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AN ALTERNATIVE APPROACH TO DECISIONS UNDER UNCERTAINTY: THE EMPIRICAL MOMENT GENERATING FUNCTION

by

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AN ALTERNATIVE APPROACH TO DECISIONS UNDER UNCERTAINTY:
THE EMPIRICAL MOMENT GENERATING FUNCTION

In their recent article, Collender and Zilberman extended the use of the expected utility (EU)-moment generating function (MGF) approach to decision making under uncertainty to the case of continuous choice variables. They demonstrated that the model has greater flexibility than the mean-variance approach since distributions other than the normal may be used with their exponential utility function. However, it is still necessary to specify a distribution, and it must have a closed-form MGF.

In the present paper, we extend the EU-MGF framework to a much broader class of optimization problems including multivariate choice problems when the relevant distributions differ or are unknown. This is accomplished by the use of the empirical moment generating function (EMGF). We require only that the MGF exist for random variables of interest; it need not be known.

Introduction

Hammond first suggested the use of the MGF for risk analysis because of its compatibility with the exponential utility function. The exponential utility function is a preferred alternative to the quadratic utility function based on its properties concerning risk aversion; the MGF approach permits nonnormal probability distributions to be included in the EU maximization problem. Yassour, Zilberman, and Rausser applied the EU-MGF framework to the discrete-choice problem of whether to adopt a new crop variety. They compared cases in which new varieties' yields followed normal and gamma distributions. Skewness of the gamma was shown to substantially affect the probability of adoption of a new variety compared with the case in which its yields were normally distributed.

Collender and Zilberman extended the model to permit choice among several continuous distributions. Under their approach, the random variables of interest must have known MGFs. They presented the cases of bivariate normal and bivariate gamma yield distributions for cotton and corn and examined the allocation of land to those two crops with yield uncertainty for each.

In this paper, we present an extension of the EU-MGF framework to the case of unknown distributions. We present both the statistical properties and applications of the EMGF, a nonparametric approach to decision making under uncertainty.

Because it is as compatible with the EU framework as a known MGF, the nonparametric approach overcomes three limitations of the parametric case:

1. Distributional Uncertainty. The choice of the appropriate distribution for returns cannot easily be addressed in the parametric case. The EMGF, since it is nonparametric, does not require that the correct family of distributions be selected; rather, it can be viewed as an estimator of the correct distribution which permits statistical tests of goodness-to-fit of various known distributions.

2. Analytical Tractability. The MGF for particular families of distributions may not exist in a convenient closed form; the EMGF is extremely simple. The difference is slight for the case of single random variables or the multivariate normal; in nearly all other cases, the EMGF is much more tractable. This is especially true when random variables of interest follow distributions from different families, such as one normal and one gamma, or when they are truncated by programs such as price supports or crop insurance.

3. Statistical Properties. The EMGF leads to several attractive statistical properties such as unbiased estimations of EU. In contrast, Pope

and Meyer have shown that unbiased estimates of EU do not follow from unbiased estimates of the parameters of the probability distribution.

The paper is organized as follows. In the following section, we review the EU-MGF approach to choice under uncertainty and introduce an alternative, nonparametric approach using the EMGF. We demonstrate that the nonparametric approach has a number of desirable properties including unbiasedness, consistency, minimum variance for its class of estimators, and a known asymptotic distribution from which confidence intervals can be derived. Next, empirical applications are presented. The first is the case analyzed earlier by Collender and Zilberman--allocation of land to cotton and corn in the Mississippi Delta. Results with the EMGF approach are consistent with earlier work. Second, we apply the nonparametric approach to a forward-contracting problem in which a participating farmer is guaranteed a price while yield risk is unchanged. This latter case is interesting because it involves distributions which are from different families. Previous models of choice under uncertainty have been able to yield only approximate solutions to this type of problem. The nonparametric approach produces an unbiased solution under the assumptions of the model.

The Expected Utility-Moment Generating Function Model of Optimal Land Allocation Among Crops Under Uncertainty

Consider a farmer who allocates L acres of land among N crops. Let l_i be the amount of land allocated to crop i; v_i, the variable cost per acre of the ith crop; and y_i, the revenue per acre of the ith crop. Assume that the revenues are randomly distributed with a joint distribution function, $F(y_1, y_2, \ldots, y_N)$, and that the farmer has a negative exponential utility function defined on profits,

(1) $$U(\pi) = -e^{-r\pi},$$

where $\pi = \Sigma_{i=1}^{N} l_i(y_i - v_i)$ is the farmer's profit.

The farmer maximizes EU subject to a land availability constraint. Let

(2) $$M_y(t_1, t_2, \ldots, t_N) = E \exp\left[\sum_{i=1}^{N} (t_i y_i)\right]$$

define the MGF of the random vector y $(y_1, y_2, \ldots, y_N)$ where E is the expectations operator. Using (1) and (2), the farmer's choice problem involves the selection of $l_1, l_2, \ldots, l_N$ to maximize

(3) $$-\exp\left[r \sum_{i=1}^{N} (l_i v_i)\right] \cdot M_y(-rl_1, -rl_2, \ldots, -rl_N)$$

subject to

(4a) $$\sum_{i=1}^{N} l_i \leq L$$

and

(4b) $$l_i \geq 0, \qquad \text{for all } i.$$

Let λ be the shadow price associated with the land constraint (4) and assume an interior solution. Using the Lagrange multiplier technique, the optimization problem has $n + 1$ first-order conditions. They are equations (4) and

(5) $$re^{r\Sigma_{i=1}^{N}[l_i v_i]} \cdot M(\cdot) \cdot \left(\frac{M_i}{M(\cdot)} - v_i\right) = \lambda$$

where M_i is the first derivative of the MGF, $M(\cdot)$, with respect to its ith element. Using (4) and (5) and assuming an interior solution, one can derive an alternative set of optimality conditions which excludes λ consisting of (4) and

$$\frac{M_i}{M} - v_i = \frac{M_1}{M} - v_1, \qquad \text{for } i = 2, N. \tag{6}$$

Collender and Zilberman assumed that returns follow a known probability density function with a closed-form MGF. They solved the first-order conditions in (6) using the method-of-moments estimates for the population parameters in the MGF. By this method, they developed explicit solutions for several analytical distributions for the case where the farmer can allocate land between two uses, only one of which yields random returns. They also presented solutions for the case of two uncertain prospects, with returns distributed either bivariate normal or bivariate gamma. While this approach is an improvement over mean-variance analysis, it requires that the MGFs be known and have a closed form which excludes a number of interesting cases.

The Empirical Moment Generating Function Solution and Its Properties

The Nonparametric Solution to the Expected Utiliy-Moment Generating Function Approach

The inflexibility of any particular analytical distribution and associated problems can be circumvented by using a nonparametric approach based on the EMGF. The sample observations are used to estimate the MGF rather than deriving the MGF from an analytical distribution. This approach permits greater flexibility in the type of problem which can be examined. For example, with the EMGF, it is possible to consider simultaneously a vector of random variables characterized by different marginal distributions.

An example of such a problem is the case of a multiple-crop enterprise with yields following different distributions; profits are random with a distribution depending on these random yields. The mean-variance approach requires that profits be normal; here, it does not matter.

The EMGF, $M_N(t)$, is defined by

$$M_N(t) = \hat{E}(e^{tX}) = N^{-1} \sum_{j=1}^{N} e^{tX_j} \tag{7}$$

for the univariate case or

$$M_N(t_1, t_2, \ldots, t_K) = \hat{E}\left(e^{\Sigma_{k=1}^{K} t_k X_{kj}}\right) = N^{-1} \sum_{j=1}^{N} e^{\Sigma_{k=1}^{K} t_k X_{kj}} \tag{8}$$

for the k-variate case. A sufficient condition for the existence of the MGF is that the range of the variable under question be limited. This is an intuitively acceptable condition in the case of economic variables, such as prices, yields, and costs, since all three are naturally bounded from below by zero. Presumably, they are bounded from above also. Furthermore, given that each of the components of profit is bounded, revenues or profits will be bounded as well.

Traditionally, the mean-variance approach involves sample estimates for the first two moments. Antle extends the approach to a third moment and links moments to inputs as well. Pope and Meyer suggest the empirical distribution function which completely characterizes the sample distribution. This has the advantage of freeing the approximation of EU from the choice of the appropriate number of moments to consider.

The EMGF also completely characterizes the sample. Unlike the historgram approach of the empirical distribution function, the EMGF easily produces unbiased estimators of all orders of moments and of EU as well. To see the comparative ease with which the EMGF fits into the decision problem at hand, we return to the first-order conditions in (6).

The necessary conditions using the EMGF are

$$(9)\qquad \frac{\sum_{j=1}^{N} x_{j1}\, e^{\Sigma_{k=1}^{K} t_k x_{jk}}}{\sum_{j=1}^{N} e^{\Sigma_{k=1}^{K} t_k x_{jk}}} - v_1 = \frac{\sum_{j=1}^{N} x_{jk}\, e^{\Sigma_{k=1}^{K} t_k x_{jk}}}{\sum_{j=1}^{N} e^{\Sigma_{k=1}^{K} t_k x_{jk}}} - v_k \qquad k = 2, \ldots, N.$$

This set of first-order conditions can be solved using a nonlinear optimization technique such as the one provided by the ZSCNT subroutine of IMSL Libraries.

Properties of the Empirical Moment Generating Function and It's Derivatives

Quandt and Ramsey used the EMGF to estimate the parameters of a mixture of two normal distributions. The statistical properties of the EMGF are derived from the following observation (Quandt and Ramsey, p. 723). Given a random sample, x_i, i = 1, 2, ..., N, of independent and indentically distributed random variables, the quantities e^{tx_i} are also independent and identically distributed for a particular t. The EMGF is simply the mean of these new random variables, $N^{-1}\,\Sigma_{i=1}^{N}\, e^{tx_i}$. Thus, the EMGF is a sample mean and is an unbiased estimator of the MGF,

$$E[e^{tx}].$$

Therefore, the EMGF solution to the EU maximization problem will be an unbiased estimator of the EU of the decision-maker, in contrast to the solution obtained with sample estimates of population parameters in some assumed analytical MGF. The Strong Law of Large Numbers guarantees consistency of the EMGF as well (Rohatgi, pp. 263-275).

In fact, it can be shown that the EMGF is a uniformly minimum variance unbiased estimator of the MGF. The proof of this property follows the discussion given by Zacks (pp. 149-155). An estimator, which is unbiased and is invariant to the order in which the sample is drawn, is a uniformly minimum variance unbiased estimator. That the EMGF is invariant to permutations of the sample order is straightforward by inspection; the sample mean of the e^{tx_i} is invariant to the ordering of the x_i.

It is not enough that the EMGF is a uniformly minimum variance unbiased estimator of the MGF since the land allocation rule relies critically on the derivatives of the MGF also. Therefore, it is necessary to show that the derivatives of the EMGF are desirable estimators of the derivatives of the MGF as well.

To show unbiasedness of the derivatives of the EMGF, recall that the EMGF is unbiased:

$$E\left[e^{tx_i} - M_x(t)\right] = 0. \tag{10}$$

Taking the partial derivative of each side of (10) with respect to t yields

$$\frac{\partial}{\partial t} E\left[e^{tx_i} - M_x(t)\right] = \frac{\partial}{\partial t}(0) = 0. \tag{11}$$

The expectation in (11) can be rewritten as

$$\frac{\partial}{\partial t} \int_x \left[e^{tx_i} - M_x(t)\right] f(x)\,dx = 0. \tag{12}$$

Since neither the limits of integration nor f(x) depends on t, the order of integration and differentiation can be interchanged. Thus, (13a) $$\int_x \frac{\partial}{\partial t}\left[e^{tx_i} - M_x(t)\right] f(x)\, dx = 0$$

or equivalently

(13b) $$E\left\{\frac{\partial}{\partial t}\left[e^{tx_i} - M_x(t)\right] f(x)\, dx\right\} = 0.$$

Taking derivatives and expectations yields

(14) $$E\left(x_i\, e^{tx_i}\right) = M_x'(t).$$

Thus, the first derivative is also unbiased and

(15) $$E\left[N^{-1} \sum_{i=1}^{N} x_i\, e^{tx_i}\right] = M_x'(t).$$

Since we have an unbiased estimator which is invariant to the ordering of the observations, our estimator of the derivatives of the MGF is also a uniformly minimum variance unbiased estimator. Also, note that this estimator is asymptotically normal by the Central Limit Theorem.

Consistency can be shown using the Strong Law of Large Numbers as long as a finite variance is assumed, i.e.,

(16) $$V(xe^{tx}) < \infty.$$

A finite variance of the estimator of the derivatives is assured by our assumption that the variables of interest have a bounded range. In fact, all moments exist. These results can be shown for all orders of derivatives which are continuous.

Finally, given that our estimator of the EMGF is normally distributed by the Central Limit Theorem, it can be shown that the statistic

$$\frac{\sqrt{N} \cdot [M_N(t) - M_x(t)]}{\sqrt{V[M_N(t)]}} \tag{17}$$

converges in distribution to a standard normal where $V[M_N(t)]$ is estimated by

$$\hat{V}[M_N(t)] = (N-1)^{-1} \left[\sum_{i=1}^{N} e^{2tx_i} - N^{-1} \left(\sum_{i=1}^{N} e^{tx_i} \right)^2 \right]. \tag{18}$$

A similar statistic can be used to test the derivatives as well.

These properties of the EMGF can be extended to the multivariate case. To do so requires that the assumption of finite range can be extended to each element of the random vector of interest.

Empirical Application of Land Allocation Rules Based on the Empirical Moment Generating Function

In the previous section, we presented the properties of the EMGF which correspond to the well-known properties of the sample mean as an estimator of the population mean. The EMGF approach was argued to be far more flexible than the parametric or mean-variance approaches. Meanwhile, it retains the desirable properties of the moment estimators in the parametric case. The EMGF is particularly useful when the MGF for the random vector of interest cannot be specified or when there is insufficient prior information to select a parametric family of distributions. It is reasonable to expect a parametric MGF, such as the normal, to perform better when that distribution can be maintained and the sample size is small.

In this section, we present empirical examples to illustrate the usefulness of the EMGF in cases for which particular distributions may be inappropriate. We first reexamine the corn and cotton land allocation problem described in Collender and Zilberman. Next, a forward contracting problem is used to illustrate the EMGF as applied to distributions of different families.

The Choice Between Cotton and Corn in the Mississippi Delta

Collender and Zilberman applied their version of the EU-MGF approach to the choice of allocating land between cotton and corn in the Mississippi Delta. They demonstrated that the assumption of normality could, under certain levels of risk aversion, lead to undervaluation of land and suboptimization of EU. In order to compare their results to the EMGF approach, we solved the same land allocation problem and tested for the statistical difference of the normal, gamma, and EMGFs at the optimal land allocation rule derived from the nonparametric approach by (17).

As table 1 shows, the land allocation decision is the same for both the nonparametric EMGF approach and the parametric approach under the gamma assumption. However, as risk aversion increases, the marginal land values and the certainty equivalents under the nonparametric and gamma assumptions diverge. These results are consistent with those of Collender and Zilberman who found that the normal and gamma solutions diverged as risk aversion increased.

The statistical tests of the MGFs and derivatives can be found in tables 2 and 3, respectively. The hypothesis that yields are gamma distributed can be rejected at every level of risk aversion. Thus, even though the gamma assumption yields the same land allocation decision in this instance, it cannot be used without reservation, especially in calculating marginal land values or certainty equivalents for the farming operation for farmers who are more than moderately risk averse.

TABLE 1

Comparison of Optimal Land in Cotton, Marginal Value of Land, and Certainty Equivalents Derived From Gamma, Normal, and Nonparametric Assumptions Given that Each Is True in Turn.

Risk aversion	Land in cotton			Marginal value of land			Certainty equivalent		
	EMGF[a]	Gamma	Normal	EMGF[a]	Gamma	Normal	EMGF[a]	Gamma	Normal
	acres			dollars					
9.21×10^{-7}	2,000	2,000	2,000.00	225.20	225.42	223.02	467,114.00	468,772.00	468,764.00
2.55×10^{-6}	2,000	2,000	2,000.00	198.70	201.04	185.69	429,787.00	439,252.00	440,776.00
5.81×10^{-6}	2,000	2,000	2,000.00	163.44	170.68	111.03	355,132.00	394,287.00	400,391.00
1.07×10^{-5}	2,000	2,000	1,589.49	133.64	145.92	48.53	253,669.00	348,396.00	360,895.00
1.72×10^{-5}	2,000	2,000	1,013.82	113.78	128.36	41.02	191,529.00	309,258.00	327,415.00
2.54×10^{-5}	2,000	2,000	710.32	100.95	116.32	31.63	153,340.00	278,508.00	300,466.00
3.52×10^{-5}	2,000	2,000	531.89	91.60	107.99	20.37	125,143.00	254,382.00	279,073.00
4.66×10^{-5}	2,000	2,000	418.47	85.39	102.10	7.22	101,238.00	235,246.00	262,054.00
5.96×10^{-5}	2,000	2,000	334.17[b]	82.19	97.84	0.00	79,848.20	220,333.00	248,397.00
7.43×10^{-5}	2,000	2,000	268.17[b]	80.89	94.68	0.00	64,076.50	208,977.00	237,319.00
9.06×10^{-5}	2,000	2,000	219.91[b]	80.46	92.28	0.00	52,544.70	200,390.00	228,229.00
1.09×10^{-4}	2,000	2,000	183.57[b]	80.33	90.42	0.00	43,861.50	193,841.00	220,688.00

[a]Empirical moment generating function.

[b]The total land farmed is less than 2,000 acres for higher magnitudes of risk aversion under the normal assumption.

Source: Computed.

TABLE 2

Test of Moment Generating Functions

Risk aversion	Standard deviation	EMGF[a]	Normal	Test statistic	Gamma	Test statistic
0.92×10^{-6}	0.17×10^{-1}	0.55	0.55	-0.51×10^{-1}	0.75	$-0.12 \times 10^{+2}$*
0.26×10^{-5}	0.16×10^{-1}	0.21	0.21	-0.32	0.49	$-0.18 \times 10^{+2}$*
0.58×10^{-5}	0.54×10^{-2}	0.35×10^{-1}	0.45×10^{-1}	$-0.17 \times 10^{+1}$	0.25	$-0.39 \times 10^{+2}$*
0.11×10^{-4}	0.89×10^{-3}	0.35×10^{-2}	0.11×10^{-1}	$-0.81 \times 10^{+1}$*	0.12	$-0.13 \times 10^{+3}$*
0.17×10^{-4}	0.84×10^{-4}	0.22×10^{-3}	0.89×10^{-2}	$-0.10 \times 10^{+3}$*	0.57×10^{-1}	$-0.67 \times 10^{+3}$*
0.25×10^{-4}	0.48×10^{-5}	0.88×10^{-5}	0.11	$-0.23 \times 10^{+5}$*	0.29×10^{-1}	$-0.60 \times 10^{+4}$*
0.35×10^{-4}	0.17×10^{-6}	0.23×10^{-6}	$0.12 \times 10^{+3}$	$-0.72 \times 10^{+9}$*	0.16×10^{-1}	$-0.92 \times 10^{+5}$*
0.47×10^{-4}	0.34×10^{-8}	0.39×10^{-8}	$0.11 \times 10^{+9}$	$-0.32 \times 10^{+17}$*	0.89×10^{-2}	$-0.26 \times 10^{+7}$*
0.60×10^{-4}	0.40×10^{-10}	0.43×10^{-10}	$0.11 \times 10^{+19}$	$-0.26 \times 10^{+29}$*	0.53×10^{-2}	$-0.13 \times 10^{+9}$*
0.74×10^{-4}	0.27×10^{-12}	0.28×10^{-12}	$0.20 \times 10^{+34}$	$0.10 \times 10^{+38}$*	0.33×10^{-2}	$-0.12 \times 10^{+11}$*
0.91×10^{-4}	0.11×10^{-14}	0.11×10^{-14}	$0.17 \times 10^{+39}$	$0.10 \times 10^{+38}$*	0.22×10^{-2}	$-0.21 \times 10^{+13}$*
0.11×10^{-3}	0.23×10^{-17}	0.23×10^{-17}	$0.17 \times 10^{+39}$	$0.10 \times 10^{+38}$*	0.15×10^{-2}	$-0.63 \times 10^{+15}$*

[a]Empirical moment generating function.

*Significant at the 5 percent level.

Source: Computed.

TABLE 3

Test of Derivatives of Normal and Gamma Moment Generating Functions

Risk aversion	Standard deviation	EMGF[a]	Normal	Test statistic	Gamma	Test statistic
0.92×10^{-6}	$0.33 \times 10^{+1}$	$0.17 \times 10^{+3}$	$0.17 \times 10^{+3}$	0.28	$0.11 \times 10^{+3}$	$0.19 \times 10^{+2}$*
0.26×10^{-5}	$0.20 \times 10^{+1}$	$0.59 \times 10^{+2}$	$0.58 \times 10^{+2}$	0.64	$0.59 \times 10^{+2}$	0.19*
0.58×10^{-5}	1.00	$0.90 \times 10^{+1}$	$0.90 \times 10^{+1}$	0.35×10^{-1}	$0.22 \times 10^{+2}$	$-0.13 \times 10^{+2}$*
0.11×10^{-4}	0.17	0.78	0.96	$-0.11 \times 10^{+1}$	$0.77 \times 10^{+1}$	$-0.41 \times 10^{+2}$*
0.17×10^{-4}	0.15×10^{-1}	0.44×10^{-1}	-0.54	$0.39 \times 10^{+2}$*	$0.27 \times 10^{+1}$	$-0.18 \times 10^{+3}$*
0.25×10^{-4}	0.83×10^{-3}	0.17×10^{-2}	$-0.27 \times 10^{+2}$	$0.32 \times 10^{+5}$*	$0.10 \times 10^{+1}$	$-0.12 \times 10^{+4}$*
0.35×10^{-4}	0.29×10^{-4}	0.42×10^{-4}	$-0.57 \times 10^{+5}$	$0.20 \times 10^{+10}$*	0.43	$-0.15 \times 10^{+5}$*
0.47×10^{-4}	0.59×10^{-6}	0.69×10^{-6}	$-0.81 \times 10^{+11}$	$0.14 \times 10^{+18}$*	0.19	$-0.33 \times 10^{+6}$*
0.60×10^{-4}	0.69×10^{-8}	0.73×10^{-8}	$-0.11 \times 10^{+22}$	$0.16 \times 10^{+30}$*	0.94×10^{-1}	$-0.14 \times 10^{+8}$*
0.74×10^{-4}	0.46×10^{-10}	0.47×10^{-10}	$-0.27 \times 10^{+37}$	$0.10 \times 10^{+38}$*	0.48×10^{-1}	$-0.10 \times 10^{+10}$*
0.91×10^{-4}	0.18×10^{-12}	0.18×10^{-12}	$0.10 \times 10^{+38}$	$0.10 \times 10^{+38}$*	0.26×10^{-1}	$-0.15 \times 10^{+12}$*
0.11×10^{-3}	0.40×10^{-15}	0.40×10^{-15}	$0.10 \times 10^{+38}$	$0.10 \times 10^{+38}$*	0.15×10^{-1}	$-0.37 \times 10^{+14}$*

[a]Empirical moment generating function.

*Significant at the 5 percent level.

Source: Computed.

Of even more interest in this case are the results with regard to the normality assumption. The normality assumption cannot be rejected for low levels of risk aversion. However, as risk aversion increases above 10^{-5}, the normal MGF becomes significantly different from the EMGF. The tests of the derivatives of the normal MGF are presented in table 3. These show that derivatives also cannot be rejected for low to moderate levels of risk aversion. As risk aversion increases and higher moments become more important to the decision-maker, normality can be rejected.

These results support Collender and Zilberman's conclusion that normality can be an unjustified and potentially damaging assumption for moderately to strongly risk averse decision-makers. Since absolute risk aversion should be decreasing as wealth increases (Arrow and Pratt), we conclude that, for levels of risk aversion likely to be associated with the wealth levels of American farmers, the assumption of normality may be adequate. However, for relatively poor farmers, such as those in many developing countries, the assumption of normality could lead to suboptimal decisions. In addition, we conclude that, while the gamma assumption of crop yield distribution appears to perform satisfactorily for slight to moderate risk aversion levels, it cannot be used without reservation. The fact that both the gamma and nonparametric approaches yield corner solutions apparently disguises the fact that the gamma distribution does not fit these data.[1]

Optimal Allocation of Land to Corn Under Forward Contracts for Nebraska Farmers

Another important use of the nonparametric approach is in choosing among options characterized by different distributions or by distributions from unstable families. To demonstrate this capability, we determined the optimal allocation of land to crops under forward contracts for a representative dryland corn farmer from Cuming County, Nebraska, based on historical yield and price distributions. We hypothesized a forward contract which offered the farmer the opportunity to remove price risk from the enrolled portion of his crop without affecting yield risk. Since revenue is the product of price and yield, we can assume a priori that the distribution of revenue when price risk is eliminated will be from a different family of distributions than the distribution of revenue when both price and yield risks are present.

The data used are a county-level time series of 58 annual observations on both price and yield of corn from the Nebraska Agricultural Statistics. Cuming County was chosen since it is one of the prime corn-growing counties in Nebraska. In order to get a stationary series, we removed the linear trend in corn yields. We tested the resulting series for randomness using runs tests and found that the hypothesis that the series was random could not be rejected at the 5 percent level. The price series was deflated by the Consumer Price Index to get constant prices in terms of 1982 dollars. We then took first differences to get a stationary process and added back a constant to restore the series to the 1982 price level. Since this process left us with two negative prices, we truncated at zero. Cost data came from the University of Nebraska Cooperative Extension Service farm budgets (Bitney) for 1979 and were converted to 1982 dollars. For the purposes of this study, we assume county-level data are close enough to farm-level data so that valid farm-level decisions can be based upon it.

The land allocation problem was solved for two contract prices--one at the 1983 support price for corn of $2.65 and the other at the target price of $2.86. For purposes of comparison, the problem was solved using both the conventional mean-variance approach and the EMGF approach. Results are presented in tables 4, 5, and 6. Table 4 gives the optimal amount of land to put under contract (of a total of 1,000 acres), the marginal land value, and the certainty equivalent of one year's production activity for each level of risk aversion. The results presented in table 4 show considerable differences between the optimal land allocation under each approach as well as differences in the certainty equivalents and implicit land values for both contract prices. It should be noted that the certainty equivalents and shadow land values presented in table 4 are based on the underlying assumption for each calculation being true. For example, the certainty equivalents under the mean-variance assumption are the certainty equivalents which would obtain if the distributions were normal or utility quadratic.

To verify that the differences presented in table 4 indicate a significant departure of the relevant revenue distributions from normality, the MGF and its derivatives were tested by (17) for each level of risk aversion at the optimal point derived under the nonparametric approach. Table 5 shows that the hypothesis that the actual MGF is the normal MGF cannot be rejected at the low to moderate levels of risk aversion for both contract prices. Furthermore, as can be seen in table 6, the values of at least one of the derivatives can be rejected at all levels of risk aversion. Thus, the test results show that the assumption that both relevant distributions were normal could be rejected at most levels of risk aversion for both MGF and its derivatives.

Finally, we performed the ex post test of the relative performance of the decisions obtained under the mean-variance and nonparametric approaches. To do this, we calculated the optimal land allocation for each method at each level of risk aversion. We then used observed yields to calculate the mean

TABLE 4

Land Under Contract, Implicit Land Values, and Certainty Equivalent Under the Mean-Variance and Empirical Moment Generating Function Approaches

Risk aversion	Mean-variance (normality)			EU-MGF[a] (nonparametric)		
	Land under contract	Land value	Certainty equivalent	Land under contract	Land value	Certainty equivalent
	acres	dollars		acres	dollars	
			Contract price: $2.65			
2.87×10^{-6}	453.75	64.51	71,270.1	1,000	72.00	74,784.5
3.21×10^{-6}	458.42	62.89	70,456.4	1,000	71.31	74,448.0
3.65×10^{-6}	463.09	60.82	69,420.4	1,000	70.44	74,018.0
4.23×10^{-6}	467.76	58.10	68,056.1	1,000	69.29	73,449.1
5.03×10^{-6}	472.43	54.35	66,177.8	1,000	67.69	72,661.6
6.20×10^{-6}	477.10	48.85	63,426.9	1,000	65.32	71,499.7
8.08×10^{-6}	481.78	40.02	59,009.9	1,000	61.47	69,615.2
1.16×10^{-5}	486.45	23.51	50,754.0	1,000	54.16	66,041.4
2.05×10^{-5}	396.24	0.00*	8,158.6[b]	1,000	35.79	56,847.5
			Contract price: $2.86			
2.87×10^{-6}	1,000.00	74.74	84,071.8	1,000	86.93	90,187.5
3.21×10^{-6}	1,000.00	72.49	82,947.8	1,000	86.14	89,794.7
3.65×10^{-6}	1,000.00	69.63	81,515.8	1,000	85.12	89,292.6
4.23×10^{-6}	993.15	65.96	79,629.9	1,000	83.77	88,628.5
5.03×10^{-6}	914.33	62.21	77,162.2	1,000	81.90	87,708.8
6.20×10^{-6}	835.50	56.70	73,821.2	1,000	79.13	86,351.5
8.08×10^{-6}	756.68	47.87	68,813.1	1,000	74.62	84,149.1
1.16×10^{-5}	677.85	31.35	59,963.3	1,000	66.09	79,972.5
2.05×10^{-5}	544.53	0.00*	25,532.3[b]	1,000	44.91	69,272.5

[a]Expected utility-moment generating function.

[b]At these levels of risk aversion, the decision-maker would not farm his entire acreage.

Source: Computed.

TABLE 5

Test of Normal Moment Generating Function

Risk aversion	Standard deviation	EMGF[a]	Normal	Test statistic
		Contract price = $2.65		
2.87×10^{-6}	0.0097	0.57	0.58	- 1.11
3.21×10^{-6}	0.010	0.53	0.54	- 1.25
3.65×10^{-6}	0.011	0.49	0.50	- 1.41
4.23×10^{-6}	0.011	0.44	0.45	- 1.64
5.03×10^{-6}	0.011	0.38	0.40	- 1.95
6.20×10^{-6}	0.011	0.30	0.33	- 2.40*
8.08×10^{-6}	0.011	0.21	0.25	- 3.16*
1.16×10^{-5}	0.0085	0.11	0.15	- 4.70*
2.05×10^{-5}	0.0037	0.03	0.06	-10.66*
		Contract price = $2.86		
2.87×10^{-6}	0.010	0.54	0.55	- 0.96
3.21×10^{-6}	0.010	0.51	0.52	- 1.07
3.65×10^{-6}	0.011	0.46	0.47	- 1.22
4.23×10^{-6}	0.011	0.41	0.42	- 1.41
5.03×10^{-6}	0.011	0.35	0.37	- 1.67
6.20×10^{-6}	0.011	0.27	0.30	- 2.05*
8.08×10^{-6}	0.010	0.19	0.22	- 2.69*
1.16×10^{-5}	0.0078	0.10	0.13	- 3.96*
2.05×10^{-5}	0.0032	0.02	0.05	- 8.67*

[a]Empirical moment generating function.

*Significant at the 5 percent level.

Source: Computed.

TABLE 6

Test of Derivatives of Normal Moment Generating Function

Risk aversion	Derivative wrt t_1: Standard deviation	Derivative wrt t_1: EMGF[a]	Derivative wrt t_1: Normal	Derivative wrt t_1: Test statistic	Derivative wrt t_2: Standard deviation	Derivative wrt t_2: EMGF[a]	Derivative wrt t_2: Normal	Derivative wrt t_2: Test statistic
				Contract price = $2.65				
2.87×10^{-6}	145.66	110.49	105.02	3.42*	1,698.13	111.66	116.31	- 0.85
3.21×10^{-6}	98.40	102.96	97.45	4.19*	1,478.86	104.14	109.28	- 1.01
3.65×10^{-6}	57.17	94.13	88.68	5.44*	1,249.54	95.32	101.06	- 1.22
4.23×10^{-6}	25.68	83.69	78.43	7.83*	1,013.73	84.88	91.33	- 1.53
5.03×10^{-6}	8.17	71.24	66.41	12.75*	777.31	72.43	79.73	- 1.98*
6.20×10^{-6}	7.69	56.39	52.35	11.00*	548.67	57.55	65.84	- 2.67*
8.08×10^{-6}	20.80	38.94	36.22	4.51*	337.12	40.02	49.32	- 3.83*
1.16×10^{-5}	28.26	19.84	18.97	1.23	150.91	20.69	30.56	- 6.07*
2.05×10^{-5}	8.15	3.99	4.32	- 0.88	20.32	4.35	13.00	-14.49*
				Contract Price = $2.86				
2.87×10^{-6}	131.48	113.83	109.11	3.11*	1,549.60	106.65	111.14	- 0.86
3.21×10^{-6}	84.26	105.50	100.80	3.87*	1,340.12	98.93	103.86	- 1.02
3.65×10^{-6}	45.21	95.79	91.20	5.15*	1,123.64	89.95	95.37	- 1.22
4.23×10^{-6}	18.16	84.40	80.07	7.68*	903.98	79.40	85.41	- 1.51
5.03×10^{-6}	6.89	70.98	67.12	11.10*	686.88	66.95	73.63	- 1.92
6.20×10^{-6}	12.66	55.21	52.15	6.49*	479.62	52.30	59.68	- 2.55*
8.08×10^{-6}	28.28	37.10	35.28	2.59*	289.32	35.41	43.40	- 3.54*
1.16×10^{-5}	31.16	18.02	17.79	- 0.31	123.12	17.49	25.44	- 5.41*
2.05×10^{-5}	6.96	3.27	3.86	- 1.69	13.98	3.33	9.41	-12.27*

[a]Empirical moment generating function.

*Significant at the 5 percent level.

Source: Computed.

and variance of utility given each decision rule for the 58 years of data. Since EU is the mean of the distribution of utility, our estimator of it, $\overline{U}$, is normally distributed by the Central Limit Theorem with variance equal to the variance of utility divided by the number of observations. This suggests the use of a one-tailed test that the EU of the land allocation from the mean-variance approach is less than the EU of the land allocation from the EMGF approach. The test statistic is

(19)
$$z = \frac{(\overline{U}_{EV} - \overline{U}_{EMGF})}{\sqrt{\frac{S^2_{U_{EV}}}{N} + \frac{S^2_{U_{EMGF}}}{N}}} .$$

u?

This is the method which Collender and Zilberman used to test the difference between utility of the normal and gamma decision rules.

The optimal allocation of land to the contract alternative, the associated (empirical) certainty equivalent, and the test statistic for each level or risk aversion are presented in table 7. While the nonparametric approach always yields utility and certainty equivalents which are at least as great as those obtained via the mean-variance approach, the test statistic never reaches statistical significance even though the land allocation rules differ markedly. Thus, although our expectation that the EMGF approach would result in higher utility than the mean-variance approach was borne out, the difference in utility is not great enough to outweigh the uncertainty due to lack of knowledge concerning the true parameters of the revenue function.

This lack of statistical significance may be related to the fact that we are using aggregate yield data. Capstick and Cochran have found that state- or even county-level time series are inadequate for risk-efficient decision

TABLE 7

Ex Post Test of Performance of Mean-Variance and Empirical Moment Generating Function Decision Criteria

Risk aversion	Land under contract: EMGF[a]	Land under contract: Normal	Certainty equivalent: EMGF[a]	Certainty equivalent: Normal	z, test statistic
	acres		dollars		
	Contract price: $2.65				
2.87×10^{-6}	1,000	453.75	74,784.58	73,678.02	0.12
3.21×10^{-6}	1,000	458.42	74,448.00	73,191.70	0.13
3.65×10^{-6}	1,000	463.09	74,017.91	72,574.23	0.15
4.23×10^{-6}	1,000	467.76	73,449.09	71,764.16	0.18
5.03×10^{-6}	1,000	472.43	72,661.55	70,653.80	0.21
6.02×10^{-6}	1,000	477.10	71,499.71	69,037.79	0.26
8.08×10^{-6}	1,000	481.78	69,615.19	66,467.94	0.33
1.16×10^{-6}	1,000	486.45	66,041.37	61,749.47	0.43
2.05×10^{-5}	1,000	396.24	56,847.55	44,705.63	1.30
	Contract price: $2.86				
2.87×10^{-6}	1,000	1,000.00	90,187.39	90,187.39	0.00
3.21×10^{-6}	1,000	1,000.00	89,794.56	89,794.56	0.00
3.65×10^{-6}	1,000	1,000.00	89,292.61	89,292.61	0.00
4.23×10^{-6}	1,000	993.15	88,628.47	88,544.20	0.01
5.03×10^{-6}	1,000	914.33	87,708.79	86,615.40	0.12
6.02×10^{-6}	1,000	835.50	86,351.43	84,149.09	0.24
8.08×10^{-6}	1,000	756.67	84,149.04	80,667.68	0.37
1.16×10^{-6}	1,000	677.85	79,972.45	74,892.04	0.51
2.05×10^{-5}	1,000	544.53	69,272.46	57,908.28	1.11

[a]Empirical moment generating function.

Source: Computed.

making. In particular, they have found that aggregation of cotton yield data at the county level for three Arkansas counties tends to average out individual differences. The usefulness of aggregate county-level data for farm decision making was also brought into question by Eisgruber and Schuman who analyzed the relationships between county- and farm-level data for several crops in central Indiana. One would expect such data to tend toward normality because of the Central Limit Theorem.

Another factor which affects the significance of the ex post test is the correlation between the two revenue choices. The correlation coefficient for the two revenue series is 0.67. Thus, diversification between these two choices will not remove the greater part of the risk and has relatively little impact on EU.

In order to test the effects of the Central Limit Theorem on our results, we solved the same decision problem using the microlevel corn data from the Mississippi Delta. We found that normality can be rejected for the MGF for both contract prices at all but the lowest levels of risk aversion. Also, normality can be rejected for at least one of the first-order derivatives of the MGF at all but one level of risk aversion. However, the ex post test of performance shows that the EMGF does not significantly outperform the mean-variance approach except at the most extreme level of risk aversion despite wide differences in the land allocation rules. Again, it is possible that yield risk dominates price risk, leaving the two revenue distributions highly correlated.

We have shown in this section that the method we propose is a significant advance for solving EU maximization problems when the distributions from which the decision-maker must choose are unknown or from different families. For some cases, it does not lead to statistically significant improvements in the ex post certainty equivalent of the corn farming in Nebraska. However, decision rules differ with the two approaches, suggesting that the EMGF should at least be used to test assumptions such as normality.

Summary and Conclusion

In this paper we have extended the application of the EU-MGF approach to decision making under uncertainty to the case where the nature of the relevant distributions is unknown or makes parametric computations impossible. To do this, we have suggested the use of the EMGF and demonstrated that both the EMGF and its derivatives are uniformly minimum variance unbiased estimators. In addition, we have proposed a test statistic to verify that a given empirical distribution fits a particular parametric form.

We have demonstrated the use of the EMGF using the case of the allocation of land between corn and cotton in the Mississippi Delta. Our tests of the actual MGF and its derivatives support Collender and Zilberman's conclusion that normality can be an unjustified and potentially damaging assumption. In addition, we conclude that, while the gamma assumption for crop yield distributions appears to perform satisfactorily for slight to moderate risk aversion levels, it cannot be used without reservation. Our tests show that the gamma distribution, as used by Collender and Zilberman, does not fit these data well.

We have demonstrated a method for allocating land between production under forward contract and unhedged production for corn farmers of Cuming County, Nebraska. While tests of the MGF and its derivatives for this case again allow us to reject normality in all instances, an ex post test of the performance of the EMGF decision against the mean-variance decision was unable to reject the hypothesis that the two decision rules yield the same EU except for the extreme levels of risk aversion.

Further applications of the EMGF technique will be of interest. Our results suggest that the technique will be useful for low-income producers, while the departures from normality do not seem to indicate significant differences in EU for wealthier farmers. The fact that optimal decisions can vary substantially, however, suggests that it will be useful in either case to test the normality assumption using the EMGF approach.

Footnotes

[1]It should be recalled that Day recommended the Pearson Type I or beta distribution. Others have suggested the use of the gamma distribution. Furthermore, the method of moments, on which the approximation of the parameters is based, is known to have less than optimal properties (Day).

References

Antle, John M. "Testing the Stochastic Structure of Production: A Flexible Moment-Based Approach." J. Bus. and Econ. Stat. 1(1983):192-201.

Arrow, Kenneth J. "The Theory of Risk Aversion." Essays in the Theory of Risk Bearing. Chicago: Markham Publishing Company, 1971.

Bitney, Larry L., ed. Estimated Crop and Livestock Production Costs. Dep. Agr. Econ., Coop. Ext. Serv., Institute of Agriculture and Natural Resources, University of Nebraska, 1978.

Capstick, Daniel F., and Mark Cochran. "Analysis of Crop Yield Distributions for Risk-Efficient Decision Criteria." Dep. Agr. Econ. and Rural Soc., University of Arkansas, 1983.

Collender, Robert N., and David Zilberman. "Optimal Land Allocation Under Uncertainty: A Generalized Portfolio Approach," Dep. Agr. and Res. Econ., Working Paper No. 292, University of California (Berkeley, 1984).

Day, Richard H. "Probability Distributions of Field Crops." J. Farm Econ. 47(1965):713-41.

Eisgruber, L. M., and L. S. Schuman. "The Usefulness of Aggregated Data in the Analysis of Farm Income Variability and Resource Allocation." J. Farm Econ. 45(1963):587-91.

Hammond, John S. "Simplification of Choice Under Uncertainty." Manage. Sci. 20(1974):1047-72.

IMSL Libraries, Inc., Ninth Edition, Houston, Texas, 1982.

Pope, Rulon, and Jack Meyer. "Minimium Variance Unbiased Estimation of Probability Distribution Functions." Paper presented at the American Agricultural Economics Association Annual Meetings, West Lafayette, July 31-August 3, 1983.

Pratt, J. W. "Risk Aversion in the Small and in the Large." Econometrica 32(1964):122-36.

Quandt, Richard E., and James B. Ramsey. "Estimating Mixtures of Normal Distributions and Switching Regressions." J. Amer. Statist. Assoc. 73(1978):730-41.

Rohatgi, V. K. An Introduction to Probability Theory and Mathematical Statistics. New York: John Wiley and Sons, Inc., 1976.

Yassour, Joseph, David Zilberman, and Gordon C. Rausser. "Optimal Choices Among Alternative Technologies with Stochastic Yield." Amer. J. Agr. Econ. 63(1981):718-24.

Zacks, S. The Theory of Statistical Inference. New York: John Wiley and Sons, Inc., 1971.