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MEASUREMENT OF CONSUMER GAINS FROM MARKET STABILIZATION

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Abstract

This paper demonstrates that there is little difference between exact measures of consumer gains from market stabilization and approximations like expected change in consumer surplus for most cases of practical significance, although if there is a difference its relative importance does not disappear as the degree of stabilization diminishes. Careful specification of the nature of stabilization is much more crucial to the accuracy of welfare measurements. It is important to determine whether the stabilization is in price or quantity, whether a partial or general equilibrium setting applies, whether supply responds to the stabilization or not, and whether the demand curve is linear or not. In any case, improved analytical approximations or easily implementable numerical methods make it unnecessary to rely on suspect measures of consumer gains.

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Unfortunately, there remains much confusion about the way to model the marketwide effects of stabilization policies and about the way to value monetarily their effects on consumers. In studies of stabilization of exchange rates in small open economies and of schemes for commodity buffer stocks, for example, the favored measure of welfare effects has been some version of expected social surplus. But Anderson (1979, p. 1165) has asserted that "No version of expected social surplus can be more than a poor approximation" to the correct measure, which he calls ex ante compensation. As apparent confirmation, Helms (1985) has presented examples where expected consumer surplus and expected equivalent variation grossly mismeasure the rigorous construct, and may even have the wrong sign. Nevertheless, the appropriate question is not whether expected surpluses can be inexact in special examples but whether there are conditions, as Willig (1976) has shown in the case of certainty, under which an approximation to the rigourous construct is acceptably close.

For typical applications, involving goods with low budget shares, there is little difference among the various measures. Using analytical approximations, we show that expected consumer surplus and expected equivalent variation converge on ex ante compensation as the budget share approaches zero. The convergence and accuracy of the expected surplus measures are confirmed with numerical examples.

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But when the budget share is sufficiently large, the sensitivity of the marginal utility of income to changes in price does become important in determining the relative error of the expected surplus measures, regardless of the change in the variance of price. In the case of a change from one certain price to another, the relative errors of different surplus measure disappear the price change approaches zero. But we show that no analogous convergence is seen as the change in variance approaches zero. Even so, when the budget share is large enough for consumers' risk aversion to matter, it may well be inappropriate to treat the stabilized market as isolated from the rest of the economy. General equilibrium effects are likely to dominate those attributable to risk aversion.

Not only can the mistake of ignoring general equilibrium effects swamp that of using the wrong measure of welfare, but so does inappropriate representation of uncertainty in the market. For example, if a mean-preserving change in the dispersion of consumption is represented as a mean-preserving change in price, the calculated effects on comsumers may easily have the wrong sign. Similarly, the error from overlooking possible changes in mean supply, as when supply is incorrectly assumed to be perfectly inelastic, can dominate that from using expected consumer surplus.

1. Measures of Consumer Compensation in Uncertain Markets

We assume that preferences of the identical consumers are nonstochastic and state independent and that markets for contingent claims do not exist. Although for simplicity the model has only one period, the effects of stabilization must be evaluated using an information set Ω that contains the distribution of states of the world $\tilde{w}$ but not the realization w. (Let a $\tilde{}$ denote a random variable.) That is, evaluation takes place "before" the realization is known. In contrast, the competitive representative consumer maximizes utility conditional on the realization w (that is, "after" w is observed), by choosing the consumption set q conditional on income y and the current price vector P. The corresponding utility function is

$$(1) \quad v(P,y) = \max_{q \geq 0} U(q) \quad \text{subject to } P'q \leq y$$

We confine our attention to cases in which the only price that is stochastic is the price of the commodity of interest (commodity j), so we can simplify notation by avoiding subscripts and denoting the price and quantity of good, j namely the jth elements of P and q, as P and q respectively. The price of good is in turn a function P(M,y) of marketed surplus M of good j, which is entirely consumed, and income y. The marketed surplus is provided by producers, a group distinct from consumers. We defer consideration of economic responses of producers till a later section. Marketed surplus is a measureable real-valued function $M(\Psi,\tilde{w})$ where Ψ is a binary variable indicating the presence (Ψ=1) or absence (Ψ=0) of uncertainty. The price of good j in the unstabilized case is denoted by $\tilde{r} = P(M(1,\tilde{w}),y)$. In the stabilized market, the price is $\tilde{s} = P(M(0,\tilde{w}),y)$.

For a given state of the world w, the equivalent variation to make consumers as well off with price r as with price s is implicitly defined by

$$(2) \quad v(r,y+EV(r,s)) = v(s,y)$$

This compensation is state-dependent, that is, conditional on w. Expected contingent equivalent variation (often denoted ex post in the literature) for a change of the price regime from $\tilde{r}$ to $\tilde{s}$ is

$$(3) \quad E[EV(\tilde{r},\tilde{s})] = \int EV(\tilde{r},\tilde{s})f(\tilde{w})dw.$$

E[] is the expectation operator, conditional on the information set Ω. One can think of E[EV] as the expected cost of insurance payments that will render the welfare of the consumer facing price regime $\tilde{r}$ unaffected by any deviation from price regime $\tilde{s}$.

The correct measure for evaluating a policy of stabilization before the realizations of w, and by extension, M and P, are observed is the "noncontingent" equivalent variation, implicitly defined as EV*,

$$(4)\quad E[v(\tilde{r},y+EV^*)] - E[v(\tilde{s},y] = 0$$

EV* is commonly denoted "<u>ex ante</u>" in the literature, a terminology that courts misinterpretation as a description of the timing of payment of compensation, which is actually irrelevant here. What is important is that EV* is conditional solely upon Ω, which does not contain the realization of $\tilde{w}$.

Noncontingent compensation holds the consumer's expected utility constant, over the comparison of price regimes, conditional on Ω.[1] But it does not prevent the change in regime from resulting in a welfare gain or loss upon the realization of $\tilde{w}$. Therefore, the consumer's expectation over the possible changes in his <u>ex post</u> marginal utility contingent on w affect his calculation of EV*. Thus, EV*, as does the corresponding noncontingent compensating variation CV*, reflects the consumer's risk aversion.

In general the noncontingent (<u>ex ante</u>) measures EV* and CV* differ from the expected state-contingent (<u>ex post</u>) measures E[EV] and E[CV]. The differ-

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[1]Note that any effect of compensation EV* on v via P is ruled out, in accordance with normal practice. Obviously, if all consumers are to be compensated, they will rationally adjust EV* to account for the effect of this increase in income on P, even though they are competitive price takers. But if any costs of stabilization to be compared with EV* are of similar magnitude, the net change in income may be negligible. Consistency in the treatment of the feedback effects of benefits and costs on P is more important than whether or not the feedback is taken into account.

ence is the "option value" in the terminology of Weisbrod (1964) and others.[2] Where s is deterministic, the option value $EV^*(\tilde{r},s) - E[EV(\tilde{r},\tilde{s})]$ is the difference between the value of free insurance against price disturbances and the expected cost of the insurance payouts. In that setting the problem with expected state-contingent compensation as a welfare indicator is conspicuous. If an individual facing a fair bet is offered risk-free "insurance" with zero return, the contingent compensation values the insurance at its expected cost of zero, whereas the noncontingent correctly include any risk premium.[3]

In previous studies of stabilization, it has been common practice to use yet a third measure, expected Marshallian consumer surplus E[A], as an approximation to the effects on consumer welfare.

$$(5) \quad E[A] = \int [\int_{\tilde{r}}^{\tilde{s}} q(\tilde{P},\tilde{y})dP] f(\tilde{w})dw$$

where A is defined as having the same sign as EV. Rogerson (1980) has shown that the conditions for E[A] to represent consumer preferences (to have the same sign as EV^* or CV^*) are the same as under certainty, namely that the marginal utility of income be unaffected by the price changes. When this condition is violated, there is no reason to believe that E[EV] or E[CV] is in general a more accurate indicator than E[A], although usually E[A] is most maligned.

But the question of the accuracy of E[A] versus E[EV] raises a more important question. When, if ever, are E[A], E[EV], and E[CV], adequate approx-

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[2]One difficulty with the literature is that the term `option price', which is, of course, distinct from the usage in finance, is rarely precisely defined. Here we use the definition implicit in Schmalensee (1972).

[3]Helms (1984) provides another demonstration that contingent measures are not true indicators of welfare. He investigates a change from $\tilde{r}$ to $\tilde{s}$ where the two random variables have identical independent distributions. If the good in question is strictly normal, E[CV] will be positive and E[EV] will be negative, even though there is no substantive change.

imations for the correct measures EV^* and CV^*? To answer this question, we derive in the next section a general analytical approximation to EV^*, and show its specific form when applied to common representations of changes in market uncertainty.

2. Analytical Approximation of Compensation Measures

Recent advances in numerical methods allow arbitrarily exact calculation of EV^* or CV^* given sufficient computational resources and knowledge of the ordinary demand curve and income risk aversion, as we demonstrate below. But first, to help understand the numerical results, we use analytical approximations to show the different welfare measures' sensitivity to various parameters under a given concept of stabilization, and to distinguish the effects of different concepts of stabilization under a given measure. It happens that these approximations are very accurate alternatives to the exact measures.

Consider a second-order Taylor series approximation to (4), which implicitly defines EV^*. We evaluate the approximation at the point $(\bar{r}, y)$, where a $\bar{\ }$ denotes the arithmetic mean of $\tilde{r}$, $E[r(\tilde{w})]$.[4] Then the approximation for a change in the distribution of $\tilde{P}$ from $\tilde{s}$ to $\tilde{r}$, compensated by EV^* is:

$$(6)\quad 0 \approx E[v_p P_q(\tilde{s}-\bar{r}) - v_p P_q(\tilde{r}-\bar{r}) - v_y EV^* + 1/2((v_{pp}P_q^2 + v_p P_{qq})(\tilde{s}-\bar{r})^2 - (\tilde{r}-\bar{r})((v_{pp}P_q^2) + v_p P_{qq})(\tilde{r}-\bar{r}) + 2v_{py}P_q EV^*) - v_{yy}(EV^*)^2)]$$

This noncontingent equivalent variation EV^* can be thought of as comprising a component from pure stabilization, EV_S^*, and another component from a deterministic change in the mean of P between regimes, EV_D^*. The same

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[4]For valid application of the Taylor series approximation to power and logarithmic utility functions, it is necessary (Loistl (1976)) to assume that the distributions of y and x are such that $0<y<2\bar{y}$, $0<p(x)<2p(\bar{x})$ for all realizations of y and x. Our numerical examples satisfy these restrictions.

second-order approximation (6) implicitly defines EV_D^*, except that all the terms involving a deviation of a random variable from its mean drop out. Then the pure stabilization component, $EV_S^* \equiv EV^* - EV_D^*$, is

$$(7) \quad v_y EV_S^* \approx 1/2\ E[(v_{pp}P_q^2 + v_p P_{qq})((\tilde{s}-\overline{s})^2 - (\tilde{r}-\overline{r})^2) - 2v_{py}P_q EV_S^*(\tilde{r}-\overline{r}) - v_{yy}(EV_S^*)^2]$$

Since $E[\tilde{r}-\overline{r}] = 0$, and EV_S^* is state-independent, the first occurrence of EV_S^* on the right-hand side of (7) can be eliminated. Since $(EV_S^*)^2$ is of higher order than the variances of prices and incomes, it can be eliminated from the last term on the right-hand side. The analogous expansion for CV_S^* shows that, to this order of approximation, $-CV_S^* = EV_S^*$. Define the value of this approximation as V_S^*. Using Roy's identity, v_{py} and v_{pp} can be expressed as:

$$(8) \quad v_{py} = -v_{yy}q - v_y \frac{\partial q}{\partial y}$$

$$(9) \quad v_{pp} = -qv_{py} - v_y \frac{\partial q}{\partial p} = q^2 v_{yy} + qv_y \frac{\partial q}{\partial y} - v_y \frac{\partial q}{\partial p}$$

Substitute (8) and (9) into (7), eliminate the compensation terms of higher order, and divide through by v_y:

$$(10) \quad V_S^* = 1/2\ E[(((q(P(\overline{r}),y))^2 \frac{v_{yy}}{v_y} + q(P(\overline{r}),y) \frac{\partial q}{\partial y} - \frac{\partial q}{\partial P}))P_q^2 - qP_{qq})(\tilde{s}-\overline{s})^2$$

These approximations (7) and (10) force one to consider how marketwide stabilization feeds back on the responses of consumers and produces, whether the partial equilibrium response of that single market or the general equilibrium response of the entire economy. That is to say, the use of V_S^* to represent the effects of arithmetic-mean-preserving stabilization of M requires that EV_D^* equals zero. Whether or not EV_D^* is in fact zero depends on the nature of demand, the type of technology, and the form of market stabilization under consideration. The connection between a particular representation of stabilization and the fundamental structural change that stabilizes the market always exists, but is usually ignored. If it is recognized explicitly, then one can better judge whether the representation is appropriate to a particular problem. In general, to estimate the total effect of market stabilization on consumers one must consider both V_S^* and EV_D^*.

One form of stabilization where EV_D^* equals zero, and hence where EV_S^* fully reflects the change in welfare is consumption stabilization when the supply elasticity of M is zero. Denote the consumer gain from such an arithmetic-mean-preserving (AMP) stabilization of quantity consumed as $V_S^*(AMPq)$,[5]

$$(11) \quad V_S^*(AMPq) = 1/2\ [(P_q)^2 \frac{\bar{r}}{P(\bar{r})} [\Upsilon(\eta^y - \rho_y) - \eta^D] - \bar{q}P_{qq}]\Delta\sigma_q^2$$

$$= \frac{P(\hat{q},y)}{2\hat{q}\eta^D} [\frac{\Upsilon}{\eta^D}(\eta^y - \rho_y) + C - 1]\Delta\sigma_q^2$$

where η^y and η^D are the income elasticity and uncompensated price elasticity of demand evaluated at $\hat{q} = \bar{r}$, ρ_y is the Pratt-Arrow coefficient of relative risk aversion with respect to income at $v(P(\hat{q},y),y)$, $\Delta\sigma_q$ is the change in the variance of quantity consumed, and Υ is the budget share,

$$(12) \quad \Upsilon = \frac{P(\hat{q},y)\hat{q}}{y}$$

C is the coefficient of relative curvature of the demand curve (analogous to the Pratt-Arrow measure of relative risk aversion with respect to income)[6]

$$(13) \quad C \equiv -qP_{qq}/P_q$$

A related expression for expected consumer surplus is obtained by holding v_y constant in (11), so that $\eta^y = \rho_y$

$$(15) \quad E[A(AMPq)] \approx \frac{P(\hat{q},y)}{2\hat{q}\eta^D} (C - 1)\Delta\sigma_q^2$$

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[5]This formula was presented in Wright and Williams (1984) footnote 3, p. 171.

[6]Wright (1979) identified the importance of C in the welfare effects of consumption stabilization.

Few previous writers have used this concept of stabilization of quantity consumed. Instead they have viewed stabilization as an AMP reduction in price dispersion. The noncontingent (<u>ex ante</u>) compensating variation for this type of stabilization, $V_S^*(AMPP)$, for this concept of stabilization is obtained in an analogous fashion,

$$(15) \quad V_S^*(AMPP) = \frac{q(\hat{P},y)}{2\bar{P}} [\gamma(\eta^y - \rho_y) - \eta^D]\Delta\sigma_p^2$$

where $\Delta\sigma_p^2$ is the change in the variance of price. (See Anderson (1979) or Turnovsky, Shalit, and Schmitz (1980).)

In the case of linear consumption demand, $\Delta\sigma_p^2 = (\eta^D)^2(q/P)^2 \Delta\sigma_q^2$, and C = 0, so (11) and (15) are equivalent. Otherwise, $V_S^*(AMPq)$ will almost always differ from $V_S^*(AMPP)$, and the concept of AMPP stabilization is, as discussed in section 4 below, inconsistent with the assumption of zero steady-state response of consumption (marketed surplus).

In the linear case, (11) shows that consumers can lose from consumption (or price) stabilization even if they are risk-averse ($\rho_y > 0$); judging from the expected surplus measure (14) one would conclude, as did Waugh (1944), that consumers definitely lose. If, on the other hand, C > 1, as in the case with demand of constant and strictly negative price elasticity, then (11) indicates that consumers can gain from consumption stabilization even if they are risk neutral with respect to income ($\rho_y = 0$), as indicated by the expected surplus calculations of Wright (1979) for the case of zero supply response. More important, comparison of (11) and (14) indicates that expected consumer surplus may be an accurate measure of the benefits of stabilization even if consumers are extremely risk averse, as long as the budget share γ is sufficiently small, a condition for accuracy of consumer surplus introduced by Hotelling (1938).

3. Relevant Range of Parameters

Thus far our discussion has been based on second-order approximations to the true noncontingent measures of the effects of stabilization. The advantage of this analytical approach is that the roles of variou[illegible] are clearly identified. But a disadvantage is that it is difficult to appreciate the accuracy of various approximations or to know how the noncontingent and contingent measures compare in typical applications. Accordingly in this section we discuss the relevant ranges of the parameters, and in the next present exact numerical calculations under those plausible parameterizations.

The methodology under discussion is typically applied to agricultural or mineral commodities. In those settings, it is important to distinguish between applications to consumption of staple foods and major fuels in less-developed economies, and all other applications.

Perhaps the easiest parameter to discuss is the income elasticity of demand, η^y. For "farm-gate" foods (stripped of services like transportation, preparation, and preservation), Engel's law dictates that income elasticities of demand will normally be low. Values of η^y from 0.0 to 0.4 probably cover the great majority of unprocessed commodities in the United States (George and King (1971)). In less-developed countries, the income elasticity of demand for the staple food may be close to unity for the poorest groups, but for all income groups typical estimates lie closer to 0.5 (George (1979), Gray (1982), and Hazell and Roell (1983)).

In the developed economies, the price elasticity of consumption demand, η^D, for the primary component of food commodities is usually closer to 0.0 than to -1.0, especially if the commodity is not so narrowly defined, as would be the case for grade differentials, that it has very good substitutes. In less-developed economies η^D for the major staple may be close to -1.0 for the poor, and some preferred foods may have much higher price elasticities.[7]

The expenditure share of most commodities in any economy is quite small. For example, in the United States the value of wheat consumed is currently less than 0.1% of GNP, and despite the recent price fluctuations for fossil fuels, their share has not risen above 6.5% of GNP. In less-developed economies the share of a staple food might be above 10% (but almost surely not above 25%), especially for the poor.[8] Nevertheless, inspection of (11) reveals that it is not the absolute magnitude of the share but the ratio γ/η^D that is relevant, and so the effect of a staple's relatively high γ tends to be moderated by its relatively high η^D.

Indentification of a plausible range for ρ_y, in contrast, is "an unresolved empirical question" (Pindyck (1984, p. 344)). Published inferences, some rather informal, based on simple single-good models with intertemporally separable consumption, suggest a range from 0.0 to about 6.0 for ρ_y, although higher values are conceivable.[9]

Evidence regarding C, the relative curvature of the demand curve, is

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[7]In Gray's study (1982) of urban consumers in Brazil, the poorest 30% had η^D of -0.80 for cereals and -0.55 for meat and fish. Middle and upper income groups had η^D of above -0.2 for these products.

[8]Hazell and Roell (1983) report that only the staple (rice and sorghum-millet respectively for Malaysian and Nigerian rural samples) had a share in excess of 10%. Gray (1982) reports shares above 10% only for the meat-fish aggregate for the poorest 30%, and for cereals for the poorest 30% of a Brazilian urban sample. In India, of all cereals only rice had a share above 10% in a national sample (with a maximum of 24% in rural areas), though for some regions obviously wheat would have had a share above 10% (George (1979)).

[9]Pindyck's (1984) model suggests a value of 5.0 to 6.0 for ρ_y. Friend and Blume (1975) infer a value greater than 2.0, while Friend and Hasbrouck (1982) estimate a value of about 6.0. Grossman and Shiller (1981) find a value greater than 4.0, but Flavin (1984) shows that their analysis, modified to account for sampling error, may be consistent with a value around 0.0.

virtually nonexistent. The issue is usually finessed in studies of the welfare effects of market stabilization, whether analytical or empirical, by arbitrary choice of either a linear or constant-elasticity specification.

Guided by the available empirical evidence, we chose as a reasonable example a (local) consumer demand elasticity $\eta^D = -0.2$ and an income elasticity of demand $\eta^y = 0.5$. The variability in output was represented by a two-point symmetic disturbance whose variance can be adjusted. When the coefficient of variation of output is set at 0.05, which would be a fairly high figure in comparison to typical U.S. agricultural commodities, the coefficient of variation of price is 0.250 in the case of linear demand or 0.258 in the constant elasticity specification. Given the inconclusiveness regarding appropriate values of ρ_y and C, we explored a wide range of those two parameters, using a family of demand curves with constant relative curvature in which the popular linear, constant elasticity, and semilog curves are special cases. We also explored a wide range of values of γ.

4. Numerical Comparisons of Alternate Measures

Given the parameters of the ordinary demand curve, we constructed an exact numerical measure of EV^*. Fortunately, recent advances make this task feasible. Hausman (1981) has recovered the ordinal indirect utility function integration for the cases of linear and constant-elasticity demand. Even if it were difficult to solve the differential equation analytically, Vartia (1983) has developed a simple numerical algorithm that yields an arbitrarily exact approximation to the ordinal indirect utility function.

From the ordinal indirect utility function $\hat{v}$, a cardinal one $v(P,y)$ is derived, generalizing Helms's approach (1985), as a monotone transformation

$$(16)\ v(P,y) = \frac{1}{1+\theta}\,[A + \hat{v}(P,y)]^{1+\theta}$$

where A is chosen so that the term in brackets is positive. The Pratt-Arrow coefficient of relative risk aversion with respect to income is then

$$(17)\ \rho_y = -\frac{\theta y \hat{v}_y}{A+\hat{v}(P,y)} - \frac{y\hat{v}_{yy}}{\hat{v}_y}$$

Thus, given ρ_y and the ordinary demand curve, the noncontingent compensation measures EV^* and CV^* are derived numerically by an iterative procedure, employing a simple Newton's method for speeding convergence.

The sensitivity of EV^*, E[EV], and E[A] to the (local) coefficient of relative risk aversion, ρ_y, is shown in Figure 1 for the cases of linear (C = 0) and constant elasticity ($C = 1-1/\eta^D = 6$) demand curves. These welfare measures indicate the effect of completely stabilizing consumption at its mean, in which case EV^* equals EV^*_S(AMPq). Although EV^* is sensitive to C, it varies little with ρ_y, at least at a budget share of 1%. For both demand curves the relation between EV^* and ρ_y is very close to that which could be derived from the analytical approximation (11), so close that a separate curve cannot be drawn for V^* under linear demand. For their part E[EV] and E[A] are virtually indistinguishable. Whether E[EV] and E[A] are good approximations to EV^* depends upon the criterion. The absolute error is small and not greatly responsive to large changes in C nor ρ_y. But the percentage error is high for the linear case, on the order of 35% for $\rho_y = 6$, because EV^* itself is so small.

For the linear demand curve, EV^*_S(AMPq) equals EV^*_S(AMPP), for equivalent degrees of price and quantity stabilization. Other numerical results confirm that EV^*_S(AMPP) is quite insensitive to demand curvature, so the curve for the linear case also closely approximates the effect of AMP stabilization of price for other specifications with higher values of C, including the constant elasticity case. Such an AMP stabilization of price is precisely what is studied in the numerical exercises of Anderson (1979) and Helms (1985). Clearly, a sign reversal in $EV_S^*(AMPP)$ occurs at some value of ρ_y; in this case it occurs at $\rho_y \approx 20$. But no sign reversal occurs in the case of constant demand elasticity if the AMP stabilization is in the amount consumed, as in the upper quadrant of Figure 1. For $C \neq 0$, AMP price stabilization requires a shift in steady-state supply, one that accounts for much of the welfare effects. EV_D is not equal to zero.

The analytical approximations (11) and (14) suggest that E[A(AMPq)] converges on $EV_S^*(AMPq)$ as γ approaches zero. Figure 2 confirms that as γ approaches 0, E[EV] and E[A] converge on EV^*. Thus, for applications to commodity markets in developed countries, the distinctions between E[EV], E[A] and EV^* are negligible. The use of expected Marshallian surplus E[A] is a sufficiently accurate approximation to EV^* for virtually all primary commodities in the United States, contrary to the implication of Helms (1985). The budget share of any one commodity is simply too small for fluctuations in expenditures on that commodity to have much effect on a consumer's total expenditures, no matter how risk averse he is to fluctuations in total expenditures. Rather than consumers' income risk aversion, the crucial issues are the distinctions between different sources of uncertainty and different demand and supply specifications. As Figure 1 shows, for $C = 6$ and supply elasticity $\eta^s = 0$, addressing AMP quantity stabilization as if it were AMP price stabilization gives results of the wrong sign.

It is true, however, that the use of E[A] or E[EV] to indicate welfare effects of stabilization may be much less acceptable if applied to a commodity with a larger share. As γ increases in Figure 2, E[EV] and E[A], diverge increasingly from the correct measure EV*. If the demand specification is constant elasticity, and $\rho_y = 1$, EV*(AMPq) and E[EV(AMPq)] are equal, and not very different from E[A(AMPq)] for moderate values of γ. But if $\rho_y = 6$, the divergence of E[A(AMPq)] from EV*(AMPq) becomes very large as γ increases beyond values typical for developed economies. Indeed use of E[A] or E[EV] as an approximation to EV*(AMPq) could lead to errors as large as those caused in the constant elasticity case by confusing an AMP stabilization of consumption with an AMP stabilization of price that causes the same change in price variance. (As in Figure 1, the effects of the latter are, for other demand specifications, very close to the results shown for the linear case.) In contrast, the quadratic approximation V* would be nearly indistinguishable from EV* over the entire range of γ shown.

In cases with high γ where E[A] diverges significantly from EV^*_S, one might suspect that the relative error would disappear as the variance of (unstabilized) consumption goes to zero. For in the deterministic case, it is well know that the proportional difference between consumer surplus and compensating and equivalent variation converges to zero as the price change approaches zero. But from (11) and (14), the ratio

$$(18)\quad \frac{E[A(AMPq)]}{V^*_S(AMPq)} = \frac{\eta^D(C-1)}{\gamma(\eta^y-\rho^y)+\eta^D(C-1)}$$

which is independent of $\Delta\sigma^2_\beta$. Consistent with this analysis based on analytical approximations, numerical results portrayed in Figure 3 show that the relative error from using expected surplus is substantially independent of the variance in price or consumption.

Thus for relatively extreme cases, namely high magnitudes of $(\eta^y-\rho^y)$ and γ, expected consumer surplus is unacceptable as an approximation to

EV^* or CV^*, regardless of the degree of stabilization involved. But the approximation V^*, which is in fact simpler to calculate, is likely to be acceptably accurate, as can be seen in Figure 3. But this does not necessarily mean that V^* should be used to evaluate stabilization policies, for two reasons. First, the extra effort to make exact numerical calculations as described here may well be justified. Second, $EV^*(AMPq)$ (or $CV^*(AMPq)$) may well be quite an inappropriate measure because of a substantial response of marketed surplus to market stabilization. This may be caused by either the consumption response of producers, which will be more important, *ceteris paribus*, when the share γ is large, or their endogenous production response.

5. Incentive-Responsive Producers

The equating of mean-preserving stabilization of consumption with mean-preserving stabilization of a production disturbance implicitly assumes three conditions: the equality of aggregate consumption with the marketed surplus of producers, zero production elasticity, and zero consumption response on the part of producers. But if producers respond to stabilization by changing their mean production or consumption, the mean consumption of nonproducers and the mean price must also change. Hence, an analysis supposing a mean-preserving decrease in the spread of consumption would not capture the true welfare effect on nonproducers of stabilization of production. The error can be so large as to make the distinctions between the correct compensation EV^* and the expected surplus $E[A]$ trivial by comparison.

We support this claim with three polar examples. In one, producers have a consumption response, in the second, producers and consumers are identical in their asset holdings as well as their other characteristics, and in the third, supply responds rationally to incentives while producers' consumption is fixed.

i) Precisely when the various *ex ante* and *ex post* measures of welfare differ the most, namely when the share of the commodity is large, it strains credulity to suppose the stabilized sector is isolated from the rest of the economy. As an illustration of the relevance of these economywide adjustments, suppose now producers consume some of what they produce, but that the welfare of the nonproducers is still the subject of interest. Further, assume that in the permanent absence of market disturbances, incomes of producers and nonproducers would be equal so the budget share, γ, determines the number of producers. Suppose in addition that producers and nonproducers have the same preferences; they differ only in their source of income, the nonproducers alone being endowed with a commodity whose output is certain and the producers alone having the rights to the production of the uncertain commodity, which has zero supply elasticity. The producers' incomes are affected not only by fluctuations in production but by the changes those fluctuations cause in price.

The producers will adjust the amount they consume of their own product as price changes alter their incomes. As a result, the variation of marketed surplus no longer reflects exactly the underlying uncertainty in output. Given positive income elasticity, a low price elasticity of demand implies a negative correlation between output and producers' consumption, and so the variability of marketed surplus increases. At income elasticities above the absolute value of the price elasticity, the general equilibrium effects can be substantial.[10] In terms of welfare, the partial equilibrium analysis can understate the value to nonproducers of stabilization by a factor of 2 or 3, as can be seen in Figure 2. Indeed, at a share of 12%, the error from ignoring general equilibrium feedbacks is larger than that from assuming $\rho_y = 1.0$ when it is actually 6.0, evidence once again that risk aversion with respect to income is not necessarily the most crucial parameter.

ii) Not only do consideration of general equilibrium effects become important as the share increases but the distinction between consumers and producers becomes implausible. In just those settings where the share could be so large as to make risk aversion with respect to income matter, the consumers are likely to be subsistence producers. In the extreme case where all individuals are alike and have identical asset holdings, regardless of supply response, the effects of market stabilization consists only of changes in aggregate welfare. These reductions in the "deadweight loss" of uncertainty are generally of substantially smaller magnitude than the gross redistribution, so the welfare significance of stabilization for consumer-producers is far less important than for pure consumers.

iii) Positive supply elasticity can dominate any mismeasurement due to risk aversion with respect to income. Assume now that producers do not consume the commodity and must commit their resources before the realized value of the production disturbance is known. Consumers, on the other hand, respond to price after $\tilde{w}$ is known. The production disturbance is multiplicative (i. e., proportional to output) and is shared by all producers. They are competitive and risk neutral and have expectations that are rational

[10]At shares not much larger than those in Figure 2, let alone as large as the larger of those illustrated by Helms (1985), no equilibrium exists for the outcome with low production.

in the Muthian sense. In this situation, supply response dampens the gain or loss to consumers from stabilization (see Wright (1979)). In Figure 4, for example, mean supply is reduced by complete stabilization of production (and consumption). Therefore, $EV_S^*(AMPq)$, which assumes zero supply response, overstates EV* increasingly as η_S increases from zero, and average steady state consumption with stabilization falls increasing below the unstabilized mean.

In general, one needs the kind of complete market model that lies behind Figure 4 to calculate the steady-state response of mean production and price from market stabilization. But if the welfare effect of the change in mean, EV_D^*, is of the same order as the effect of stabilization, EV_S^*, one can ignore the change in the mean in second-order terms in (6) and approximate EV_D^* as

$$(19)\quad V_D^* = \frac{v_p}{v_y} P_x(\overline{r}-\overline{s}) + (\overline{K}-\overline{m}) = -\overline{r}\Delta P$$

Thus, a shorthand approximation to EV* is, from (11) and (19),

$$(20)\quad V_D^* + V_S^* = \frac{P(\hat{q})}{2\hat{q}\eta^D} [(\gamma/\eta^D)(\eta y-P_y) + C-1]\, \Delta\sigma_q^2 - \hat{q}\Delta P$$

This shorthand approximation is very accurate over a wide range of supply elasticities, given knowledge of the changes in the mean and variance of consumption.

6. The Challenge of Defining Stabilization

The foregoing discussion has included several ways to solve the problem of measuring welfare. But in so doing, it has exposed the crucial problem of defining stabilization, a problem largely ignored in the existing literature. The pioneering conceptualization of market stabilization by Waugh (1944) as an AMP decrease in the dispersion of price, has been shared by virtually all subsequent writers in that tradition for almost three decades, including prominently Oi (1961) and Massell (1969). This view stems naturally from focusing on the individual agent as a decision maker who takes prices as exogenous. But price is not exogenous to the commodity market as a whole; price fluctuations are not fundamental (Samuelson (1972)). Unless both demand and supply are linear, and production disturbances are additive, a change in mean consumption must accompany an AMP decrease in price dispersion.

Using invariance to choice of numeraire as their criterion, Flemming, Turnovsky, and Kemp (1977) have argued for the representation of a decrease in uncertainty as a geometric mean-preserving (GMP) decrease in the spread of price. But invariance to the choice of numeraire is an odd criterion. Following Samuelson, it would make more sense to relate the concept of stabilization to the fundamental source of the change reflected in the price distribution. For instance with Leontief technology, given the conditions of other prices being fixed, the one factor of production specific to the commodity in question having fixed supply, and producers' income fluctuations not affecting demand, the appropriate concept would be an AMP in price if the change in the distribution of output/input ratio were AMP. Or if the change preserves the mean of the input/output ratio, then the appropriate price stabilization measure would be harmonic-mean-preserving (HMP). Only if the change preserved the geometric mean of the input/output ratio would the appropriate concept for price stabilization be GMP.

Many actual stabilization schemes involve storage. Here too it is important to relate the concept of stabilization to the fundamental change in uncertainty. Simulations by Wright and Williams (1982) show that in a comparative-statics analysis storage does not, in general, cause either AMP or GMP changes in the dispersion of price, even if the production disturbance is AMP and supply elasticity is zero. If supply response is negligible, it makes more sense to consider stabilization as an AMP decrease in the spread of consumption, for their numerical results indicate that storage has approximately that effect.

7. Conclusion

Several recent papers have argued that conventional measures like expected consumer surplus are inappropriate indicators of the welfare effects of stabilization, primarily because expected consumer surplus does not account for consumers' risk aversion. But these criticisms apply only to examples implausibly extreme for typical commodities in developed economies. For such commodities the effects of ignoring risk aversion and, more generally, changes in the marginal utility of income, tend to be small enough to render the distinctions among expected contingent compensation, expected consumer surplus, and the theoretically correct measure negligible. On the other hand, the widely ignored demand-curvature parameter, C, is crucial.

For stabilization of markets for staple foods of consumers in poorer countries, expected surplus may be an unacceptably inaccurate measure. Fortunately, recent advances in numerical integration of demand curves make it possible to calculate the correct measure arbitrarily accurately at an acceptable cost. Moreover, we have derived here an analytical approximation that is much more accurate than expected consumer surplus. Since it is so simple to use, there is no reason for analysts to use expected surplus as an approximation if the demand function is known, even if they are reluctant to make the modest computational commitment associated with the exact numerical solution. If, on the other hand, only the (local) demand parameters are known, our results suggest that our approximation is robust to unidentifiable variations in the demand specification. We believe this is usually the situation in practice. If so, "exact" integration has no obvious advantage over an "approximation." Unfortunately, even the approximation relies crucially on a parameter of demand curvature that at present awaits accurate empirical evaluation.

The major issue in measuring the welfare effects of stabilization on consumers is not the method of measurement, but the specification of the fundamental nature of the stabilization and identification of the crucial parameters of the model. These include the curvature of demand and the output and consumption responses of producers to stabilization. Only if a commodity's budget share is larger than seen in developed countries is risk aversion with respect to income likely to be an important parameter. But even then, ignoring the consumption and output responses of producers may well be a much larger source of error.

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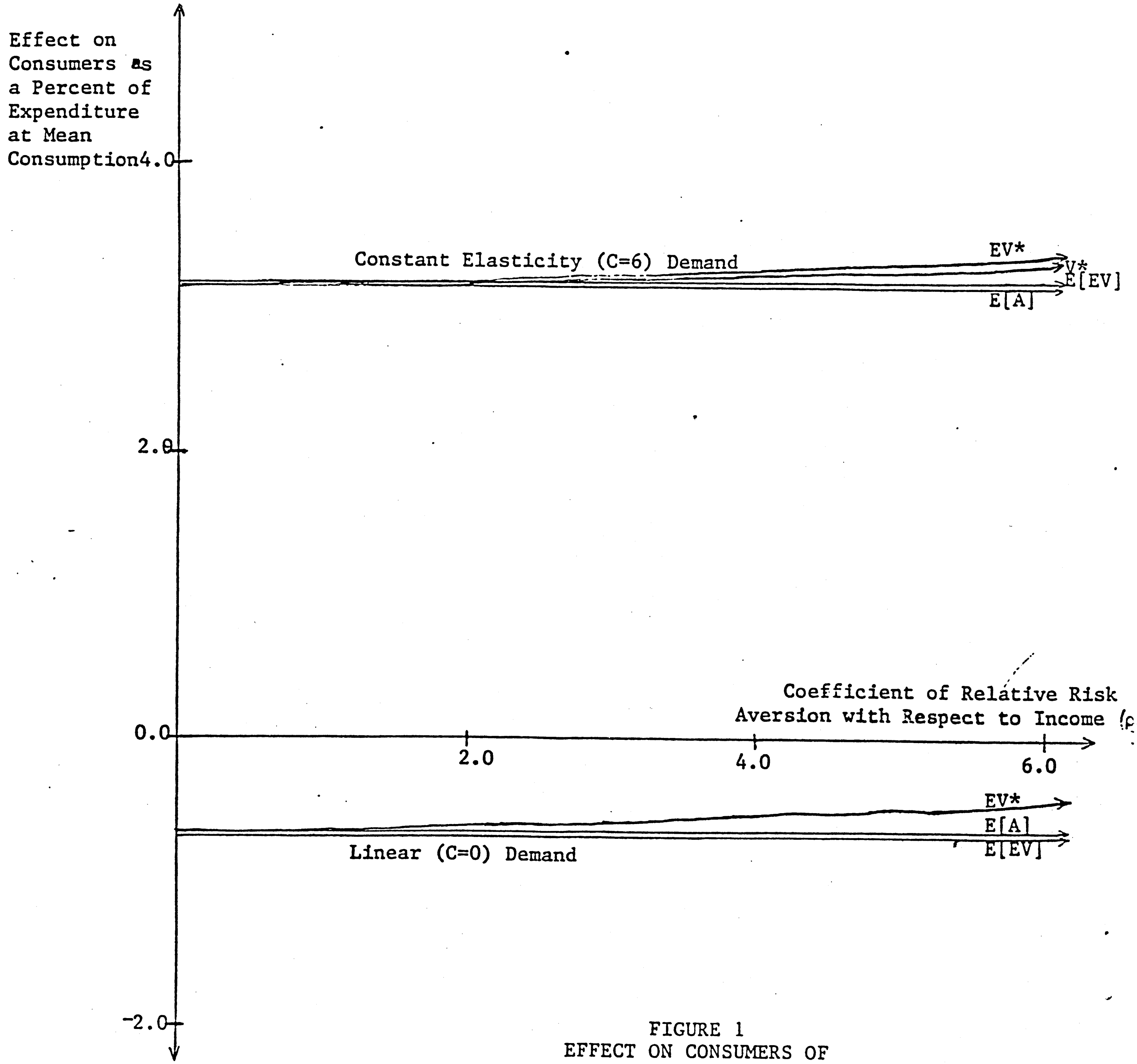

FIGURE 1
EFFECT ON CONSUMERS OF
COMPLETE STABILIZATION OF CONSUMPTION

$(\eta^D = -0.2,\ \gamma = 0.01,\ \eta^y = 0.5,\ \sigma_q = 0.05)$

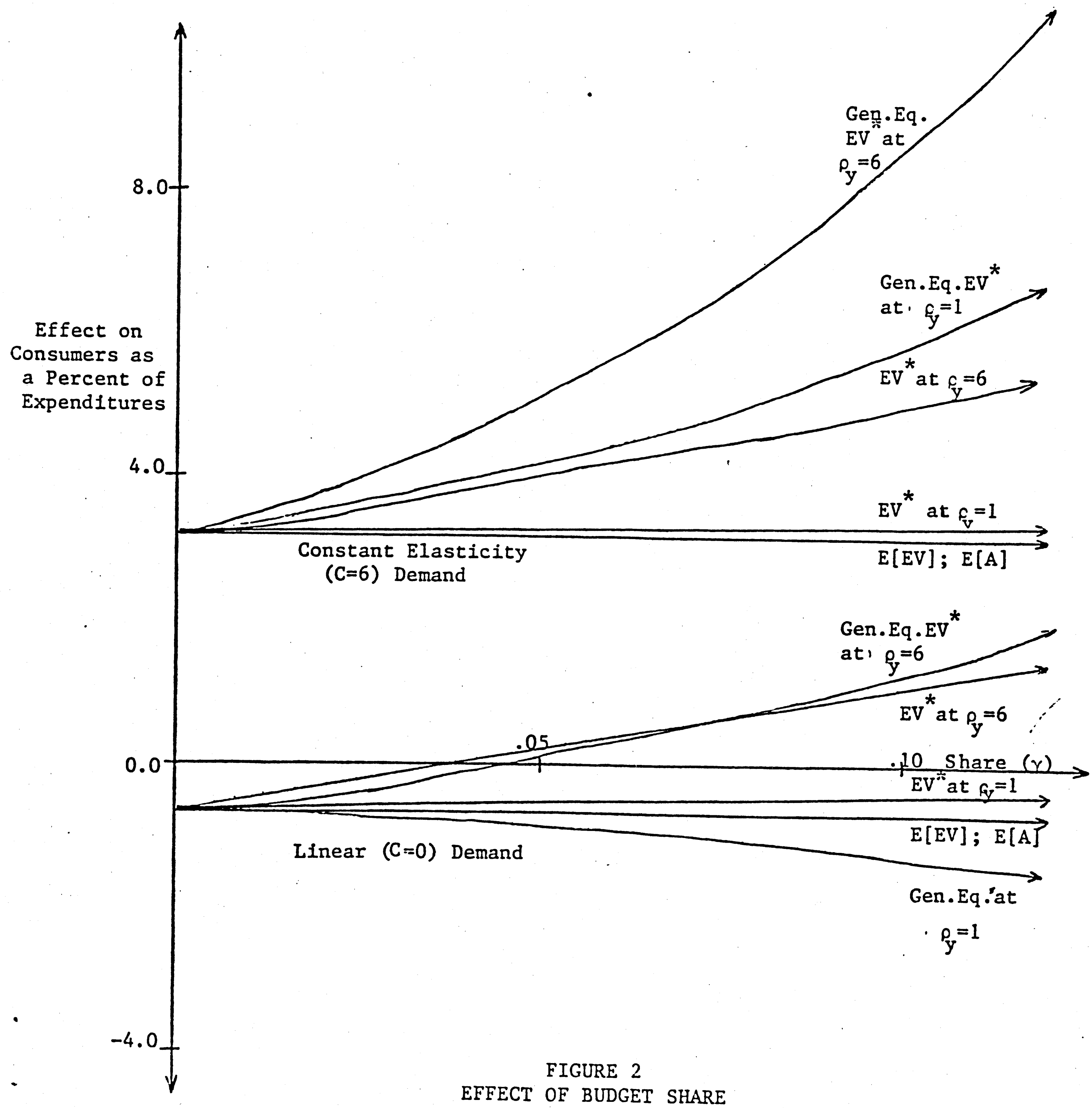

FIGURE 2
EFFECT OF BUDGET SHARE
ON WELFARE MEASURES
(η^D=-0.2, η^y=0.5, σ_q=0.05)

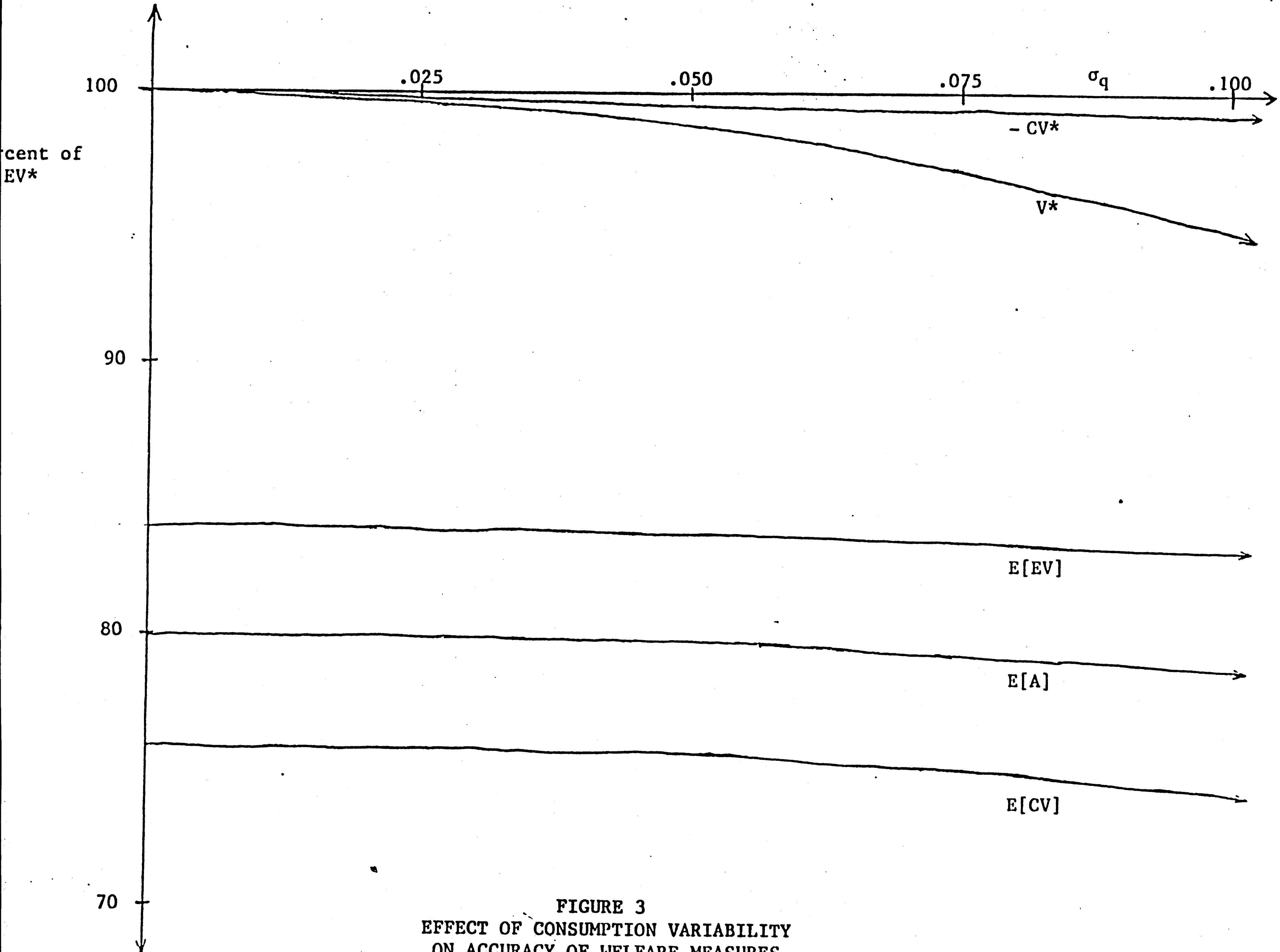

FIGURE 3
EFFECT OF CONSUMPTION VARIABILITY
ON ACCURACY OF WELFARE MEASURES
(η^D=-0.2, C=6.0, γ=0.10, ρ_y=3.0, η^y=0.5)

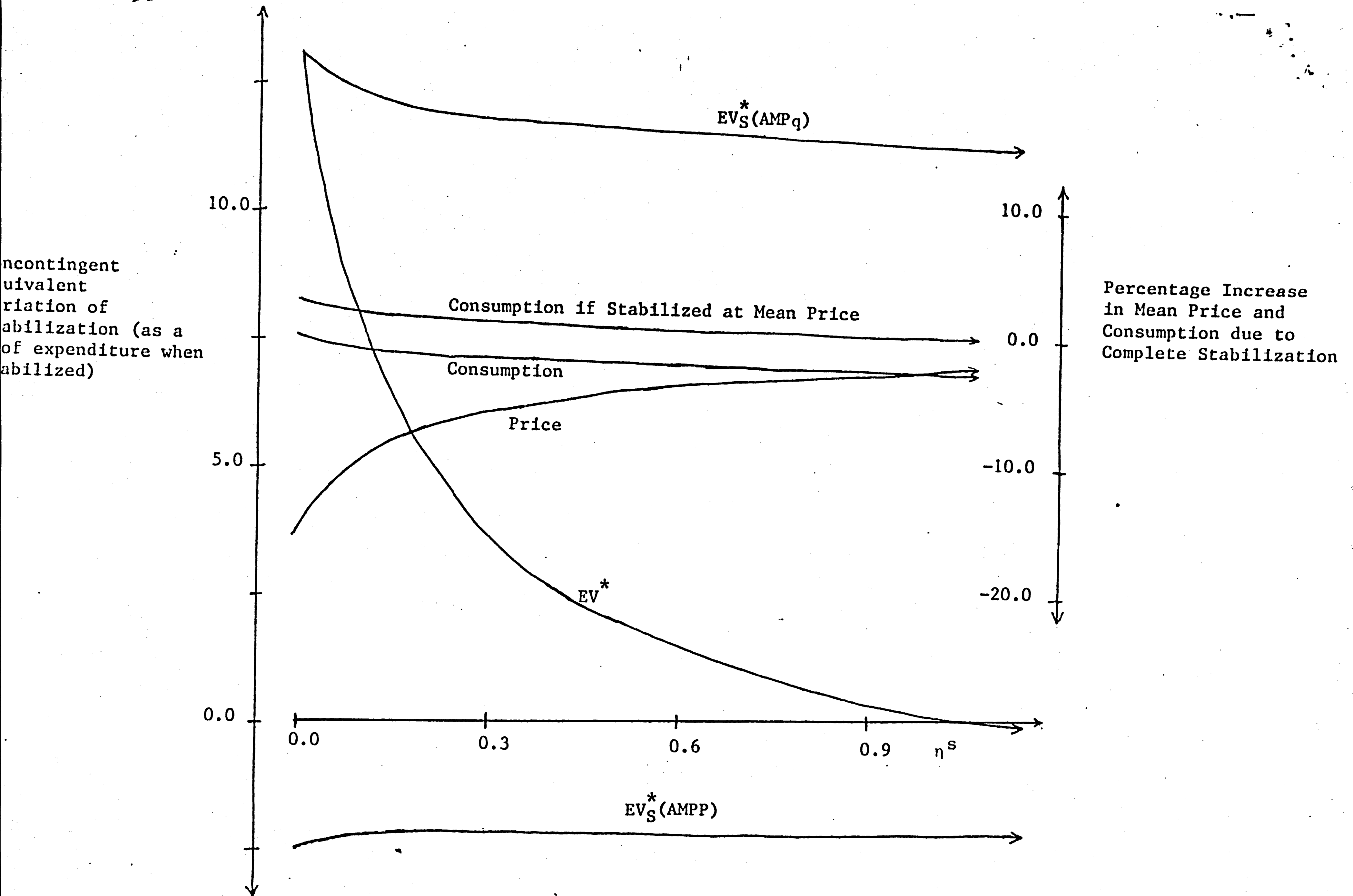

FIGURE 4
EFFECT OF ELASTIC SUPPLY
(η^D=-0.2, η^y=0.5, C=6.0, γ=0.01, ρ_y=3.0, σ_q=0.1)