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UNCERTAINTY, INSTABILITY, AND THE COMPETITIVE FIRM

by

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UNCERTAINTY, INSTABILITY, AND THE COMPETITIVE FIRM[1]

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Competitive behavior under price uncertainty is compared to competitive behavior under price instability to show that risk-averse firms do not always reduce output under uncertainty.

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1. Introduction

For the competitive firm, price instability represents uncertainty if prices are unknown to the firm at the time decisions are made. However, if prices are known prior to the firm's decisions (e.g., through information-gathering activities), the firm faces certainty in the presence of price instability. Oi (1961) and, later, Tisdell (1963) initiated the investigation of firm behavior under price instability and certainty and focused on normative aspects. Tisdell (1978), Hanoch (1974), and Newbery and Stiglitz (1979) extended the analysis by distinguishing between price instability and uncertainty and investigated the welfare impacts of firm behavior under price uncertainty.

Paralleling the above work, the theory of the firm under price uncertainty has been studied by Sandmo (1971), Leland (1972), and McCall (1971). They concluded that, when prices are uncertain, the output of a risk-averse firm is smaller than that of a risk-neutral firm (which operates as if prices were fixed at their average level). Viewed differently, these models compared the behavior of a risk-averse firm under uncertainty with its behavior when prices are stabilized at their average level.

To complement those studies, this paper compares the behavior of the firm under uncertainty with its behavior when prices are known but unstable. In this framework there is a substantial gain from information since the removal of uncertainty allows firms to take advantage of price instability. This additional information enables the firm to adjust output to accommodate changes in prices while, in the Sandmo-Leland (price uncertainty) case, the firm produces the same output each period (the quantity that corresponds to its certainty equivalent price).

2. The competitive behavior of the firm under price uncertainty

As in Sandmo (1971), the price-taking firm maximizes its expected utility of profit,

$$Eu(\pi) = Eu[p \cdot y - c(y)], \tag{1}$$

where $u(\pi)$ is a utility function of profits, p is price of output assumed to be a random variable with a density function $f(p)$ and expected value $\overline{p}$, y is output produced and instantaneously sold, and $c(y)$ is the variable cost function.

The first- and second-order conditions for a maximum are obtained by differentating equation (1) with respect to y:

$$E\left\{u'(\pi) \cdot [p - c'(y)]\right\} = 0 \tag{2}$$

$$E\left\{u''(\pi) \cdot [p - c'(y)]^2 - u'(\pi) \cdot c''(y)\right\} < 0. \tag{3}$$

Condition (3) suggests that risk aversion and increasing marginal cost are sufficient conditions for optimality.

The impact of risk aversion on the firm's optimal output in a world of uncertainty is analyzed by comparing conditions (2) and (3) with the conditions obtained from the "certainty equivalent" case in which the price is known to equal $\overline{p}$. This analysis is also relevant for comparing the behavior of the firm under price uncertainty with its behavior under price stability. However, a valid comparison of the firm's decisions under price uncertainty with its actions under price certainty should not assume away price instability since the removal of uncertainty for one firm (e.g., by information-gathering activities) does not change the fluctuation of prices for a competitive industry.[2]

3. Price instability versus price uncertainty

Oi (1961) and Tisdell (1978) consider firms that operate under price instability with full certainty. In their models optimal output is decided upon once prices are known. Thus, given the price density function f(p), the firm's optimal welfare is denoted by

$$E \max_{y} u[p \cdot y - c(y)]. \tag{4}$$

The conditions for an optimum are simply

$$p - c'(y) = 0 \qquad \text{for all } p \sim f(p) \tag{5}$$

and

$$c''(y) > 0 \qquad \text{for all } p = c'(y). \tag{6}$$

It is apparent that availability of information will enable the firm to adjust its output to changes in prices using the traditional marginal pricing rules.

Some interesting comparisons of the behavior under price uncertainty versus that under price instability with full information can be made by comparing the Sandmo-Leland model with that developed by Oi. Under price instability, the firm's optimal output will be variable whereas, under price uncertainty, output will be constant. Thus, if on wants to study the output level in the two cases, one should compare the output level under uncertainty with the expected level of output under certainty as done in the following analysis.

Under uncertainty, the optimal output level, y_u, is determined by

$$Eu'(\pi) \cdot [p - c'(y_u)] = 0. \tag{7}$$

Since y_u is invariant, equation (7) becomes

$$Eu'(\pi) \cdot p = c'(y_u)\ Eu'(\pi). \tag{8}$$

However,

$$Eu'(\pi) \cdot p = \text{cov}\ [p, u'(\pi)] + \overline{p} \cdot Eu'(\pi); \tag{9}$$

hence,

$$c'(y_u) - \overline{p} = \frac{\text{cov}\ [p, u'(\pi)]}{Eu'(\pi)}. \tag{10}$$

Since $u'(\pi) > 0$ for all levels of π, $Eu'(\pi) > 0$. Moreover, since $\partial\pi/\partial p > 0$,

$$\text{cov}\ [p, u'(\pi)] \begin{Bmatrix} \geq \\ = \\ < \end{Bmatrix} 0 \quad \text{if } u''(\pi) \begin{Bmatrix} \geq \\ = \\ < \end{Bmatrix} 0; \tag{11}$$

therefore,

$$c'(y_u) \begin{Bmatrix} \geq \\ = \\ < \end{Bmatrix} \overline{p} \quad \text{if } u''(\pi) \begin{Bmatrix} \geq \\ = \\ < \end{Bmatrix} 0. \tag{12}$$

It appears that, for a risk-averse firm $[u''(\pi) < 0]$ whose marginal cost function is increasing, the level of output under uncertainty will be smaller than the optimal output level chosen by the firm when the output price equals its expected value, $\overline{p}$. This result was developed by Sandmo (1971) and Leland (1972). However, if one compares y_u not to the certainty equivalent output but to the expected value of output under instability with certainty, the answer is more ambiguous.

Under price certainty, the firm produces y such that, for all p,

$$p = c'(y) \tag{13}$$

and

$$E(p) = Ec'(y). \tag{14}$$

However, following Jensen's inequality,

$$Ec'(y) \begin{Bmatrix} \geq \\ = \\ < \end{Bmatrix} c'(\bar{y}) \quad \text{if } c''' \begin{Bmatrix} \geq \\ = \\ < \end{Bmatrix} 0; \tag{15}$$

therefore,

$$c'(\bar{y}) \begin{Bmatrix} \geq \\ = \\ < \end{Bmatrix} \bar{p} \quad \text{if } c''' \begin{Bmatrix} \leq \\ = \\ > \end{Bmatrix} 0. \tag{16}$$

When prices are unstable but certain, the firm adjusts its optimal output to the known price. This adjustment depends on the structure of the cost function; the curvature of the marginal cost function determines how the average output under price instability compares with the output when prices are stabilized. Condition (16) indicates that the average output under instability is smaller (larger) than output under stability when the marginal cost curve is convex (concave).

To compare the average output $(\bar{y})$ under price instability and full certainty with the output under uncertainty (y_u), combine equations (12) and (16). The relationship depends on the risk preference of the firm and the curvature of its marginal cost curve. Table 1 summarizes the outcomes for all possible situations. The results in this table indicate that, given price instability, risk-neutral firms with increasing convex marginal cost curves produce, on the average, more output under price uncertainty than under price certainty. Moreover, risk aversion does not always imply that output under uncertainty is lower than the average output under certainty, and risk loving does not imply that the output under uncertainty is higher. Risk aversion results in lower output under uncertainty if the marginal cost function is not convex; however, <u>assuming an increasing and convex marginal cost function, output under uncertainty might be higher than the average output under</u>

TABLE 1

Comparison of Output Under Uncertainty with Average Output Under Certainty and Instability

Marginal cost curve	Risk loving $u'' > 0$	Risk neutral $u'' = 0$	Risk averse $u'' < 0$
Convex, $c''' > 0$	$\bar{y}_u > y$	$\bar{y}_u > y$	a/
Linear, $c''' = 0$	$\bar{y}_u > y$	$\bar{y}_u = y$	$\bar{y}_u < y$
Concave, $c''' < 0$	a/	$\bar{y}_u < y$	$\bar{y}_u < y$

a/ Cannot be determined *a priori*.

certainty. This is the case when the degree of risk aversion of the firm is small enough with respect to the curvature of the marginal cost function.

4. Conclusions

This paper has focused on the output behavior of a perfectly competitive firm in an environment where price instability exists and the firm uses investment in information to move from a world of uncertainty to a situation of certainty with price instability. The added information will change the production policy of the firm. It will change its output at each period to counteract changes in prices. However, it is shown that the average output under price certainty is not necessarily higher than under price uncertainty; and the relationship between the two depends on the risk preference of the firm and the curvature of its marginal cost function.

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Footnotes

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[2] This analysis is, moreover, consistent with the reservation that firms facing uncertain prices have costs and production structures different from those of firms acting in stable environments. Thus, the comparison between the behavior under price uncertainty and the behavior under price stability is incomplete because it does not consider the long-run changes.