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ON CONSUMPTION INDIVISIBILITIES, THE DEMAND FOR DURABLES, AND INCOME DISTRIBUTION

by

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I. Introduction

The limited range of issues addressed by traditional theory motivated Becker and Lancaster to introduce new approaches to consumer theory. The new approaches are capable of analyzing issues such as quality changes among goods and consumers' reaction to new goods. These approaches depart from traditional theory by rejecting the assumption that consumers derive utility from goods and services purchased in the market per se. Instead, they assume that utility is obtained from entities which are produced by the family itself with purchased market goods, services, and the time of some members of the family.

Becker and Lancaster use different specifications of the family-production technologies. Lancaster's model assumes multiproduct-linear technologies (each activity generates several characteristics keeping fixed input output relationship). In Becker's work each activity produces one output following a neoclassical production function.

This paper uses the family production-function approach to explain the differences and interdependencies between the demand for durable and nondurable goods. As in Becker, each production activity is assumed to generate one commodity. However, the family-production technology is assumed to have putty-clay properties. Namely, each commodity can be produced via several processes. Each process has its own fixed proportion between variable inputs and the commodity and may require the use of a specific capital good. In the analysis durables play the role of capital goods while nondurable goods are the variable inputs. The choice of durable goods determines the fixed nondurables-output ratios while the actual amount of the commodity consumed is determined by the amounts of nondurables used. Unlike the traditional putty-clay model, however, there is a relatively small number of durables for producing each commodity, and the amount of commodities produced by each durable is assumed to be unconstrained (or, alternatively, the productive capacities of the durable goods are above the range of practical levels of consumption). Thus, the selection of durable goods are the results of discrete choices.

Home appliances--such as washers, dryers, stoves, dishwashers, furnances, etc.--are examples of durables which suggest the model developed here.[1] Washing machines and dryers, for example, are usually utilized by households only for relatively small periods of time, and their variable costs are approximately constant per load of clothes washed, dried, etc. In most cases a family has to make a discrete choice whether to purchase a washer and dryer to clean its clothes incurring a fixed cost and relatively low variable cost, or to use a laundromat not making any investment but paying higher variable cost (in terms of time and operation cost).

Using this model, the first part of the paper analyzes the individual consumer's demand functions for durables, commodities, and goods as functions of prices and income.[2] The model is developed for the simple case of two commodities--one a composite commodity and the other a specific commodity (such as clean clothes). Two processes can be used to produce the specific commodity; one requires a purchase of a durable and the other does not require such purchase. This simplified model allows graphical presentations and proofs using some of the traditional tools of graphical analysis of consumer behavior (income consumption curves, price consumption curves, etc.) and thus is useful for instructional purposes.

The second part of the paper presents a tractable approach for generating durable demand relationships given income distributions and utility function specifications. The results suggest that several income distribution parameters, rather than simply average income level, are essential in deriving demand for durables.

II. The Model

A consumer derives his utility from consuming y_0 units of a numeraire commodity and y_1 units of a specific commodity each period. The consumer's utility function is traditionally increasing in both commodities, concave and twice differentiable, and it is denoted by $U(y_0, y_1)$.[3] Commodity 1 can be produced using $K + 1$ alternative processes denoted by $k = 0, 1 . . ., K$. The consumer has to purchase a specific durable in order to use each of the processes with $k > 0$. The zero process does not require purchase of a durable. (It may use an already owned durable.) Let δ_k be a dichotomous variable taking the value one when the k^{th} durable is used to produce commodity 1 and zero otherwise. Assuming that a family is employing only one of the processes to produce the specific commodity, the dichotomous variable, δ_k, is constrained by

$$(1) \qquad \sum_{k=0}^{K} \delta_k = 1.$$

There are m nondurable goods (including time) which are used in the production of commodity 1. The amount of good j required to produce one unit of commodity 1 using process k is denoted by β_{jk}. The price of the j^{th} nondurable is P_j. Thus, the average variable cost of consuming commodity 1 using technology k is

$$\pi_k = \sum_{j=1}^{m} P_j \beta_{jk} \qquad k = 0, \ldots, K. \tag{2}$$

π_k will be referred to as the price of commodity 1 under technology k. The fixed annual cost associated with the use of process k is denoted by rI_k, where I_k is the purchase price of the k^{th} durable, and r is the sum of the interest and amortization rate. Since process 0 does not involve purchase of a durable, $I_0 = 0$.

The permanent income of the family is denoted by R. Income is spent on periodical payment for the durable purchase, the purchase of nondurables used to produce commodity 1, and purchases associated with consumption of commodity 0 (numeraire commodity which price is 1).[4]

Thus, the budget constraint of a family is given by

$$\sum_{k=0}^{K} \pi_k y_1 \delta_k + y_0 + r \sum_{k=0}^{K} \delta_k I_k = R. \tag{3}$$

The consumer choice problem is

$$\max_{\delta_0, \ldots, \delta_K, y_0, y_1} U(y_0, y_1) \tag{4}$$

subject to (1) and (3) where δ_k is either 0 or 1.

The consumer problem can be solved in two steps. First compute the optimal consumption pattern and the resulting utility under each process then select the process (and the durable) that maximizes utility. To simplify the graphical analysis, consider the case where commodity 1 can be produced only by two processes (K = 1). One is a technology which does not require the purchase of a new durable, and the associated price of the commodity is π_0. The alternative technology involves the purchase of a durable and requires fixed cost of rI_1 dollar per period and average variable cost of π_1 dollar per unit of commodity consumed.

The optimal consumption choice will be determined by comparing V_0, the maximum utility derived under technology 0 with V_1, the maximum utility derived when durable 1 is installed where

(5) $$V_0 = \max_{y_0,\, y_1} U(y_0, y_1)$$

subject to

$$y_1 \pi_0 + y_0 = R$$

and

(6) $$V_1 = \max_{y_0,\, y_1} U(y_0, y_1)$$

subject to

$$y_1 \pi_1 + y_0 = R - rI_1.$$

The choice problem is illustrated graphically in Figure 1. The budget constraint, when one is restricted to operate without the durable, is GF. The budget constraint, when the durable is used, is DE. The two budget lines intersect at C.

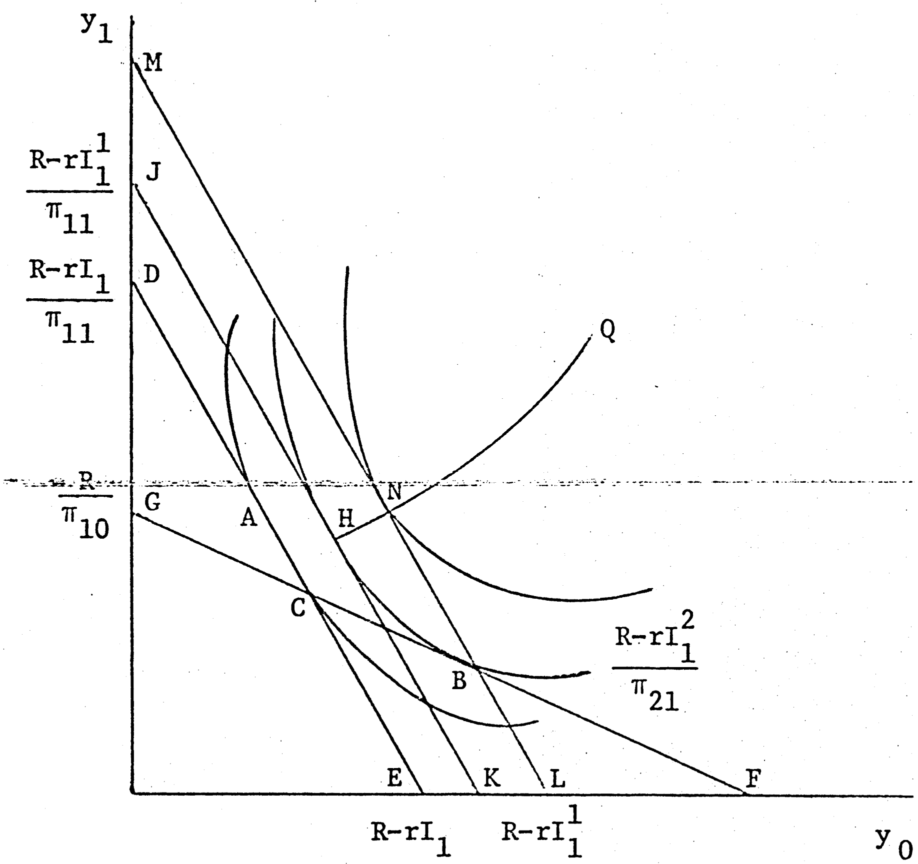
y1
M
R-rI11 / π11
J
R-rI1 / π11
D
Q
R / π10
G
A
H
N
C
R-rI12 / π21
B
E
K
L
F
R-rI1
R-rI11
y0

Figure 1.

To find the optimal solution, one has to compare the utility index at the tangency points of the indifference curves with the budget lines. In Figure 1 these intersection points are A and B and, since the indifference curve at B represents a higher utility level, point B will denote the optimal utility choice.

Figure 1 represents only a certain type of outcome for the consumption choice problem. Different budget lines or preference orderings may result in other types of outcomes. That is, the budget constraint of the consumer generates a set of efficient points. This set includes all the feasible commodity bundles where the consumption of one commodity cannot be increased without a reduction in the consumption of the other commodity. In the case illustrated in Figure 1, the set of efficient points is denoted by the broken line, DCF. The globally optimal consumption choices belong to the set of efficient points. All of the efficient consumption bundles can be optimal choices except the point of intersection of the two budget lines. This point, while efficient, cannot be optimal since, even when it is the best choice under one of the processes, the budget line of the other process will be tangent to a higher indifference curve and that tangency point will be the global optimum.

A unique solution to the consumer choice problem always exists in cases where the optimal consumption pattern under one of the processes belongs to the set of efficient consumption choices, but the optimal choice under the other process does not belong to the set. This happens when the optimal consumption of y_1 under both processes is greater or smaller than at the intersection point of the budget line. When it is greater, process 1 is superior and, in terms of Figure 1, the global optimum belongs to DC. When it is less, the process without the fixed cost is more desirable, and the optimum belongs to CF in Figure 1.

If the optimal consumption choices under both processes belong to the efficient set, one may have a multiple solution when the same indifference curve is tangent to both budget lines. This, however, will not be the usual case and, when both processes belong to the efficient set, one has a unique solution almost always. Figure 1 demonstrates this point. Given the consumption technology, nondurable prices, and the consumer's income, only a certain price of the durable goods will result in a multiple solution. This happens when the price of the durable is I_1^1; the budget line, when the durable is used, is JK; and both points H and B are optimal choices. When the price of the durable is greater than I_1^1 (as in the case of Figure 1 when the durable used is associated with DE), the consumer will not use the durable, and the optimal consumption point will be at B. However, in cases where the price of the durable is smaller than the critical price I_1^1, the consumer will prefer the process which uses the durable, and the final consumption pattern will be determined accordingly. In these cases a reduction in the price of the durable has the effect of an increase in income in a traditional consumption choice problem. Thus, the consumption points, when durable prices are below I_1^1, belong to curve HNQ which has all the usual properties of an income consumption curve.

This analysis can be extended to derive the properties of several interesting relationships. They include the individual's demand for the durable as a function of its price, the commodities' prices (good prices) and income, the demand for commodity 1 as a function of its prices under the different technologies and income, the demand for the nondurables, and the indirect utility function. The following sections will analyze the effects various parametric changes have on these relationships.

III. The Effects of Changes in the Durable and Commodity 1 Prices on the Demand for the Durable and Commodity 1

Let $\tilde{V}(\pi_1, R)$ be the indirect utility function associated with a traditional choice problem

(7) $$\tilde{V}(\pi, R) = \max_{y_0, y_1} U(y_0, y_1)$$

subject to

$$y_0 + \pi_1 y_1 = R.$$

The analysis associated with Figure 1 suggests a qualitative choice model for durable demand. Specifically, using (7) to combine (5) and (6) yields the formulation of the demand for durable

(8) $$\delta_1 = \delta_1^D(\pi_{10}, \pi_{11}, I_1, R) = \begin{cases} 1 \text{ if } \tilde{V}(\pi_{11}, R - rI_1) > \tilde{V}(\pi_{10}, R) \\ 0 \text{ if } \tilde{V}(\pi_{11}, R - rI_1) < \tilde{V}(\pi_{10}, R). \end{cases}$$

When equality holds, we do not have a unique solution; and the consumer is indifferent to purchasing or not purchasing the durable.

To define the demand for commodity 1, let $D(\pi_1, R)$ be the traditional demand for y_1 derived from solving the optimization problem in (7). The demand for y_1 in our case becomes

(9) $$y_1 = y_1^D(\pi_{10}, \pi_{11}, I_1, R) = \delta_1 D(\pi_{11}, R - rI_1) + (1 - \delta_1) D(\pi_{10}, R).$$

As (8) and Figure 1 indicate, the demand curve for the durable (given income, R, and commodity prices, π_{10}, π_{11}) is a step function which equals to one when the durable price is smaller than some critical level (I_1^1 in Figure 1) and zero when the price of the durable is greater than I_1^1. The demand for the durable at price, I_1^1, is indeterminant.

Changes in the durable goods price will affect the consumption of commodity 1 only when the durable is used. In this case (9) suggests that a reduction in the durable goods price has the same affect as increasing income by r times the amount of the reduction. This and Figure 1 yields a graphical presentation which relates the consumption of commodity 1 to the price of the durable. Such a graph is derived in Figure 2. The relationship consists of two disconnected parts. For prices which are greater than I_1^1, the consumption of commodity 1 is constant and equal to y_1^B (the consumption at point B in Figure 1). The shape of the second part of the graph, which corresponds to prices smaller than I_1^1, is determined by the income elasticity of the traditional demand for commodity 1 (when income is $R - rI_1^1$). Four possible shapes are depicted in Figure 2. A luxury commodity, with income elasticity greater than one, has a negatively sloped and convex curve relating consumption of y_1 to durable price (given durable price smaller or equal to I_1^1). This case is depicted by BF in Figure 2. The negatively sloped linear curve, BE, corresponds to cases of unitary income elasticity. Normal commodities (with income elasticities between zero and one) have negatively sloped and concave curves like BD. Finally, inferior commodities have positively sloped curves (BC).

Suppose that commodity 1 prices under both processes are independent (i.e., the processes use different nondurable) and consider how changes in

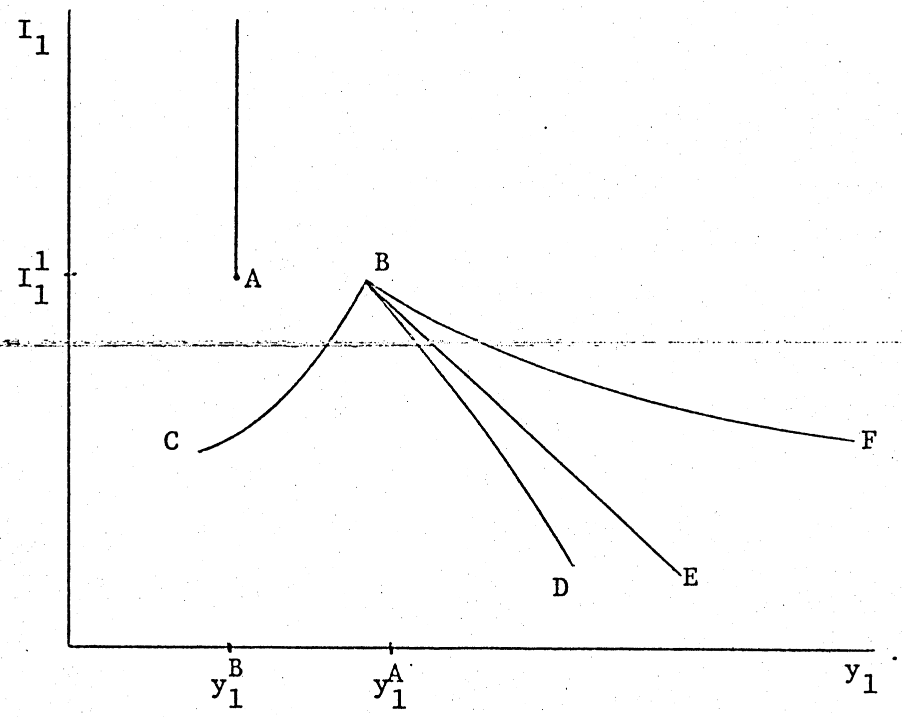

Figure 2.

these prices affect the choice of the durable and quantity demanded of commodity 1.[5] Here again, Figure 1 and (8) suggest that the demand for durable is a step function of π_{10} given R, π_{11}, and I_1^1. The durable will be purchased for all π_{10} levels above the critical level and will not be purchased for lower prices. Similarly, the demand for the durable is a step function of π_{11} (given π_{10}, R, I_1^1).

Using (9), the relationship between the price of commodity 1 under each of the processes and the quantity of the commodity demanded is investigated in Figure 3.

The starting point for the analysis is a commodity 1 of prices combination (π_{10}^0, π_{11}^0) which result in a multiple solution given R and I_1. It is also assumed that commodity 1 is a normal commodity; thus, the demand curve, $D(\pi_{10}, R)$, is above $D(\pi_{11}, R - rI_1)$. From (9), it is concluded that, when the price of commodity 1 under process 0 is π_{10}^0, the graph of the relationship between quantity demanded of commodity 1 and its price under process 1 consists of two disconnected parts. The first, for prices greater than π_{11}^0 is denoted by the line, CE. For these π_{11} levels, the durable is not used; instead, process 0 is used and commodity 1 price under this process determines y_1 demand level to be $y_1^D = D(\pi_{10}^0, R)$. When π_{11} is smaller than the critical level, π_{11}^0, the durable good is used, the consumption of y_1 is determined according to $D(\pi_{11}, R - rI_1)$, and the second part of the demand curve for y_1 (when π_{11} varies) consists of the point B and all the points of $D(\pi_{11}, R - rI_1)$ to the right of B. When $\pi_{11} = \pi_{11}^0$, we have a multiple solution, and the demand for y_1 can be either y_1^A or y_1^B. The demand for y_1 as a function of π_{10} gives

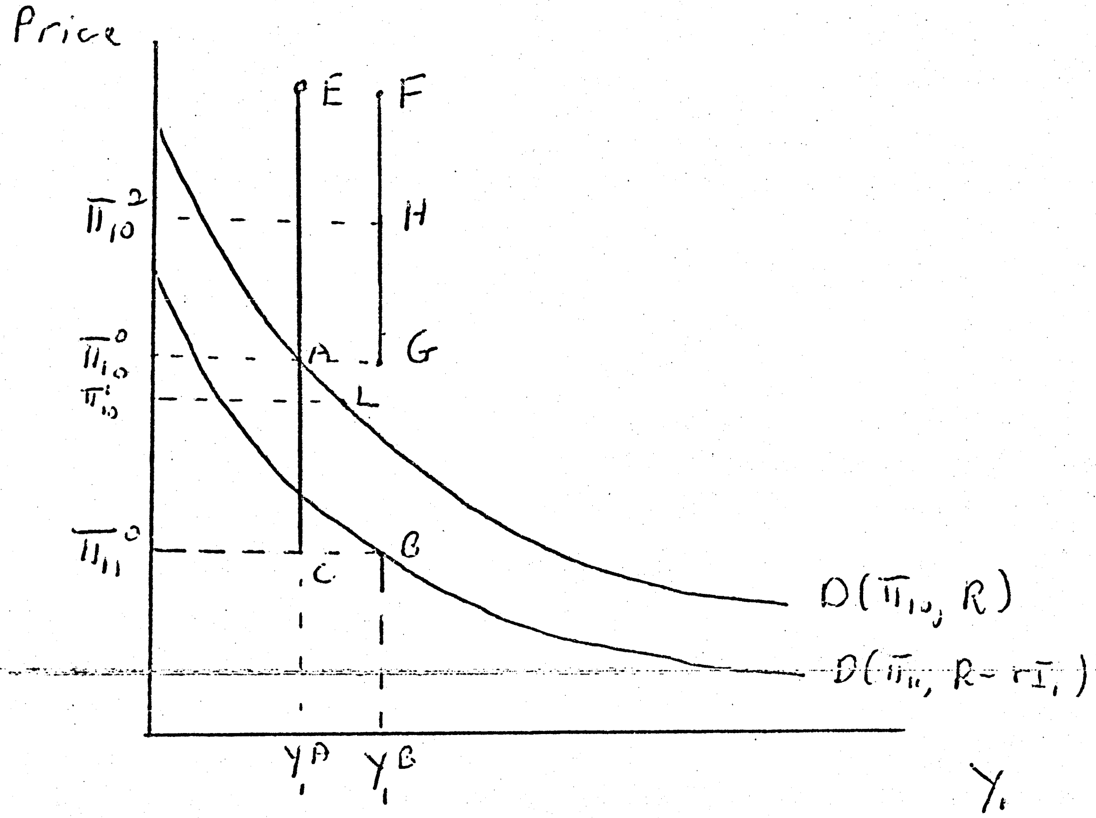

Figure 3.

R, I_1, and $\pi_{11} = \pi_{11}^0$ is determined similarly. When π_{10} is greater than the critical level, π_{10}^0, the durable is used, commodity 1 demand is equal to y_1^B, and the corresponding part of the demand curve is denoted by FG. When commodity 1 prices under process 0 are smaller than π_{10}, this process is used, and the corresponding segment of the demand curve for π_{11} consists of A and all the points, $D(\pi_{10}, R)$, to the right of A. Note that an increase in the price of commodity 1 under process 0 from, let's say, π_{10}^1 to π_{10}^2 will generate a move from L to H, and actual demand will increase. The availability of a durable with lower commodity 1 prices allows a switch that increases consumption of commodity 1 when the price of commodity 1, under the traditional technology, is rising above a critical level.

The results do not change essentially when commodity 1 is an inferior commodity. The only difference is that the curve, $D(\pi_{11}, R - rI_1)$, is above $D(\pi_{10}, R)$ in this case. Both demand relationships of y_{11}, however, have two discontinuous segments. The demand for y_{11} cannot increase when π_{11} increases ($\pi_{10} = \pi_{10}^0$) while it may increase when π_{10} increases and π_{11} is kept constant at π_{11}^0.

IV. The Effects of Changes in Income on the Demand for the Durable and Commodity 1

It is of interest to find how changes in income affect the consumption of commodities and the use of durables. This subject can be analyzed using Figure 4. In Figure 4 the budget lines are drawn for two cases with the same prices of commodities and durable but different income levels. Note that, in both cases, the budget line for the process using the durable and the budget

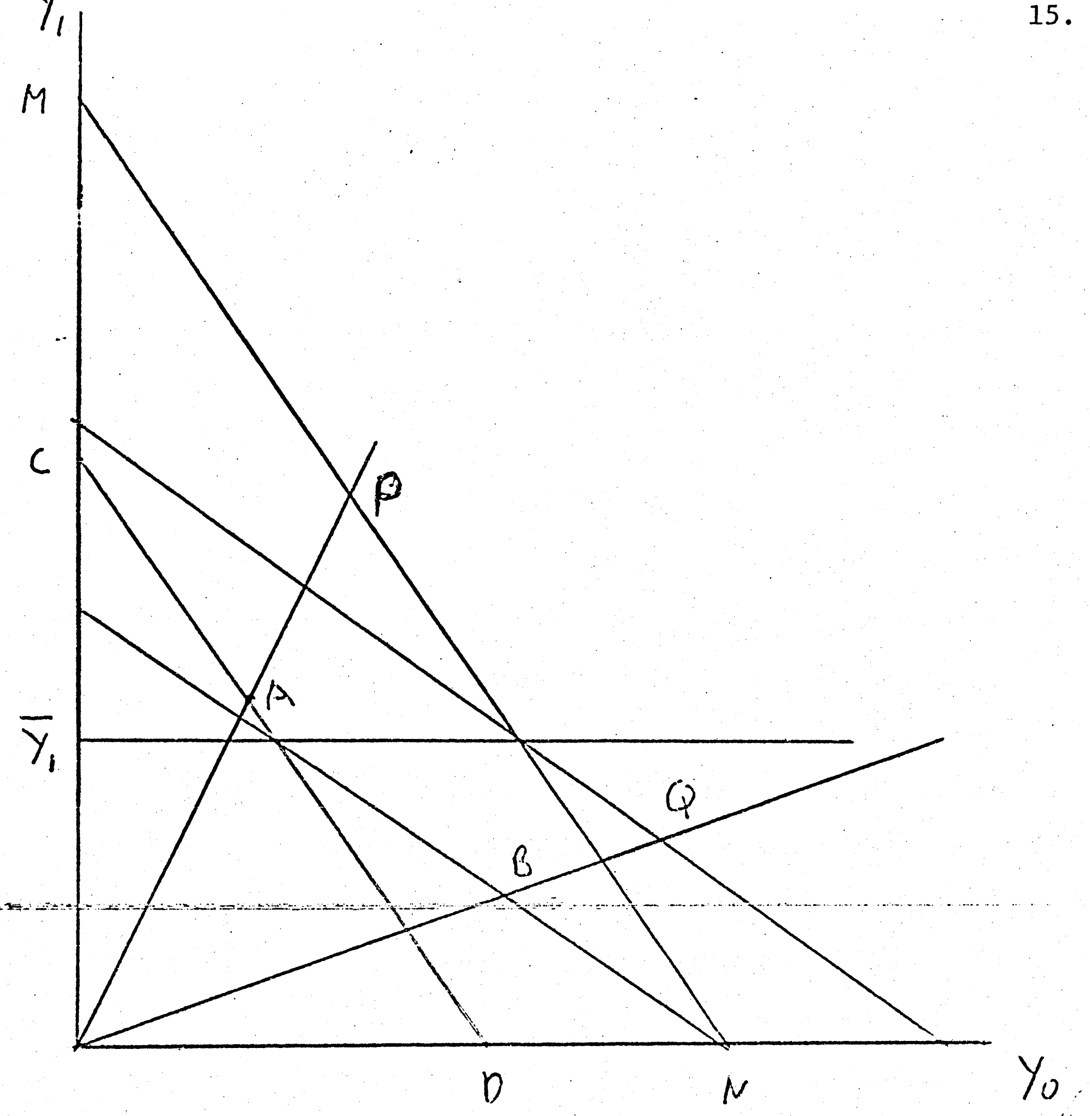

Figure 4.

line for consumption without the durable intersect always on the same level of y_1 denoted by $\overline{y}_1$ where $\overline{y}_1 = rI_1/(\pi_{10} - \pi_{11})$. This happens because an increase in income causes the same horizontal shift in both budget lines, i.e., the efficient set is homogeneous of degree 1 in income. This result and the fact that the process using the durable (without the durable) is superior when the optimal outcomes under both processes are above (below) the intersection points of the budget lines lead to some interesting insights.

These results indicate, for example, that poor people will not use the durable. This is the case when the budget lines do not intersect (thus both budget lines are below the line $y_1 = \overline{y}_1$) and the budget line for the process without the durable dominates the one with the durable. It can also be deduced that the durable will always be purchased above a certain income if the commodity 1 is not inferior and its income elasticity is always positive.[6] Under these conditions, there must be an income level above which the optimal solutions under both processes are above the line $y_1 = \overline{y}_1$. This condition also implies that, for given prices, there is at least one critical income level when the two budget lines are tangent to the same indifference curve. Moreover, if preferences are homothetic, then there is only one critical point which separates lower income levels for which it is optimal not to use the durable and higher levels for which the use of the durable is optimal.[7] This property of homothetic preferences results from the fact that income consumption curves are rays from the origin and that, if the consumer is indifferent to two consumption combinations, he is also indifferent to the combination generated by multiplying both of them by the same scalar. Let the points A and B in Figure 4 be two optimal outcomes for a given income R_0, and let them lie on the same indifference curve. The points P and Q will be the outcomes associated with increasing the income by ΔR dollars. The transformation from A to P is a scalar multiplication of A by $1 + \Delta R/(R_0 - rI_1)$, and the transformation from B to Q is a scalar multiplication of B by $1 + \Delta R/R_0$. Since the first scalar is larger, P is preferred over Q. Hence, the consumer will prefer to use the durable at all income levels above the one at which the consumer is indifferent between the two processes. Similarly, one can prove that a reduction of income below the critical income level (R_0) will cause the consumer to prefer the process without the durable.

V. The Effects in Changes in Nondurable Price

This section will analyze the effects of changes in nondurable good prices on the choice of the durable and their own demand curves. First, distinguish between goods that are used only in one of the processes (specialized goods) and goods which are used in both processes. For specialized goods, the analysis is rather simple. When their prices are very high, they make their processes less desirable and the consumer will not use the processes. For each specialized good, there is a critical level; and, once it is below the level (other variables kept constant), its process is adopted. The demand curve for a specialized good has two disconnected segments. It is 0 for all prices above the critical level when the specialized good process is not used. For all prices below the critical level, the demand is negatively sloped (assuming commodity 1 is not a Giffen commodity).

The slope of these demand curves can be derived from the demand for commodity 1. Let good j be a specialized input in process 1 with input output coefficient β_{j1}. When process 1 dominantes, the price elasticity of good j is simply the price elasticity of the demand for commodity 1 times the share of the expenses on good j in the variable cost of producing commodity 1. Let η_{x_j} be the price elasticity of good j and $\eta_{y_{11}}$ the price elasticity of the demand for commodity 1 under process 1, then[8]

$$\text{(10)} \qquad \eta_{x_j} = \frac{P_j}{\pi_{11}} \beta_{j1} \, \eta_{y_{11}} .$$

Equation (10) transfers to consumption theory a familiar condition from production theory stating that elasticity of derived demand of a good is lower as the share of this good in variable cost is lower.

The analysis is much more complicated for goods which participate in both processes. To simplify somewhat, consider first the case when the good's input-commodity coefficients under the two processes are proportional to the price of commodity 1 under both processes (i.e., for good j, $\beta_{j0}/\beta_{j1} = \pi_{10}/\pi_{11}$). This will be the case, for example, when only one nondurable is used in producing commodity 1 in both processes. In this case the change of the price of good j has the same proportional effect on the prices of commodity 1 under both processes, and the new commodity prices keep the same proportions as the technical coefficients. Therefore, the effects of changes in good j prices in this case can be derived by analyzing the optimal choices when both prices of commodity 1 are changed in the same proportion. The lines AB and DE in Figure 5 are budget constraints for one set of commodity 1 prices. The budget lines CB and FE are derived by a proportional changes in the initial prices. Note that, in both cases, the intersection points of the budget lines have the same y_0 value denote it by $\bar{y}_0$. Thus,

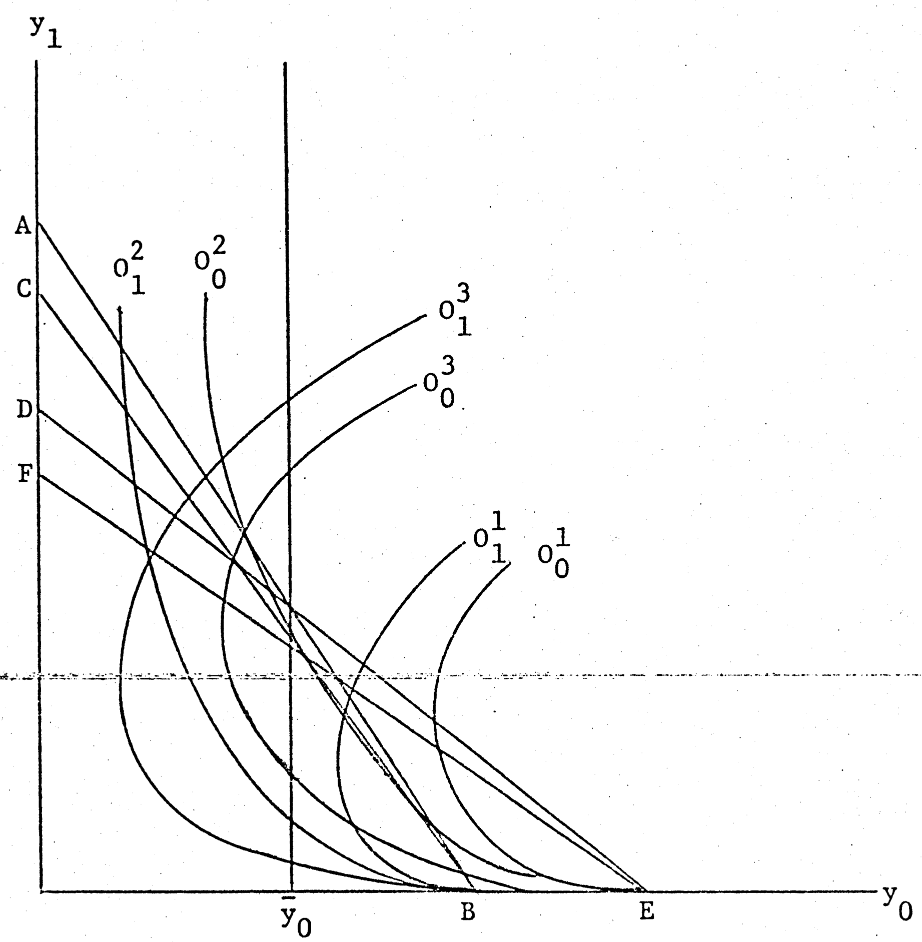

Figure 5.

in both cases, process 1 is preferred if both optimal outcomes are left to the line $y_0 = \bar{y}_0$; process 0 (the durable is not used) is preferred when both outcomes are to the right of the critical y_0 level

$$(11) \qquad \bar{y}_0 = rI_1 \left[\frac{\beta_{j0}}{\beta_{j0} - \beta_{j1}} - \frac{R}{rI_1} \right]$$

where $\beta_{j1}/\beta_{j0} = \pi_{11}/\pi_{10}$.

More insight into the behavior of the optimal outcomes can be gained by constructing the price possibility curves (offer curves) for both processes.[9] Two points, one on each curve, correspond to each pair of commodity 1 prices associated with the given proportions. The offer curves are convex to the origin. They are also negatively sloped when the price of the commodity is high but may reverse slopes for lower prices (for normal goods).[10] The relationship between these offer curves and the line $y_0 = \bar{y}_0$ determines the choice of optimal technology when prices are changed, but the ratio of the two prices is kept constant.

One possibility is that, under all prices, the use of the process without the durable will be preferred. This is the case, for example, where the offer curves are O_0^1 and O_1^1 in Figure 5. A second possibility is that the process without the durable is preferred when the prices of commodity 1 are very high. But when the prices of commodity 1 become lower, use of a durable to increase the consumption of commodity 1 is preferable. This is the case when the offer curves are O_0^2 and O_1^2 in Figure 5. Both intersect the line $y_0 = \bar{y}_0$ only once. Such is typically the case with luxury goods or normal goods with high income elasticity. Another possibility is that low prices of commodities result in the use of process 0; higher prices cause a switch to the process without the durable, but very high prices will result in a reversal of technology and the use of the process without the durable. This is the case when the offer curves are O_1^3 and O_0^3, and it may occur for some normal commodities or in cases where the commodity is inferior for some income levels.[11] In these cases high prices of commodity 1 result in a low level of consumption of the commodity, and the use of the durable is not justified. When prices become lower, the substitution effect will increase the consumption of commodity 1 and encourage use of the durable. When prices become very low, however, the income effect will cause an increase in the demand for other goods, and the money spent to pay the fixed cost of the durable good can yield higher utility in other uses.

The relationship between the quantity consumed and the price of commodity 1 under process 1 when prices keep fixed proportions is especially interesting for the latter case (with O_1^3 and O_2^3). This relationship is described in Figure 6 where the segments AB and EF correspond to prices for which process 0 is preferred while the segment CD corresponds to prices for which the use of the durable is preferred. This demand relationship is peculiar since it results in situations where an increase in both prices of commodity 1 (they keep fixed proportions) implies an increase in its consumption (i.e., movement from M to N).

The results for cases when both prices of commodity 1 keep a fix proportion suggest several patterns of durable choice when a single same good, x_j, is the variable input in both processes. The critical value, $\bar{y}_0$, is an indicator, independent of the consumer taste, of a durable choice pattern a consumer may have. Consumers with negative $\bar{y}_0$ will not purchase the durable at any price (their $a_1 = 0 \; \forall \; P_j$). From (11), this condition applies when

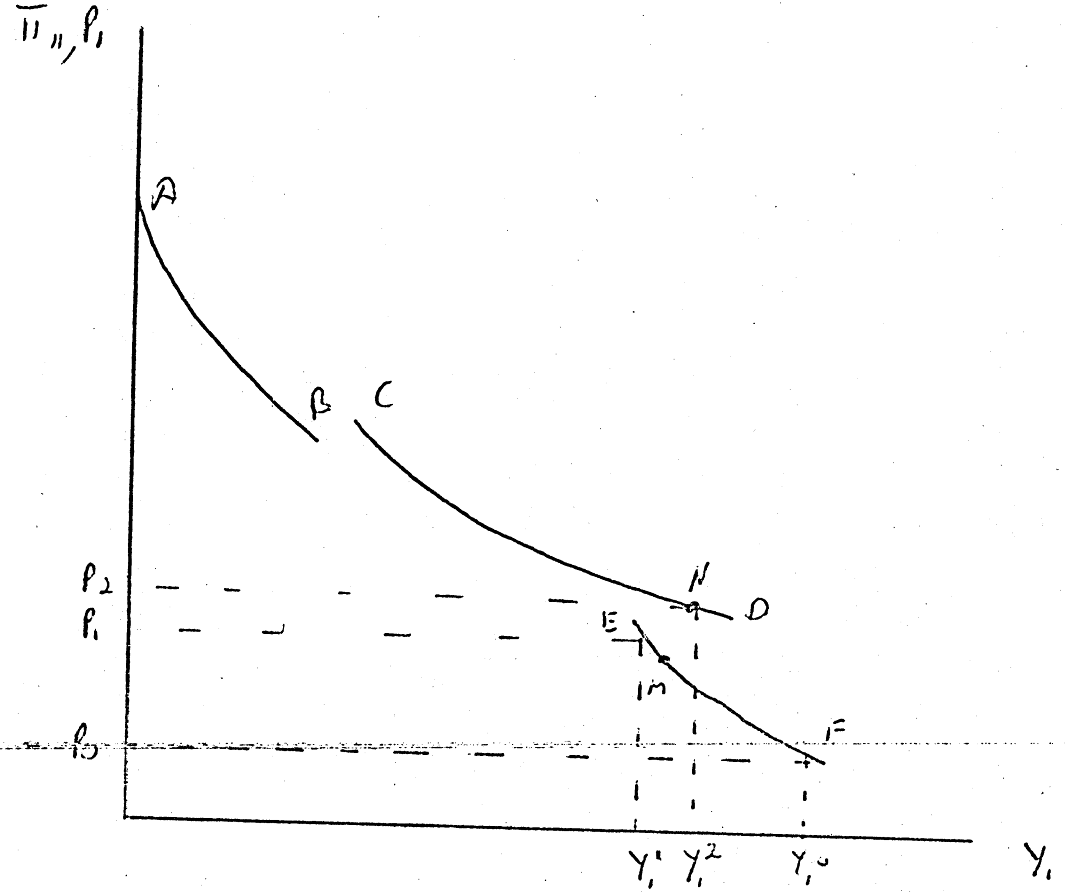

Figure 6.

the share of the fix durable cost in total income is larger than the relative efficiency gain from the durable purchase [i.e., $rI/R > (\beta_{j0} - \beta_{j1})/\beta_{j0}$)]. Thus, we should not expect consumers with less than the critical income level, i.e., with $R < rI_1 \beta_{j0}/(\beta_{j0} - \beta_{j1})$ to purchase the more efficient durable even if the price of the nondurable used to produce commodity 1 is rising very much. The purchase of the durable is also unlikely for all P_j by consumers with small positive $\bar{y}_0$. For consumer with a higher $\bar{y}_0$, durable choice will be a step function of the nondurable prices. As Figure 5 suggests, the durable will not be purchased for high nondurable prices. There will be a segment of lower nondurable prices which will result in purchase of the nondurable, and a third segment of reswitching might occur if commodity 1 is not a luxury commodity (its elasticity is less than 1) in the relevant segment.

From (9) the demand for commodity 1 as a function of the nondurable price is given by

$$y_1^D = \delta_1 D(\beta_{j1} P_j, R - rI_1) + (1 - \delta_1) D(\beta_{j0} P_{j1}, R).$$

The demand curve for commodity 1 as a function of the nondurable good price will be well behaved and negatively sloped for consumers who do not switch to use the durable for any P_j (low-income consumers with negative y_0). It will have a point of discontinuity but will continue to suggest that a reduction in a good price will increase the demand for the commodity it produces for consumers who switch to the durable good when P_j is smaller than a certain critical level. For consumers who reswitch, this demand curve will behave like the one in Figure 6, and there will be a range of prices when increase in the nondurable price may increase commodity 1 consumption. This type of consumer behavior is not unrealistic. For example, suppose commodity 1 is home heating, both technologies use natural gas, and the durable is a new more efficient furnace. The initial price of natural gas may be very low (like P_0), and the consumer will be at F. An increase of natural gas price from P_0 to P_1 will reduce natural gas consumption to y_1^1, however, an additonal increase will cause a purchase of the more efficient furnace and increase in heating consumption (from y_1^1 to y_1^2).

The demand for the nondurable good, in our case, is derived from (2) and (9) to be

$$x_j = \delta_1 \beta_{j1} D(P_j \beta_{j1}, R - rI_1) + (1 - \delta_1) \beta_{j0} D(P_j \beta_{j0}, R). \tag{12}$$

Using (12), one can see that the price elasticity of the demand for the non-durable, x_j, is equal to the price elasticity of the demand for commodity 1 with respect to the price of commodity 1 for the process used, i.e.,

$$\eta_{x_j} = \delta_1 \eta_{y_{11}} + (1 - \delta_1) \eta_{y_{10}} \tag{13}$$

where

$$\eta_{y_{10}} = \frac{\partial D(\pi_{10}, R)}{\partial \pi_{10}} \frac{\pi_{10}}{y_{10}} .$$

Like other demand curves, the demand for the nondurable as a function of its price is likely to have discontinuity points at critical prices as one switches from one technology to another. However, unlike the demand for y_1 as a function of P_j, the demand for x_j will not necessarily increase at the critical price as one switches from technology 0 to 1. In many cases the switch to the use of a durable may reduce the nondurable consumption; and, in some rare cases, the consumption of x_j at the critical price will not be affected by the technological switch (the demand for x_j at this point will be continuous but not differentiable).

To illustrate and comprehend better how technological switch affects the consumption of x_j at a given P_j level, consider the case where the consumer has a C.E.S. utility function

$$U(y_0, y_1) = A\left(e_0 y_0^{-\rho} + e_1 y_1^{-\rho}\right)^{\frac{-1}{\rho}} \tag{14}$$

where A is a scale parameter, e_0 and e_1, share coefficient and $\sigma = 1/(1 + \rho)$ elasticity of substitution between commodity 0 and 1 of the utility function. Using (12) for this case, the demand for x_j for the C.E.S. utility function will be

$$x_j^D = \delta_1 \frac{R - rI_1}{P_j\left[(e_0/e_1)^{\sigma} \beta_{j1}^{\sigma-1} P^{\sigma-1} + 1\right]} + \frac{(1 - \delta_1)R}{P_j\left[(e_0/e_1)^{\sigma} \beta_{j0}^{\sigma-1} P^{\sigma-1} + 1\right]}. \tag{15}$$

From (15), one derives that, given prices and income, the demand for good j under technology 0 (x_{j0}) is greater than under technology 1 if the income share of the fixed cost associated with technology one exceeds the income share of commodity 1 under technology 0 times $[1 - (\beta_{1j}/\beta_{0j})^{\sigma-1}]$, i.e.,

$$x_{j0} \gtrless x_{j1} \text{ if } \frac{rI_1}{R} \gtrless \left[1 - \left(\frac{\beta_{j1}}{\beta_{j0}}\right)^{\sigma-1}\right]\left[1 + \left(\frac{e_1}{e_0}\right)^{\sigma} \beta_{j0}^{1-\sigma} P_j^{1-\sigma}\right]^{-1}. \tag{16}$$

Thus, for the important special case of the Cobb-Douglas utility function ($\sigma = 1$), (16) suggests that consumption of good j will decline as one switches from technology 0 to technology 1. Condition (16) also indicates that this behavior will always occur when the elasticity of substitution is smaller than one and for cases of higher elasticity of substitution when the initial income share of commodity zero is small.

The lesson from these results is that, when the elasticity of substitution in consumption between commodities 0 and 1 is small, a reduction in the price of commodity 1, resulting from purchasing the durable, will reduce the demand for x_j although the demand for y_1 will increase. The effect of the lower variable input requirement per unit associated with the new technology will overcome the effect of the increase in y_1 associated with the lower π_1. Condition (16) also suggests that, when the elasticity of substitution is large and a large share of income was spent initially on commodity 0, the increase in demand for y_1 associated with a switch from technology 0 to 1 is large enough to increase the demand for x_j in spite of the reduction of the input requirement per commodity unit. The likelihood of increase in x_j with a switch to technology 1 is higher as the fixed cost associated with technology 1 and its relative input requirement (β_{j1}/β_{j0}) are smaller.

The generalization suggested by the C.E.S. results is that, when the substitution effect is not strong enough to overcome the increase in efficiency of the nondurable associated with the new technology, the demand for x_j may have the shape depicted in Figure 7. In this case the consumer buys less of the nondurable while consuming more of commodity 1 as the nondurable price is declining and the consumer purchases the durable good (movement from P_{j0} to P_{j1} will result in a switch from A to B in Figure 7). If Figure 7 describes the demand for energy used for heating (following an early example), an

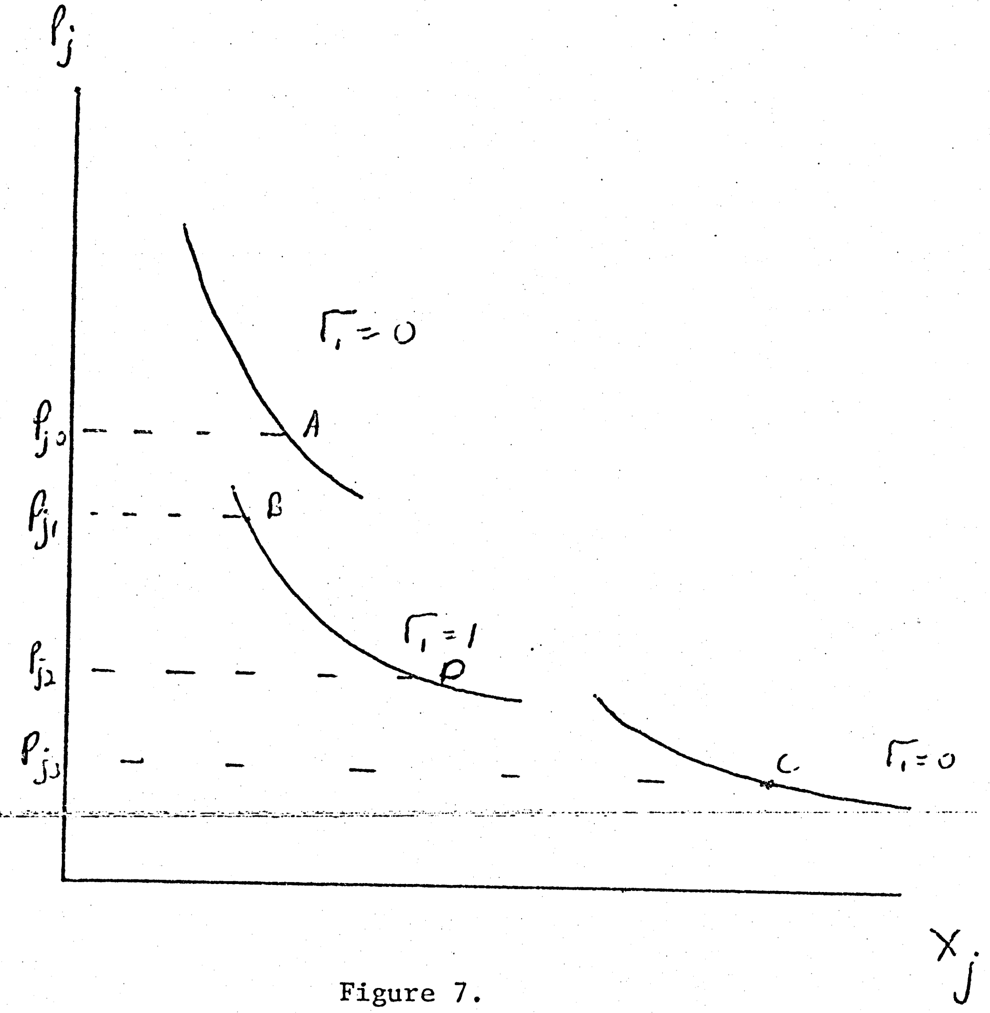
ρ_j
$\Gamma_1 = 0$
ρ_{j0}
A
ρ_{j1}
B
$\Gamma_1 = 1$
ρ_{j2}
D
ρ_{j3}
C
$\Gamma_1 = 0$
X_j

Figure 7.

increase in energy price from P_{j3} to P_{j2} will reduce energy use; but, as Figure 6 indicates, it may increase consumption of heat because of a switch to more energy-efficient technology.

However, note that when the substitution effect is very strong, the demand curve for x_j can have the same general shape as the demand for y_1 as P_j changes. Namely, a reduction in x_j price, causing a switch from technology 0 to 1 will increase consumption of y_1 and demand for x_j; and an additional reduction in prices, which results in a reswitch, may reduce the demand for x_j as well as y_j.

The effects of a change in the price of a good which participates in both processes should be analyzed according to a sum of two changes. The first is a proportional change in both prices of commodity 1, and the second is a change in the price of commodity 1 under the process when the good's share in variable cost are higher. For example, if the price of good j is increased by ΔP_j and the initial ratio $\beta_{j0}/\pi_{10} > \beta_{j1}/\pi_{11}$, then the effects of the change are the sum of: (1) an increase of

$$\Delta P_j \frac{\beta_{1j}}{\pi_{11}}$$

in the price of commodity 1 under both processes and (2) an increase of

$$\Delta P_j \frac{\beta_{j0} - \beta_{j1}}{\pi_{10} + \beta_{j1} \Delta P_j}$$

in the price of commodity 1 under process 0. The total effect of the price change can be analyzed using the results of this and the previous sections.

VI. The Behavior of the Indirect Utility Function

Let $V(\pi_{10}, \pi_{11}, I_1, R)$ be the indirect utility function of the durable choice problem considered here. For each price income combination, it will be equal to the indirect utility function under the selected technology, i.e.,

$$V(\pi_{10}, \pi_{11}, I_1, R) = \delta_1 \tilde{V}(\pi_{11}, R - rI_1) + (1 - \delta_1) \tilde{V}(\pi_{10}, R). \tag{17}$$

The indirect utility function is continuous since each of the functions generating it is continuous, and they are equal at the switching point. However, it is not always differentiable since, at the switch points, the indirect utility functions under each of the technologies have different gradients. Moreover, the indirect utility function is not concave in income and is not convex in prices. Again, the behavior of the function near the switch point is the reason for the irregularities.

To better understand the function's behavior, consider Figure 8. The curve ABC depicts indirect utility as a function of income for the likely case of one switch. The segment, AB, corresponds to technology 0 and segment, BC, to the use of the durable. At the switching point, B, the marginal utility of income, when the durable is used, is greater than when it is not used (otherwise, there is no reason to switch). Therefore, marginal utility is not declining with income; and, in some cases, marginal utility may be larger at higher income levels (as comparison of the marginal utility at points D and E demonstrates). Thus, assuming identical preferences, it might happen that an individual with higher income (above the critical switching level, R_B) will

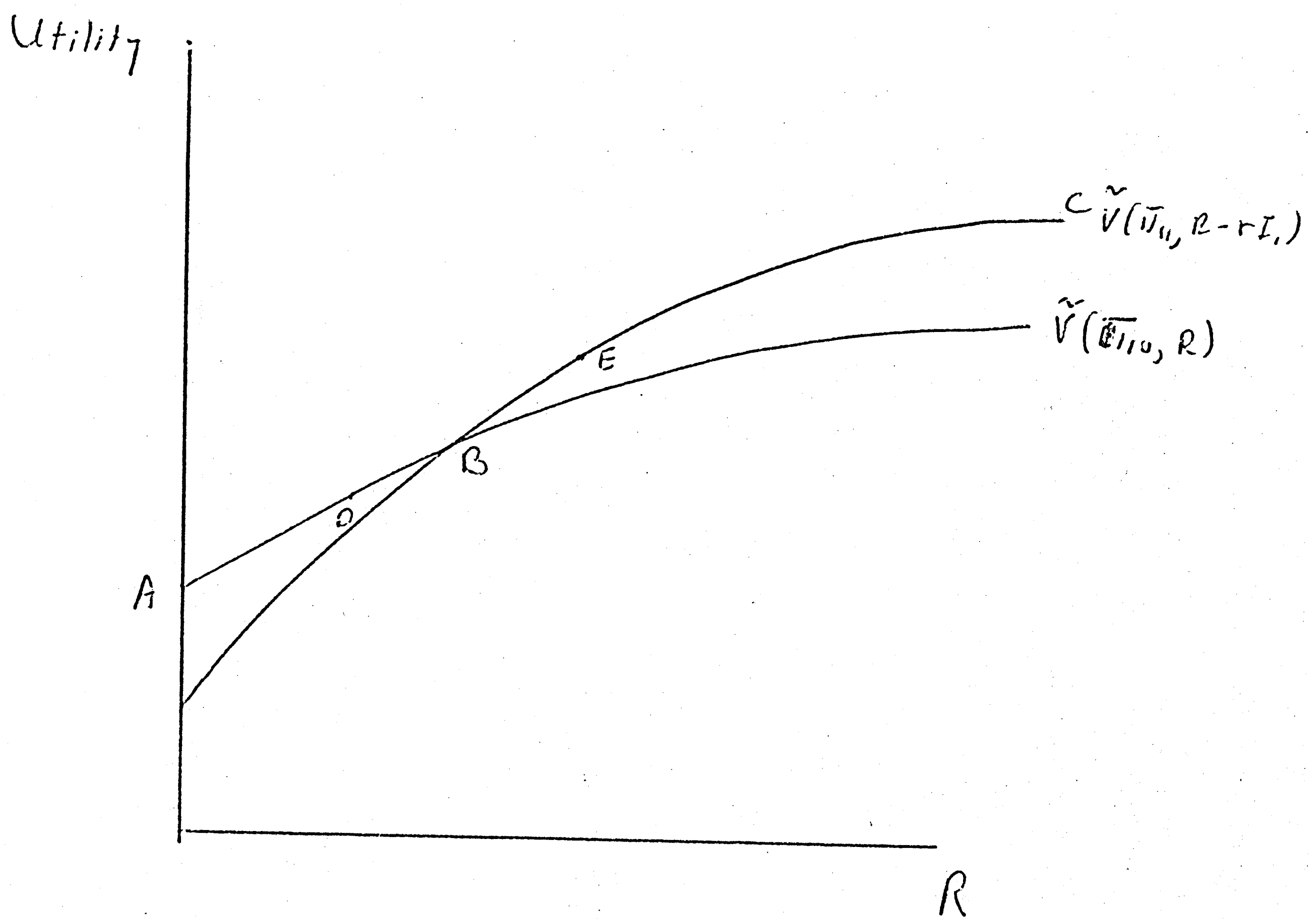
Utility
A
B
C
D
E
R
$\tilde{V}(\overline{\pi}_{11}, R - rI_1)$
$\tilde{V}(\overline{\pi}_{10}, R)$

Figure 8.

enjoy more from a given increase in income than a poorer individual who does not own the durable and uses a less efficient consumption technology.

VII. Aggregation of Demand

Thus far, this paper has analyzed the behavior of an individual consumer. This section builds on the previous results to derive aggregate demand relationships. It is assumed that all consumers have identical tastes but differ in their income. It is also assumed that their preferences can be expressed by homothetic utility functions.

Most of the analysis will be for the case where commodity 1 can be produced with only two processes--one of which requires the use of a durable good. Some of the results will be extended to a case where several durables producing commodity 1 are available.

Four aggregate relationships will be derived: the desired stock of durable good k at time t, $S_k(t)$; the aggregate demand for durable good k at time t, $Q_k(t)$; the aggregate demand for commodity 1 at time t, $y_1(t)$; and the aggregate demand for nondurable j for the production of commodity 1 in period t, $X_j(t)$.

VIII. The Demand for Durable Good Stock

Consider the case where only one durable is available. Recall that, when utilities are homothetic, there is a cutoff income level which separates lower incomes which do not lead to use of the durable and higher income levels which cause consumers to prefer the use of the durable good. This cutoff income level is a function of commodity 1 and the durable good prices. Let this cutoff level be denoted by $R_1 = R_c(\pi_{10}, \pi_{11}, rI_1)$. From (8), the critical income is determined by solving

$$\tilde{V}(\pi_{10}, R_c) = \tilde{V}(\pi_{11}, R_c - rI_1). \tag{18}$$

Thus, given prices, the desired quantity of the durable good in the economy is equal to the number of consumers with income above the cutoff level. Therefore, knowledge of the income distribution is necessary for deriving the demand for durables.

Let the function, $f_t(R)$, be the income density function at time t such that $f_t(R)\Delta R$ is the fraction of households in the total population with income between R and R + ΔR. This density function is defined on positive income levels above a minimum level, R_m. The function $F_t(R)$ denotes the share of households whose income does not exceed R and is derived by integration of $f_t(R)$. Let the total population size be denoted by T, thus $TF_t(R)$ is the number of households whose income does not exceed R.

Using these definitions and the critical income defined by (18), the total quantity of the durable good desired at time t is a function of population size, income distribution parameters, and prices and is given by

$$S_1(t) = T \int_{R_c(\pi_{10}, \pi_{11}, I_1)}^{\infty} f(R)dR = T\left\{1 - F_t\left[R_c(\pi_{10}, \pi_{11}, I_1)\right]\right\}. \tag{19}$$

To demonstrate the suggested approach, consider the case where income has Pareto distribution and consumers have Cobb-Douglas utility functions. The Pareto distribution has been found to fit empirical data rather well (Champernowne). The density function is given by

(20) $$f(R) = \gamma R_m^{\gamma} R^{-\gamma-1} \text{ for } 0 \leq R_m \leq R \quad \gamma > 0$$
$$0 \qquad \text{otherwise.}$$

This income distribution assumes that the minimal income in the economy is R_m; this income level has the highest population density, and population density declines as income is rising.[12] Moreover, this income distribution approximates the share of population with income above a critical level, R, to be equal to $(R_m/R)^{\gamma}$. The parameter, γ, is the elasticity of the population share of the high income group, and it approximates the percentage change in the fraction of population with income above a critical level when this level is reduced by 1 percent.

By introducing (20) into (19), one finds

(21) $$S_1(t) = TR_m^{\gamma} [R_c(\pi_{10}, \pi_{11}, rI_1)]^{-\gamma}.$$

Consider the Cobb-Douglas utility function

(22) $$U(y_0, y_1) = y_0^{\alpha_0} y_1^{\alpha_1}$$

where $\alpha_1 + \alpha_0 \leq 1 \quad \alpha_0, \alpha_1 \geq 0$.

The indirect utility function associated with (21) is

(23) $$\tilde{V}(\pi, R) = (R\alpha_1)^{\alpha_1} (R\alpha_0)^{\alpha_0} (\alpha_1 + \alpha_0)^{-(\alpha_1 + \alpha_0)} \pi^{-\alpha_1}.$$

From (18), the critical income is

(24) $$R_c(\pi_{10}, \pi_{11}, I_1) = rI_1 \left[1 - \left(\frac{\pi_{11}}{\pi_{10}}\right)^{\alpha_1/(\alpha_1 + \alpha_0)}\right]^{-1}.$$

To simplify the expression later, assume $\alpha_1 + \alpha_0 = 1$ and denote $\alpha_1 = \alpha$. Introducing (24) into (21) obtains the optimal stock of durable 1 at time t

$$(25) \qquad S_1(t) = T\left\{R_m\left[1 - \left(\frac{\pi_{11}}{\pi_{10}}\right)^{\alpha}\right]\right\}^{\gamma}(rI_1)^{-\gamma}.$$

Both the price elasticity (in absolute value) and the mode income elasticity of the stock demand for the durable are equal to the elasticity of population share of the high-income group. The increase in stock demand resulting from a decline in the variable cost of the modern technology relative to the old technology (π_{11}/π_{10}) is higher as the elasticity of population share of high income group and the share of commodity 1 in current expenses (α) increase. Increase in the elasticity of population share of the high-income group will reduce aggregate stock demand, and the impact will be greater as the critical income which results in durable purchase is lower.

When $\gamma \geq 1$, the average income in the economy is given by $\bar{R} = \gamma R_m/(\gamma - 1)$.[13] Thus, the demand for the durable good stock can be rewritten as

$$(26) \qquad S_1(t) = T\left\{\frac{\gamma - 1}{\gamma}\left[1 - \left(\frac{\pi_{11}}{\pi_{10}}\right)^{\alpha_1}\right]\bar{R}\right\}^{\gamma}(rI_1)^{-\gamma}.$$

Again, the average income elasticity of stock demand of the durable is equal to γ.

IX. The Aggregate Flow Demand for the Durable Good

While $S_1(t)$ denotes the desired stock of capital good 1 at time t, the actual demand for new durables at time t is the difference between the demanded stock and the stock accumulated prior to time t. For simplicity, let us exclude the possibility of physical deterioration of the durable and thus replacement demand. Let us also assume that the actual stock of the durable is adjusting instantaneously to increase in stock demand. Thus, the flow demand for the durable good is equal to the change in the stock demand for the durable over time. If changes in parameters (prices, income distribution, etc.) cause reduction in the desired stock level of the durable good, the stock demand will be equal to zero.

Considering the cases when stock demand for the durable is rising over time and assuming differentiability of the relevant functions with respect to time, the flow demand for capital is derived by differentiating (19) to yield

$$(27)\quad Q_1(t) = \dot{S}_1(t) = T\left\{S_1(t)\,\frac{\dot{T}}{T} - \frac{\partial F_t}{\partial t} - \frac{\partial F_t}{\partial R_c}\left[\frac{\partial R_c}{\partial \pi_{11}}\,\dot{\pi}_{11} - \frac{\partial R_c}{\partial \pi_{10}}\,\dot{\pi}_{10} + \frac{\partial R_c}{\partial rI}\,(r\dot{I})\right]\right\}$$

where the upper dot denotes differentiation with respect to time. The flow demand for the durable good is an increasing function of the rate of population growth, the change over time in the fraction of population with income above the critical level, the decline over time of the variable cost associated with the modern technology compared to the old technology, and the decline in the price of the durable good.

Using the example when income is Pareto distributed and the utility function is Cobb-Douglas, the flow demand for the capital good is derived (assuming $\gamma > 1$) to be

$$(28)\qquad Q_1(t) = S_1(t)\left\{\frac{\dot{T}}{T} + \gamma\left[\frac{\dot{R}_m}{R_m} - \left(\frac{r\dot{I}_1}{rI_1}\right) - \alpha\,\frac{(\pi_{11}/\pi_{10})^{\alpha}}{1-(\pi_{11}/\pi_{10})^{\alpha}}\left(\frac{\dot{\pi}_{11}}{\pi_{10}}\right)\bigg/\left(\frac{\pi_{11}}{\pi_{10}}\right)\right]\right\}.$$

The flow demand for the durable is a product of the stock demand and the rate of change in the desired stock of the durable good (the expression in the brackets). The rate of change in the desired stock of the durable is linearly dependent on the rate of decline in the price of the durable and the interest rate, and the rate of increase in the minimum (average) income elasticity of the population share of the high income group in the linear coefficient. The effect of the rate of reduction in the variable cost associated with technology 1 relative to technology 0 on the rate of change in desired durable stock is proportional to the product of the elasticity of the population share of the high income group, the share of coefficient of commodity 1 in current expenses and an increasing function of $(\pi_{11}/\pi_{10})^{\alpha}$.

X. The Aggregate Demand for Commodity 1 and Nondurables

The aggregate demand for commodity 1 for cases where there is one critical income above which consumer purchase the durable (i.e., homothetic utility functions) is derived from (9) to yield

$$(29)\qquad Y_1(t) = \left\{ T \int_0^{R_c(\pi_{10},\, \pi_{11},\, rI_1)} D(\pi_{10},\, R)\, f(R)dR + \int_{R_c(\pi_{10},\, \pi_{11},\, rI_1)}^{\infty} D(\pi_{11},\, R - rI_1)\, f(R)dR \right\}.$$

For the case of Cobb-Douglas utility function,

$$(30)\qquad D(\pi,\, R) = \frac{\alpha R}{\pi}.$$

Introducing (24) and (30) to (29) for the case of Pareto income distribution defined in (20) with $\gamma > 1$ yields

$$(31)\quad Y_1(t) = T\,\frac{\alpha\bar{R}}{\pi_{10}}\left\{1 + \left[\frac{R_m}{rI_1}\left(1 - \left[\frac{\pi_{11}}{\pi_{10}}\right]\right)^{\alpha}\right]^{\gamma-1}\left(\left[\frac{\pi_{10}}{\pi_{11}}\right]^{1-\alpha} + \frac{1}{\gamma}\left(1 - \left[\frac{\pi_{11}}{\pi_{10}}\right]\right)^{\alpha}\frac{\pi_{10}}{\pi_{11}} - 1\right)\right\}.$$

The aggregate demand for commodity 1 in (31) equals to aggregate demand for commodity 1 under technology 0, $(T\alpha\bar{R}/\pi_{10})$, plus an additional element that expresses the effect of introducing technology 1 on the demand of commodity 1. While average income is the only income distribution parameter required to compute aggregate demand for commodity 1 when only technology 0 is available (and we have a "traditional" demand model), the availability of durable good 1 and the discrete choice it implies requires more than one income distribution parameter to compute aggregate demand. As (31) indicates, the aggregate demand for commodity 1 is an increasing function of the minimum income, population size and the share of commodity 1 in current expenses (α). It is a decreasing function of the elasticity of the population share of the high-income group, the price of commodity 1 under technology 1 relative to its price under technology 0, and the price of commodity 1 under technology 0 (when π_{11}/π_{10} is kept constant).

The derivation of aggregate demand for nondurable j used in producing commodity 1 is similar to the derivation of the demand for the commodity. Using (12) and (28), the aggregate demand for nondurable j is

$$(32)\quad X_j(t) = \int_0^{R_c(\pi_{10}, \pi_{11}, rI_1)} \beta_{10} D(\pi_{10}, R)\, f(R)dR + \int_{R_c(\pi_{10}, \pi_{11}, rI_1)}^{\infty} \beta_{11} D(\pi_{11}, R - rI_1)\, f(R)dR.$$

Consider the case where commodity 1 is the only nondurable used in both technologies where the utility function is Cobb-Douglas and income is Pareto distributed. In this case (32) beomes

$$(33)\qquad X_j(t) = \frac{\alpha T\overline{R}}{P_j}\left\{1\left(\frac{R_m}{rI_1}\right)^{\gamma-1} \frac{\gamma - 1}{\gamma}\left[1 - \left(\frac{\pi_{11}}{\pi_{10}}\right)^{\alpha}\right]^{\gamma}\right\}.$$

The aggregate demand for nondurable good j is equal to the demand for good j when only technology 0 is available ($\alpha T\overline{R}/P_j$) minus an additional element reflecting the contribution of technology 1 in saving good j. Note that in our examples, the availability of the new durable will increase the aggregate demand for commodity 1 while reducing the aggregate demand of the nondurable.

XI. The Case of More than One Durable

The analysis thus far can be easily extended to the case where more than one durable exists for generating commodity 1. When there are two durables, for example, it can be concluded that, if the record durable is more expensive than the first but requires lower variable cost (i.e., $I_2 > I_1$ but $\pi_{12} < \pi_{11}$), then the consumer with homothetic preferences will have two critical income levels, R_c^1 and R_c^2. The level of income for which the consumer will switch from no durable to durable 1 is R_c^1, and the level where the consumer switches from durable 1 to 2 is R_c^2 and R_c^2 will be larger than R_c^1. Thus population with identical tastes will be segmented according to income to three groups. The low-income group ($R \leq R_c^1$) will not use any durable; the middle income group ($R_c^1 < R \leq R_c^2$) will use durable good 1; and the high-income group will use durable good 2. Assuming a two-parameter Pareto income distribution $S_1(t)$, the aggregate stock demand for durable 1 at time t, for example, is given by

$$(34) \qquad S_1(t) = TR_m^{\gamma}\left[(R_c^1)^{-\gamma} - (R_c^2)^{-\gamma}\right].$$

Using (23), the critical values for a Cobb-Douglas utility function are

$$(35) \qquad R_c^1 = \frac{rI_1}{1 - (\pi_{11}/\pi_{10})^{\alpha}}$$

$$R_c^2 = \frac{rI_2 - [1 - (\pi_{12}/\pi_{11})^{\alpha}]\, I_1}{1 - (\pi_{12}/\pi_{11})^{\alpha}}.$$

Thus, the aggregate stock demand for durable good 1 becomes

$$(36) \qquad S_1(t) = T\,\frac{R_m}{r}\left\{\left[\frac{1 - (\pi_{11}/\pi_{10})^{\alpha}}{I_1}\right]^{\gamma} - \left[\frac{1 - (\pi_{12}/\pi_{11})^{\alpha}}{I_2 - (\pi_{12}/\pi_{11})\, I_1}\right]^{\gamma}\right\}.$$

Similarly, the aggregate stock demand for durable 2 becomes

$$S_2(t) = T\left(\frac{R_m}{r}\right)\left[\frac{1 - (\pi_{12}/\pi_{11})^{\alpha}}{I_2 - (\pi_{12}/\pi_{11})^{\alpha} I_1}\right]^{\gamma} . \tag{37}$$

One can extend the analysis to derive aggregate flow demands for the durables, and aggregate demand functions for the nondurables and commodity 1.

XII. Conclusions

Explicit consideration of the indivisibilities caused by the availability of consumer durables modifies significantly the behavioral pattern predicted by consumer theory and the nature of its basic relationship. Individual consumers' demand relationships will have points if noncontinuity reflecting technological switches, and changes in product prices may result in total realignment of the consumers' durable mix which will drastically change the nature of demand for all goods.

Recent development in econometrics allow estimation of simultaneous discrete and continuous choices made by consumers (see, for example, Heckman). Indeed, frameworks similar to the one presented here have been applied to estimate demand relationships to durables like air conditioners (Hausman) and refrigerators (Brownstone). The results of individual demand estimations can be used for estimating aggregate demand for durables when estimators of joint distribution of key variables that affect consumer choices (i.e., income) are available. These probability measures will be used as weights in generating the aggregate demand relationships following an aggregation procedure similar to the one taken in the latest part of this paper. These later reactions have developed aggregate demand relationships for durables and nondurable analytically. These aggregates depend on more than one income distribution parameter even when consumers have identical homothetic preferences (such preferences allow aggregation with the knowledge of mean income only when traditional consumer behavior models are used).

While the framework introduced here is useful to understand durable and nondurable choices in many cases, it is still limited and requires improvement and generalizations. Two particular elements that have to be incorporated in the analysis are product quality differences[14] and labor leisure choices, in particular, the effects of different sources of income (wage earning and returns from assets) on durable choices.

FOOTNOTES

†Giannini Foundation Paper No. (reprint identification only).

[1]One element which is not included in this work and should be incorporated in future research is quality changes among commodities. Many times the durable good is the source of differences in quality. The commodity transformation service has different quality when one moves in a new Cadillac or an old Pinto. Moreover, in many cases capacity can be analyzed as an additional quality characteristic.

[2]Small and Rosen and Hanemann have analyzed the demand for variables selected by discrete choice as part of thier analysis of the welfare impact of quantal choice models. Their analysis does not allow changes in other variables simultenously with the discrete choice (less time is required for washing when washers and dryers are bought than when a laundromat is used), and they do not analyze extensively income effects. The aggregate relations they consider assume constant income level (or constant marginal utility of income), while we aggregate here over income.

[3]It is assumed that $U'_{y_i}(y_0, y_1) = \infty$ when $y_i = 0$, $\forall i$. This assumption ensures that all commodities are consumed.

[4]The income R can also be interpreted as Becker's "full income," i.e., income from profit and all potential income from labor. In this case the model can be extended to include liesure as one of the commodities in the utility function and it is produced only by labor.

[5]Of course, the prices of commodity 1 under the two processes are interdependent. Some inputs are used in both processes (on different proportions),

and change in these inputs will affect the prices of commodity 1 under both processes. These cases will be investigated later using the results derived here.

[6]The notion of an inferior commodity used here is the traditional one. A commodity is inferior under a given consumption technology if its consumption is reduced when income is increased and prices do not change. Note that one can define here a new notion of inferiority and say that a commoditiy is in- ferior if its consumption is reduced when income is changed. A commodity might be normal in the traditional way and inferior under the new notion.

[7]Actually, this result is more general; here it will be proven only for homothetic utilities.

[8]Let $y_{11} = D(\pi_{11}, R - rI_1)$, then using (9)

$$x_j = \delta_1 \beta_{j1}^2 y_{11}.$$

Since $\partial\pi_1/\partial P_j = \beta_{j1} \partial x_j/\partial P_j = \delta_1 \beta_{j1} \partial y_{11}/\partial\pi_{11}$. Using

$$\eta_{y_{11}} = \frac{\partial y_{11}}{\partial\pi_{11}} \frac{\pi_{11}}{y_{11}}$$

one derives

$$\eta_{x_j} = \frac{\partial x_j}{\partial P_j} \frac{P_j}{x_j} = \delta_1 \frac{\beta_{j1}^2 y_{11} P_j}{x_j \pi_{11}} \eta_{y_{11}} = \eta_{y_{11}} \frac{P_j}{\pi_{11}} \beta_{j1}.$$

[9]Each curve is the locus of all optimal y_0, y_1 combinations under each process resulting from change in commodity 1 price under the process given the other parameters.

[10]A special behavior pattern occurs when the elasticity of demand for commodity 1 is always unitary. In this case the PCC lines are parallel to the y_1 axes.

[11]Another special case is when the elasticity of demand is unitary under both processes. In these cases the optimal process choice is not affected by a price change given that both prices of commodity 1 follow fixed proportions.

[12]Actually, the Pareto distribution is approximating very well the behavior of the tail end of income distribution, with income above the mode level. When it is used empirically, one has to accommodate of the exclusion of the very low-income groups from the analysis.

[13]If R is bound from above, one can express $S_1(t)$ as a function of $\bar{R}$ for $\gamma < 1$.

[14]The approach taken by Rosen and Novshek and Sonnenschein seems to be useful for extending the analysis to consider differences in product qualities. This seems especially appropriate in situations where there is a large variety of durables with small differences in production coefficients and quality properties.

REFERENCES

Becker, G. S. "A Theory of Allocation of Time." Econ. J. 75 (September 1965): 493-517.

Brownstone, D. "Econometric Models of Choice and Utilization of Energy Using Durables." EFRI Workshop on the Choice and Utilization of Energy Using Durables, Boston, Massachusetts, November 1-2, 1979.

Champernowne, D. G. The Distribution of Income Between Persons. Cambridge: Cambridge University Press, 1973.

Hanemann, W. M. "Applied Welfare Analysis with Quantal Choice Models." Working Paper No. 173, Department of Agricultural and Resource Economics, University of California, Berkeley, June 1981.

Hausman, J. A. "Individual Discount Rates and the Purchase and Utilization of Energy-Using Variables." Bell J. Econ. 10 (Spring 1979): 33-54.

Heckman, J. J. "Dummy Endogenous Variables in a Simultaneous Equations Systems." Econometrica 46 (March 1978): 679-694.

Lancaster, K. J. "A New Approach to Consumer Theory." J. Polit. Econ. 74 (April 1966): 132-157.

Novshek, W., and Sonnenschein, H. "Marginal Consumers and Neoclassical Demand Theory." J. Polit. Econ. 87, no. 6 (1979): 1368-1376.

Rosen, S. "Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition." J. Polit. Econ. 82, no. 1 (January/February 1976): 34-55.

Small, K. A., and Rosen, H. S. "Applied Welfare Economics with Discrete Choice." Econometrica 49 (January 1981): 105-130.