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# BRANNAN PLAN AGRICULTURAL SUPPORTS IN A WORLD OF UNCERTAINTY AND INCOMPLETE MARKETS: PARETO SUPERIORITY AND DISTRIBUTION

by

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## I. INTRODUCTION

A government target price / deficiency payment program (the Brannan Plan) pays farmers the difference between a given target price and the prevailing market price for their crop. In a world of certainty, such a program benefits producers, hurts taxpayers/consumers and causes a net welfare loss to society as a whole (i.e., compensation cannot be made so as to preserve or increase competitive equilibrium utilities of all agents). The object of this paper is to show that when there is uncertainty and markets are incomplete, all of these conclusions can be reversed: producers can be worse off, consumers better off and society better off.

It is well known that when markets are incomplete, competitive equilibrium is not, in general, Pareto optimal (Borch (1962)), even in a constrained sense (Newbery and Stiglitz (1981,1982a), Hart). This observation has spawned extensive literatures on the welfare effects of commodity price stabilization (see, for example, Wright, Schmitz, Newbery and Stiglitz (1981,1982b), Turnovsky) and optimal trade policy (e.g., Young and Anderson, Newbery and Stiglitz (1984), and Eaton and Grossman). However, to my knowledge, its relevance to the welfare effects of a Brannan Plan program is yet to be explored. This paper aims to begin this exploration in the context of a simple two good (food and numeraire) two state model with a representative farmer, a representative consumer/taxpayer, no market for state contingent claims, ex-ante producer selection of inputs and stochastic output which is the only source of uncertainty.

The suboptimality of competitive equilibrium in an incomplete markets setting is due to inequality between agents' marginal rates of substitution between income in different states of the world. Hence, any policy which induces an exchange of state-contingent income in the lens of mutual advantage is Pareto-improving. The Brannan Plan generates state-contingent income transfers between consumers and producers via two mechanisms:

(1) Positive supply response to the program causes market prices of the supported commodity to fall, increasing consumers' real income and, in states which are characterized by high price and, thus, no deficiency payments, reducing farmer profits.[1]

(2) In low price states, the program transfers income from consumers (as taxpayers) to producers.

These observations suggest that with positive supply response, the Brannan Plan can be Pareto improving when an exchange of low-price-state-income (consumers to produc-

1 When demand is price inelastic, increases in output lower total revenue and increase farmer costs. When demand is elastic, the farmer's first order condition (see (3) below) implies that marginal profit in the high price state is negative. Hence, in both cases, farmer profits in the high price state decline with increased output.

ers) for high-price-state-income (producers to consumers) is in the lens of mutually beneficial trade. This speculation is formalized and confirmed below.

The distributional effects described above also have intuitive explanations. Adverse effects on farmers result from supply response. When price is random, a target price cuts a lefthand tail off the price distribution. In the absence of any other effects, this truncation leads to a profit distribution for any production choice which first order stochastically dominates the corresponding profit distribution with no target price. However, if the Brannan Plan program induces a supply response, they do more than chop a tail off the price distribution; they also lead to shifts in the distribution. If farmers are competitive, they do not consider the effects which their actions have on prices in determining their supply decisions. Hence, under certain circumstances, producer supply responses to a target price can produce an equilibrium price distribution which is less desirable for farmers than the original competitive equilibrium.

Consumers, on the other hand, can be made better off by a target price program because they don't pay for the price drop which results from equilibrium supply response in low output (high price) states. In the certainty case, consumers must pay for the single-state price drop via the tax mechanism and this cost always exceeds the benefits which they receive from lower price. However, when there is more than one state, the target price may not be effective in all states; there may be states in which the market clearing price remains higher than the target level. In these states, consumers pay nothing in taxes for the support mechanism but benefit from the lower price which supply response produces. These "free" benefits can, under some circumstances, exceed the excess costs paid by consumers in high output (low price) states.

The remainder of this paper endeavors to shed some light on the conditions under which these outcomes can occur. Section II describes the model in more depth. Sections III and IV discuss effects on producers and consumers, respectively. Section V examines overall welfare implications. Finally, Section VI presents a numerical example in which key parameters are varied.

## II. THE MODEL

Consider a static two good economy in which the two goods are a food commodity (x) and a numeraire (y).

### Production

Assume that there exists a representative (aggregate) farmer who can be characterized as follows :[2]

1. Preferences are defined on profits and satisfy the rationality axioms of Von Neumann and Morgenstern (see Borch (1968)). The representative farmer's utility can then be represented by an expected utility function, $EU(\tilde{\pi})$ where E denotes the expectation operator over states of nature, $\tilde{\pi}$ the state dependent profit and U(.) the state-specific (ex-post) utility function (assumed twice differentiable).

2. The ex-post utility function is not state-dependent (i.e., $\tilde{U}=U$). Further, let $U'>0$ and $U''\leq 0$ .

3. He/she has a production technology defined by a twice differentiable cost function C(z) (where cost is measured in units of the numeraire) and an output

2 A sufficient condition for the existence of a representative farmer is that all farmers are identical, a standard assumption in models of welfare under uncertainty (e.g., Newbery and Stiglitz).

function, $x=\theta z$ , where z is the input choice which must be made before the state is revealed and $\theta$ is a state-dependent output coefficient. Assume $C'>0$ , $C''>0$ , and $E(\theta)=1$ .

4. The farmer is a price taker and has rational expectations in the sense that the price he/she expects in state s is the equilibrium price in that state.

## Consumers

Assume that there exists a representative consumer whose indirect utility function is V(P,Y), where P is the price of food, Y is aggregate consumer income and V( ) is a twice differentiable state-independent function. Assume $V_P<0$ , $V_Y>0$ , and $V_{YY}\leq 0$ . Let this consumer also obey the standard rationality axioms of choice under uncertainty, so that his/her utility can be represented by EV(P,Y) . Further, suppose that in the absence of taxes to pay for deficiency transfers Y is constant across states. Finally, assume that consumers pay the full cost of the Brannan Plan via a lump sum (ex-post) tax.[3]

3 One simple story for this economy is as follows: Suppose there is an endowment, Y, of the numeraire, owned by the representative consumer, and some fixed factor of production in the agricultural sector, k, which is now usable for production of the numeraire -- say, land and/or human capital. The inputs in agricultural production are the numeraire good (y) and the fixed factor (k) so that the ex-ante agricultural production function can be represented as follows:

$$z=f(y,k)\equiv f^*(y) \quad , \quad f^{*\prime}>0 \quad , \quad f^{*\prime\prime}<0$$

Since the cost of z is also in terms of the numeraire, this production function implies the following derivatives of the cost function:

$$C'=(f^{*\prime})^{-1}>0 \quad , \quad C''=-f^{*\prime\prime}(f^{*\prime})^{-3}>0$$

General

Assume that there is perfectly symmetric information and that equilibrium is stable in a Walrasian sense.[4] Further, to simplify the algebra, suppose that there are two equi-probable states and $\theta_1 > \theta_2$. Finally, since target price levels below the no-program competitive equilibrium price in state 1 (denoted $P_1^{ce}$) will not be effective in either state, only target prices larger than this level will be considered.[5]

With this construction, farmer profits in state s are:

$$\pi_s = \max(P_s, P^*)\theta_s z - C(z) \tag{1}$$

where $P_s$ is the market price of food prevailing in state s and $P^*$ is the target price. The farmer's utility maximization problem can therefore be written:

$$\max_z \; .5[U(P^*\theta_1 z - C(z)) + U(\max(P_2, P^*)\theta_2 z - C(z))] \tag{2}$$

with first order condition (assuming an interior solution):

$$.5[U'_1(P^*\theta_1 - C') + U'_2(\max(P_2, P^*)\theta_2 - C')] = 0 \tag{3}$$

where $U'_i$ denotes the state i derivative. Clearly, the farmer's optimal z, z*, is a function of received prices in all states, $\{\max(P_s, P^*)\}$. Therefore, given rational farmer expectations, market prices are determined by the equilibrium conditions (using Roy's identity):

$$x^d(P_s, Y_s) \equiv -\frac{V_P(P_s, Y_s)}{V_Y(P_s, Y_s)} = \theta_s z^*(\{\max(P_s, P^*)\}) \qquad s=1,2 \tag{4}$$

---

4 Walrasian stability implies that, with price responding positively to excess demand, excess demand declines (increases) as price rises (falls) from equilibrium.

5 Throughout the balance of this paper, reference to $P^*$ (the target price) and properties of functions of $P^*$ will relate only to $P^* \geq P_1^{ce}$.

where

$$Y_s = Y - (P^* - \min(P_s, P^*))\theta_s z^*(\{\max(P_s, P^*)\})$$

and $x^d(\ )$ denotes consumer demand, assumed downward sloping in price. Let $P_1(P^*)$ and $P_2(P^*)$ denote the solutions to (4), assumed existent, unique, continuous everywhere and differentiable at all points other than where $P^* = P_2(P^*)$.[6]

The solution to (4) gives market prices as a function of the target price. Therefore, the equilibrium producer input choice can be represented as a function of $P^*$ alone:

$$z^{**}(P^*) \equiv z^*(\{\max(P_s(P^*), P^*)\}) \qquad (5)$$

Note that $z^{**}(P^*)$ is continuous everywhere and differentiable at all points other than $\bar{P}_s^*$ which satisfies $\bar{P}_s^* = P_2(\bar{P}_s^*)$.[7]

## III. PRODUCERS

In this section, the effects of a Brannan Plan program on producers will be investigated using arguments based on the diagrams depicted in Figure 1. Here, the supply curve, $z^{**}(P^*)$, is diagrammed with $P^{*f}(z)$, the farmer indifference curve corresponding to the competitive equilibrium utility level -- that is, the set of $(z, P^*)$ points (where z implies a state 2 price) such that farmer utility is the same as at competitive equilibrium with no Brannan Plan program. At the moment, the shapes of and relationship between these two curves is purely speculative. However, their significance is apparent from the following observation:

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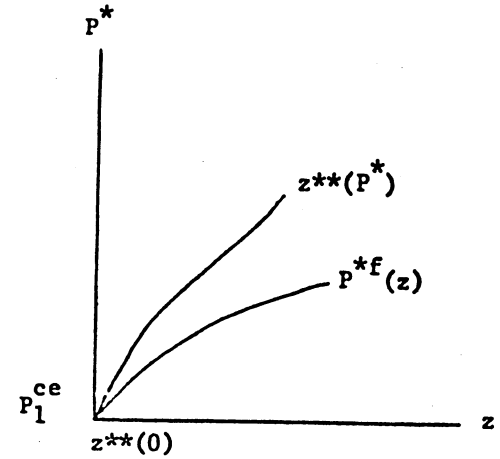
6 At these points, the functional relationship between $P^*$ and farmer first order conditions change (see (3)).

7 Twice differentiability of U and C imply (from the implicit function theorem) that $z^*$ is a differentiable function of its arguments. Given assumptions made on $\{ P_s(P^*) \}$, the continuous and composite mapping theorems (Marsden, p. 84 and p. 168) therefore imply the properties of $z^{**}(P^*)$ stated here.

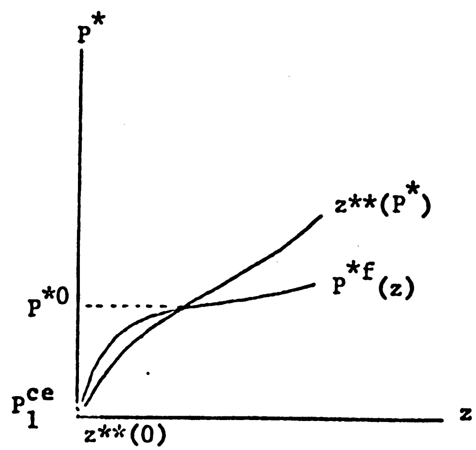
FIGURE 1

z Supply and Farmer Indifference Curves in $(z, P^*)$ Space




(a)

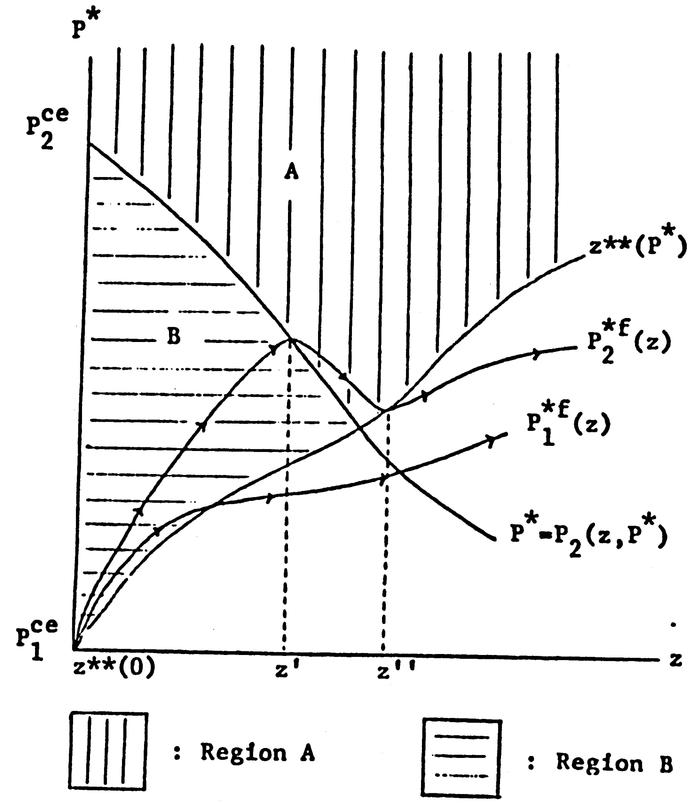
The farmer is better off with any target price in Case (a).




(b)

The farmer is worse off with $P^*$ in $(P_1^{ce}, P^{*0})$ and better off with $P^*$ greater than $P^{*0}$.

FIGURE 2



Observation 1 (O1): Given z, farmer utility increases with a higher target price. Therefore, higher indifference curves correspond to higher utility.

Proof: Partially differentiate farmer expected utility with respect to $P^*$:

$$\partial EU/\partial P^* = .5[U'_1\theta_1 z + \delta U'_2\theta_2 z] > 0 \text{ where } \delta = \begin{cases} 0 & \text{when } P^*<P_2 \\ 1 & \text{when } P^*>P_2 \end{cases} \qquad (6)$$

Given (O1), there will be target prices which make producers worse off than at competitive equilibrium if and only if $P^*f(z)$ lies somewhere above the supply curve, as in Figure 1 case (b). The rest of this section is dedicated to determining if and when this can happen. First, the slopes of the two curves are calculated, based upon which a simple condition is derived for the existence of target prices which make farmers worse off. This condition is then interpreted analytically and given economic content.

At the outset, one other observation should be made:

Observation 2 (O2): A high enough target price (e.g., $P^* \geq P_2^{ce}$) will make farmers better off than they were at competitive equilibrium. Therefore, there is always a target price level beyond which the indifference curve lies everywhere below the supply curve.

Proof: With $P^* \geq P_2^{ce}$, the worst possible price distribution for the farmer is $P^*$ holding in both states. Since farmer utility with $z^{**}(0)$ and this price distribution is higher than at competitive equilibrium, the utility-maximizing choice of z must give the farmer a higher utility than $\overline{U}^{ce}$, the competitive equilibrium farmer utility level.

The Farmer Indifference Curve

The competitive equilibrium farmer indifference curve, $P^{*f}(z)$, is defined by the equation:

$$.5[U(P^*\Theta_1 z-C(z))+U(\max(P_2(z,P^*),P^*)\Theta_2 z-C(z))] = \bar{U}^{ce} \tag{7}$$

where the function $P_2(z,P^*)$ solves the following market-clearing condition:

$$\Theta_2 z = x^d(P_2, Y-(P^*-\min(P_2,P^*))\Theta_2 z)) \tag{8}$$

As with the supply curve, it can be shown that $P^{*f}(z)$ is continuous everywhere and differentiable at all points other than $\bar{z}_f$ which satisfies the equality :[8]

$$P^{*f}(\bar{z}_f) = P_2(\bar{z}_f, P^{*f}(\bar{z}_f))$$

Since the functional relationship between $P^*$ and z defined in (7) changes at $P^*=P_2$, two cases must be considered:

Case 1: $P_2>P^*$ . Totally differentiating (7) with respect to z and $P^*$ and substituting for $dP_2/dz$ from (8):

$$\frac{dP^{*f}}{dz} = -\frac{x_p^d(P_2,Y)[U'_1(P^*\Theta_1-C')+U'_2(P_2\Theta_2-C')]+U'_2\Theta_2^2 z}{U'_1\Theta_1 z x_p^d(P_2,Y)} \tag{9}$$

$$= -\frac{U'_2\Theta_2^2}{U'_1\Theta_1 x_p^d(P_2,Y)} > 0 \quad \text{when } P^{*f}(z^{**}(P^*))=P^*$$

where the second equality follows from the F.O.C. (3). Note that when $z>z^{**}(P^*)$ ,[9]

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8 Twice differentiability of V, continuity of $Y_2$ , and differentiability of $Y_2$ at all points other than where $P_2(z,P^*)=P^*$ imply (from the implicit function theorem) that $P_2(z,P^*)$ is continuous in its arguments and differentiable at all points other than where it equals $P^*$. This result, in addition to the differentiability of U, imply (from the implicit function theorem and the continuous and composite mapping theorems) the properties of $P^{*f}(z)$ stated above.

$$U'_1(P^*\theta_1-C')+U'_2(P_2\theta_2-C') < 0 \quad (10)$$

Therefore, $dP^{*f}/dz > 0$ when $z>z^{**}(P^*)$ and $P_2>P^*$.

Case 2: $P_2<P^*$. Again totally differentiating (7):

$$\frac{dP^{*f}}{dz} = -\frac{U'_1(P^*\theta_1-C')+U'_2(P^*\theta_2-C')}{U'_1\theta_1 z+U'_2\theta_2 z} \quad (11)$$

Note that when $z>(<)z^{**}(P^*)$ , $dP^{*f}/dz$ in (11) is positive (negative).

## The Supply Curve

Differentiating (5) with respect to $P^*$ :

$$\frac{dz^{**}}{dP^*} = \begin{cases} z^*_1+z^*_2(dP_2/dP^*) & \text{when } P_2>P^* \text{ (Case 1)} \\ z^*_1+z^*_2 & \text{when } P_2<P^* \text{ (Case 2)} \end{cases} \quad (12)$$

where $z^*_i$ denotes the partial derivative of $z^*$ with respect to the farmer received price in state i. As it turns out (see (03) below), only Case 1 is important to the analysis here. Totally differentiating equation system (4):

$$\frac{dP_2}{dP^*} = \frac{\theta_2 z^*_1}{x_p^d(P_2,Y)-\theta_2 z^*_2} \quad (13)$$

$$\rightarrow \frac{dz^{**}}{dP^*} = z^*_1\left|\frac{x_p^d(P_2,Y)}{x_p^d(P_2,Y)-\theta_2 z^*_2}\right| \quad (14)$$

Walrasian stability implies that the denominator of (14) is negative. Hence, for Case 1, $dz^{**}/dP^*$ has the same sign as $z^*_1$.

---

9 Inequality (10) can be proven as follows: For large enough z, say $z^0$, $(P^*\theta_1-C')$ and $(\max(P_2,P^*)\theta_2-C')$ are both negative, implying that (10) is satisfied. Hence, if there were a $z'<(z^{**}(P^*),z^0)$ for which (10) were violated, then, given continuity, the intermediate value theorem implies that there will be a $z''<[z',z^0)$ which satisfies (3), contradicting the assumption of a unique equilibrium.

The discussion above gives rise to the following observation:

Observation 3 (O3): A necessary and sufficient condition for the existence of target prices which make farmers worse off is that the slope of the farmer indifference curve ( $(P^{*f}/dz)^{-1}$ ) be less than the slope of the supply curve ( $dz^{**}/dP^{*}$ ) at some point of intersection at which $P^{*}<P_2$ (Case 1) .[10]

Proof: See Appendix A.

Informal Argument: Consider Figure 2 where $P_1^{*f}(z)$ and $P_2^{*f}(z)$ denote two possible farmer indifference curves and $P_2(z,P^{*})$ is defined in (8). From (O1), the farmer is worse off with a Brannan Plan program only when his indifference curve lies in region A or region B. From the diagram, sufficiency of the above condition is clear. Necessity requires that the condition in (O3) is satisfied whenever there is a target price which makes producers worse off, implying (from (O1)) that

$$P^{*f}(z^{**}(P^{*})) > P^{*} \tag{15}$$

First consider a target price, $P^{*1}$, which puts the indifference curve in region B of Figure 2. Since (15) is satisfied, both curves are continuous, and they both start at the competitive equilibrium point, $(P_1^{ce},z^{**}(0))$, the indifference curve must either rise above the supply curve from the competitive equilibrium point, as shown in Figure 2, or cross the supply curve from below at some output level, z, which is between $z^{**}(0)$ and $z^{**}(P^{*1})$. In either case, the condition in (O3) will be satisfied. Now consider a target price which puts the indifference curve in region A. In this region, $P^{*f}$ is negatively sloped from (11). Therefore, given continuity, $P^{*f}$ crosses the $P^{*}=P_2$ curve at some point

10 $(dP^{*f}/dz)^{-1}$ exists at these points since, from (9), $dP^{*f}/dz>0$.

above the supply curve and there exists a target price which puts the indifference curve in region B. Hence, from the preceding arguments, the condition in (O3) will be satisfied.

Algebraically, the condition in (O3) is:

$$\left|\frac{dP^{*f}}{dz}\right|^{-1} = -\left|\frac{U'_1\theta_1 x_p^d(P_2,Y)}{U'_2\theta_2^2}\right| < z^*_1\left|\frac{x_p^d(P_2,Y)}{x_p^d(P_2,Y)-\theta_2 z^*_2}\right| = \frac{dz^{**}}{dP^*} \quad (16)$$

Rearranging terms and substituting for $z^*_1$ and $z^*_2$ (obtained by totally differentiating (3)), (16) can be written:

$$-x_p^d(P_2,Y) < QU'_2 z[\delta_2(P_2\theta_2-C')-\delta_1(P^*\theta_1-C')] \quad (17)$$

where:

$$Q \equiv \theta_2^2/(U'_1C''+U'_2C''-U''_1(P^*\theta_1-C')^2-U''_2(P_2\theta_2-C')^2) > 0$$

$$\delta_i \equiv -U''_i/U'_i = \text{index of absolute risk aversion in state } i$$

From (16) and (17), the following proposition can be deduced:

Proposition 1 : Farmers are always better off with a Brannan Plan program than without if (a) $z^*_1 \leq 0$ at points of intersection between $P^{*f}(z)$ and $z^{**}(P^*)$, (b) demand is price elastic for $P<(P_1^{ce},P_2^{ce})$, or (c) farmers are risk neutral.

Proof:

Part (a): $z^*_1 \leq 0$ implies that the right hand side of (16) is non-positive. Since the left hand side is positive, (16) cannot be satisfied. QED (a).

Part (b): Given (a), only the case of $z^*_1>0$ needs to be considered. Elastic demand implies that $P_1^{ce}\theta_1>P_2^{ce}\theta_2$ (i.e., total revenue is higher when price is lower). Since the target price is always higher than $P_1^{ce}$ and, given $z^*_1>0$, $P_2(P^*)$ is always lower than $P_2^{ce}$, $P^*\theta_1>P_2\theta_2$. The F.O.C. (3) therefore implies that $(P^*\theta_1 - C') > 0$ and $(P_2\theta_2 - C') < 0$, making the right hand side of (17) non-positive (since $\phi_2$ and $\phi_1$ are non-negative by assumption). But the left hand side of (17) is always positive, making satisfaction of the inequality impossible. QED (b).

Part (c): If farmers are risk-neutral, $\phi_1 = \phi_2 = 0$, making the right hand side of (17) equal to zero. Since the left hand side is strictly positive, the inequality cannot hold. QED (c).

Proposition 1 is supported by the following intuition. When demand is elastic, producer supply response to a target price induces only a small price drop in state 2; thus, the cost of supply response in state 2 is small relative to the gain from deficiency payments in state 1. When farmers are risk neutral, output, utility and the mean effective price of ex ante output z, $.5(P^*\theta_1 + P_2\theta_2) \equiv \overline{P}$, move together; since a decline in output implies an increase in $\overline{P}$ (via the market mechanism), output, $\overline{P}$ and utility necessarily increase when a Brannan Plan program is adopted.

Though Proposition 1 points out two cases for which condition (17) cannot hold, casual inspection reveals that there are circumstances under which (17) does hold. For example, as $x_p^d(P_2, Y)$ approaches zero, with demand inelastic in the relevant region and farmers risk averse, the left hand side approaches zero but the right hand side remains strictly positive; hence, (17) will be satisfied. Further description of these circumstances is left to the numerical analysis in Section VI; this section derives parameter value ranges which produce adverse farmer effects and provides an indication of the extent to which farmers can be hurt by a Brannan Plan program.

## IV. CONSUMERS

As with producers, a useful device for consumer utility analysis is a graph of the consumer indifference curve for competitive equilibrium utility, $P^{*c}(z)$, and of the equilibrium supply curve in $(z,P^*)$ space (Figure 3). Again, the shapes of and relationship between these curves is speculative at the moment and will be the subject of the subsequent analysis. At the outset, note the following:

Observation 4 (O4): Given z and recalling that consumers must pay the full cost of a Brannan Plan program, a higher target price makes consumers worse off. (With z fixed, market price does not fall with a higher target price but tax costs of the program increase.) Therefore, higher indifference curves correspond to lower consumer utility.

Proof: Partially differentiating consumer expected utility (see (19)) with respect to $P^*$ and using Roy's identity to simplify:

$$\frac{\partial EV}{\partial P^*} = \begin{cases} -.5V_{1Y}\theta_1 z<0 & \text{when } P^*<P_2 \\ -.5(V_{1Y}\theta_1 z+V_{2Y}\theta_2 z)<0 & \text{when } P^*>P_2 \end{cases} \tag{18}$$

where $V_{iY}$ denotes $V_Y$ in state i.

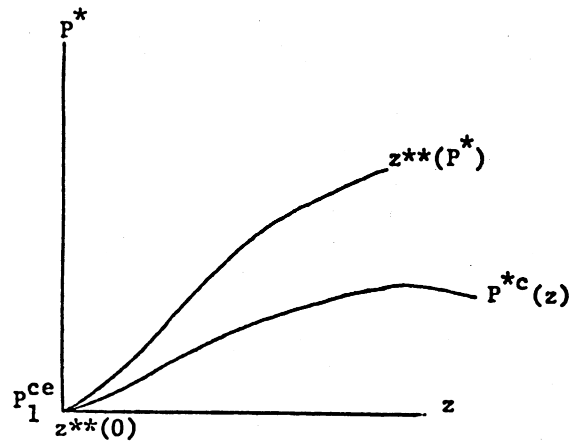
Given (O4), there will be target prices which make consumers better off if and only if the consumer indiffernce curve for the competitive equilibrium utility level lies somewhere above the supply curve, as in Figure 3, case (b). To determine if and when this can happen, some analytics are needed.

### The Consumer Indifference Curve

The competitive equilibrium consumer indifference curve, $P^{*c}(z)$, is defined by the equation:

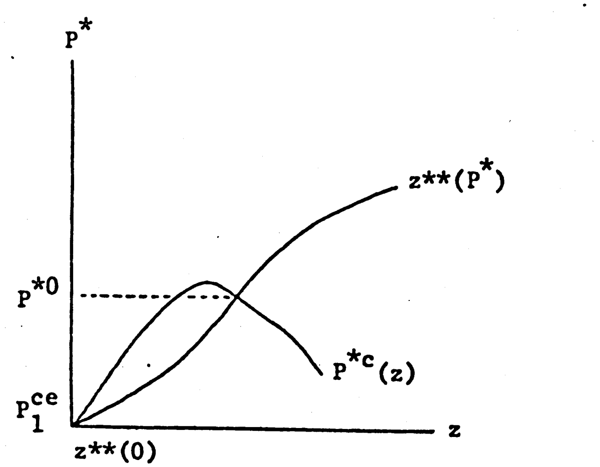
FIGURE 3

z Supply and Consumer Indifference Curves in $(z,P^*)$ Space




(a)

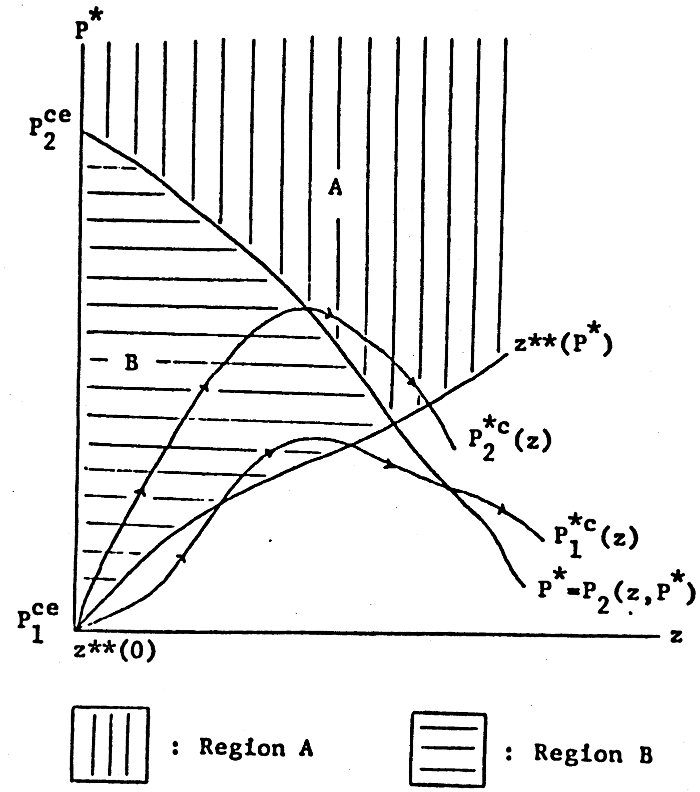
The consumer is worse off with any target price in Case (a).




(b)

The consumer is better off with $P^*$ in $(P_1^{ce}, P^{*0})$ and worse off with $P^*$ greater than $P^{*0}$.

FIGURE 4



$$EV = .5(V(P_1(z,P^*),Y-(P^*-P_1(z,P^*))\theta_1 z) \tag{19}$$

$$+\ V(P_2(z,P^*),Y-(P^*-\min(P^*,P_2(z,P^*)))\theta_2 z)) = \overline{V}^{ce}$$

where $\overline{V}^{ce}$ is competitive equilibrium expected consumer utility and the functions $P_1(z,P^*)$ and $P_2(z,P^*)$ solve (20) and (21):

$$\theta_1 z = x^d(P_1, Y-(P^*-P_1)\theta_1 z) \tag{20}$$

$$\theta_2 z = x^d(P_2, Y-(P^*-\min(P_2,P^*))\theta_2 z) \tag{21}$$

Note that $P^{*c}(z)$ is continuous everywhere and differentiable at all points other than $\overline{z}_c$ which satisfies the equality :[11]

$$P^{*c}(\overline{z}_c) = P_2(\overline{z}_c, P^{*c}(\overline{z}_c))$$

Consider two cases:

Case 1: $P_2 > P^*$. Totally differentiating (19), simplifying using Roy's identity and substituting for $dP_2/dz$ from (21):

$$\left|\frac{dP^{*c}}{dz}\right| = -\left|\frac{V_{2Y}\theta_2^2 z + (P^*-P_1)\theta_1 V_{1Y} x_p^d(P_2,Y)}{V_{1Y}\theta_1 x_p^d(P_2,Y)z}\right| \tag{22}$$

$$= -\left|\frac{V_{2Y}\theta_2^2}{V_{1Y}\theta_1 x_p^d(P_2,Y)}\right| > 0 \text{ at } (z^{**}(0), P_1^{ce})$$

Case 2: $P_2 < P^*$. Using similar algebra:

$$\left|\frac{dP^{*c}}{dz}\right| = -\left|\frac{V_{1Y}(P^*-P_1)\theta_1 + V_{2Y}(P^*-P_2)\theta_2}{V_{1Y}\theta_1 z + V_{2Y}\theta_2 z}\right| < 0 \tag{23}$$

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11 Proof of these properties for $P^{*c}(z)$ follows arguments exactly analagous to those presented in footnotes 6 and 7.

Equations (22) and (23) give rise to the following observation:

Observation 5 (O5): A necessary and sufficient condition for the existence of target prices which make consumers better off is that the slope of the consumer indifference curve ( $(dP^{*c}/dz)^{-1}$ ) is less than the slope of the supply curve ( $dz^{**}/dP^{*}$ ) at some point of intersection at which $dP^{*c}/dz>0$ (and, therefore, given (23), $P^{*}<P_2$).

Proof: See Appendix B.

Informal Argument: Consider Figure 4, where $P_1^{*c}(z)$ and $P_2^{*c}(z)$ denote two possible consumer indifference curves. Given (O4) and (23), the argument for (O5) is exactly analagous to that for (O3).

The above condition can be stated in equation form (using (22) and (14)):

$$\left|\frac{dP^{*c}}{dz}\right|^{-1} = -\left|\frac{V_{1Y}\theta_1 z x_p^d(P_2,Y)}{V_{2Y}\theta_2^2 z+(P^{*}-P_1)\theta_1 V_{1Y}x_p^d(P_2,Y)}\right| \tag{24}$$

$$< z^{*}_1\left|\frac{x_p^d(P_2,Y)}{x_p^d(P_2,Y)-\theta_2 z^{*}_2}\right| = \frac{dz^{**}}{dP^{*}} \quad \text{where } \frac{dP^{*c}}{dz} > 0$$

Now note that inequality (24) implies the following:

$$-\left|\frac{V_{1Y}\theta_1 x_p^d(P_2,Y)}{V_{2Y}\theta_2^2}\right| < z^{*}_1\left|\frac{x_p^d(P_2,Y)}{x_p^d(P_2,Y)-\theta_2 z^{*}_2}\right| \tag{25}$$

Rewriting this inequality and substituting for $z^{*}_1$ and $z^{*}_2$ :

$$- x_p^d(P_2,Y) < Q[U'_1[(V_{2Y}/V_{1Y})-(U'_2/U'_1)]+U''_1(P^{*}\theta_1-C')z(V_{2Y}/V_{1Y}) \tag{26}$$

$$-U''_2(P_2\theta_2-C')z]$$

where Q is as defined in (17). (O5), (25), and (26) lead to the following proposition:

<u>Proposition 2</u> : Consumers are always worse off with a Brannan Plan program than without if: (a) $z^*_1 \leq 0$ at points of intersection between $P^{*c}(z)$ and $z^{**}(P^*)$, (b) demand is price elastic for $P<(P_1^{ce}, P_2^{ce})$ and either condition (27) or condition (28) (sufficient but not necessary) is satisfied, or (c) the representative farmer is risk neutral and condition (27) (necessary and sufficient) is satisfied. Conditions (27) and (28) are as follows:

$$V_{1Y} = V_Y(P_1(P^*), Y-(P^*-P_1(P^*))\theta_1 z^{**}(P^*)) \geq V_Y(P_2(P^*), Y) = V_{2Y} \tag{27}$$

$$\frac{V_{2Y}}{V_{1Y}} \leq \frac{U'_2}{U'_1} \tag{28}$$

$$\text{for all } P^* : P^* = P^{*c}(z^{**}(P^*)) < P_2(P^*)$$

Proof:

Part (a): When $z^*_1 \leq 0$, the right hand side of (25) is non-positive and the left hand side is positive. Hence, inequality (25) cannot hold. QED (a).

Part (b): In view of part (a), only the case of $z^*_1 > 0$ needs to be examined. Elastic demand implies that $P_1^{ce}\theta_1 > P_2^{ce}\theta_2$. Since $z^*_1 > 0$ , for arbitrary target price $P^{*0} > P_1^{ce}$,

$$P_2(P^{*0})\theta_2 < P_2^{ce}\theta_2 < P_1^{ce}\theta_1 < P^{*0}\theta_1$$

The F.O.C. (3) therefore implies that $(P^*\theta_1 - C') > 0$ and $(P_2\theta_2 - C') < 0$. Hence, the last two terms of (26) are non-positive. Further, profits are higher in state 1, making $U'_1 \leq U'_2$. Therefore, with either condition (27) or (28), elastic demand implies that the right hand side of (26) is non-positive at relevant points. But the left hand side is always positive and the inequality cannot be satisfied. QED (b).

Part (c): If farmers are risk-neutral, $U''_1=U''_2=0$ and $U'_1=U'_2$. Hence, condition (27) implies that the left hand side of (26) is non-positive at relevant points, making satisfaction of this inequality impossible. QED (c).

To interpret condition (27), note that it will be satisfied if $V_{YP}\leq 0$ (or $V_{YP}\approx 0$) and $V_{YY}\leq 0$. Since

$$V_{YP} = (V_Y/P)\alpha(\delta^*-\eta)$$

where $\delta^*$ is the consumer index of relative risk aversion, $\eta$ is the income elasticity of demand, and $\alpha$ is the expenditure share for the commodity of interest (see Newbery and Stiglitz (1982b)), $V_{YP}\leq 0$ (or $V_{YP}\approx 0$) implies that $\eta\geq\delta^*\geq 0$ or $\alpha\approx 0$. Hence, (27) can be justified by an approximately constant $V_Y$ in the relevant range of P and Y (which implies $\eta\approx 0$ (Just, et. al. (1982))), a high income elasticity of demand (relative to $\delta^*$), or a small expenditure share.

In the context of Proposition 2, condition (28) requires that, given an elastic demand, the proportional increase in the farmer's marginal utility of income from state 1 to state 2 (attributable to the lower farmer income in state 2) is greater than the proportional change in the consumer's marginal utility of income (attributable to the higher market price and higher consumer income in state 2). Since effects on consumers are diffuse relative to those on farmers, this condition is not implausible.

Though Proposition 2 indicates some conditions under which consumers are never better off with target prices, inequality (26), when evaluated at competitive equilibrium, gives a sufficient condition for the existence of target prices which benefit consumers. Examples which satisfy this inequality are not difficult to construct. For instance, if demand is price inelastic in the relevant region, farmers strictly risk averse, and, evaluated at competitive equilibrium, $V_{2Y}=V_{1Y}$, $V_{2YP}=0$

and $V_{2PP} \simeq 0$, then there exist target prices which make consumers better off. (These conditions imply that the right hand side of (26) is strictly positive and that the left hand side (expanded by differentiating Roy's identity) is approximately zero.) For further discussion on the circumstances under which consumers are better off with target prices, see the numerical analysis in section VI.

## V. WELFARE

When considering the overall welfare effects of a Brannan Plan program, two sets of questions should be asked: (1) If compensation is not made to anyone, what are the distributional (consumer vs. producer) effects of target prices and can all agents be made better off? (2) Can compensation be made so as to make everyone better off with a Brannan Plan program? If so, is there an optimal target price and what is it?

Figure 5 sheds some light on the first set of questions. It depicts the following four cases:

Case (a): In this case, farmers are always better off with a Brannan Plan program; consumers are also better off with $P^* \in (P_1^{ce}, P^{*1})$, but worse off with $P^* > P^{*1}$. In light of equations (9), (14) and (22), case (a) occurs when:

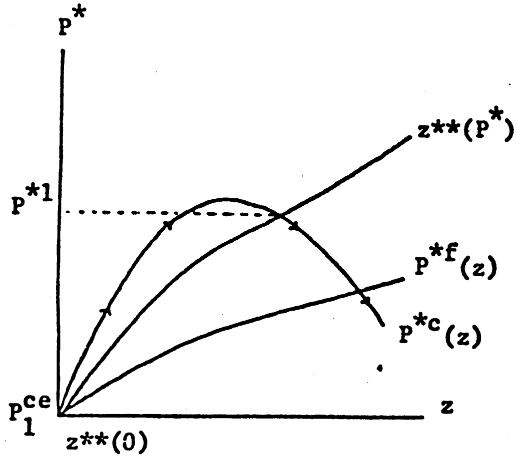
$$\left|\frac{dP^{*c}}{dz}\right|^{-1} = -\left|\frac{V_{1Y}\theta_1 x_p^d(P_2, Y)}{V_{2Y}\theta_2^2}\right| < z^*_1\left|\frac{x_p^d(P_2, Y)}{x_p^d(P_2, Y) - \theta_2 z^*_2}\right| \tag{29}$$

$$= \left|\frac{dz^{**}}{dP^*}\right| < -\left|\frac{U'_1\theta_1 x_p^d(P_2, Y)}{U'_2\theta_2^2}\right| = \left|\frac{dP^{*f}}{dz}\right|^{-1}$$

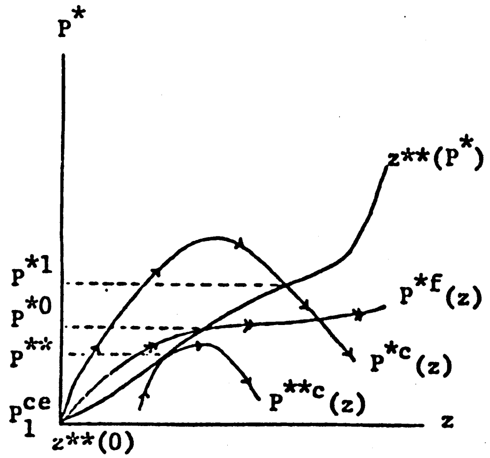
evaluating all terms at competitive equilibrium. Clearly, this inequality can only be satisfied when:

$$V_{1Y}/V_{2Y} < U'_1/U'_2 \tag{30}$$

FIGURE 5



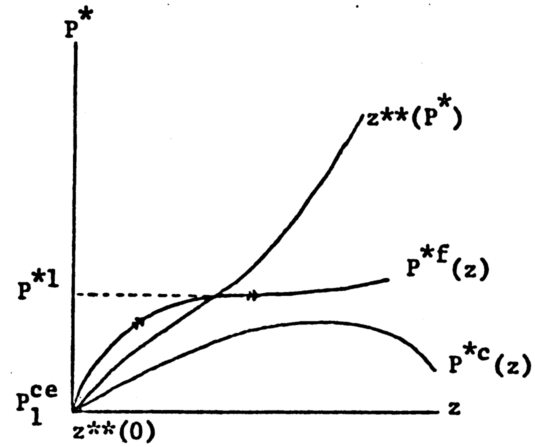

(a)



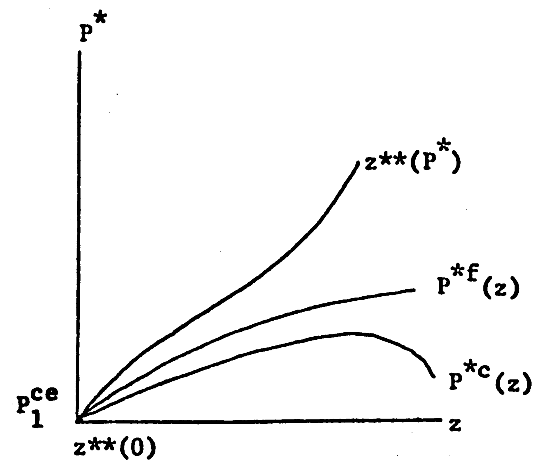

(b)

Here, $P^{**}$ denotes the consumer's optimal choice of $P^*$ and $P^{**c}(z)$ denotes the consumer indifference curve for the utility level generated by equilibrium with $P^* = P^{**}$.

FIGURE 5 (CONTINUED)




(c)




(d)

at competitive equilibrium. When demand is inelastic and farmers risk averse, this condition will be satisfied so long as $V_{YP}$ is not negative and large in absolute value. Though satisfaction of (30) is not implausible, note that (29) requires $x_p^d(P_2,Y)$ to fall in a particular range:

$$z^*_1\left|\frac{\theta_2^2 V_{2Y}}{\theta_1 V_{1Y}}\right| - \theta_2 z^*_2 > -x_p^d(P_2,Y) > z^*_1\left|\frac{\theta_2^2 U'_2}{\theta_1 U'_1}\right| - \theta_2 z^*_2 \qquad (31)$$

The smaller is $z^*_1$, the closer $(V_{2Y}/V_{1Y})$ is to $(U'_2/U'_1)$ and the smaller is $\theta_2$, the narrower is this range and the less plausible is case (a).

Case (b): As drawn in Figure 5, there are three target price ranges to consider for this case: (i) when $P^* \in (P_1^{ce}, P^{*0})$, consumers are better off and farmers worse off with a Brannan Plan program than without; (ii) when $P^* \in (P^{*0}, P^{*1})$, both consumers and farmers are better off; and (iii) for $P^* > P^{*1}$, consumers are worse off and farmers better off. Though, as drawn, there exist target prices which benefit all agents (in range (ii)), note that if consumers (as taxpayers) choose the target price, their choice will make farmers worse off. Section VI verifies that these outcomes are possible but depend on parameter specifications.

Case (b) can occur when conditions (17) and (30) are satisfied. For a food commodity which has low price and income elasticities, these conditions are not implausible and this case is of particular interest.

Case (c): This case is just the opposite of case (a): Consumers are always worse off with a Brannan Plan; farmers are also worse off with $P^* \in (P_1^{ce}, P^{*1})$, but better off with $P^* > P^{*1}$. A necessary condition for this construction is that (28) be satisfied at competitive equilibrium. Further, from Proposition 1, this case can only occur when demand is price inelastic and farmers are risk averse. Therefore if:

$$V_{YP} > 0 \quad \text{or} \quad V_{YP} \doteq 0 \tag{32}$$

then (30) will be satisfied and (28) violated. Either a small income elasticity of demand or a small expenditure share imply (32) and, hence, rule out case (c).

Case (d): Here, consumers are always worse off and farmers always better off with a Brannan Plan. This case occurs under a variety of circumstances suggested in Sections III and IV. For example, if condition (27) is satisfied and either farmers are risk neutral or demand is price elastic, case (d) will result.

In summary, the cases in Figure 5 indicate that any distributional outcome is possible and, in particular, the distributional implications of the standard neoclassical model under certainty do not generalize to a world of uncertainty and incomplete markets. In fact, for a food commodity, it is likely that the distributional effects of some target prices are exactly the opposite of those implied by the neoclassical model.

To address the second set of questions, consumer and producer compensating variations (CS and PS, respectively) are defined as follows (where prices and outputs represent compensated equilibrium outcomes):

$$\sum_{i=1}^{2} .5[V(P_i(P^*), Y-(P^*-\min(P^*, P_i(P^*)))\theta_i z^{**}(P^*)-CS)] = \bar{V}^{ce} \tag{33a}$$

$$\sum_{i=1}^{2} .5[U(\max(P^*, P_i(P^*))\theta_i z^{**}(P^*)-C(z^{**}(P^*))-PS)] = \bar{U}^{ce} \tag{33b}$$

Case 1: $P^* < P_2$. Differentiating with respect to $P^*$ and solving for the change in CS and PS:

$$dCS/dP^* = .5[(1/E(V_Y))(-V_{1Y}\theta_1 z^{**}(P^*) \tag{34a}$$

$$-V_{1Y}(P^*-P_1)\theta_1(dz^{**}/dP^*)-V_{2Y}\theta_2 z^{**}(P^*)(dP_2/dP^*))]$$

$$dPS/dP^* = .5[(1/E(U'))(U'_1\theta_1 z^{**}(P^*)+U'_2\theta_2 z^{**}(P^*)(dP_2/dP^*))] \quad (34b)$$

Summing (34a) and (34b) gives the change in societal compensating variation:

$$\frac{dW}{dP^*} = \frac{dCS}{dP^*} + \frac{dPS}{dP^*} = .5[\theta_1 z^{**}(P^*)\left|\frac{U'_1}{E(U')} - \frac{V_{1Y}}{E(V_Y)}\right| \quad (35)$$

$$+ \theta_2 z^{**}(P^*)\left|\frac{U'_2}{E(U')} - \frac{V_{2Y}}{E(V_Y)}\right|\left|\frac{dP_2}{dP^*}\right| - \left|\frac{V_{1Y}}{E(V_Y)}\right|(P^*-P_1)\theta_1\left|\frac{dz^{**}}{dP^*}\right|]$$

Case 2: $P^*>P_2$. Analogs to equations (34a), (34b), and (35) are :

$$\frac{dCS}{dP^*} = E\left|\frac{V_Y}{E(V_Y)}(-\theta z^{**}(P^*)-(P^*-P)\theta\frac{dz^{**}}{dP^*})\right| \quad (36a)$$

$$\frac{dPS}{dP^*} = E\left|\frac{U'}{E(U')}\theta z^{**}(P^*)\right| \quad (36b)$$

$$\frac{dW}{dP^*} = \left|\frac{Cov(U',\theta)}{E(U')} - \frac{Cov(V_Y,\theta)}{E(V_Y)}\right| z^{**}(P^*) - E\left|\frac{V_Y}{E(V_Y)}(P^*-P)\theta\frac{dz^{**}}{dP^*}\right| \quad (37)$$

These equations imply the following propositions:

<u>Proposition 3</u> : If, for all $P^*$ satisfying $P^*>P_2(P^*)$ (Case 2), (a) $dz^{**}/dP^* \geq 0$, and (b) $V_{1Y} \geq V_{2Y}$, then, with compensation, the optimal target price satisfies the condition: $P^* < P_2(P^*)$.

Proof: $dW/dP^*$ in (37) is non-positive since $Cov(U',\theta) \leq 0$ (due to higher profits in state 1 and $U'' \leq 0$), $Cov(V_Y,\theta) \geq 0$ due to (b), and the second term is non-positive due to (a). QED.

<u>Proposition 4</u> :

(i) If (a) $dz^{**}/dP^* \geq 0$ and $dP_2/dP^* \leq 0$ for all $P^*$ , then a necessary condition for the existence of a welfare-improving target price is that condition (30) be satisfied at equilibria for some interval of target price levels.

(ii) If (b) $dP_2/dP^* \leq 0$ at $P^*=P_1^{ce}$, then a sufficient condition for the existence of a welfare-improving target price is that (30) be satisfied at the no-program competitive equilibrium.

Proof:

Part (i): Note that:

$$\frac{Q_2}{Q_1} = 2\frac{E(Q)}{Q_1} - 1 = (2\frac{E(Q)}{Q_2} - 1)^{-1} \qquad (38)$$

where $Q_i \equiv U'_i$ or $V_{iY}$. Hence, (30) is true if and only if:

$$\frac{U'_1}{E(U')} > \frac{V_{1Y}}{E(V_Y)} \text{ and } \frac{U'_2}{E(U')} < \frac{V_{2Y}}{E(V_Y)} \qquad (39)$$

Therefore, given (a), if (30) is violated, $dW/dP^*$ in (35) is negative (or non-positive at competitive equilibrium). Further, rewriting the first term of (37) and substituting from (38):

$$E\left|\theta\left|\frac{U'}{E(U')} - \frac{V_Y}{E(V_Y)}\right|\right| z^{**}(P^*) = \qquad (40)$$

$$.5z^{**}(P^*)\left|\frac{\theta_1+\theta_2(U'_2/U'_1)}{1+(U'_2/U'_1)} - \frac{\theta_1+\theta_2(V_{2Y}/V_{1Y})}{1+(V_{2Y}/V_{1Y})}\right|$$

Since $\theta_1 > \theta_2$, let $\delta \equiv \theta_1 - \theta_2 > 0$ and rewrite (40):

$$.5z^{**}(P^*)\delta[(1+(U'_2/U'_1))^{-1} - (1+(V_{2Y}/V_{1Y}))^{-1}] \qquad (41)$$

which is positive if and only if (30) is satisfied. Again, given (a), $dW/dP^*$ is negative if (30) is violated. QED (i).

Part (ii): Given (b), $dW/dP^*$ in (35) is positive when $P^*=P_1^{ce}$ and (30) is satisfied. QED (ii).

In light of Proposition 4, condition (30) makes the role of incomplete markets explicit. If markets existed for Arrow-Debreu securities, the marginal rates of substitution for income in the two states would be equated between producers and consumers in every market equilibrium, violating (30). With incomplete markets, condition (30) implies that state 2 income can be transferred from producers to consumers in exchange for transfer of state 1 income from consumers to producers, making everyone better off. But this exchange is precisely what a compensated Brannan Plan program accomplishes. Consumers get more real income in state 2 (with a lower food price) while they transfer income to producers in state 1 via the deficiency payments; conversely, producers forego income in state 2 (lower price) in exchange for higher income in state 1.

To interpret Propositions 3 and 4 further, it will be useful to derive analytical expressions for $dz^{**}/dP^*$ and $dP_2/dP^*$, which are derivatives of the compensated equilibrium supply and second state price. In this compensated economy setting, the farmer's optimal choice of z will be a function of the received prices in the two states and the ex-ante compensation, $PS(P^*)$. Denoting this function $z^*(P^*, \max(P_2, P^*), PS(P^*))$, $z^*_i$ will again be used to represent the partial derivative of $z^*$ with respect to its ith argument. As usual, two cases must be considered:

Case 1: $P^* < P_2$.

$$\frac{dP_2}{dP^*} = \frac{\theta_2(z^*_1 + z^*_3(dPS/dP^*) + x^d_{2Y}(dCS/dP^*)}{x^d_{2p} - \theta_2 z^*_2} \tag{42}$$

$$= \frac{\theta_2 z^*_1 + z^*_3 AU'_1\theta_1 - B_1}{x^d_{2p} - \theta_2 z^*_2 - z^*_3 AU'_2\theta_2 + B_2}$$

where $x^d_{2Y}$ and $x^d_{2p}$ denote partial derivatives of the demand function in the second state and

$$A = .5z^{**}(P^*)/E(U')[\theta_2 - .5x^d_{2Y}(V_{1Y}/E(V_Y))(P^* - P_1)\theta_1]$$

$$B_i = .5x^d_{2Y}/E(V_Y)[V_{iY}\theta_i z^{**}(P^*) + V_{1Y}(P^* - P_1)\theta_1 z^*_i]$$

Note that sufficient conditions for $dP_2/dP^*<0$ are: (a) $z^*_1>0$, (b) $z^*_3\geq 0$, and (c) $x^d_{2Y}\simeq 0$. $z^*_3$ can be expanded as follows:

$$z^*_3 = -(Q/\theta_2^2)[\delta_1 U'_1(P^*\theta_1-C')+\delta_2 U'_2(P_2\theta_2-C')] \tag{43}$$

Hence, if the farmer has constant absolute risk aversion, $z^*_3$ will be zero; if he/she has decreasing absolute risk aversion, $z^*_3$ will be positive.

With respect to $dz^{**}/dP^*$:

$$\frac{dz^{**}}{dP^*} = (z^*_1+.5z^*_3\theta_1 z^{**}(U'_1/E(U')))+(z^*_2+.5z^*_3\theta_2 z^{**}(U'_2/E(U')))\frac{dP_2}{dP^*} \tag{44}$$

$$= \frac{x^d_{2p}(z^*_1+.5z^*_3\theta_1 z^{**}(U'_1/E(U')))}{x^d_{2p}-\theta_2 z^*_2-.5z^*_3\theta_2^2 z^{**}(U'_2/E(U'))} \quad \text{if } x^d_{2Y}=0$$

Therefore, $dz^{**}/dP^*>0$ if $z^*_1>0$, $z^*_3\geq 0$ and $x^d_{2Y}\simeq 0$.

Case 2: $P^*>P_2$. For this case, only $dz^{**}/dP^*$ is of interest:

$$dz^{**}/dP^*=z^*_1+z^*_2+z^*_3E(\theta z^{**}(P^*)(U'/E(U'))) \tag{45}$$

Sufficient for $dz^{**}/dP^*\geq 0$ is $z^*_1\geq 0$, $z^*_2\geq 0$, and $z^*_3\geq 0$.

Having established some general conditions under which these derivatives have the desired signs, the following corollaries can be derived:

<u>Corollary 4.1</u> : If, for all $P^*$, (a) $z^*_i\geq 0$, i=1,2, (b) the farmer has non-increasing absolute risk aversion, and (c) $V_{YP}$ and $V_{YY}$ are small and non-positive, then: when (i) demand is price elastic for $P<[P_1^{ce},P_2^{ce}]$, or (ii) farmers are risk neutral (which implies (a) and (b)), any Brannan Plan program causes a net welfare loss to society.

Proof:

Part (i): From (c), $x^d_{2Y} \simeq 0$. From (b), $z^*_3 \geq 0$. Therefore, given (a), $dz^{**}/dP^* \geq 0$ and $dP_2/dP^* \leq 0$ for all $P^*$, satisfying the prior conditions for Propositions 3 and 4. Due to Proposition 3, only equation (35) needs to be examined. Price elastic demand and $dP_2/dP^* \leq 0$ imply:

$$P_2(P^*)\theta_2 \leq P_2^{ce}\theta_2 < P_1^{ce}\theta_1 < P^*\theta_1$$

Therefore, profits in state 1 are higher than in state 2, making $U'_2/U'_1 \geq 1$. From (c), $V_{2Y}/V_{1Y} \leq 1$. Hence, (30) cannot be satisfied, proving part (i) from Proposition 4 (i). QED (i).

Part (ii): When farmers are risk-neutral, $U'_2/U'_1 = 1$. Therefore, given (c), (30) cannot be satisfied. QED (ii).

Corollary 4.2 : If (a) demand is price inelastic for $P \in [P_1^{ce}, P_2^{ce}]$, (b) farmers are strictly risk averse with non-increasing absolute risk aversion, and (c) $\eta$ (the income elasticity of demand) is aproximately zero for $P \in [P_1^{ce}, P_2^{ce}]$, $Y_s = Y$ $(s=1,2)$, then a positive target price, $P^* > P_1^{ce}$, will be socially optimal.

Proof: From (a), $z^*_1 > 0$ at $P^* = P_1^{ce}$, which implies (with (b) and (c)) that $dP_2/dP^* < 0$ at $P^* = P_1^{ce}$. From (a) and (b), $U'_2/U'_1 < 1$ at the no-program equilibrium. From (b) and (c), $V_{YP}(P,Y) \geq 0$ for $P \in [P_1^{ce}, P_2^{ce}]$, implying that $V_{2Y}/V_{1Y} \geq 1$ at the no-program equilibrium. Hence, the requirements of Proposition 4 (ii) will be met. QED.

This last corollary is the key result of this section. It indicates that under circumstances which are plausible for a food commodity, a Brannan Plan program will be welfare-improving.

It remains to show that the observations above can imply more than trivial effects in agricultural markets. This is done by way of a numerical example in the next section.

## VI. A NUMERICAL EXAMPLE

Consider the following example:

(1) The farmer has a constant absolute risk aversion utility function,

$$U(\pi) = -e^{-\delta\pi} \quad , \delta > 0 \tag{46}$$

and a constant elasticity cost function,

$$C(z) = z^{\eta} \quad , \eta > 1 \tag{47}$$

(2) The consumer indirect utility function takes the form,

$$V(P,Y) = \frac{P^{1-\gamma}}{\gamma-1} + Y \tag{48}$$

which implies constant price elasticity, zero income elasticity aggregate demand,

$$x^{d}(P,Y) = P^{-\gamma} \quad , \gamma > 0 \tag{49}$$

These assumptions are particularly convenient for welfare analysis for the following reasons: They make the compensated and uncompensated equilibria identical; farmer output choices do not depend on fixed income transfers; and consumer demand is income-independent. Further, the correct (and unique) money measure of consumer utility change from a target price is the expected change in consumer surplus minus the expected tax cost. To see this, let $P^{ce}$ denote competitive equilibrium prices, $P'$ post-program prices and T the tax costs of the program. Then:

$$CS = E(V(P',Y-T)) - E(V(P^{ce},Y)) = E\left[\frac{(P^{ce})^{1-\gamma}}{1-\gamma} - \frac{(P')^{1-\gamma}}{1-\gamma}\right] - E(T) \tag{50}$$

Since the marginal utility of income is 1, the right hand side of (50) is a money measure of utility change. Finally, the correct (and unique) money measure of farmer utility change is:

$$PS = \ln\left|\frac{E(U(\pi_0))}{E(U(\pi_1))}\right|/\delta \qquad (51)$$

where $\pi_0$ = competitive equilibrium profit, $\pi_1$ = post-program profit, and ln denotes the natural logarithm.

There are four parameters in this problem, which were varied as follows:

(1) $\delta$ . Based on extant theoretical and empirical evidence ,[12] relative risk aversion was approximately varied between values of 1 and 5. In particular, the certainty competitive equilibrium problem was solved (given the other parameters), giving a profit level $\pi^*$. $\delta$ was then varied so that:

$$\delta\pi^* \in \{1,2,3,4,5\}$$

(2) $\gamma$ . The price elasticity of demand was varied from .2 to .9 (by increments of .1).

(3) $\eta$ . Two values of the cost elasticity were considered: 2 and 3.

(4) $\theta_2$ . (Recall that $\theta_1$=2-$\theta_2$.) The production coefficient was varied between .7 and .9.

In addition, the target price level was varied between the competitive equilibrium prices in the two states. In particular, the target price was set at a linear combination of these two prices,

$$P^* = (1-q)P_1^{ce} + qP_2^{ce}$$

12 Arrow shows that, given boundedness of the utility function, the relative risk aversion coefficient must be less than one for small levels of wealth (profit) and greater than one for high wealth levels. Friend and Blume, Friend and Hasbrouck and Grossman and Shiller provide empirical evidence that American household coefficients are between zero and six. Binswanger provides evidence on these coefficients for farmers in rural India, indicating that they lie between .1 and 10.

where q was varied between 0 and 1 by increments of .02.

Since this problem has no closed form solution, a modified bisection algorithm was used to solve for equilibrium outcomes. (See Appendix C for a complete description of the method used.)

## Numerical Results

Tables 1 and 2 present selected results of the numerical analysis .[13] Table 1 presents, for a variety of parameter values, selected characteristics of competitive equilibrium, equilibrium with the price support which maximizes consumer utility, and equilibrium with the target price which is socially optimal given compensation. Table 2 indicates how high target prices can go with farmers still worse off, consumers better off or society better off. By way of interpretation, these tables give rise to the following comments:

1) Adverse Farmer Effects. Table 1 reveals that farmers can be much worse off with target prices either chosen to maximize consumer utility or chosen "optimally" for society. Four cases are illustrative:

Case 1) $\eta=2$ , $\gamma=.2$ , $\theta_2=.9$ , and $\delta\pi^*=1$ .

Case 2) $\eta=2$ , $\gamma=.2$ , $\theta_2=.8$ , and $\delta\pi^*=1$ .

Case 3) $\eta=3$ , $\gamma=.2$ , $\theta_2=.9$ , and $\delta\pi^*=1$ .

Case 4) $\eta=3$ , $\gamma=.2$ , $\theta_2=.8$ , and $\delta\pi^*=1$ .

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13 Due to space limitations, only a small subset of the numerical results are presented. For example, since variation of $\eta$ did not qualitatively alter outcomes, only results for $\eta=2$ are shown in the tables.

The following statistics for these cases highlight the adverse effects on farmers:

| Statistics | Case 1 | Case 2 | Case 3 | Case 4 |
|---|---|---|---|---|
| $\pi_1^{ce}$ | .5414 | .6150 | .7047 | .7289 |
| $E(\pi^{ce})$ | 1.3256 | 3.1317 | 1.5683 | 3.4322 |
| $SOPS/\pi_1^{ce}$ | -.1474 | -.2190 | .1087 | .1719 |
| $SOPS/E(\pi^{ce})$ | -.0602 | -.0430 | .0488 | .0365 |
| $COPS/\pi_1^{ce}$ | -.3315 | -.2190 | -.1571 | -.2487 |
| $COPS/E(\pi^{ce})$ | -.1354 | -.0430 | -.0706 | -.0528 |

where SOPS and COPS denote producer surplus with, respectively, the socially optimal target price and the optimal target price for consumers, and $\pi^{ce}$ denotes competitive equilibrium profit. For Case 1, producers would be willing to give up a third of their state 1 competitive equilibrium profit to avoid imposition of the consumer optimal target price and 15% of this profit to avoid the socially optimal target price. Of course, these large percentages depend crucially on certain parameter specifications; in particular, all four of the above cases have low demand elasticities (.2) and low risk aversion coefficients (1). When either of these parameters is increased in value, adverse effects on farmers are still possible (see Table 1), but they become small relative to profits and are not observed at the socially optimal price support level.

2) Favorable Consumer Effects. Table 1 Part A ( $\gamma$=.2 ) indicates that consumer gains from a Brannan Plan program can be tremendous. While these gains grow dramatically with production risk ( $\theta_2$ ), they are very large even with the lowest risk level examined. Further, Table 2 shows that these favorable effects persist with very high target price levels. Though consumer gains are still possible with demand

levels which are consistent with higher consumer utility are, not surprisingly, very sensitive to the specification of $\gamma$.

3) Distribution. Even when farmers are better off with a given target price, their gain can be small relative to profits and/or the consumer gain. When $\gamma$=.2, for example, the producer gain is always small relative to the consumer benefit. When $\gamma$=.8, consumers tend to lose a little with target prices and farmers to gain a little. Neither case justifies the standard characterization of Brannan Plan programs as farm subsidies/bail-outs. In the former case, the most plausible for agriculture, these programs would be better characterized as consumer subsidies. In the latter case, the effects of supply response on farmer profits curtail the producers' utility gains.

4) Tax Costs. Large consumer and social gains from a Brannan Plan program can be associated with large tax costs in state 1. The following cases (for all of which $\gamma$=.2 , $\sigma_{\pi}$*=3 , and EQU=SO) illustrate this point:

| $\theta_2$ | $\eta$ | Tax Cost in State 1 | CS | W | Commodity Expenditure in State 1 |
|---|---|---|---|---|---|
| .9 | 2 | .9972 | .6677 | .6960 | .8188 |
| .8 | 2 | 1.6390 | 2.2970 | 2.5195 | .4222 |
| .7 | 2 | 2.1402 | 6.1106 | 6.6155 | .2278 |
| .9 | 3 | 1.2538 | .5350 | .7057 | 1.0055 |
| .8 | 3 | 2.0730 | 2.0599 | 2.6604 | .5534 |
| .7 | 3 | 3.5654 | 5.5079 | 6.8787 | .2902 |

5) Pareto Superiority. Since any target price between the PWO and CBO levels in Table 2 makes both producers and consumers better off than at competitive equilibrium, this table indicates that there is a wide range of circumstances under which a Brannan Plan program leads to an allocation which is Pareto superior to competitive equilibrium, even in the absence of compensation. Note, however, that as $\gamma$ rises, the range of target prices for which Pareto superiority holds in the absence

of compensation narrows. Table 1 indicates that a socially optimal target price (optimal with compensation) can lead to both a Pareto superior allocation without compensation and large social gains, though both of these attributes depend on parameter specifications.

6) Potential Pareto Superiority. Table 1 shows that the social gains from target prices can be enormous (e.g, when $\gamma$=.2). Not surprisingly, the key parameters effecting the magnitude of social gains are, in order of importance, the demand elasticity (negatively related) and the production risk (positively related). Table 2 shows that social gains persist over a wide range of target price levels, even when $\gamma$ is at the high end of the range considered here.

In summary, the numerical analysis reveals that all of the effects discussed in the earlier sections can be large and can persist over a wide range of target price levels, particularly when the price elasticity of demand is low.

## VII. CONCLUSION

This paper has shown that under conditions commonly thought to hold in agriculture (i.e., farmer risk aversion, price inelastic demand, and incomplete markets), a Brannan Plan program can be used to induce equilibrium allocations which Pareto dominate competitive equilibrium. Further, in the absence of compensation, the distributional effects of such a program can be just the opposite of those implied by conventional thinking: farmers can be worse off and consumers/taxpayers better off. As the numerical example shows, all of these effects can be of considerable magnitude.

Of course, the foregoing analysis raises many more questions than it answers. For example, do the results derived above extend to more general economic settings? In particular, do they extend to an open economy setting, an intertemporal model with storage, loans and technical change, or a multiple-farmer economy in which outputs are imperfectly correlated and scope for insuring against aggregate price risk is provided by commodity options? Further, the effects and merits of a Brannan Plan program should not only be examined in a policy vacuum in which policy choice is exogenous. The effects and merits of production controls, consumer subsidies, fixed price contracts, taxes and trade policies, to name a few, should also be examined and contrasted in both single-instrument and multiple-instrument environments in which government behavior is modelled explicitly. Finally, the model should be empiricized for individual markets and the implications of actual policy choices examined. The analysis presented above suggests that these lines of inquiry promise to be fruitful in enhancing understanding of the agricultural policy choice setting.

TABLE 1

Competitive Equilibrium (CE), Equilibrium With The Socially Optimal Price Support (SO), and Equilibrium With the Consumer Optimal Target Price (CO)

A) $\eta=2$ , $\gamma=.2$ , $\sigma_\pi^*=$ 1, 3, and 5 , and $\theta_2$ = .9, .8, and .7

| $\theta_2$ | $\sigma_\pi^*$ | EQU | $P^*$ | z | $P_1$ | $P_2$ | PS | CS | W |
|---|---|---|---|---|---|---|---|---|---|
| .9 | 1 | CE | 0.0 | .8557 | 1.3531 | 3.6904 | 0.0 | 0.0 | 0.0 |
| | | SO | 1.7271 | .9555 | .7794 | 2.1259 | -.0798 | .4197 | .3399 |
| | | CO | 1.4933 | .9474 | .8137 | 2.2192 | -.1795 | .5058 | .3263 |
| .8 | 1 | CE | 0.0 | .7899 | 1.3071 | 9.9259 | 0.0 | 0.0 | 0.0 |
| | | SO | 1.6519 | 1.0326 | .3423 | 2.5590 | -.1347 | 2.2760 | 2.1413 |
| | | CO | 1.6519 | 1.0326 | .3423 | 2.5590 | -.1347 | 2.2760 | 2.1413 |
| .7 | 1 | CE | 0.0 | .7479 | 1.1506 | 25.4192 | 0.0 | 0.0 | 0.0 |
| | | SO | 1.6360 | 1.1290 | .1468 | 3.2433 | .0939 | 6.1875 | 6.2814 |
| | | CO | 1.6360 | 1.1290 | .1468 | 3.2433 | .0939 | 6.1875 | 6.2814 |
| .9 | 3 | CE | 0.0 | .8363 | 1.5178 | 4.1396 | 0.0 | 0.0 | 0.0 |
| | | SO | 1.7275 | .9557 | .7789 | 2.1243 | .0284 | .6677 | .6960 |
| | | CO | 1.6751 | .9555 | .7798 | 2.1267 | -.0003 | .6943 | .6940 |
| .8 | 3 | CE | 0.0 | .7889 | 1.3149 | 9.9847 | 0.0 | 0.0 | 0.0 |
| | | SO | 1.6617 | 1.0337 | .3404 | 2.5853 | .2225 | 2.2970 | 2.5195 |
| | | CO | 1.6617 | 1.0337 | .3404 | 2.5853 | .2225 | 2.2970 | 2.5195 |
| .7 | 3 | CE | 0.0 | .7479 | 1.1506 | 25.4192 | 0.0 | 0.0 | 0.0 |
| | | SO | 1.6360 | 1.1134 | .1574 | 3.4777 | .5049 | 6.1106 | 6.6155 |
| | | CO | 1.6360 | 1.1134 | .1574 | 3.4777 | .5049 | 6.1106 | 6.6155 |
| .9 | 5 | CE | 0.0 | .8361 | 1.5200 | 4.1458 | 0.0 | 0.0 | 0.0 |
| | | SO | 1.7301 | .9557 | .7787 | 2.1239 | .1004 | .6698 | .7702 |
| | | CO | 1.7301 | .9557 | .7787 | 2.1239 | .1004 | .6698 | .7702 |
| .8 | 5 | CE | 0.0 | .7889 | 1.3149 | 9.9847 | 0.0 | 0.0 | 0.0 |
| | | SO | 1.8351 | 1.0338 | .3403 | 2.5840 | .3908 | 2.1899 | 2.5807 |
| | | CO | 1.6617 | 1.0246 | .3559 | 2.7027 | .3247 | 2.2559 | 2.5806 |
| .7 | 5 | CE | 0.0 | .7479 | 1.1506 | 25.4192 | 0.0 | 0.0 | 0.0 |
| | | SO | 1.6360 | 1.1134 | .1574 | 3.4778 | .5516 | 6.1106 | 6.6622 |
| | | CO | 1.6360 | 1.1134 | .1574 | 3.4778 | .5516 | 6.1106 | 6.6622 |

TABLE 1 (CONTINUED)

| $\theta_2$ | $\sigma\pi^*$ | EQU | $P^*$ | z | $P_1$ | $P_2$ | PS | CS | W |
|---|---|---|---|---|---|---|---|---|---|
| B) $\eta$=2 , $\gamma$=.5 , $\sigma\pi^*$= 1 and 5, and $\theta_2$= .9, .8, and .7 | | | | | | | | | |
| | | CE | 0.0 | .7910 | 1.3208 | 1.9731 | 0.0 | 0.0 | 0.0 |
| .9 | 1 | SO | 1.4643 | .8179 | 1.2355 | 1.8456 | .0303 | -.0191 | .0112 |
| | | CO | 1.3339 | .7999 | 1.2916 | 1.9295 | -.0060 | .0098 | .0038 |
| | | CE | 0.0 | .7826 | 1.1339 | 2.5514 | 0.0 | 0.0 | 0.0 |
| .8 | 1 | SO | 1.3891 | .8496 | .9521 | 2.1647 | .0573 | -.0077 | .0496 |
| | | CO | 1.1623 | .8155 | 1.0443 | 2.3495 | -.0244 | .0497 | .0253 |
| | | CE | 0.0 | .7673 | 1.0051 | 3.4664 | 0.0 | 0.0 | 0.0 |
| .7 | 1 | SO | 1.3004 | .8817 | .7611 | 2.6251 | .0676 | .0626 | .1302 |
| | | CO | 1.0543 | .8416 | .8354 | 2.8813 | -.0469 | .1332 | .0863 |
| | | CE | 0.0 | .7752 | 1.3752 | 2.0543 | 0.0 | 0.0 | 0.0 |
| .9 | 5 | SO | 1.4838 | .8206 | 1.2274 | 1.8335 | .0291 | .0283 | .0574 |
| | | CO | 1.3888 | .8075 | 1.2675 | 1.8935 | -.0104 | .0502 | .0398 |
| | | CE | 0.0 | .7484 | 1.2399 | 2.7898 | 0.0 | 0.0 | 0.0 |
| .8 | 5 | SO | 1.4259 | .8550 | .9500 | 2.1375 | .0922 | .1030 | .1952 |
| | | CO | 1.2709 | .8318 | 1.0036 | 2.2582 | .0065 | .1458 | .1523 |
| | | CE | 0.0 | .7274 | 1.1183 | 3.572 | 0.0 | 0.0 | 0.0 |
| .7 | 5 | SO | 1.3922 | .8939 | .7405 | 2.5539 | .1929 | .1842 | .3771 |
| | | CO | 1.1731 | .8610 | .7982 | 2.7531 | .0356 | .2590 | .2946 |
| C) $\eta$=2 , $\gamma$=.8 , $\sigma\pi^*$= 1, and $\theta_2$= .9, .8, and .7 | | | | | | | | | |
| | | CE | 0.0 | .7350 | 1.3044 | 1.6763 | 0.0 | 0.0 | 0.0 |
| .9 | 1 | SO | 1.3416 | .7421 | 1.2888 | 1.6562 | .0091 | -.0085 | .0006 |
| | | CO | 0.0 | .7350 | 1.3044 | 1.6763 | 0.0 | 0.0 | 0.0 |
| | | CE | 0.0 | .7353 | 1.1694 | 1.9412 | 0.0 | 0.0 | 0.0 |
| .8 | 1 | SO | 1.2466 | .7520 | 1.1370 | 1.8875 | .0213 | -.0190 | .0023 |
| | | CO | 1.1848 | .7426 | 1.1549 | 1.9172 | .0009 | .0002 | .0011 |
| | | CE | 0.0 | .7357 | 1.0572 | 2.2920 | 0.0 | 0.0 | 0.0 |
| .7 | 1 | SO | 1.1560 | .7613 | 1.0131 | 2.1964 | .0296 | -.0242 | .0054 |
| | | CO | 1.0819 | .7491 | 1.0336 | 2.2409 | .0019 | .0011 | .0030 |

TABLE 2

Equilibrium With The Highest Target Price Which Makes:
1) Producers Worse Off (PWO), 2) Consumers Better Off (CBO),
And 3) Society Better Off (SBO)

A) $\eta$=2 , $\gamma$=.2 , $\sigma_\pi^*$= 1, 3, and 5, and $\theta_2$= .9, .8, and .7

| $\theta_2$ | $\sigma_\pi^*$ | CASE | $P^*$ | z | $P_1$ | $P_2$ | PS | CS | W |
|---|---|---|---|---|---|---|---|---|---|
| .9 | 1 | PWO | 1.9140 | .9587 | .7666 | 2.0908 | -.0036 | .3346 | .3310 |
| | | CBO | 2.1945 | 1.0659 | .4514 | 1.2311 | .1805 | .0196 | .2001 |
| | | SBO | 2.2193 | 1.0896 | .4043 | 1.1026 | .2742 | -.2564 | .0178 |
| .8 | 1 | PWO | 1.8242 | 1.0352 | .3380 | 2.5670 | -.0425 | 2.1802 | 2.1377 |
| | | CBO | 3.2032 | 1.3467 | .0907 | .6890 | .9383 | .2695 | 1.2079 |
| | | SBO | 3.8927 | 1.5902 | .0395 | .3001 | 1.7755 | -1.5528 | .2227 |
| .7 | 1 | PWO | 0.0 | .7479 | 1.1506 | 25.4192 | 0.0 | 0.0 | 0.0 |
| | | CBO | 5.0336 | 1.7640 | .0158 | .3484 | 2.5435 | .0796 | 2.6231 |
| | | SBO | 6.0043 | 2.1024 | .0066 | .1449 | 3.8570 | -3.6354 | .2216 |
| .9 | 3 | PWO | 1.6751 | .9555 | .7798 | 2.1267 | -.0003 | .6943 | .6940 |
| | | CBO | 2.3567 | 1.0901 | .4034 | 1.1001 | .3920 | .0555 | .4475 |
| | | SBO | 2.4282 | 1.1723 | .2805 | .7650 | .4338 | -.4202 | .0136 |
| .8 | 3 | PWO | 0.0 | .7889 | 1.3149 | 9.9847 | 0.0 | 0.0 | 0.0 |
| | | CBO | 3.2222 | 1.2893 | .1126 | .8548 | 1.0384 | .4285 | 1.4668 |
| | | SBO | 4.0892 | 1.6361 | .0343 | .2603 | 2.0531 | -2.0247 | .0284 |
| .7 | 3 | PWO | 0.0 | .7479 | 1.1506 | 25.4192 | 0.0 | 0.0 | 0.0 |
| | | CBO | 5.0336 | 1.7625 | .0158 | .3498 | 2.5444 | .0869 | 2.6313 |
| | | SBO | 6.0043 | 2.1024 | .0066 | .1449 | 3.8570 | -3.6353 | .2217 |
| .9 | 5 | PWO | 0.0 | .8361 | 1.5200 | 4.1458 | 0.0 | 0.0 | 0.0 |
| | | CBO | 2.3603 | 1.0715 | .4397 | 1.1992 | .4227 | .0851 | .5077 |
| | | SBO | 2.5420 | 1.2081 | .2413 | .6582 | .5406 | -.4902 | .0504 |
| | | PWO | 0.0 | .7889 | 1.3149 | 9.9847 | 0.0 | 0.0 | 0.0 |
| .8 | 5 | CBO | 3.2222 | 1.2892 | .1128 | .8569 | 1.0388 | .4302 | 1.4691 |
| | | SBO | 4.0892 | 1.6361 | .0343 | .2603 | 2.0531 | -2.0247 | .0284 |
| .7 | 5 | PWO | 0.0 | .7479 | 1.1506 | 25.4192 | 0.0 | 0.0 | 0.0 |
| | | CBO | 5.0336 | 1.7625 | .0158 | .3498 | 2.5444 | .0869 | 2.6313 |
| | | SBO | 6.0043 | 2.1024 | .0066 | .1449 | 3.8570 | -3.6353 | .2217 |

TABLE 2 (CONTINUED)

B) $\eta=2$ , $\gamma=.5$ , $\sigma_\pi^*=$ 1 and 5, and $\theta_2=$ .9, .8, and .7

| $\theta_2$ | $\sigma_\pi^*$ | CASE | $P^*$ | z | $P_1$ | $P_2$ | PS | CS | W |
|---|---|---|---|---|---|---|---|---|---|
| .9 | 1 | PWO | 1.3469 | .8017 | 1.2859 | 1.9209 | -.0020 | .0071 | .0051 |
| | | CBO | 1.3730 | .8053 | 1.2744 | 1.9037 | .0058 | .0016 | .0074 |
| | | SBO | 1.6511 | .8328 | 1.1916 | 1.7801 | .0893 | -.0892 | .0001 |
| .8 | 1 | PWO | 1.2189 | .8240 | 1.0228 | 2.3012 | -.0009 | .0369 | .0360 |
| | | CBO | 1.3607 | .8453 | .9718 | 2.1866 | .0490 | .0004 | .0494 |
| | | SBO | 1.7473 | .8731 | .9111 | 2.0499 | .1922 | -.1911 | .0011 |
| .7 | 1 | PWO | 0.0 | .7673 | 1.0051 | 3.4664 | 0.0 | 0.0 | 0.0 |
| | | CBO | 1.3989 | .8949 | .7389 | 2.5486 | .1039 | .0245 | .1283 |
| | | SBO | 1.9396 | .9031 | .7254 | 2.5021 | .3295 | -.3113 | .0182 |
| .9 | 5 | PWO | 1.4024 | .8093 | 1.2617 | 1.8947 | -.0032 | .0472 | .0441 |
| | | CBO | 1.5518 | .8223 | 1.2222 | 1.8258 | .0545 | .0002 | .0547 |
| | | SBO | 1.7343 | .8262 | 1.2108 | 1.8087 | .1233 | -.1225 | .0008 |
| .8 | 5 | PWO | 0.0 | .7484 | 1.2399 | 2.7898 | 0.0 | 0.0 | 0.0 |
| | | CBO | 1.6119 | .8552 | .9496 | 2.1365 | .1537 | .0078 | .1615 |
| | | SBO | 1.9089 | .8594 | .9402 | 2.1154 | .2349 | -.2346 | .0003 |
| .7 | 5 | PWO | 0.0 | .7274 | 1.1183 | 3.8572 | 0.0 | 0.0 | 0.0 |
| | | CBO | 1.6661 | .8940 | .7403 | 2.5533 | .2620 | .0252 | .2872 |
| | | SBO | 2.1591 | .8942 | .7400 | 2.5521 | .2694 | -.2611 | .0083 |

C) $\eta=2$ , $\gamma=.8$ , $\sigma_\pi^*=$ 1, and $\theta_2=$ .9, .8, and .7

| $\theta_2$ | $\sigma_\pi^*$ | CASE | $P^*$ | z | $P_1$ | $P_2$ | PS | CS | W |
|---|---|---|---|---|---|---|---|---|---|
| .9 | 1 | PWO | 0.0 | .7350 | 1.3044 | 1.6763 | 0.0 | 0.0 | 0.0 |
| | | CBO | 0.0 | .7350 | 1.3044 | 1.6763 | 0.0 | 0.0 | 0.0 |
| | | SBO | 1.3788 | .7472 | 1.2777 | 1.6420 | .0194 | -.0192 | .0002 |
| .8 | 1 | PWO | 0.0 | .7353 | 1.1694 | 1.9412 | 0.0 | 0.0 | 0.0 |
| | | CBO | 1.1848 | .7426 | 1.1549 | 1.9172 | .0009 | .0002 | .0011 |
| | | SBO | 1.3083 | .7612 | 1.1197 | 1.8588 | .0397 | -.0392 | .0005 |
| .7 | 1 | PWO | 0.0 | .7357 | 1.0572 | 2.2920 | 0.0 | 0.0 | 0.0 |
| | | CBO | 1.0819 | .7491 | 1.0336 | 2.2409 | .0019 | .0011 | .0030 |
| | | SBO | 1.2548 | .7774 | .9369 | 2.1397 | .0615 | -.0605 | .0010 |

APPENDIX A

Proof of Observation 3

Sufficiency: Suppose

$$\left|\frac{dP^{*f}}{dz}\right|^{-1} < \left|\frac{dz^{**}}{dP^{*}}\right| \quad \text{at } (z^0, P^{*f}(z^0)) \tag{A1}$$

where $z^0 = z^{**}(P^{*f}(z^0))$. Since from (9) $dP^{*f}/dz>0$ at $z^0$, (A1) implies $dz^{**}/dP^{*}>0$ and, from the inverse function theorem, $z^{**-1}(z) \underline{=} P^{*s}(z)$ exists in a neighborhood of this point. Hence, (A1) can be written:

$$\left|\frac{dP^{*f}}{dz}\right| > \left|\frac{dP^{*s}}{dz}\right| \quad \text{at } z^0 \tag{A2}$$

Using the definition of the derivative and recalling that $P^{*f}(z^0)=P^{*s}(z^0)$, (A2) can be written:

$$\lim_{h\to 0} \frac{P^{*f}(z^0+h)-P^{*s}(z^0+h)}{h} > 0 \tag{A3}$$

Recalling (O1), (A3) implies that there exists an $h>0$ such that farmers are worse off with target price $P^{*s}(z^0+h)$ than at competitive equilibrium without target prices.

Necessity: Suppose

$$P^{*f}(z^{**}(P^{*\prime})) > P^{*\prime} \tag{A4}$$

for some $P^{*\prime}$ so that, by (O1), the farmer is worse off with this target price than at competitive equilibrium. It must be shown that (A4) implies the condition in (O3).

Suppose not. Then whenever

$$z=z^{**}(P^{*f}(z)) \text{ and } P^{*f}(z)<P_2(z,P^{*f}(z))$$

$$\left|\frac{dP^{*f}}{dz}\frac{dz^{**}}{dP^{*}}\right| \leq 1 \tag{A5}$$

Given (A5), the initial condition, $P^{*f}(z^{**}(P_1^{ce}))=P_1^{ce}$, and the continuity/differentiability properties of the two functions $z^{**}$ and $P^{*f}$,

$$P^{*f}(z^{**}(P^{*})) \leq P^{*} \tag{A6}$$

whenever

$$P^{*f}(z^{**}(P^{*}) \leq P_2(z^{**}(P^{*}),P^{*f}(z^{**}(P^{*}))) \tag{A7}$$

Since (A6) contradicts (A4),

$$P^{*f}(z^{**}(P^{*'})) > P_2(z^{**}(P^{*'}),P^{*f}(z^{**}(P^{*'}))) \tag{A8}$$

Given continuity of

$$P^{*f}(z^{**}(P^{*})) - P_2(z^{**}(P^{*}),P^{*f}(z^{**}(P^{*})))$$

which is less than zero at $P^{*}=P_1^{ce}$ ( $P_1^{ce}<P_2^{ce}$ ) and greater than zero at $P^{*}=P^{*'}$ ((A8)), there exists a $P^{*}\in(P_1^{ce},P^{*'})$ which satisfies:

$$P^{*f}(z^{**}(P^{*})) = P_2(z^{**}(P^{*}),P^{*f}(z^{**}(P^{*}))) \tag{A9}$$

from the Intermediate Value Theorem (IVT). Let $P^{*''}$ satisfy (A9) and, if there is more than one $P^{*}$ satisfying (A9), then, w.l.o.g., let $P^{*''}$ also satisfy the condition:

$$P^{*f}(z^{**}(P^{*})) > P_2(z^{**}(P^{*}),P^{*f}(z^{**}(P^{*}))) \quad \text{for all } P^{*}\in(P^{*''},P^{*'}] \tag{A10}$$

At $P^{*''}$, (A7) is satisfied, implying that (A6) holds. Hence, in order for (A4) to be satisfied, $P^{*f}(z^{**}(P^{*}))-P^{*}$, a continuous function, must rise from a non-positive number at $P^{*''}$ to a positive number at $P^{*'}$. From the IVT, there exists a $P^{**}\in[P^{*''},P^{*'})$ such that (A6) is satisfied with equality. From (A9) and (A10) (which imply that $P^{*}\geq P_2$ for $P^{*}$ in the relevant range), $P^{*f}(z^{**}(P^{*}))-P^{*}$ is differentiable for all $P^{*}\in(P^{*''},P^{*'})$ and at least differentiable from the right at $P^{*''}$. Therefore, at some $P^{**}$, the right hand derivative of $P^{*f}(z^{**}(P^{*})-P^{*}$ must be positive in order for (A4) to hold:

$$\left|\frac{dP^{*f}(z^{**}(P^{**}))}{dz}\frac{dz^{**}(P^{**})}{dP^{*}}\right| - 1 > 0 \tag{A11}$$

However, from (11), $dP^{*f}(z^{**}(P^{**}))/dz = 0$ at any $P^{**}$, contradicting (A11), implying that the supposition is false and proving necessity.

APPENDIX B

Proof of Observation 5

Sufficiency: Suppose

$$\left|\frac{dP^{*c}}{dz}\right|^{-1} < \left|\frac{dz^{**}}{dP^{*}}\right| \quad \text{at } (z^0, P^{*f}(z^0)) \tag{B1}$$

where $z^0 = z^{**}(P^{*c}(z^0))$ and $dP^{*c}/dz > 0$. Since $dz^{**}/dP^{*} > 0$ by (B1), $z^{**}(.)$ can be inverted and $P^{*c}$ can be substituted for $P^{*f}$ in (A2) and (A3) to complete the proof (using (O4)).

Necessity: Suppose

$$P^{*c}(z^{**}(P^{*\prime})) > P^{*\prime} \tag{B2}$$

for some $P^{*\prime}$ so that, by (O4), the consumer is better off with the target price than an competitive equilibrium. It must be shown that (B2) implies the condition in (O5).

Suppose not. To determine the implications of this supposition, the following Lemma needs to be established:

Lemma: If

$$\left|\frac{dP^{*c}}{dz}\frac{dz^{**}}{dP^{*}}\right| > 1 \tag{B3}$$

$$P^{*c}(z^{**}(P^{*0}))=P^{*0} \text{ and } \frac{dP^{*c}(z^{**}(P^{*0}))}{dz} < 0$$

then (B3) holds at $P^{*+}$ where

$$P^{*c}(z^{**}(P^{*+}))=P^{*+} \text{ and } \frac{dP^{*c}(z^{**}(P^{*+}))}{dz} > 0$$

Proof of Lemma: Suppose the Lemma is false. Then

$$\left|\frac{dP^{*c}}{dz}\frac{dz^{**}}{dP^{*}}\right| \leq 1 \tag{B4}$$

whenever $dP^{*c}/dz>0$ at a point of intersection. Let $n=1,\ldots,N$ index the following two intervals (defining $P^{*++0} \equiv P_1^{ce}$):

$$\text{(1) for all } P^{*} \in [P^{*++(n-1)}, P^{*+(n)}] \text{ , } \frac{dP^{*c}(z^{**}(P^{*}))}{dz} \geq 0$$

$$\text{(2) for all } P^{*} \in (P^{*+(n)}, P^{*++(n)}) \text{ , } \frac{dP^{*c}(z^{**}(P^{*}))}{dz} < 0$$

Since $P^{*c}$ is continuously differentiable at all but a finite number of points (at which $P^{*c}(z)=P_2(z,P^{*c}(z))$) and $z^{**}$ is continuous, these intervals are non-degenerate when they exist. Further, (22) and (B3) imply that intervals (1) and (2) exist for n=1 under the given supposition. Finally, note that there exists an N (possible infinite) such that the set of these intervals covers the space of $P^{*} \geq P_1^{ce}$.

For each n, the following property can be established (given the above supposition):

$$\left|\frac{dP^{*c}(z^{**}(P^{*x}))}{dz}\,\frac{dz^{**}(P^{*x})}{dP^{*}}\right| \leq 1 \tag{B5}$$

$$\text{for all } P^{*x} \in [P_1^{ce}, P^{*++(n)}) : P^{*c}(z^{**}(P^{*x}))=P^{*x} \tag{B6}$$

Proof of this property requires the usual two step inductive argument:

Step 1: (B5) holds for n=1. The interval $[P_1^{ce}, P^{*++(1)})$ can be partioned into (1) and (2) above (where n is set equal to one). For interval (1), (B4) implies (B5). For interval (2), suppose that there is a $P^{*x}$ satisfying (B6). Now note that:

(i) $z^{**}(P^{*x}) > z^{**}(P_1^{ce})$. (If not, then consumers are indifferent between $(P^{*x}, z^{**}(P^{*x}))$ and $(P_1^{ce}, z^{**}(P_1^{ce}))$ where $P^{*x} > P_1^{ce}$ and $z^{**}(P^{*x}) \leq z^{**}(P_1^{ce})$. Given (22) and continuity of the indifference curve, the IVT implies that there then exists a $P^* \epsilon (P_1^{ce}, P^{*x}]$ such that the consumer is indifferent between $(P^*, z^{**}(P_1^{ce}))$ and $(P_1^{ce}, z^{**}(P_1^{ce}))$. But this condition violates (04).)

(ii) $[z^{**}(P_1^{ce}), z^{**}(P^{*+(1)})]$ and $(z^{**}(P^{*+(1)}), z^{**}(P^{*++(1)}))$ are disjoint. ($dP^{*c}/dz$ must be either non-negative or negative but not both.)

(iii) $P^{*x} \epsilon (P^{*+(1)}, P^{*++(1)})$. (Definition of interval (2).)

From (i), (ii) and (iii):

$$z^{**}(P^{*x}) > z^{**}(P^{*+(1)}) > z^{**}(P_1^{ce}) \tag{B7}$$

Further, since $dP^{*c}/dz < 0$ for all $P^* \epsilon (P^{*+(1)}, P^{*x}]$,

$$P^{*c}(z^{**}(P^{*x})) < P^{*c}(z^{**}(P^{*+(1)})) \leq P^{*+(1)} < P^{*x} \tag{B8}$$

where the second inequality follows from the initial condition, $P^{*c}(z^{**}(P_1^{ce})) = P_1^{ce}$, and the satisfaction of (B5) for interval (1). But (B8) contradicts the definition of $P^{*x}$, completing Step 1.

Step 2: Given that (B5) holds for $n = n^* \leq N-1$, (B5) holds for $(n^*+1)$. Again partition $[P_1^{ce}, P^{*++(n^*+1)})$ into two intervals:

$$[P_1^{ce}, P^{*+(n^*+1)}] \text{ and } (P^{*+(n^*+1)}, P^{*++(n^*+1)})$$

The above premise, (B4) and the definition of interval (1) $(n = n^*+1)$ imply that

(B5) is satisfied for all $P^*$ in the first of these intervals. With respect to the second interval, note that since $P^{*c}(z^{**}(P_1^{ce}))=P_1^{ce}$, (B4) and satisfaction of (B5) for the first interval imply:

$$P^{*c}(z^{**}(P^*)) \leq P^* \quad \text{for all } P^* \in [P_1^{ce}, P^{*}+(n^*+1)] \tag{B9}$$

Now suppose that (B3) is satisfied at some $P^{*0}$ in the second interval. Without loss of generality, let $P^{*0}$ also satisfy the condition:

$$P^{*c}(z^{**}(P^*)) \leq P^* \quad \text{for all } P^* \in [P_1^{ce}, P^{*0}] \tag{B10}$$

Since $dP^{*c}(z^{**}(P^{*0}))/dz<0$, (B3) implies that $dz^{**}(P^{*0})/dP^*<0$, implying that for $g>0$ small enough, $z^{**}(P^{*0}-g)>z^{**}(P^{*0})$. Given continuity of $z^{**}$ and $z^{**}(P^{*0})>z^{**}(P_1^{ce})$ (due to the definition of $P^{*0}$ and the arguments in (i) above (see Step 1)), the IVT implies that there exists a $P^{*xx} \in (P_1^{ce}, P^{*0}-g)$ such that $z^{**}(P^{*xx})=z^{**}(P^{*0})$. With the definition of $P^{*0}$, this implies that

$$P^{*c}(z^{**}(P^{*xx})) = P^{*0} > P^{*xx} \tag{B11}$$

But (B11) contradicts (B10), implying that (B3) cannot be satisfied at some $P^{*0}$ in the second interval and, hence, completing Step 2.

Since Steps 1 and 2 prove (B5) for all n, (B3) cannot hold, contradicting the supposition and proving the Lemma.

The supposition that (B2) holds but the condition in (05) does not hold implies (B4). But, from the Lemma, (B4) implies

$$\frac{dP^{*c}(z^{**}(P^*))}{dz}\frac{dz^{**}(P^*)}{dP^*} \leq 1 \quad \text{for all } P^* : P^{*c}(z^{**}(P^*))=P^* \tag{B12}$$

Since $P^{*c}(z^{**}(P_1^{ce}))=P_1^{ce}$ and $P^{*c}(z^{**}(P^*))-P^*$ is everywhere continuous and right hand differentiable, (B12) implies that

$$P^{*c}(z^{**}(P^*)) \leq P^* \quad \text{for all } P^* \geq P_1^{ce} \tag{B13}$$

Since (B13) contradicts (B2), the supposition is false and necessity is proven.

## APPENDIX C

### Calculation of Competitive Equilibrium

Based on the construction in Section VI, the farmer's utility maximization problem is:

$$\max_{z} .5(-e^{-\delta\pi_1} - e^{\delta\pi_2}) \tag{C1}$$

where $\pi_i \equiv P_i\theta_i z - z^{\eta}$. (C1) gives the following first order condition:

$$e^{-\delta\pi_1}(\delta P_1\theta_1 - \delta\eta z^{\eta-1}) + e^{-\delta\pi_2}(\delta P_2\theta_2 - \delta\eta z^{\eta-1}) = 0 \tag{C2}$$

The equilibrium conditions are:

$$P_1^{-\gamma} = \theta_1 z \tag{C3}$$

$$P_2^{-\gamma} = \theta_2 z \tag{C4}$$

Substituting from (C3) and (C4) into (C2) gives the single equilibrium equation to be solved:

$$F(z) \equiv C(A_1B_1 + A_2B_2) = 0 \tag{C5}$$

where:

$$C = \delta e^{\delta z^{\eta}} > 0 \ , \ A_i = e^{-\delta(z\theta_i)^{(\gamma-1)/\gamma}} > 0 \ , \ B_i = (z^{-1/\gamma}\theta_i^{(\gamma-1)/\gamma} - \eta z^{\eta-1}) \quad , \ i=1,2$$

Now note that when $\gamma<1$, $B_2>B_1$. Further, when $\eta>1$, $\partial B_2/\partial z < 0$. Therefore, $F(z)<0$ for all $z>z_1$ where $z_1$ solves $B_2=0$, implying that any equilibrium z must be less than $z_1$. Similarly, $F(z)>0$ for all $z>z_0$ where $z_0$ solves $B_1=0$. Thus, $z_0$ and $z_1$ give starting values for an equilibrium search.

To solve for equilibrium (z), a modified bisection algorithm was used (see Vandergraft). The method can be summarized by the following steps:

(1) Set $z=.5(z_0+z_1)$ .

(2) Calculate $F(z)$.

(3) If $F(z)>0$, then set $z_0=z$. If $F(z)<0$, then set $z_1=z$. If $F(z)=0$ then do the following: Calculate $F'(z)$. If $F'(z)<0$, then you are done (set $z_0=z_1=z$). If $F'(z)\geq 0$ then set $z_0=z$.

(4) Return to (1) and repeat (unless the loop has been executed a specified number of times, 15 in this analysis).

<u>Calculation of Equilibrium With Brannan Plan</u>

For each target price level ( $P^*$ ), equilibrium was calculated assuming that $P_1<P^*$ and $P_2>P^*$ (Case 1). If the resulting equilibrium state 2 price was less than $P^*$, then equilibrium was recalculated assuming $P_1<P^*$ and $P_2<P^*$ (Case 2).

For Case 1, equation (C5) becomes:

$$F(z) = C(A_1^*B_1^* + A_2B_2) \qquad \text{(C5')}$$

where:

$$A_i^* = e^{-\delta P^*\Theta_i z}\ ,\ B_i^* = P^*\Theta_i - \eta z^{\eta-1}$$

By arguments similar to those above, the starting values (upper and lower limits for z) were:

$$z_0 = \min(z_0^*,z_1^*)\ ,\ z_1 = \max(z_0^*,z_1^*)$$

where $z_0^*$ solves $B_1^*=0$ and $z_1^*$ solves $B_2=0$. Equilibrium was calculated using the steps above.

For Case 2, equation (C5) becomes:

$$F(z) = C(A_1^* B_1^* + A_2^* B_2^*) \tag{C5''}$$

Starting values were $z_0$ and $z_1$ which solve $B_2^*=0$ and $B_1^*=0$ respectively. Equilibrium was calculated as above with one modification: If $F(z)=0$ in Step (3), then set $z_0=z_1=z$, since $F'(z)<0$ when $F(z)=0$ for this case.

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