u_1 is the value of the test statistic (3.2), and c_1^2 is the critical value of the test for a particular α level. The risk for the pretest estimator (4.1) is

$$\begin{aligned}\rho(\theta,\delta_1(g)) &= E[\{(g^*-\theta)-I(0 \leq u_1 \leq c_1^2)g^*\}'\{(g^*-\theta)-I(0 \leq u_1 < c_1^2)g^*\}] \\ &= E[(g^*-\theta)'(g^*-\theta)] - 2E[(g^*-\theta)'I(0 \leq u_1 \leq c_1^2)g^*] \\ &\quad + E[I(0 \leq u_1 \leq c_1^2)g^{*\prime}g^*]\end{aligned} \tag{4.2}$$

where the first term on the right-hand side of (4.2) is the risk of the inequality restricted estimator (2.6). In the two-parameter case, the pretest estimator is in reference to Figure 2

$$\delta_1(\underset{\sim}{g}) = I(g_1 \geq 0)I(g_2 - ag_1 > 0)I(g_1^2 + g_2^2 \geq c_1^2)\begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \qquad \text{Region A}$$

$$+ I(g_1 < 0)I(g_2 > c_1)\begin{bmatrix} 0 \\ g_2 \end{bmatrix} \qquad \text{Region B}$$

$$+ I(g_1 < 0)I(g_2 + (1/a)g_1 < 0)\begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \text{Region C}$$

$$+ I(g_2 + (1/a)g_1 \geq 0)I(g_2 - ag_1 < 0)I(w_1^2 + w_2^2 \geq c_1^2)\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \qquad \text{Region D} \tag{4.3}$$

where $w_1 = (g_1 + ag_2)/(1+a^2)$ and $w_2 = (ag_1 + a^2 g_2)/(1+a^2)$.

The risk for the corresponding pre-test estimator is

$\rho(\underset{\sim}{\theta}, \delta_1(\underset{\sim}{g})) =$

$$E[I(g_1 \geq 0)I(g_2 - ag_1 \geq 0)I(g_1^2 + g_2^2 \geq c_1^2)(\underset{\sim}{g} - \underset{\sim}{\theta})'(\underset{\sim}{g} - \underset{\sim}{\theta})] \qquad \text{Region A and reject}$$

$$+ E[I(g_1 < 0)I(g_2 \geq c_1)((g_2 - \theta_2)^2 + \theta_1^2)] \qquad \text{Region B and reject}$$

$$+ E[I(g_2 + (1/a)g_1 \geq 0)I(g_2 - ag_1 < 0)$$

$$\times I(w_1^2 + w_2^2 \geq c_1^2)((w_1 - \theta_1)^2 + (w_2 - \theta_2)^2)] \qquad \text{Region D and reject} \tag{4.4}$$

$$+ E[I(g_1 > 0)I(g_2 - ag_1 > 0)I(g_1^2 + g_2^2 < c_1^2)]\underset{\sim}{\theta}'\underset{\sim}{\theta} \qquad \text{Region A and do not reject}$$

$$+ E[I(g_1 \leq 0)I(0 \leq g_2 \leq c_1)]\underset{\sim}{\theta}'\underset{\sim}{\theta} \qquad \text{Region B and do not reject}$$

$$+ E[I(g_2 \leq 0)I(g_2+(1/a)g_1<0)\underset{\sim}{\theta}'\underset{\sim}{\theta}$$ Region C and do not reject

$$+ E[I(g_2-ag_1<0)I(g_2+(1/a)g_1>0)I(w_1^2+w_2^2<c_1^2)\underset{\sim}{\theta}'\underset{\sim}{\theta}$$ Region D and do not Reject

The various components of the risk may be identified in Figure 2.

4.2 One-Sided Inequality Hypothesis Pre-test Estimator, $H_o: \; A\underset{\sim}{\theta} \geq \underset{\sim}{0}$

When the hypothesis structure is $H_o: \; A\underset{\sim}{\theta} \geq \underset{\sim}{0}$ versus $H_a: \; \underset{\sim}{\theta} \in R^2$, a pretest estimator, using the test statistic (3.6), is

$$\delta_2(\underset{\sim}{g}) = I(0<u_2<c_2^2)\underset{\sim}{g}^* + I(c_2^2 \leq u_2<\infty)\underset{\sim}{g} \tag{4.5}$$

$$= \underset{\sim}{g}^* + I(c_2^2 \leq u_2<\infty)(\underset{\sim}{g}-\underset{\sim}{g}^*).$$

Correspondingly, the risk for the pretest estimator (4.5) is

$$\begin{aligned} \rho(\underset{\sim}{\theta},\delta_2(\underset{\sim}{g})) &= E[\{(\underset{\sim}{g}^*-\underset{\sim}{\theta}) + I(c_2^2 \leq u_2<\infty)(\underset{\sim}{g}-\underset{\sim}{g}^*)\}' \\ &\quad \{(\underset{\sim}{g}^*-\underset{\sim}{\theta}) + I(c_2^2 \leq u_2<\infty)(\underset{\sim}{g}-\underset{\sim}{g}^*)\}] \\ &= E[(\underset{\sim}{g}^*-\underset{\sim}{\theta})'(\underset{\sim}{g}^*-\underset{\sim}{\theta})] + 2E[(\underset{\sim}{g}^*-\underset{\sim}{\theta})'I(c_2 \leq u_2<\infty)(\underset{\sim}{g}-\underset{\sim}{g}^*)] \\ &\quad + E[I(c_2^2 \leq u_2<\infty)(\underset{\sim}{g}-\underset{\sim}{g}^*)'(\underset{\sim}{g}-\underset{\sim}{g}^*)] \end{aligned} \tag{4.6}$$

where again the first term on the right-hand side of (4.6) is the risk of the inequality restricted estimator.

In the case of two parameters, the pretest estimator in line with Figure 3 is

$$\delta_2(\underset{\sim}{g}) = I(g_2 \geq 0)I(g_1< -c_2)\begin{bmatrix} g_1 \\ g_2 \end{bmatrix}$$ Region B and reject

$$+ I(g_2<0)I(g_2+(1/a)g_1<0)I(g_1^2+g_2^2>c_2^2)\begin{bmatrix} g_1 \\ g_2 \end{bmatrix}$$

Region C and reject

$$+ I(g_2-ag_1< -c_2\sqrt{1+a^2})I(g_2+(1/a)g_1\geq 0)\begin{bmatrix} g_1 \\ g_2 \end{bmatrix}$$

Region D and reject

$$+ I(g_1\geq 0)I(g_2-ag_2\geq 0)\begin{bmatrix} g_1 \\ g_2 \end{bmatrix}$$

Region A and do not reject

(4.7)

$$+ I(g_2\geq 0)I(0\geq g_1> -c_2)\begin{bmatrix} 0 \\ g_2 \end{bmatrix}$$

Region B and do not reject

$$+ I(g_2+(1/a)g_1\geq 0)$$

$$\times\ I(-c_2\sqrt{1+a^2} \leq g_2-ag_1<0)\begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

Region D and do not reject

where w_1 and w_2 were defined in conjunction with (4.3).

The risk corresponding to $\delta_2(\underset{\sim}{g})$ is

$$\rho(\underset{\sim}{\theta},\delta_2(\underset{\sim}{g})) = E[I(g_2>0)I(g_1< -c_2)(\underset{\sim}{g}-\underset{\sim}{\theta})'(\underset{\sim}{g}-\underset{\sim}{\theta})]$$

Region B and reject

$$+ E[I(g_2<0)I(g_2+(1/a)g_1<0)I(g_1^2+g_2^2\geq c^2)(\underset{\sim}{g}-\underset{\sim}{\theta})'(\underset{\sim}{g}-\underset{\sim}{\theta})]$$

Region C and reject

$$+ E[I(g_2-ag_1< -c_2\sqrt{1+a^2})I(g_2+(1/a)g_1\geq 0)(\underset{\sim}{g}-\underset{\sim}{\theta})'(\underset{\sim}{g}-\underset{\sim}{\theta})]$$

Region D and reject

$$+ E[I(g_1\geq 0)I(g_2-ag_1>0)(\underset{\sim}{g}-\underset{\sim}{\theta})'(\underset{\sim}{g}-\underset{\sim}{\theta})]$$

Region A and do not reject

(4.8)

$$+ E[I(g_2\geq 0)I(0\geq g_1> -c_2)((g_2-\theta_2)^2+\theta_1^2)]$$

Region B and do not reject

$$+ E[I(g_2+(1/a)g_1 \geq 0)I(-c_2\sqrt{1+a^2} < g_2 - ag_1 \leq 0)$$

$$\times\{(w_2-\theta_2)^2+(w_1-\theta_1)^2\}]$$

Region D and do not reject

$$+ E[I(g_2<0)I(g_2+(1/a)g_1<0)I(g_1^2+g_2^2<c_2^2)\underset{\sim}{\theta}'\underset{\sim}{\theta}]$$

Region C and do not reject

The various components of the risk can be identified in Figure 3.

5. RISK EVALUATIONS

As a means of providing an intuitive base for determining, under a squared error loss measure, the characteristics of the risk functions for the two pretest estimators, consider the two parameter case and the hypothesis design matrix A given in (2.7). The unweighted squared error loss criterion in the $\underset{\sim}{\theta}$ space

$$\rho(\underset{\sim}{\theta},\hat{\underset{\sim}{\theta}}) = E[(\hat{\underset{\sim}{\theta}}-\underset{\sim}{\theta})'(\hat{\underset{\sim}{\theta}}-\underset{\sim}{\theta})] = E[(\hat{\underset{\sim}{\beta}}-\underset{\sim}{\beta})'X'X(\hat{\underset{\sim}{\beta}}-\underset{\sim}{\beta})] \tag{5.1}$$

yields a weighted risk function in terms of the $\underset{\sim}{\beta}$ space (2.3) with the weight matrix equal to X'X, i.e., the mean squared error of prediction criterion. Alternatively, interest often centers on an unweighted loss function in the $\underset{\sim}{\beta}$ space and thus the following risk function:

$$\rho(\underset{\sim}{\beta},\hat{\underset{\sim}{\beta}}) = E[(\hat{\underset{\sim}{\beta}}-\underset{\sim}{\beta})'(\hat{\underset{\sim}{\beta}}-\underset{\sim}{\beta})] = E[(\hat{\underset{\sim}{\theta}}-\underset{\sim}{\theta})'QS^{-1/2}S^{-1/2}Q'(\hat{\underset{\sim}{\theta}}-\underset{\sim}{\theta}) \tag{5.2}$$

$$= E[(\hat{\underset{\sim}{\theta}}-\underset{\sim}{\theta})'W(\hat{\underset{\sim}{\theta}}-\underset{\sim}{\theta})]$$

Consequently, the measure is changed depending on whether we focus on prediction (5.1) or parameter estimation (5.2). Numerical integration procedures of the type reported by Milton (1972) were used to evaluate the pretest risk functions (4.4) and (4.8).

In the interest of space and to be consistent with the GHM (1982) formulations, we use a general restrictive matrix to remain in the $\underset{\sim}{\theta}$ space, and report results for the mean squared error prediction criterion (5.1). Risk functions in the $\underset{\sim}{\beta}$ space add little if any additional information since the risk characteristics and statistical implications are similar to in the $\underset{\sim}{\theta}$ space.

Although the hypothesis test structure is different, as a basis of comparison the risk for the conventional pretest estimator $\delta_o(\underset{\sim}{g})$ (Judge and Bock, 1978), that is based on testing H_o: $A\underset{\sim}{\theta} = \underset{\sim}{0}$ against the alternative H_a: $A\underset{\sim}{\theta} \neq \underset{\sim}{0}$, is also included. This traditional equality pretest estimator may be specified (Judge and Bock, 1978) as $\delta_o(\underset{\sim}{g}) = I(c_o^2 < u_o < \infty)(\underset{\sim}{g})$, where the conventional test statistic $u_o \sim \chi^2$ and c_o the critical value of the test is determined by $\int_{c_o^2}^{\infty} d\chi^2_{(K)} = \alpha$.

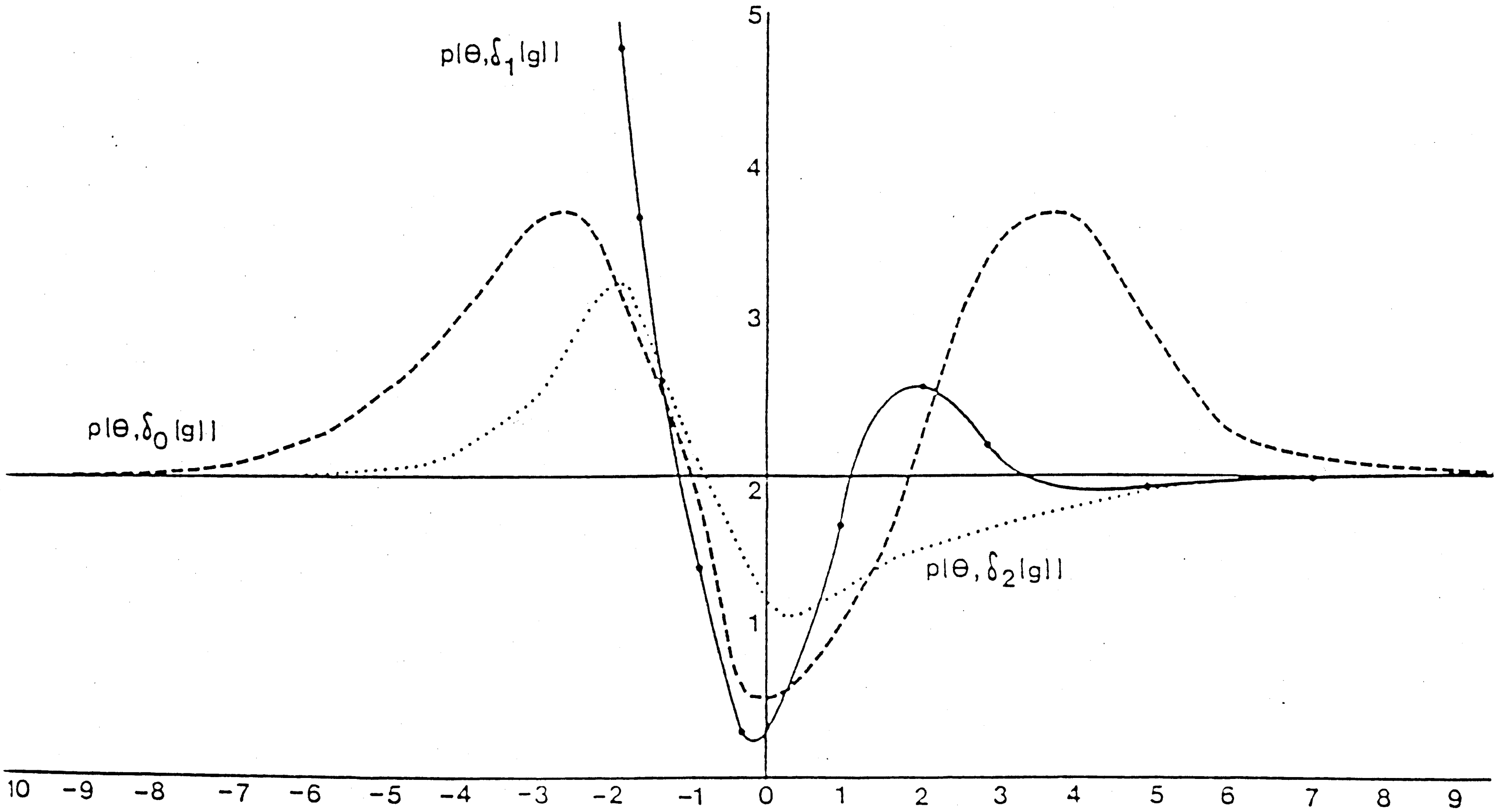
5.1 Pretest Risks $\rho(\underset{\sim}{\theta}, \delta_i(\underset{\sim}{g}))$, $\lambda(\theta_2 = 2.081\theta_1)$

In developing the risk functions, let $\lambda(\cdot) = \sqrt{\underset{\sim}{\theta}'\underset{\sim}{\theta}}$ define the magnitude of the inequality hypotheses errors. In order to trace out the largest and smallest risk values for a particular λ we first examine the risks for the one-sided pretest estimators when the direction of the hypotheses are either all correct or all incorrect. An $A = \begin{bmatrix} 1 & .0 \\ -.8 & 1 \end{bmatrix}$ implies a correlation between g_1 and g_2 of .6247. Under this specification, risks were examined along the line $\theta_2 = 2.081\theta_1$ which bisects the angle in cone A in Figure 1. The line $\theta_2 = 2.081\theta$ as chosen as we find the ttal is more powerful along this line in the rejection regions than along any other line. Risk outcomes over $\lambda(\cdot)$, for $\alpha = 0.05$, are given in Table 1, and the corresponding risk functions are given in Figure 4.

$\lvert\theta_1\rvert$	$\rho(\underset{\sim}{\theta}, \delta_1(\underset{\sim}{g}))$		$\rho(\underset{\sim}{\theta}, \delta_2(\underset{\sim}{g}))$		$\rho(\underset{\sim}{\theta}, \delta_o(\underset{\sim}{g}))$
	$+\theta_1$	$-\theta_1$	$+\theta_1$	$-\theta_1$	
0.0	0.277	0.277	1.115	1.115	0.400
0.10	0.416	0.251	1.060	1.229	0.425
0.20	0.653	0.347	1.052	1.408	0.499
0.30	0.963	0.564	1.078	1.650	0.621
0.40	1.313	0.904	1.127	1.943	0.787
0.50	1.676	1.360	1.189	2.268	0.995
0.75	2.374	3.003	1.363	2.995	1.652
1.00	2.562	5.330	1.528	3.258	2.406
1.25	2.361	8.327	1.669	2.992	3.097
1.50	2.103	11.990	1.782	2.539	3.576
1.75	1.971	16.320	1.865	2.207	3.753
2.00	1.946	21.316	1.920	2.056	3.636
2.50	1.978	33.306	1.977	2.002	2.926
3.00	1.995	47.960	1.995	2.000	2.311
3.50	1.999	65.279	1.999	2.000	2.064
4.00	2.000	85.263	2.000	2.000	2.008
5.00	2.000	133.223	2.000	2.000	2.000

Table 1: Risk values $(E[(\hat{\underset{\sim}{\theta}}-\underset{\sim}{\theta})'(\hat{\underset{\sim}{\theta}}-\underset{\sim}{\theta})] = E[(\hat{\underset{\sim}{\beta}}-\underset{\sim}{\beta})'X'X(\hat{\underset{\sim}{\beta}}-\underset{\sim}{\beta})])$ for the pretest estimators $\delta_o(\underset{\sim}{g})$, $\delta_1(\underset{\sim}{g})$ and $\delta_2(\underset{\sim}{g})$ over the parameter space $\lambda(\theta_2=2.081\theta_1) = (\theta'\theta)^{\frac{1}{2}}$, and $\alpha = 0.05$, and $A = \begin{bmatrix} 1 & .0 \\ -.8 & 1 \end{bmatrix}$.

Figure 4. Risk Functions for the Pretest Estimators $\delta_o(\underset{\sim}{g})$, $\delta_1(\underset{\sim}{g})$ and $\delta_2(\underset{\sim}{g})$, for $\alpha = 0.05$ and over $\lambda(\theta_2 = 2.081\,\theta_1)$.



At the origin ($\lambda=0$), the pretest estimator $\delta_1(\underset{\sim}{g})$, corresponding to the simple null hypothesis H_o: $A\underset{\sim}{\theta} = \underset{\sim}{0}$, has a risk of 0.277, while the risk for the pretest estimator $\delta_2(\underset{\sim}{g})$ corresponding to the composite null hypothesis, H_o: $A\underset{\sim}{\theta} \geq \underset{\sim}{0}$, is 1.115. Thus, as might be expected, at the origin, $\delta_1(\underset{\sim}{g})$ is risk superior to $\delta_2(\underset{\sim}{g})$. Also, as might be expected, at the origin the traditional equality restricted pretest estimator $\delta_o(\underset{\sim}{g})$ has risk .400 and is thus less than the risk for $\delta_2(\underset{\sim}{g})$ and greater than the risk for $\delta_1(\underset{\sim}{g})$. The risk outcomes for $\delta_o(\underset{\sim}{g})$ are given in the last column of Table 1.

As λ increases along $\theta_2 = 2.081\theta_1$, the risk for $\delta_1(\underset{\sim}{g})$ increases, intersects the risks for $\delta_2(\underset{\sim}{g})$ and the ML estimator $\underset{\sim}{g}$ from below, rises to a maximum of 2.56 around $\lambda = 1.00$ and finally declines and becomes asymptotic to the risk of $\underset{\sim}{g}$ for $\lambda \geq 4.0$. The pretest estimator $\delta_o(\underset{\sim}{g})$ is equal to or risk superior to that of $\delta_1(\underset{\sim}{g})$ over a large range of the positive $\lambda(\theta_2=2.081\theta_1)$ parameter space.

The risk function for the pretest estimator $\delta_2(\underset{\sim}{g})$ is a much different story. When the direction of both hypotheses are correct, over the range of the parameter space $0 \leq \lambda(\theta_2=2.081\theta_1) < \infty$, the risk of the pretest estimator $\delta_2(\underset{\sim}{g})$ is like that of the inequality restricted estimator $\underset{\sim}{g}^*$ (Judge and Yancey (1986)) and is uniformly risk superior to that of the ML estimator $\underset{\sim}{g}$ (see Table 1 and Figure 4). Along $\theta_2 = 2.081\theta_1$ with $.35 \leq \lambda$, the pretest estimator $\delta_2(\underset{\sim}{g})$ is risk superior to the multivariate one-sided pretest estimator $\delta_1(\underset{\sim}{g})$. Also along $\theta_2 = 2.081\theta_1$ $.75 < \lambda$, the pretest estimator $\delta_2(\underset{\sim}{g})$ is risk superior to the traditional equality restricted pretest estimator $\delta_o(\underset{\sim}{g})$.

When the directions of the inequality hypotheses are incorrect along $\theta_2 = 2.081\theta_1$, the pretest estimator $\delta_1(\underset{\sim}{g})$ performs very badly; in fact, as λ approaches $-\infty$, its risk is unbounded and becomes asymptotic to that of the equality restricted ML estimator $\underset{\sim}{g}^+ = \underset{\sim}{0}$. For $\lambda \leq -2.89$, the risks of the equality restricted estimator $\underset{\sim}{g}^+$ and the pretest estimator $\delta_1(\underset{\sim}{g})$ coincide. Only near $\underset{\sim}{\theta} = \underset{\sim}{0}$ does the pretest estimator $\delta_1(\underset{\sim}{g})$ give a satisfactory risk performance when compared to risks for $\underset{\sim}{g}$, $\delta_o(\underset{\sim}{g})$ and $\delta_2(\underset{\sim}{g})$.

Alternatively, along $\theta_2 = 2.081\theta_1$ the risk for the pretest estimator $\delta_2(\underset{\sim}{g})$ increases as λ becomes negative, intersects the risk of $\delta_1(\underset{\sim}{g})$ at about $\lambda(\theta_2{=}2.081\theta_1) = -1.73$, and intersects the maximum likelihood risk at about $\lambda = -0.92$. The risk of the pretest estimator $\delta_2(\underset{\sim}{g})$ reaches a maximum of 3.258 when $\lambda = -2.31$ and decreases until it is equal to the ML risk when $\lambda \geq -4.52$. Consequently, over a large part of the line $\theta_2 = 2.081\theta_1$, the risk for the pretest estimator $\delta_2(\underset{\sim}{g})$ is superior to that of the multivariate one-sided pretest estimator $\delta_1(\underset{\sim}{g})$. Alternatively, the inequality pretest estimator $\delta_2(\underset{\sim}{g})$ is risk inferior to the equality pretest estimator $\delta_o(\underset{\sim}{g})$ over a part of the $-2.89 \leq \lambda \leq 1.73$ parameter space.

5.2 Pretest Risks, $\lambda(\theta_2{=}-.4806\theta_1)$ and $\lambda(\theta_1{=}0, \theta_2{>}0)$

In order to investigate risk performance for another part of the $\underset{\sim}{\theta}$ parameter space, let $\theta_2 = -.4806\theta_1$, a line orthogonal to $\theta_2 = 2.081\theta_1$. Consequently, for H_o $A\theta \geq 0$, one inequality hypothesis is correct and the other is incorrect, and we move along a line that cuts the second and fourth quadrants in the $\underset{\sim}{\theta}$ space. Under this scenario, the risk outcomes for each of the pretest estimators are given in Table 2, and the risk functions are graphed in Figure 5.

Over the line $\theta_2 = -.4806\theta_1$, the inequality estimator $\delta_1(\underset{\sim}{g})$ has unbounded risk for both large values of λ, and it is inferior to the risks of $\underset{\sim}{g}$, $\delta_o(\underset{\sim}{g})$ and $\delta_2(\underset{\sim}{g})$ over a large range of the parameter space. Alternatively, for the pretest estimator $\delta_2(\underset{\sim}{g})$, the risk functions along $\theta_2 = -.4806\theta_1$ are symmetrical around $\lambda = 0$ and reach a maximum at $|\lambda| = 2.77$ and then become asymptotic to the ML risk for $|\lambda| \geq 6.68$. The risk of the pretest estimator $\delta_2(\underset{\sim}{g})$ is inferior to those of the ML estimator and the $\delta_o(\underset{\sim}{g})$ pretest estimator over a large part of the line $\theta_2 = -.4806\theta_1$.

Finally, the risk functions for inequality pre-test estimators for the part of the parameter space with $(\theta_1=0, \theta_2>0)$, behave in predictable ways. For example, in the case of the pretest estimator $\delta_2(\underset{\sim}{g})$, the risk function becomes asymptotic to the risk value 1.5999 as λ increases. For $\lambda\theta_2 \leq 0$, the risk function is similar in shape to the risks when $\theta_2 = .g\theta_1$. However, for comparable values of λ the risks of $\delta_2(\underset{\sim}{g})$ are lower than the risk, reaching the risk when $\lambda = 4.00$. Correspondingly, for $\lambda = \theta_2 \geq 0$ the risk function for $\delta_1(\underset{\sim}{g})$ reaches a maximum of 2.59 at $\theta_2 = 2.00$ and finally becomes asymptotic to the risk value 1.5.

5.3 Impact of the Choice of α

The risk outcomes for $\delta_1(\underset{\sim}{g})$ with $\alpha = .01$ and $.10$ are given in Table 3 and the corresponding risk functions are given in Figure 6. As α

changes, the risks at the origin and over the range of the risk function change. Under the hypothesis structure H_o: $A\theta = 0$, and along the line $\theta_2 = 2.081\theta_1$, the risk of the pretest estimator $\delta_1(g)$ at the origin is .086 for $\alpha = 0.01$ and .427 for $\alpha = .10$. Correspondingly, as α increases the maximum risk value decreases.

Under the H_o: $A\theta \geq 0$ hypothesis structure, the risk of the pretest estimator $\delta_2(g)$ at the origin is 0.882 for $\alpha = .01$ and 1.314 for $\alpha = .10$. For $\lambda < 0$ along $\theta_2 = 2.081\theta_1$ and $\alpha = .01$ and .10, the risk of the pretest estimator $\delta_2(g)$ has maxima of 3.874 and 2.229, respectively, and then approaches the risk of the ML estimator. In other words, its risk behavior is similar to that of the traditional, equality restricted pretest estimator for alternative values of α.

These results indicate that, since the risk outcome of either pretest estimator varies with the choice of α, the cavalier way that the level of the test is sometimes chosen in applied work may have rather severe risk consequences. Unfortunately, as is the case with the equality pretest estimator $\delta_o(g)$ (Judge and Bock (1978)) the question of the optimal level of the test remains unanswered.

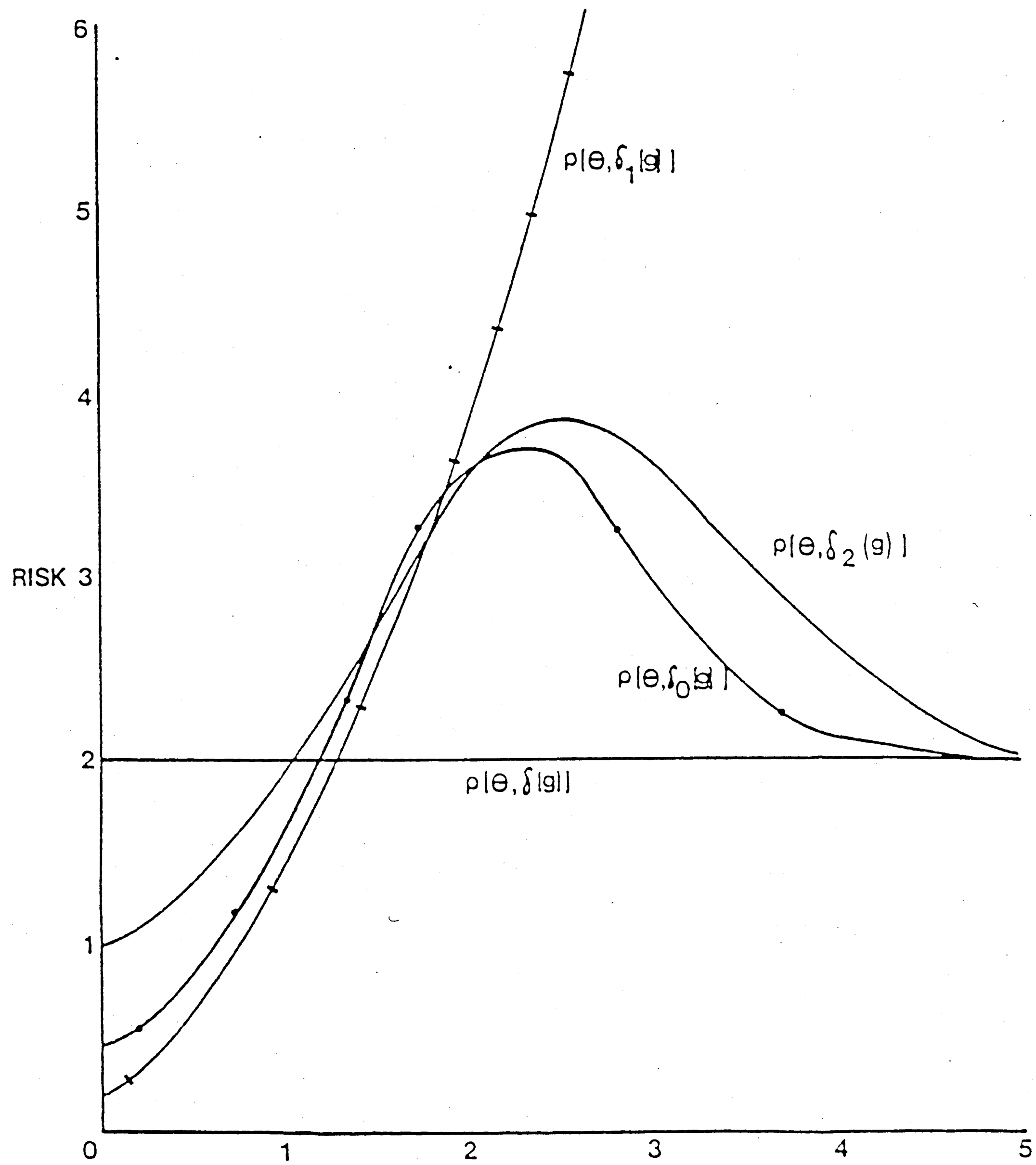
6. SUMMARY AND EXTENSIONS

Within the setting of a general linear model and general linear inequality hypotheses, we have, using likelihood ratio test statistics corresponding to null hypotheses H_o: $A\theta = 0$ or H_o: $A\theta \geq 0$ in a parameter space of dimension two, demonstrated the alternative acceptance and rejection regions and how to obtain the critical values

Table 2. Risk Values for Pretest Estimators $\delta_1(\underset{\sim}{g})$ and $\delta_2(\underset{\sim}{g})$, for Values in the Parameter Space $\theta_2 = -.4806\theta_1$ and $\theta_1 = 0, \theta_2 \in R^1$ and $\alpha = 0.05$

θ_1	$\theta_2 = -.4806\theta_1$		θ_2	$\theta_1 = 0, \theta_2 \in R^1$	
	$\rho(\underset{\sim}{\theta}, \delta_1(\underset{\sim}{g}))$	$\rho(\underset{\sim}{\theta}, \delta_2(\underset{\sim}{g}))$		$\rho(\underset{\sim}{\theta}, \delta_1(\underset{\sim}{g}))$ $\theta_2 > 0$	$\rho(\underset{\sim}{\theta}, \delta_2(\underset{\sim}{g}))$ $\theta_2 > 0$
0	.277	1.115	0	.277	1.115
0.10	.290	1.126	.111	.326	1.087
0.20	.328	1.160	.222	.401	1.072
0.30	.391	1.215	.333	.498	1.070
0.40	.480	1.291	.444	.617	1.078
0.50	.594	1.387	.555	.754	1.095
0.75	.980	1.692	.832	1.157	1.163
1.00	1.536	2.114	1.110	1.599	1.251
1.25	2.235	2.566	1.387	2.014	1.338
1.50	3.080	3.008	1.664	2.340	1.414
1.75	4.066	3.384	1.942	2.536	1.462
2.00	5.187	3.646	2.219	2.589	1.519
2.50	7.813	3.735	2.774	2.358	1.570
3.00	10.909	3.340	3.329	1.970	1.590
3.50	14.429	2.823	3.883	1.685	1.597
4.00	18.344	2.356	4.438	1.553	1.599
5.00	27.330	2.034	5.548	1.504	1.599
6.00	37.984	2.000	6.657	1.500	1.599
8.00	65.253	2.000	8.876	1.500	1.599
10.00	101.025	2.000	11.095	1.500	1.599

Figure 5. Risk Functions for the Pretest Estimators $\delta_o(\underset{\sim}{g})$, $\delta_1(\underset{\sim}{g})$ and $\delta_2(\underset{\sim}{g})$, for $\alpha = 0.05$ and $\theta_2 = -.4806\,\theta_1$.



of the tests for a given α level. In order to understand the statistical implications of using these test structures we have, under a squared error loss measure, developed and numerically evaluated the risk functions for the corresponding pretest estimators and compared their sampling performance along several lines in the hypothesis specification error parameter space.

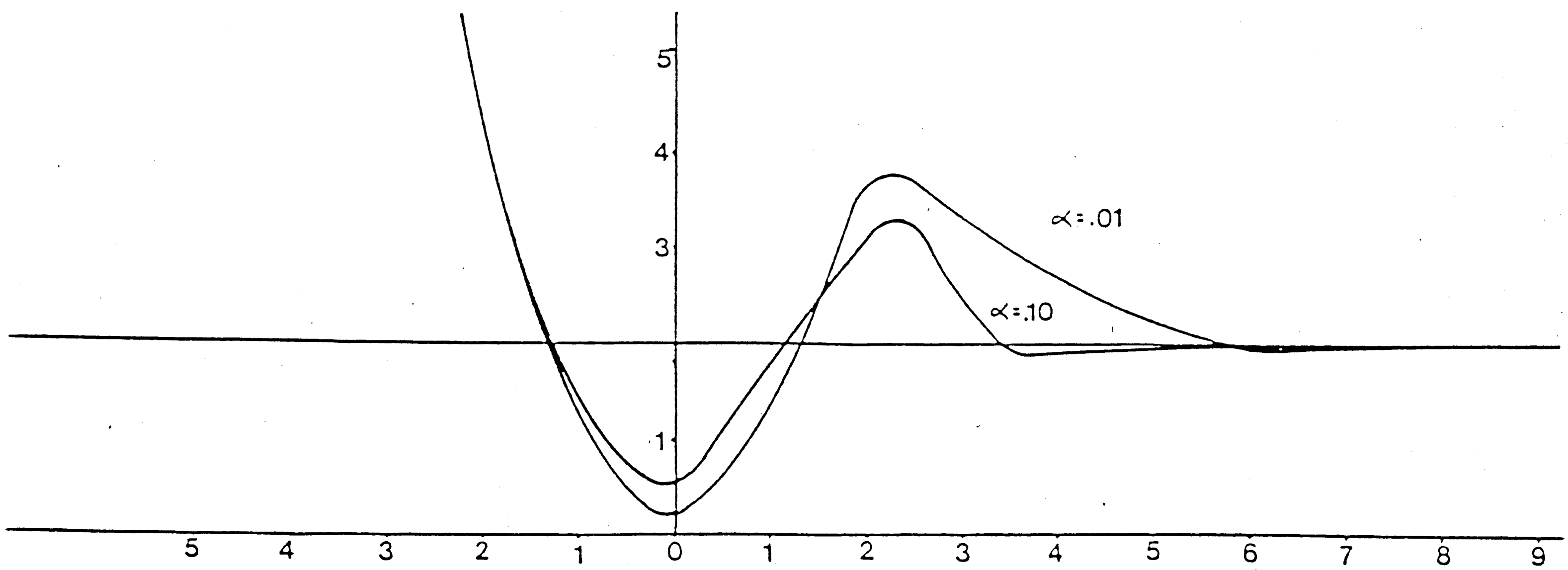
Extension of these results to a K-dimensional space, with J linear inequality hypotheses, and an unknown scale parameter follows directly. With an unknown scale parameter, the distributions of the LR test statistics become a weighted sum of $F_{(\cdot)}$ random variables instead of Chi-square random variables, and the degrees of freedom for the F random variables are changed in a natural way (Judge and Bock 1978). In evaluations in the $\underset{\sim}{\beta}$ space under risk measure (5.2), the numerical integration procedures become a bit more difficult, but the general characteristics of the risk functions remain essentially the same.

Of special note is the unsatisfactory performance of the pretest estimator $\delta_1(\underset{\sim}{g})$ associated with the simple null test structure, H_o: $\underset{\sim}{A}\underset{\sim}{\theta} = \underset{\sim}{0}$. When the direction of the alternative hypothesis is correct, the risk of the pretest estimator $\delta_1(\underset{\sim}{g})$ is inferior to those of the ML, the equality pretest estimator $\delta_o(\underset{\sim}{g})$ and alternative inequality pretest estimator $\delta_2(\underset{\sim}{g})$ over a large part of the parameter space. When the null hypotheses are true, $\underset{\sim}{A}\underset{\sim}{\theta} = \underset{\sim}{0}$, and $\lambda(\cdot) = 0$, the multivariate one-sided pretest estimator $\delta_1(\underset{\sim}{g})$ risk is smaller than those of its pretest competitors $\delta_o(\underset{\sim}{g})$ and $\delta_2(\underset{\sim}{g})$. When the direction of the

Table 3. Risk Values for Pretest Estimators $\delta_1(\underset{\sim}{g})$ and $\delta_2(\underset{\sim}{g})$ When $\theta_2 = 2.081\theta_1$, for $\alpha_1 = 0.01$ and 0.10

$\|\theta_1\|$	$\rho(\underset{\sim}{\beta},\delta_1(\underset{\sim}{g}))$				$\rho(\underset{\sim}{\beta},\delta_2(\underset{\sim}{g}))$			
	$\alpha = .01$		$\alpha = .10$		$\alpha = .01$		$\alpha = .10$	
	$+\theta_1$	$-\theta_1$	$+\theta_1$	$-\theta_1$	$+\theta_1$	$-\theta_1$	$+\theta_1$	$-\theta_1$
0.00	0.086	0.086	0.427	0.427	0.882	0.882	1.314	1.314
0.10	0.185	0.106	0.568	0.383	0.873	0.948	1.236	1.440
0.20	0.402	0.242	0.784	0.452	0.906	1.094	1.282	1.613
0.30	0.730	0.496	1.046	0.644	0.967	1.328	1.202	1.825
0.40	1.154	0.891	1.321	0.960	1.043	1.655	1.228	2.035
0.50	1.650	1.336	1.579	1.398	1.126	2.071	1.270	2.294
0.75	2.952	2.998	2.005	3.013	1.381	3.331	1.408	2.720
1.00	3.790	5.329	3.304	5.332	1.512	4.354	1.552	2.754
1.50	3.156	11.990	1.899	11.970	1.779	3.874	1.788	2.229
2.00	2.143	21.316	1.927	21.316	1.921	2.378	1.922	2.017
2.50	1.987	33.307	1.978	33.306	1.978	2.021	1.978	2.000
3.00	1.995	47.960	1.995	47.960	1.995	2.000	1.995	2.000
3.50	1.999	65.279	1.999	65.279	1.999	2.000	1.999	2.000
4.00	2.000	85.262	2.000	85.263	2.000	2.000	2.000	2.000
5.00	2.000	133.222	2.000	133.233	2.000	2.000	2.000	2.000
6.00	2.000	191.840	2.000	191.840	2.000	2.000	2.000	2.000

Figure 6. Pretest Risk Functions for $\delta_1(\underset{\sim}{g})$, for $\alpha = 0.01$ and 0.10 and $\theta_2 = 2.0818\theta_1$.



inequality hypotheses are incorrect, the risk of the both θ_1 and θ_2 are negative, pretest estimator $\delta_1(\underset{\sim}{g})$ has outcomes much like those of the unbounded risk of the equality restricted estimator. Thus, except near the origin, the pretest estimator $\delta_1(\underset{\sim}{g})$ is risk inferior to those of the pretest estimators $\delta_o(\underset{\sim}{g})$ and $\delta_2(\underset{\sim}{g})$ and the ML estimator $\underset{\sim}{g}$.

For the composite null hypothesis structure H_o: $A\underset{\sim}{\theta} \geq \underset{\sim}{0}$, when the direction of the hypothesis is correct, along $\theta_2 = 2.081\theta_1$ and that is the pretest estimator $\delta_2(\underset{\sim}{g})$ has risk equal to or less than that of the ML estimator $\underset{\sim}{g}$. Also along θ_2=$2.081\theta_1$ and $\theta_1 > 0$, the pretest estimator $\delta_2(\underset{\sim}{g})$ is risk superior to the equality pretest estimator $\delta_o(\underset{\sim}{g})$. When the null hypothesis is incorrect along $\theta_2 = 2.081\theta_1$ and $\theta_1 < 0$, the risk of the pretest estimator $\delta_2(\underset{\sim}{g})$ is inferior to that of the ML estimator over a large part of the parameter space. But as λ approaches infinity with $\theta_1 < 0$, the risk of $\delta_2(\underset{\sim}{g})$ approaches the risk of the ML estimator and, thus, guards against large losses when the direction of the inequality hypothesis is incorrect. Under a squared error measure and for comparable levels of the test α, except for values of λ near the origin, $\delta_2(\underset{\sim}{g})$ is risk superior to $\delta_o(\underset{\sim}{g})$ and $\delta_1(\underset{\sim}{g})$ along $\theta_2 = 2.081\theta_1$ for most values of the parameter space.

As in the case of the equality hypothesis pretest estimator $\delta_o(\underset{\sim}{g})$, the level of the test α has, for the pretest estimators $\delta_1(\underset{\sim}{g})$ and $\delta_2(\underset{\sim}{g})$, an important impact on their sampling performances. Also, as in the conventional equality hypothesis pretest case, the question concerning the optimum level of the test is unresolved.

In the case of two parameters and a squared error loss measure all of the inequality pretest estimators are inadmissible. For three or more parameters the inadmissability of all three pretest estimators can be demonstrated (Judge, Yancey and Bock (1984)). In the case of three or more parameters, it is conjectured that the conventional positive-part Stein estimator will, except near the origin, be risk superior to both inequality type pretest estimators.

Yancey, Bohrer, and Judge (1982) have, for the case when A is diagonal so the powers of the tests of for $H_o: A\theta = 0$ and $H_o: A\theta \geq 0$ are the same, made power comparisons between inequality and equality hypothesis testing structures. Each of the tests guards against, to a different degree, failing to discover that parameter values are in a particular region of the parameter space. The inequality likelihood ratio test is best when one places the highest priority on discovering that both claims in the null are incorrect. The equality likelihood ratio is best when testing $\theta_1 = \theta_2 = 0$ and the priority is on discovering that $\theta_i \geq 0$ and $\theta_j < 0$. Goldberger (1986) has, in the case of two parameters and a nonorthogonal setting, used numerical integration procedures to develop power comparisons of the two one sided tests. Alternatively, the authors have used numerical integration procedures to investigate the power surfaces for the two parameter nonorthogonal case. For example, when $A = \begin{bmatrix} 1 & 0 \\ -.8 & 1 \end{bmatrix}$, some outcomes of the power functions for two multivariate one-sided tests are reported in Appendix Table 1. Along the line $\theta_2 = 2.081\theta_1$, the one sided test $H_o: A\theta = 0$ dominates the one sided test $H_o: A\theta \geq 0$. Along the line $\theta_2 = -.4806\theta_1$, the reverse is true. These results point up the difficulty of using the power criterion in interpreting and making a choice between tests in practice.

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θ_1	$H_o: \mathbf{A}\boldsymbol{\theta} = \mathbf{0}$ $H_o: \mathbf{A}\boldsymbol{\theta} \geq \mathbf{0}$	$H_o: \mathbf{A}\boldsymbol{\theta} \geq \mathbf{0}$ $H_o: \mathbf{A}\boldsymbol{\theta} \not> \mathbf{0}$	$H_o: \mathbf{A}\boldsymbol{\theta} = \mathbf{0}$ $H_o: \mathbf{A}\boldsymbol{\theta} \geq \mathbf{0}$	$H_o: \mathbf{A}\boldsymbol{\theta} \geq \mathbf{0}$ $H_o: \mathbf{A}\boldsymbol{\theta} \not> \mathbf{0}$
	$\theta_2 = 2.081\theta_1$		$\theta_2 = -.4806\theta_1$	
0.0	0.050	0.050	0.050	0.050
0.1	0.077	0.071	0.051	0.051
0.2	0.115	0.102	0.051	0.053
0.3	0.164	0.138	0.053	0.058
0.4	0.225	0.186	0.055	0.063
0.5	0.296	0.246	0.058	0.071
0.75	0.511	0.435	0.067	0.099
1.00	0.722	0.648	0.080	0.143
1.25	0.875	0.824	0.096	0.197
1.50	0.957	0.932	0.117	0.268
1.75	0.989	0.980	0.141	0.351
2.00	0.998	0.996	0.169	0.445
2.5	1.000	1.000	0.236	0.637

Appendix Table 1: Power outcomes for two one sided tests when $\theta_2 = 2.081\theta_1$ and $\theta_2 = -.4806\theta_1$ and $\alpha = .05$.