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DETERMINATION OF THE PREDOMINANCE OF VARIOUS EXPECTATIONS PATTERNS IN COMMODITY FUTURES AND SPOT MARKETS

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I. Introduction

Most empirical models of storable commodities skirt issues of specifying internally consistent dynamic representations for intertemporal markets. Gardner, in an excellent treatment of public and private stocks, argues that the demand function for stocks ". . . cannot (to the author's knowledge) be derived analytically even under simple specifications of other equations." Subotnik and Houck model the determination of one-quarter ahead futures prices within the same model that produces current quarter spot prices. Their formulation excludes altogether any consideration of expectations; future prices in their model are determined exclusively on the basis of current period supply-demand conditions. Both of these models and numerous other models that have been advanced in the literature are based on Working's "theory of storage." This framework provides a simple specification of intertemporal price spreads based upon current stocks.

In a dynamic world of uncertainty, however, the Working formulation is in essence a self-contained but static theory of intertemporal price relationships. The conceptual inconsistency in Working's hypothesis was demonstrated first by Weymar who used the Muth rational expectation hypothesis to show that the spread between future prices for two different dates of delivery should depend upon expected stocks rather than stocks already in existence. In contrast, Working has stated that "it is only supplies already in existence which have any significant bearing . . . on current intertemporal price relationships. . . ." Only a static theory would support such a statement.

Available empirical evidence on the relevance of the Working static framework versus an internally consistent rational expectation formulation is indeed unclear. Nevertheless, some studies (e.g., Pearson and Houck) have found that information concerning future supply-demand conditions and future expected stock levels do influence current period spot prices. Hence, models which failed to properly conceptualize and measure this influence are expected to generate inferior forecasts. In other words, if tractable dynamic representations of these influences can be captured, it is expected that their forecasting accuracy will dominate models that are currently available in the literature.

The focus of this paper is on dynamic representations of intertemporal markets for storable commodities. Our purpose is to develop a general theoretical framework that will allow us to determine estimatable dynamic equations which can be used to distinguish between various static and dynamic representations which are imparted by alternative conditional expectation formation patterns. Much of dynamic economic modeling suffers from the lack of sufficiently rich data sets to discriminate across alternative expectation formation patterns. Generally, economists impose the expectation formation pattern as part of their maintained hypothesis. However, given the rich data sets that are available for both spot and future commodity markets, this is perhaps the most likely area of application where real empirical progress can be made in discriminating across expectation formation patterns.

Another motivation for the structure of the theoretical model advanced in this paper relates to the notion of "rationally" expected prices. In the original formulation of rational expectations by Muth and its subsequent use by economists, rationality has been defined only in terms of benefits. That is, the cost of collecting information to formulate rationally expected prices has been neglected. It can be shown theoretically that, for some economic environments, naive expectations are, in fact, rational. This apparent paradox results from the failure of rational expectations as defined in the economic literature to incorporate the costs of collecting information on critical random variables.

The theoretical model developed in this paper for storable commodities presumes active futures and spot markets. Uncertainty, risk aversion, and basis risk are formally incorporated in the model representation. Numerous authors have dealt with risk aversion and uncertainty in future and spot market prices, but most all authors neglect basis risk and production uncertainty (e.g., Turnovsky, Sarris, Feder, et al.). As usual, speculators are presumed to transact only in the futures market while hedgers are assumed to transact in both the futures and spot markets.

For the above model, dynamic representations of both the futures and spot prices are derived. Each of these representations is based on expected spot prices for period t conditional on information available at t - 1. Six different formulations of the conditional expectations can be investigated by the formulations. The six expectation-formation patterns are:

1. Rational expectations
2. Adaptive expectations
3. Naive expectations
4. Future market prices
5. Normal expectations
6. Various convex combinations of 1-5.

For each expectation formulation, the price dynamics for both the futures and spot markets are compared and contrasted. An econometric test is developed to discriminate among these formations.

II. The Microeconomic Framework

In terms of individual agents, the behavior of four separate trading groups are identified and investigated in this section: producer/hedger, storer/hedger, forward-contracting hedger, and speculator. Some of these groups are involved in both the current spot and future markets, some in future markets, and some in future markets and forward-contracting markets for export or processed goods. Behavior of other groups is summarized by the spot-market demand and forward-contracting demand for export or processed goods. Interaction of these demands with the behavior of the four explicit groups then gives rise to three markets for which equilibrium conditions must be satisfied:

1. The futures market
2. The spot market
3. The forward-contracting market.

Individuals are assumed not to migrate among groups as perceived short-run profitability changes due to asset fixities associated with all groups except speculators. Each decision-maker explicitly included in the model faces a two-stage decision problem in which, first, any spot-market plans for time period t and futures market positions with delivery date t are decided in time period t - 1 and, second, at time period t any futures position with delivery date t can be closed out or not depending on spot and futures prices at the delivery date. The first-stage decision is assumed to maximize expected utility of income. In each case, expected utility is approximated locally by a linear mean-variance relationship following the arguments of Just and Zilberman. The second-stage decision simply maximizes income since all random elements become known at time t. The consideration of the latter decision is not usually found in papers of this nature and suggests distinctly different results as shown below. The reason for the difference is that if basis risk is small relative to overall price risk, then the risk faced by the decision-maker can be made relatively inconsequential if actual delivery/acceptance on the futures market is considered as an alternative at contract termination. That is, some profit or loss can be locked in at the initial decision stage (except for speculators) so the decision-maker only faces the smaller risk related to the basis at contract termination. The existance of a certain outcome in portfolio selection models has been shown in the finance literature to distinctly alter the role of risky alternatives.

For notational purposes, let

p_t = spot-market price at time t

p_t^c = forward contracting price at time t - 1 for delivery at time t (this may be a raw product equivalent price for a processed commodity or a price at which the commodity will be exported)

p_t^f = futures price at time t - 1 for contracts with maturity at time t

$\tilde{p}_t^f$ = futures price at time t for contracts with maturity at time t

p_{ti} = spot-market price for time t expected by decision-maker i at time t - 1 except in the case of speculators where p_{ti} is the decision-maker's expectation for $\tilde{p}_t^f$.

f_{ti} = futures market position taken at time t - 1 in contracts with maturity at time t (positive for sales, negative for purchases)

$\tilde{f}_{ti}$ = futures market transactions at time t in contracts with maturity at time t (negative for sales, positive for purchases)

σ_i = variance of p_{ti} with respect to p_t, i.e., $E_{t-1}(p_{ti} - p_t)^2$ where E_{t-1} is the expectation operator at time t - 1 except in the case of speculators where $\sigma_i = E_{t-1}(p_{ti} - \tilde{p}_t^f)^2$

ϕ_i = absolute risk aversion of decision-maker i

Q_{ti} = production planned by producer i at time t - 1 for time t

$I_{t-1,i}$ = inventory held by storer i out of supply at time t - 1 for release at time t

$X_{t-1,i}$ = raw product quantity required by processor/exporter i at time t to honor commitments made at time t - 1.

The Producer/Hedger.--Consider first the case of a producer/hedger i who uses the futures market to hedge against price declines during the production period. Suppose his cost of production is quadratic and given by $\alpha_{0i} Q_{ti} + (1/2)\alpha_{1i} Q_{ti}^2$. The associated utility of income is

$$U_i(\pi_{ti}) = U_i[p_t(Q_{ti} + e_{ti} - f_{ti} + \tilde{f}_{ti}) - \alpha_{0i} Q_{ti} - \frac{1}{2\alpha_{1i}} Q_{ti}^2 + p_t^f f_{ti} - \tilde{p}_t^f \tilde{f}_{ti}]$$

where e_{ti} is a random disturbance in production unknown at time t - 1 but known at time t, $E_{t-1}(e_t) = 0$. Also, consistent with competition, producers are assumed not to perceive the effect of their own production on price or correlation of their production with price, $E_{t-1}(p_t e_{ti}) = 0$. Suppose also that, due to basis risk,

$$\tilde{p}_t^f - p_t \sim N(0, 2\tilde{\sigma}) \tag{1}$$

where $E[(\tilde{p}_t^f - p_t)\, e_{ti}] = 0$ and $\tilde{\sigma} > 0$. Then the producer has a two-stage decision problem where at time t - 1 he chooses expected production Q_{ti} and an initial futures market position f_{ti}. At time t he then decides how much of his futures position to close out.

Using the optimality principle of dynamic programming, the problem can first be solved at the second stage given the first stage decisions and then at the first stage after substituting second-stage decision functions. At the second stage (time t), all random forces become known so the problem is one of certainty or simple profit maximization where profit is

$$\pi_{ti} = \pi_{ti}^* + \Delta\pi_{ti} \tag{2}$$

where

$$\pi_{ti}^* = p_t\,(Q_{ti} + e_{ti} - f_{ti}) - \alpha_{0i}Q_{ti} - \frac{1}{2\alpha_{1i}} Q_{ti}^2 + p_t^f\, f_{ti} \tag{3}$$

$$\Delta\pi_{ti} = (p_t - \tilde{p}_t^f)\, \tilde{f}_{ti}. \tag{4}$$

Since π_{ti}^* is completely determined at time t, the decision problem is to maximize $\Delta\pi_{ti}$ subject to $0 \leq \tilde{f}_{ti} \leq f_{ti}$ assuming $f_{ti} \geq 0$; the solution is

$$\tilde{f}_{ti} = \begin{cases} f_{ti} & \text{if } p_t > \tilde{p}_t^f \\ 0 & \text{if } p_t \leq \tilde{p}_t^f. \end{cases} \tag{5}$$

Next, substituting (5) into (4) and using (1) to take expectations obtains

$$E_{t-1}(\Delta\pi_{ti}) = \sigma^* f_{ti} \tag{6}$$

$$V_{t-1}(\Delta\pi_{ti}) = c\tilde{\sigma}\ f_{ti}^2 \tag{7}$$

where

$$\sigma^* = \sqrt{\frac{\tilde{\sigma}}{\pi}},$$

$$c = 1 - \frac{1}{\pi},$$

and V_{t-1} is the variance operator at time t - 1 (see Patel and Read for moments of the half normal distribution which support these results). Thus, using (2)-(4) and approximating expected utility at time t - 1 with a mean-variance function obtains

$$EU_{ti} \equiv E_{t-1}[U_i(\pi_{ti})] \doteq p_{ti}(Q_{ti} - f_{ti}) - \alpha_{0i}\, Q_{ti} - \frac{1}{2}\alpha_{1i}\, Q_{ti}^2 + (p_t^f + \sigma^*)\ f_{ti}$$
$$- \frac{\phi_i}{2}[\sigma_i(Q_{ti} - f_{ti})^2 + (\sigma_i + p_{ti}^2)\ V_{t-1}\ (e_t) + c\tilde{\sigma}\ f_{ti}^2].$$

First-order conditions for expected utility maximization yield

$$Q_{ti} = \frac{p_{ti} - \alpha_{0i} + \phi_i\ \sigma_i\ f_{ti}}{\alpha_{1i} + \phi_i\ \sigma_i} = \frac{p_{ti} - \bar{\alpha}_{0i} + \phi_i\ \sigma_i f_{ti}}{\alpha_{1i} + \phi_i \sigma_i} + \varepsilon_{ti}^{\alpha} \tag{8}$$

$$f_{ti} = \frac{p_t^f - p_{ti} + \sigma^* + \phi_i\ \sigma_i\ Q_{ti}}{\phi_i\ (\sigma_i + c\tilde{\sigma})}, \tag{9}$$

where $\alpha_{0i} = \bar{\alpha}_{0i} + \tilde{\varepsilon}_{ti}^{\alpha}$, $\varepsilon_{ti}^{\alpha} = \tilde{\varepsilon}_{ti}^{\alpha}/(\alpha_{1i} + \phi_i\ \sigma_i)$, and $\tilde{\varepsilon}_{ti}^{\alpha}$ represents random variation in production costs from time to time which are anticipated at production planning time, $E(\tilde{\varepsilon}_{ti}^{\alpha}) = E(\varepsilon_{ti}^{\alpha}) = 0$. Second-order conditions for a maximum can be shown to hold if $\alpha_{1i} > 0$ and $\phi_i > 0$, i.e., if the production cost curve is upward bending and the decision-maker is risk averse.

The Store/Hedger.--Consider next the case of a storer of the commodity who also has the option of hedging against price declines during the period of storage. Suppose his cost of storage is quadratic and is given by $\beta_{0i}I_{t-1,i}$ + (1/2) $\beta_{1i}I_{t-1,i}$. The associated utility of income is

$$(10)\quad U_i(\pi_{ti}) = U_i[p_t(I_{t-1,i} - f_{ti} + \tilde{f}_{ti}) - p_{t-1}\, I_{t-1,i} - \beta_{0i}\, I_{t-1,i} - \frac{1}{2}\beta_{1i}\, I^2_{t-1,i} + p^f_t\, f_{ti} - \tilde{p}^f_t\, \tilde{f}_{ti}].$$

Considering this case as a two-stage decision problem as for the producer case, the storer decides at time t how much of his futures position to close out after observing $\tilde{p}^f_t$ and p_t and given initial decisions $I_{t-1,i}$ and f_{ti}. Representing profit as in (2) where

$$\overset{*}{\pi}_{ti} = p_t(I_{t-1,i} - f_{ti}) - p_{t-1}I_{t-1,i} - \beta_{0i}I_{t-1,i} - \frac{1}{2}\beta_{1i}\, I^2_{t-1,i} + p^f_t\, f_{ti}$$

and $\Delta\pi_{ti}$ is given by (4) makes this second-stage problem mathematically equivalent to the producer case so that close out decisions follow (5) and the mean and variance of $\Delta\pi_{ti}$ follow (6) and (7).

Substituting this decision function in (10) and approximating expected utility at time t - 1 with a mean-variance function obtains

$$EU_{ti} \equiv E_{t-1}[U_i(\pi_{ti})] \doteq p_{ti}(I_{t-1,i} - f_{ti}) - p_{t-1}\, I_{t-1,i} - \beta_{0i}\, I_{t-1,i} - \frac{1}{2}\beta_{1t}\, I^2_{t-1,i} + (p^f_t + \sigma^*)\, f_{ti} - \frac{\phi_i}{2}\left[\sigma_i(I_{t-1,i} - f_{ti})^2 + c\tilde{\sigma}\, f^2_{ti}\right].$$

First-order conditions for expected utility maximization yield

$$(11) \quad I_{t-1,i} = \frac{p_{ti} - p_{t-1} - \beta_{0i} + \phi_i \sigma_i f_{ti}}{\beta_{1i} + \phi_i \sigma_i} = \frac{p_{ti} - p_{t-1} - \bar{\beta}_{0i} + \phi_i \sigma_i f_{ti}}{\beta_{1i} + \phi_i \sigma_i} + \varepsilon_{ti}^{\beta}$$

$$(12) \quad f_{ti} = \frac{p_t^f - p_{ti} + \sigma^* + \phi_i \sigma_i I_{t-1,i}}{\phi_i(\sigma_i + c\tilde{\sigma})},$$

where $\beta_{0i} = \bar{\beta}_{0i} + \tilde{\varepsilon}_{ti}^{\beta}$, $\varepsilon_{ti}^{\beta} = \tilde{\varepsilon}_{ti}^{\beta}/(\beta_{1i} + \phi_i \sigma_i)$, and $\tilde{\varepsilon}_{ti}^{\beta}$ represents random changes in storage costs from time to time which are anticipated at the time of storage decisions, $E(\tilde{\varepsilon}_{ti}^{\beta}) = E(\varepsilon_{ti}^{\beta}) = 0$. Second-order conditions can be shown to hold if $\beta_{1i} > 0$ and $\phi_i > 0$, i.e., the storage cost curve is upward bending and the storer is risk averse.

The Exporter-Processor/Hedger.--A third distinctly different group of decision-makers is the one that forward contracts a delivery of commodity possibly in processed form and then uses the futures market to hedge against price increases before the commodity is actually purchased to prepare for contracted delivery. Suppose the cost of processing is quadratic and given by $\gamma_{0i} X_{t-1,i} + (1/2) \gamma_{1i} X_{t-1,i}$. Alternatively, these costs can represent an effect on revenue due to quadratic demand for the product or a loss rate incurred in handling. The utility of income is

$$U_i(\pi_{ti}) = U_i[p_t^c X_{t-1,i} - p_t(X_{t-1,i} + f_{ti} - \tilde{f}_{ti}) + p_t^f f_{ti} - \gamma_{0i} X_{t-1,i}$$

$$- \frac{1}{2} \gamma_{1i} X_{t-1,i}^2 - \tilde{p}_t^f \tilde{f}_{ti}]$$

(recall $f_{ti} < 0$ for purchases and $\tilde{f}_{ti} < 0$ for sales).

Considering a two-stage decision problem as in previous cases, the exporter-processor decides at time t how much of his futures position to close out after observing p_t^f and p_t given initial decisions $X_{t-1,i}$ and f_{ti}. Representing profit as in (2) where

$$\pi_{ti}^* = p_t^c X_{t-1,i} - p_t(X_{t-1,i} + f_{ti}) + p_t^f f_{ti} - \gamma_{0i} X_{t-1,i} - \frac{1}{2}\gamma_{1i} X_{t-1,i}^2$$

and $\Delta\pi_{ti}$ is given by (4), the second-stage problem becomes one of maximizing $\Delta\pi_{ti}$ subject to $f_{ti} \leq \tilde{f}_{ti} \leq 0$ assuming $f_{ti} \leq 0$; the solution is

$$\tilde{f}_{ti} = \begin{cases} f_{ti} & \text{if} \quad p_t^f > p_t \\ 0 & \text{if} \quad p_t^f \leq p_t. \end{cases} \tag{13}$$

Substituting (13) into (4) and using (1) obtains

$$E_{t-1}(\Delta\pi_{ti}) = -\sigma^* f_{ti} \tag{14}$$

$$V_{t-1}(\Delta\pi_{ti}) = c\tilde{\sigma}f_{ti}^2. \tag{15}$$

Thus, using (13)-(15) and approximating expected utility by a mean-variance function obtains

$$EU_{ti} \equiv E[U_i(]_{ti})\pi \doteq p_t^c X_{t-1,i} - p_{ti}(X_{t-1,i} + f_{ti}) + p_t^f f_{ti} - \gamma_{0i} X_{t-1,i}$$

$$- \frac{1}{2}\gamma_{1i} X_{t-1,i}^2 - \sigma^* f_{ti} - \frac{\phi_i}{2}[\sigma_i(X_{t-1,i} + f_{ti})^2 + c\tilde{\sigma}f_{ti}^2].$$

First-order conditions for expected utility maximization yield

$$X_{t-1,i} = \frac{p_t^c - p_{ti} - \gamma_{0i} - \phi_i\sigma_i f_{ti}}{\gamma_{1i} + \phi_i\sigma_i} = \frac{p_t^c - p_{ti} - \bar{\gamma}_{0i} - \phi_i\,\sigma_i\,f_{ti}}{\gamma_{1i} + \phi_i\,\sigma_i} + \varepsilon_{ti}^{\gamma} \tag{16}$$

(17) $$f_{ti} = \frac{p_t^f - p_{ti} - \sigma^* - \phi_i \sigma_i X_{t-1,i}}{\phi_i(\sigma_i + c\tilde{\sigma})},$$

where $\gamma_{0i} = \overline{\gamma}_{0i} + \tilde{\varepsilon}^{\gamma}_{ti}$, $\varepsilon^{\gamma}_{ti} = \tilde{\varepsilon}^{\gamma}_{ti}/(\gamma_{1i} + \phi_i + \sigma_i)$, and $\tilde{\varepsilon}^{\gamma}_{ti}$ represents random changes in processing costs from time to time which are anticipated at the time processing decisions are made, $E(\tilde{\varepsilon}^{\gamma}_{ti}) = E(\varepsilon^{\gamma}_{ti}) = 0$. Second-order conditions for a maximum can be shown to hold if $\gamma_{1i} > 0$ and $\phi_i > 0$.

<u>The Speculator</u>.--Finally, the fourth component of involvement in the futures market comes strictly from speculation. The utility of income for speculator i is

$$U_i(\pi_{ti}) = U_i[(p_t^f - \tilde{p}_t^f)\, f_{ti}]$$

and assumes that the speculator has no involvement in the spot market. Thus, approximating expected utility by a mean-variance function yields

$$EU_{ti} = E[U_i(\pi_{ti})] \doteq (p_t^f - p_{ti})\, f_{ti} - \frac{\phi_i}{2}\sigma_i f_{ti}^2.$$

First-order conditions for expected utility maximization imply

(18) $$f_{ti} = \frac{1}{\phi_i \sigma_i}(p_t^f - p_{ti}),$$

and second-order conditions hold if $\phi_i > 0$.

Using the decision functions in (8), (9), (11), (12), and (16)-(18), the following section develops a market model by aggregating decisions over individuals. This is done assuming that basis risk is small compared to overall spot-market price risk. That is, if $\tilde{\sigma}$ is small compared to σ_i, then $\sigma_i + c\sigma = \tilde{\sigma}_i$ since c is not a large constant. With this approximation and

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Then, using (19)-(25), spot-market production supply is

(28). $$S_t = \sum_{i\in I_p} (Q_{ti} + e_{ti}) = \alpha_0 + \alpha_1 p_t^f + e_t,$$

spot-market inventory demand is

(29) $$I_{t-1} = \beta_0 + \beta_1(p_t^f - p_{t-1}) + \delta_t \qquad E(\delta_t) = 0,$$

spot-market demand at time t to fill forward contracted commitments at time t - 1 and supply for the forward contracting market at time t - 1 is

(30) $$X_{t-1} = \gamma_0 + \gamma_1(p_t^c - p_t^f) + \eta_t \qquad E(\eta_t) = 0,$$

and futures market excess supply (supply minus demand) is

(31) $$f_t = \sum_{i\in I} \frac{1}{\phi_i\sigma_i} (p_t^f - p_{ti}) + Z\,\sigma^* + Q_t + I_{t-1} - X_{t-1} + \nu_t,$$

where

(32) $$\alpha_0 = \sum_{i\in I_p} \frac{1}{\alpha_{1i}} (\sigma^* - \alpha_{0i}) \qquad \alpha_1 = \sum_{i\in I_p} \frac{1}{\alpha_{1i}}$$

(33) $$\beta_0 = \sum_{i\in I_i} \frac{1}{\beta_{1i}} (\sigma^* - \beta_{0i}) \qquad \beta_1 = \sum_{i\in I_i} \frac{1}{\beta_{1i}}$$

(34) $$\gamma_0 = \sum_{i\in I_c} \frac{1}{\gamma_{1i}} (\sigma^* - \gamma_{0i}) \qquad \gamma_1 = \sum_{i\in I_c} \frac{1}{\gamma_{1i}}$$

(35) $$e_t = \sum_{i\in I_p} (e_{ti} + \varepsilon_{ti}^{\alpha}) \qquad \delta_t = \sum_{i\in I_i} \varepsilon_{ti}^{\beta}$$

(36) $$\eta_t = \sum_{i\in I_c} \varepsilon_{ti}^{\gamma} \qquad Q_t = \alpha_0 + \alpha_1 p_t^f$$

$$(37) \qquad Z = \sum_{i\in I_p} \frac{1}{\phi_i\sigma_i} + \sum_{i\in I_i} \frac{1}{\phi_i\sigma_i} - \sum_{i\in I_c} \frac{1}{\phi_i\sigma_i} \qquad \nu_t = \sum_{i\in I} \nu_{ti}.$$

The market model is closed by market equilibrium conditions for the three markets:

$$(38) \qquad I_t + D_t + X_{t-1} = S_t + I_{t-1}$$

in the spot market,

$$(39) \qquad F_{t-1} = X_{t-1}$$

in the processed good/export (forward contracted) market, and

$$(40) \qquad f_t = 0$$

in the futures market. Relationships (26)-(40) can be used to solve for price dynamics in both the spot and futures markets as in the following section.

III. Dynamic Price Implications

This section considers the dynamic behavior of prices implied by the market model of the previous section. This is done by classifying decision-makers into four different groups depending on the kind of price expectations they hold. Since price expectations appear nowhere in equations (24)-(40) other than in (31) where they are summed over all decision-makers, this grouping can be done without regard to the grouping by the specified trading activities.

In particular, let

I_n = set of indexes representing decision-makers with naive price expectations, $p_{ti} = p_{t-1}$,

I_f = set of indexes representing decision-makers who use the futures market price for a price expectation, $p_{ti} = p_t^f$,[1]

I_a = set of indexes representing decision-makers who hold adaptive expectations, $p_{ti} = \Sigma_{k=0}^{\infty} (1 - \theta)\, \theta^k\, p_{t-k-1} \equiv p_t^a$,

I_r = set of indexes representing decision-makers who hold rational expectations, $p_{ti} = E_{t-1}\,(p_t)$,

I_x = set of indexes representing decision-makers with normal expectation, $p_{ti} = 0$,[2]

and

$I = I_n \cup I_f \cup I_a \cup I_r \cup I_x$; I_n, I_f, I_a, I_n, I_x, disjoint.

Furthermore, for notational convenience, define

$$\lambda_n = \sum_{i \in I_n} \frac{1}{\phi_i\, \sigma_i}, \qquad \lambda_f = \sum_{i \in I_f} \frac{1}{\phi_i\, \sigma_i},$$

$$\lambda_a = \sum_{i \in I_a} \frac{1}{\phi_i\, \sigma_i}, \qquad \lambda_r = \sum_{i \in I_r} \frac{1}{\phi_i\, \sigma_i},$$

$$\lambda_x = \sum_{i \in I_x} \frac{1}{\phi_i\, \sigma_i}, \qquad \lambda = \lambda_n + \lambda_f + \lambda_a + \lambda_r + \lambda_x.$$

In this context, one can regard λ_i/λ as a share of market behavior due to each expectation group, i = n, f, a, r, x. It is not a share of all decision-makers holding the respective type of expectation since each individual is weighted by the inverse of the product of risk aversion and mean-squared error of expectation. For example, the naive expectations group may be very large but contributes little to market behavior because of high risk aversion or a high mean-squared error associated with its particular form of expectation.

Before solving for price dynamics, matters are simplified by partially reducing the model. Since F_{t-1}, X_{t-1}, and p_t^c are not observable from standard data sources, they must be eliminated from the model for empirical purposes. To do this, use (27), (30), and (39) to find

$$p_t^c = \tilde{C} + \tilde{\gamma}\, p_t^f + \frac{1}{\gamma_1 + c}(v_{t-1} - \eta_t), \tag{41}$$

where

$$\tilde{C} = \frac{C - \gamma_0}{\gamma_1 + c} \qquad \tilde{\gamma} = \frac{\gamma_1}{\gamma_1 + c}.$$

Thus,

$$F_{t-1} = X_{t-1} = \tilde{C}_0 - \tilde{C}_1\, p_t^f + \tilde{\gamma}\, v_{t-1} + \tilde{\eta}_t, \tag{42}$$

where

$$\tilde{C}_0 = \frac{\gamma_1 C + \gamma_0 c}{\gamma_1 + c} \qquad \tilde{C}_1 = c\tilde{\gamma} \qquad \tilde{\eta}_t = \frac{c}{\gamma_1 + c}\eta_t.$$

This reparameterization is valid with full information since, after elimination of (27) and (30), the parameters C, c, γ_0, γ_1, and the disturbance η_t appear nowhere else. Thus, substituting (42) into (26), (31), and (38) and further using (31) in (40), the model is

$$D_t = A - ap_t - b\tilde{C}_0 + b\tilde{C}_1\, p_t^f + \tilde{U}_t \tag{43}$$

$$S_t = \alpha_0 + \alpha_1\, p_t^f + e_t \tag{44}$$

$$I_{t-1} = \beta_0 + \beta_1(p_t^f - p_{t-1}) + \delta_{t-1} \tag{45}$$

$$S_t + I_{t-1} - I_t - D_t = \tilde{C}_0 - \tilde{C}_1\, p_t^f + \tilde{v}_{t-1} \tag{46}$$

$$Q_t + I_t = \bar{C}_0 - (\lambda - \lambda_f)\, p_t^f + \lambda_n\, p_{t-1} + \lambda_a\, p_t^a + \lambda_r\, E_{t-1}(p_t) - \tilde{C}_1\, p_t^f + \tilde{v}_{t-1} - \nu_t \tag{47}$$

where

$$\bar{C}_0 = \tilde{C}_0 - Z\sigma^* \qquad \tilde{v}_{t-1} = \tilde{\gamma}\, v_{t-1} - \tilde{\eta}_t \qquad \tilde{u}_t = u_t - b\, \tilde{v}_{t-1}.$$

To examine price dynamics, use (29) and (35) in (47) to find

$$p_t^f = \tilde{\lambda}^{-1}\,[K_1 + (\lambda_n + \beta_1)\, p_{t-1} + \lambda_a\, p_t^a + \lambda_r\, E_{t-1}(p_t) + \tilde{v}_{t-1} - \delta_t] \tag{48}$$

where

$$\tilde{\lambda} = \lambda - \lambda_f + \alpha_1 + \beta_1 + \tilde{\gamma}\, c$$

$$K_1 = \gamma_1\, \tilde{C} - Z\, \sigma^* - \alpha_0 - \beta_0 + \gamma_0.$$

Then substitute (43)-(45) into (46) to obtain

$$\beta_1\, p_{t+1}^f - \tilde{\beta} p_t^f - (\beta_1 + a)\, p_t + \beta_1 p_{t-1} + K_2 + u_t - e_t + (1 - b)\, \tilde{v}_{t-1} + \delta_t - \delta_{t-1} = 0 \tag{49}$$

where

$$\tilde{\beta} = \alpha_1 + \beta_1 + (1 - b)\, c\tilde{\gamma}$$

$$K_2 = A + (1 - b)\,(\gamma_0 + \gamma_1\, \tilde{C}) - \alpha_0.$$

Thus, using (48) in (49) obtains

$$(50)\ (\beta_1\lambda_n - B_1)\, p_t + (B_2 - \lambda_n\tilde{\beta})\, p_{t-1} + \beta_1\lambda_a p^a_{t+1} - \tilde{\beta}\lambda_a p^a_t + \beta_1\lambda_r E_t(p_{t+1})$$

$$- \tilde{\beta}\lambda_r E_{t-1}(p_t) + K_3 + \tilde{\lambda}(u_t - e_t - \delta_{t-1}) + \beta_1\tilde{\gamma}v_t$$

$$+ [\tilde{\lambda}(1 - b) - \tilde{\beta}]\, \tilde{v}_{t-1} + (\tilde{\lambda} - \tilde{\beta})\, \delta_t = 0$$

where

$$B_1 = \tilde{\lambda}\, (\beta_1 + a) - \beta_1^2$$

$$B_2 = \beta_1\, (\tilde{\lambda} - \tilde{\beta})$$

$$K_3 = \tilde{\lambda}K_2 + K_1(\beta_1 - \tilde{\beta}).$$

To study the price dynamics in (50), the rational expectations must be expressed in terms of spot prices. To do this, suppose decision-makers who hold rational expectations formulate them as though all other decision-makers were also rational.[3] Thus, for the moment, consider $\lambda = \lambda_r$ so $\lambda_n = \lambda_f = \lambda_a = \lambda_x = 0$ and take expectations in (50) at t - 1 using $E_{t-1}[E_t(p_{t+1})] = E_{t-1}(p_{t+1})$ to obtain

$$(51)\qquad \beta_1\lambda E_{t-1}\,(p_{t+1}) - (B_1 + \lambda\tilde{\beta})\, E_{t-1}(p_t) + B_2\, E_{t-1}(p_{t-1})$$

$$+ K_3 + [\tilde{g}(1 - b) - \tilde{\beta}]\, \tilde{\gamma}E_{t-1}(v_{t-1}) - E_{t-1}(\delta_{t-1}) = 0.$$

Note that in equation (51),

$$E_{t-1}(p_{t-1}) = p_{t-1} \qquad E_{t-1}(v_{t-1}) = v_{t-1} \qquad E_{t-1}(\delta_{t-1}) = \delta_{t-1}.$$

That is, p_{t-1}, v_{t-1}, and δ_{t-1} are observed at time t - 1, e.g., $v_{t-1} = (\gamma_1 + c)$ $(p_t^c - C - \gamma\, p_t^f)$. Alternatively, advancing equation (50) j time periods and taking expectations at time t - 1 obtains

$$(52) \qquad \beta_1\lambda\, E_{t-1}(p_{t+j+1}) - (B_1 + \lambda\tilde{\beta})\, E_{t-1}(p_{t+j}) + B_2 E_{t-1}\,(p_{t+j-1}) = 0, \qquad j = 1, 2, \ldots,$$

where $K_3 = 0$ follows from the assumption that prices are expressed in deviations from the long-run mean price (i.e., substituting $\overline{p}$ for price expectations in (52) implies $\overline{K}_3 = 0$).

Following Turnovsky, equation (52) can be viewed as a second-order difference equation which has solution

$$E_{t-1}(p_{t+j-1}) = H_1 r_1^j + H_2 r_2^j\,, \qquad j = 1, 2, \ldots,$$

where r_1 and r_2 are roots of the quadratic equation

$$(53) \qquad \beta_1\lambda r^2 - (B_1 + \lambda\tilde{\beta})\, r + K_5 = 0.$$

After some manipulation, one can show

$$(54) \qquad (B_1 + \lambda\tilde{\beta})^2 - 4\beta_1\lambda K_5 > 0$$

assuming $b \leq 1$ and that the law of supply and demand operates in the sense that

$$\alpha_1 = \frac{\partial S_t}{\partial p_t^f} > 0, \qquad \beta_1 = \frac{\partial I_{t-1}}{\partial p_t^f} > 0,$$

$$\tilde{\lambda} = \frac{\partial f_t}{\partial p_t^f} > 0, \qquad a = -\frac{\partial D_t}{\partial p_t} > 0,$$

$$\gamma_1 = \frac{\partial X_{t-1}}{\partial p_t^c} > 0, \qquad c = -\frac{\partial F_{t-1}}{\partial p_t^c} > 0.$$

Thus, both roots of (53) are real. Furthermore,

$$\frac{B_1 + \lambda\tilde{\beta}}{2\beta_1\lambda} > 1.$$

Thus, one of the roots, say, r_2, must be larger than unity and correspond to diverging expectations. Ruling out this implausible possibility following the arguments of Turnovsky ($H_2 = 0$), the solution of (52) is

(55) $$E_{t-1}(p_{t+j-1}) = H_1 r^j$$

where

$$r = \frac{1}{2\beta_1\lambda}\{B_1 + \lambda\tilde{\beta} - [(B_1 + \lambda\tilde{\beta})^2 - 4\beta_1\lambda B_2]^{1/2}\} > 0.$$

The latter inequality follows from B_1, $B_2 > 0$ which holds under the conditions above.

Finally, to solve for rational expectations, use (55) to find

$$E_{t-1}(p_{t+1}) = r\, E_{t-1}(p_t);$$

then substitute into (51) and solve for $E_{t-1}(p_t)$ recalling that $K_3 = 0$,[4]

(56) $$E_{t-1}(p_t) = \beta^* B_2 p_{t-1} + \beta^*\tilde{\gamma}[\tilde{\lambda}(1 - b) - \tilde{\beta}]\, v_{t-1} - \beta^*\lambda\tilde{\delta}_{t-1}$$

where

$$\beta^* = (B_1 + \lambda\tilde{\beta} - \beta_1\lambda_r)^{-1}.$$

Thus, returning to (47), the equation can be transformed for estimable purposes using (36), (44), and (56),

$$S_t + I_t = \bar{C}_0 - (\lambda - \lambda_f - \tilde{C}_1)\, p_t^f + (\lambda_n + \lambda_r \beta^* B_2)\, p_{t-1} + \nu_t^* \tag{57}$$

where

$$\nu_t^* = \lambda_r \beta^* \tilde{\gamma}[\tilde{\lambda}(1 - b) - \tilde{\beta}]\, v_{t-1} - \lambda_r \beta^* \tilde{\lambda} \delta_{t-1} + \tilde{v}_{t-1} - \nu_t . \tag{58}$$

Alternatively, returning to (50) and again considering $\lambda_r < \lambda$, an estimable dynamic spot-price equation is obtained by substituting (56):

$$p_t = a_1\, p_{t-1} + a_2\, p_t^a + \tilde{\nu}_t \tag{59}$$

where

$$a_1 = B_3(\tilde{\beta}\lambda_n + \tilde{\beta}\lambda_r \beta^*\, B_2 - B_2) \tag{60}$$

$$a_2 = B_3 \lambda_a (\beta_1\, \theta - \tilde{\beta}) \tag{61}$$

$$B_3 = \{\beta_1[\lambda_n + \lambda_a(1 - \theta) + g_r\, \beta^*\, B_2] - B_1\}^{-1} \tag{62}$$

and

$$\begin{aligned} \tilde{\nu}_t = {} & B_3\{\beta_1 \tilde{\gamma}[1 + \lambda_r \beta^* \tilde{\lambda}(1 - b) - \lambda_r \beta^* \tilde{\beta}]\, v_t - \tilde{\gamma}[\tilde{\lambda}(1 - b) - \tilde{\beta}]\, [1 + \tilde{\beta}\lambda_r \beta^*]\, v_{t-1} \\ & - (\beta^* \tilde{\lambda} + \tilde{\beta} - \tilde{\lambda})\, \delta_t + (\tilde{\beta}\lambda_r \beta^* - 1)\, \tilde{\lambda}\delta_{t-1} - [\tilde{\lambda}(1 - b) - \tilde{\beta}]\, \tilde{\eta}_t + \tilde{\lambda}(v_t - e_t)\}. \end{aligned} \tag{63}$$

In addition to the estimable equation for spot-market price in (59), one can also consider an estimable futures market price equation by substituting (56) into (48),

(64) $$p_t^f = b_0 + b_1\, p_{t-1} + b_2\, p_t^a + \omega_t$$

where

(65) $$b_0 = \tilde{\lambda}^{-1}(\gamma_1 \tilde{C} - Z\sigma^* - \alpha_0 - \beta_0 + \gamma_0)$$

(66) $$b_1 = \tilde{\lambda}^{-1}(\lambda_n + \beta_1 + \lambda_r \beta^* B_2)$$

(67) $$b_2 = \tilde{\lambda}^{-1}\, \lambda_a$$

(68) $$\omega_t = \tilde{\lambda}^{-1}\{\tilde{\gamma}[1 + \lambda_r \beta^* \tilde{\lambda}(1 - b) - \lambda_r \beta^* \beta]\, v_{t-1} - \tilde{\eta}_t - \delta_t - \lambda_r \beta^* \tilde{\lambda} \delta_{t-1}\}.$$

With these results, one obtains several alternative five-equation systems that can be used for estimation. For example, one is given by (43), (44), (45), (46), and (51); another is given by (43), (44), (45), (59), and (64).

Footnotes

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[1]The case of futures market expectations is interesting since it is the one case in the micro decision equations of section I where producers, storers, and forward contractors do not hedge their spot-market behavior. Many producers, for example, are observed in reality not to hedge their spot-market decision. The micro model suggests that such behavior is optimal with futures market expectations and that spot-market decisions are based on futures prices even though the decision-makers do not hedge to transfer risk and avail themselves of the futures price.

[2]Here we assume without loss of generality that all prices are represented as deviations from the long-run average spot-market price.

[3]Rational expectations with knowledge of all other decision-maker expectation mechanisms could also be considered, but such an assumption seems unrealistic and greatly complicates the algebra.

[4]Note that one cannot simply set $H_1 = p_{t-1}$ as Turnovsky does since p_{t-1} is a realization and not an expectation; it contains some current random disturbances that may not be involved in future expectations.

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