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A THEORY OF THE BIAS IN FUTURES MARKETS OF STORABLE COMMODITIES

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I. INTRODUCTION

Over the last 20 years, there has been much controversy regarding the validity of the theory as to which futures (or forward) prices of storable commodities are downward-biased estimates of the market expectations of subsequent spot prices. The issue is important because, if the alleged "bias" is nonexistent, then commodity futures prices offer a directly observable measure of expected prices and could be used for planning purposes. Empirical work on this subject to date [e.g., Telser, 1958, 1960; Cootner, 1960a, 1960b; Gray, 1961; Dusak, 1975] has failed to definitively accept or reject the theory. However, all empirical tests were based on the theoretical assumption that futures prices are always below expected prices irrespective of whether there is a shortage or surplus in the cash market; and no attempt has been made to construct a theory regarding the determination of the bias.

The purpose of this paper is to exhibit a theory of the simultaneous determination of cash, futures, and expected prices and to use it in illustrating the theoretical sign and magnitude of the futures market bias. In the process the difference between the concept of the bias and the concept of the so-called "price of storage" [Working, 1949] will be clarified.

The theory of the futures market bias is frequently attributed to Keynes [1930, pp. 142-47]. Keynes, however, in developing his theory of "normal backwardation," had in mind something quite different than the difference between the futures price and the expected spot price. To quote him:

"If there are no redundant liquid stocks, the spot price may (emphasis added) exceed the forward price, i.e., in the language of the market, there is a 'backwardation'" [Keynes, 1930, p. 143].

Later, he states:

". . . the existence of surplus stocks must cause the forward price to rise above the spot price" (emphasis added) [Keynes, 1930, p. 144].

Clearly, Keynes was concerned about the difference of the futures price from the spot or cash price and not the expectation of the spot price. In other words, he dealt with the difference between two observable prices. Working [1949] termed the difference between the forward price and the spot or near price the "price of storage" and empirically verified and clarified, in terms of commodity theory, Keynes' conjecture according to which backwardation would consistently exist in periods of low stocks, while the opposite--namely, a futures price above the spot price (which was termed by Keynes "contango")--would exist in periods of surplus stocks. However, Keynes went beyond the statement quoted above to state that "in normal conditions (emphasis added) the spot price exceeds the forward price, i.e., there is a backwardation."

Hicks [1946, p. 138] interpreted the words "normal conditions" in Keynes' quote above to mean a situation where expected spot prices are the same as current spot prices. Hence, he could invoke Keynes' support for his theory which stated that futures prices must be below expected spot prices.

"Futures prices are, therefore, nearly always made partly by speculators who seek a profit by buying futures when the futures price is below the spot price they expect to rule (emphasis added) on the corresponding date. . . . He (the speculator) will therefore be willing to go on buying futures so long as the futures price remains definitely below the spot price he expects (emphasis added), for it is the difference between these prices which he can expect to receive as a return for his risk bearing" [Hicks, 1946, p. 138].

It is, thus, to Hicks that the theory of futures price bias must be attributed. Hicks supported his theory by arguing that, in a futures market, hedging is most likely to be net short (hedging sales of futures exceed the hedging purchases of futures).

"Now there are quite sufficient technical rigidities in the process of production to make it certain that a number of entrepreneurs will want to hedge their sales; supplies in the near future are largely governed by decisions taken in the past so that if these planned supplies can be covered by forward sales, risk is reduced. But although the same thing sometimes happens with planned purchases as well, it is almost inevitably rarer. . . . Thus, while there is likely to be some desire to hedge planned purchases, it tends to be less insistent than the desire to hedge planned sales. If forward markets consisted entirely of hedgers, there would always be a tendency for a relative weakness on the demand side; a smaller proportion of planned purchases than of planned sales would be covered by forward contracts" [Hicks, 1946, p. 137].

However, while this statement might be empirically true in the market for a finished or semifinished good, it might not hold in the market for a relatively homogeneous raw material of the kind for which futures markets currently exist. Consider, for instance, the case of wheat. According to Hick's argument quoted above, the demand for short hedging should come from the producers. It is, however, known that farmers are not the major hedgers in the sense outlined above, namely, by selling futures.[1] Most of the demand for hedging comes from the merchandizing firms and the wheat processors. In such a system the demand for hedging could very well be on the long side. In a period of wheat shortage, for instance, bakeries and other final users of wheat--if they have commitments to deliver their products at fixed prices--would want to secure flour supplies and would try to hedge by forward contracting for flour supplies with millers at fixed prices. They, in turn, would try to hedge their commitments to sell flour at fixed prices by buying long in the futures market their anticipated milling needs of wheat. In such a situation, net hedging could very well be long; hence, by applying Hicks' reasoning, the current futures price would be above the subsequently expected spot price.

From the above discussion, it is clear that, to describe the dynamics of the spot and futures prices, the theory should incorporate the four main market participants, namely, physical producers, stockholders, and users of the commodity as well as the futures market speculators. This is accomplished in the model that is analyzed below. The model describes the desired cash and futures market positions of all the market participants. While much theoretical literature to date has independently analyzed these simultaneous decisions for different classes of market actors, no one has put all these decisions together in a complete dynamic model of price determination. The explicit derivation of cash, futures, and expected prices is achieved by imposing a mean-variance utility for the different market participants rather than a more general utility indicator. Given, however, the clarification that is possible within this framework, the simplification seems justified and the results should be applicable to more general situations.

II. A DYNAMIC COMMODITY MARKET MODEL

The storable commodity market will be composed of four kinds of participants. Producers (h), inventory stockholders (c), and processors (d) will deal in both the spot and the futures contract (acting as hedgers) and futures market speculators (s). The letters will be used to denote the above market actors.

The representative members of each class when making a decision for period t will be assumed to maximize a mean-variance utility of profits of the type

(1) $$E_{t-1}(\pi_{kt}) - \phi_k \, Var_{t-1}(\pi_{kt}) \qquad k = h, c, d, s$$

where $E_{t-1}(\cdot)$ and $Var_{t-1}(\cdot)$ are notations for the conditional expectation and variance at time $t - 1$, π_{kt} are t-period profits, and ϕ_k denote firm-specific positive risk parameters. (Under exponential utilities and normal random variables, the above will be the coefficients of absolute risk aversion of every representative market participant.)

In every period t - 1, each cash market participant will make an ex ante commitment in the spot commodity for which he will incur a short-run cost. The profit, however, from the spot commitment will not be realized until period t when the price of that period is revealed. To offset the price risk, the cash market participants, whose profit or loss also will be realized in period t, will take in period t - 1 a futures position as well. The futures market speculator, in turn, takes a position only in the futures market. The short-run cost functions of the cash market actors will all be assumed quadratic and symmetric about some fixed value for analytical convenience (instead of more general convex functions). Thus, implicitly it is assumed that firms are operating at their long-run minimum cost outputs, and deviations from those outputs incur quadratic short-run costs. No cost will be assumed for speculating in futures markets. This can be justified by the empirical fact that interest-bearing notes can be posted as margins in futures trading accounts and that broker commissions are very small compared to the profits or losses from price fluctuations.

Denote by h_t the representative producer's planned production in period t; by c_{t-1}, the end of period t - 1 inventory carry-over holdings; and by d_t, the planned consumption of the commodity by the processor in period t. Let $f_{k,t-1}$ (k = h, c, d, s) denote the commitments in futures contracts of the representative member of the kth class of futures market participants in period t - 1. A positive value for f_k denotes a commitment to sell (a short position), while a negative value denotes a commitment to buy (a long position). Denote by $p^f_{t,t-1}$ the t - 1 period quoted price of a futures contract that expires in period t and by p_t the spot price in period t. With this notation, the profits of the various market participants which are realized in period t are given by the following expressions.

Producers

$$\pi_{ht} = p_t h_t - A(a) - \frac{(h_t - a)^2}{2\alpha} + f_{h,t-1}\left(p^f_{t,t-1} - p_t\right). \tag{2}$$

Processors

$$\pi_{dt} = -p_t d_t + B(b) - \frac{(d_t - b)^2}{2\beta} + f_{d,t-1}\left(p^f_{t,t-1} - p_t\right). \tag{3}$$

Inventory stockholders

$$\pi_{ct} = (p_t - p_{t-1}) c_{t-1} - C(c) - \frac{(c_{t-1} - c)^2}{2\gamma} + f_{c,t-1}\left(p^f_{t,t-1} - p_t\right). \tag{4}$$

Futures market speculators

$$\pi_{st} = f_{s,t-1}\left(p^f_{t,t-1} - p_t\right). \tag{5}$$

In the above expressions, A(a) and C(c) denote the long-run average costs of the representative producer and inventory stockholder, respectively, while B(b) denotes the long-run average profit (net of input costs of the commodity under study) of the representative processor. The positive parameters, α, β, and γ are specific to the short-run cost functions of the cash market participants. It also has been implicitly assumed that the price of a futures contract at expiration will be equal to the then-prevailing spot price.

Denote by $p^e_{t,t-1}$ the conditional expectation in period t - 1 of the spot price that will prevail in period t. All market participants will be assumed to have the same market-determined expectations about the spot price. Also, denote by σ_1^2 the one-period conditional variance of the spot price.

(6) $$\sigma_1^2 \equiv E_{t-1}\left(p_t - p_{t,t-1}^e\right)^2 .$$

Given the profit expressions (2)-(5), the cash and futures market commitments of each class of traders can be found by maximizing the quadratic utility function in (1) for each class. The results are as follows:

Producers

(7a) $$h_t = a + \alpha\, p_{t,t-1}^f .$$

(7b) $$f_{h,t-1} = a + \alpha\, p_{t,t-1}^f + \frac{p_{t,t-1}^f - p_{t,t-1}^e}{\phi_h\, \sigma_1^2} .$$

Processors

(8a) $$d_t = b - \beta\, p_{t,t-1}^f .$$

(8b) $$f_{d,t-1} = -b + \beta\, p_{t,t-1}^f + \frac{p_{t,t-1}^f - p_{t,t-1}^e}{\phi_d\, \sigma_1^2} .$$

Inventory stockholders

(9a) $$c_{t-1} = c + \gamma\left(p_{t,t-1}^f - p_{t-1}\right) .$$

(9b) $$f_{c,t-1} = c + \gamma\left(p_{t,t-1}^f - p_{t-1}\right) + \frac{p_{t,t-1}^f - p_{t,t-1}^e}{\phi_c\, \sigma_1^2} .$$

Futures market speculators

$$f_{s,t-1} = \frac{p^{f}_{t,t-1} - p^{e}_{t,t-1}}{\phi_s \, \sigma_1^2}. \qquad (10)$$

Equations (7a) and (7b) are special cases of more general equations that have been derived from considerations of optimal one-period hedging strategies for competitive producers with general utility functions [Holthausen, 1979; Feder, Just, and Schmitz, 1980]. Similarly, equations (9a), (9b), and (10) are special cases of equations derived for hedger and speculator behavior from more general utilities [e.g., Stein, 1961; Schrock, 1971; Rutledge, 1972; Johnson, 1960]. Equations (8a) and (8b) also have been derived in slightly more general forms by Anderson and Danthine [1981]. Note that, in equations for the cash market participants, it is the futures price which dictates its cash commitments; and when the futures price is unbiased, namely, when $p^{f}_{t,t-1} = p^{e}_{t,t-1}$, all cash positions are fully hedged in futures.

The cash and futures prices will be determined by equilibrium in the cash and futures markets. These conditions are:

Cash market equilibrium in period t

$$h_t + u_t + c_{t-1} = d_t + v_t + c_t. \qquad (11)$$

Futures market equilibrium in period t - 1

$$f_{h,t-1} + f_{c,t-1} + f_{d,t-1} + f_{s,t-1} = 0. \qquad (12)$$

In (11), u_t and v_t are zero mean and uncorrelated random variables, over time, which affect additively the supply and demand for the commodity in period t.

III. CASH AND FUTURES PRICE DYNAMICS

If the expressions (7b), (8b), (9b), and (10) for the futures market commitments are substituted in the futures market-clearing equation (12), then the futures price can be obtained as a function of the t - 1 period spot price and the expectation in period t - 1 of the t-period spot price as follows:

$$(13) \qquad p^f_{t,t-1} = \left(\delta + \frac{1}{\phi\sigma_1^2}\right)^{-1} \left(b - a - c + \gamma p_{t-1} + \frac{p^e_{t,t-1}}{\phi\sigma_1^2}\right)$$

where the parameters δ and ϕ are defined below:

$$(14) \qquad \delta = \alpha + \beta + \gamma$$

$$(15) \qquad \frac{1}{\phi} = \frac{1}{\phi_h} + \frac{1}{\phi_c} + \frac{1}{\phi_d} + \frac{1}{\phi_s} .$$

When the expressions (7a), (8a), and (9a) are substituted in the cash-market equilibrium equation (11), the following dynamic price equation is obtained.

$$(16) \qquad \gamma p^f_{t+1,t} - \delta p^f_{t,t-1} - \gamma(p_t - p_{t-1}) + b - a + w_t = 0$$

where $w_t = v_t - u_t$.

Substituting expression (13) for the futures price in the cash market equilibrium relation (16) and rearranging terms, we obtain the following equation:

$$\gamma p^e_{t+1,t} - \delta p^e_{t,t-1} - \gamma\left[1 + (\alpha + \beta)\ \phi\sigma_1^2\right] p_t + \gamma p_{t-1}$$

(17)

$$+ (b - a)\left(1 + \gamma\phi\sigma_1^2\right) + c(\alpha + \beta)\ \phi\sigma_1^2 + \varepsilon w_t = 0$$

where $\varepsilon = 1 + \delta\phi\sigma_1^2$.

From (17), the unconditional expected spot price (or what might be termed the long-run average price) can be found by setting $p^e_{t+1,t} = p^e_{t,t-1} = p_t = p_{t-1} \equiv \bar{p}_s$ and $w_t = 0$.

(18)
$$\bar{p}_s = \frac{b - a}{\alpha + \beta} + \frac{c}{\gamma + \frac{1}{\phi\sigma_1^2}} \ .$$

Define the deviations of current expected or spot prices by tildes. For instance, $\tilde{p}_t = p_t - \bar{p}_s$. We then obtain from (17) a dynamic spot market price equation in the deviations of spot prices from $\bar{p}_s$ as follows:

(19)
$$\gamma\tilde{p}^e_{t+1,t} - \delta\tilde{p}^e_{t,t-1} - \gamma\left[1 + (\alpha + \beta)\ \phi\sigma_1^2\right] \tilde{p}_t + \gamma\tilde{p}_{t-1} + \varepsilon w_t = 0.$$

To solve (19), we assume that expectations are formed rationally in the Muthian sense. Applying standard rational expectations algebra [see, e.g., Turnovsky, 1979], it can be shown that

(20)
$$\tilde{p}^e_{t+i,t} = r^i\ \tilde{p}_t.$$

(21)
$$\tilde{p}_t = r\tilde{p}_{t-1} + \frac{\varepsilon w_t}{\gamma\left[1 - r + (\alpha + \beta)\ \phi\sigma_1^2\right]}$$

where r is the smallest root (which is positive and smaller than one) of the algebraic equation

(22) $$\gamma r^2 - \left[2\gamma + (\alpha + \beta)\left(1 + \gamma\, \phi\sigma_1^2\right)\right] r + \gamma = 0.$$

Since we have assumed rational expectations, the one-period price variance, σ_1^2, should also be consistent with (20) and (21). Applying (20) and (21) to the definition of σ_1^2 in (6), we obtain the following:

(23) $$\sigma_1^2 = \frac{\varepsilon^2\, \sigma_w^2}{\gamma^2\left[1 - r + (\alpha + \beta)\, \phi\sigma_1^2\right]^2}$$

where $\sigma_w = \mathrm{Var}(w_t)$.

Equation (23) defines an implicit equation in σ_1^2. Recently, McCafferty and Driskill [1980] have raised the possibility that equations such as (23) might not have any unique solutions. That a solution exists is easy to see in this model. Take the square root of (23) (since $0 < r < 1$ and the expression in the brackets of the denominator (23) is positive) and rewrite it as

(24) $$G(\sigma_1) = \frac{\sigma_1\left[1 - r + (\alpha + \gamma)\, \phi\sigma_1^2\right]}{1 + \delta\phi\sigma_1^2} = \frac{\sigma_w}{\gamma}\ .$$

The function $G(\sigma_1)$ above is continuous for nonnegative values of σ_1 and has the following properties:

(25) $$G(0) = 0, \quad G(\infty) = \infty.$$

Hence, by the mean-value theorem, a positive solution of (24) exists. It is possible, however, that this solution might not be unique. This could be shown by examining the derivative of G with respect to σ_1 and seeing that it is not necessarily positive. In the case of nonuniqueness, one must invoke some adjustment process by which the market finds the rational expectations equilibrium and a stability argument to specify which of the possible equilibria will eventually be reached. Since, however, it is only the existence of some rational expectations equilibrium that is required for the argument of this paper, this line of analysis is not pursued here.

IV. FUTURES MARKET BIAS AND THE PRICE OF STORAGE

Recall that Keynes argued that, under normal conditions, the futures price is below the spot price. In the context of the model developed here, normal conditions could be interpreted to mean cases were $v_t - u_t = 0$, and the spot price is at its long-run average value. What is the corresponding long-run average value of the futures price? By substituting the value for p_s in place of p_{t-1} and $p^e_{t,t-1}$ in the expression (13) that defines the futures price, we obtain the long-run average futures price (denoted by p_f) as follows:

(26) $$\bar{p}_f = \frac{b - a}{\alpha + \beta}.$$

Comparing (26) and (18), it can be seen that

(27) $$\bar{p}_f - \bar{p}_s = - \frac{c}{\gamma + \frac{1}{\phi\sigma_1^2}} < 0.$$

$\bar{p}_f < \bar{p}_s$

The amount of normal backwardation is seen to be a direct function of the amount of normal carry-over stocks and an inverse function of the parameter, γ, which could be interpreted as the inverse of the slope of the supply of storage function (9a) (for a theory of the supply of storage, see Brennan [1958]). The larger the amount of average carry-over stocks and the steeper the supply of storage function (i.e., the smaller the value of γ), the larger will be the magnitude of the "normal" or long-run average bias. Also, note that, when all market participants are risk neutral so that $\phi = 0$, the normal bias is zero.

The value of the bias in some time period, t - 1, is different than what is exhibited in (27). Write (13) with all prices as deviations from their respective long-run mean.

$$\tilde{p}^f_{t,t-1} = \left(\delta + \frac{1}{\phi\sigma_1^2}\right)^{-1} \left(\gamma\, \tilde{p}_{t-1} + \frac{\tilde{p}^e_{t,t-1}}{\phi\sigma_1^2}\right) \quad (28)$$

where $\tilde{p}^f_{t,t-1} = p^f_{t,t-1} - \bar{p}_f$, $\tilde{p}_{t-1} = p_{t-1} - \bar{p}_s$, and $\tilde{p}^e_{t,t-1} = p^e_{t,t-1} - \bar{p}_s$.

Using (20), we obtain after some algebra.

$$\tilde{p}^f_{t,t-1} - \tilde{p}^e_{t,t-1} = \left(\delta + \frac{1}{\phi\sigma_1^2}\right)^{-1} (\gamma - r\delta)\, \tilde{p}_{t-1} \quad (29)$$

or

$$p^f_{t,t-1} - p^e_{t,t-1} = \bar{p}_f - \bar{p}_s + \left(\delta + \frac{1}{\phi\sigma_1^2}\right)^{-1} (\gamma - r\delta)\, \tilde{p}_{t-1}. \quad (30)$$

The term $\gamma - r\delta$ can be written with the help of (22) as follows:

$$\gamma - r\delta = \gamma r\left[1 - r + (\alpha + \beta)\ \phi\sigma_1^2\right] > 0. \tag{31}$$

The futures price bias in any period is thus composed of two terms--one which is always negative and represents the amount of normal backwardation and another which has the same sign as p_{t-1}, the deviation of current spot price from its long-run average value. When there is a situation of excess supplies, then spot prices are bound to be below their average values and, hence, $\tilde{p}_{t-1} < 0$. In this case the bias is unambiguously negative, namely, the futures price is definitely below the subsequently expected spot price. When, however, there is a commodity shortage making current price rise above its long-run average value, the bias might become positive.

The above observation might seem at first sight to contradict Keynes. This, however, is not true. Recall from the quotations in the introduction that Keynes' conjecture was that the difference between the futures and the current spot price must be negative when there is a shortage, while it may be positive when there is a surplus. What has been shown above, instead, is that the difference between the futures and the expected spot price may be positive when there is a shortage and must be negative when there is a surplus. The difference between the futures and current spot price (the price of storage), with which Keynes was concerned, can be expressed in this model as follows:

$$p_{t,t-1}^f - p_{t-1} = \bar{p}_f - \bar{p}_s - \left(1 + \delta\phi\sigma_1^2\right)^{-1} \left[1 - r + (\alpha + \beta)\ \phi\sigma_1^2\right] \tilde{p}_{t-1}. \tag{32}$$

Since the expression multiplying $\tilde{p}_{t-1}$ is negative, it is clear that the price of storage behaves in this model exactly as Keynes postulated. In a period of shortage, $\tilde{p}_{t-1} > 0$; hence, from (27) and (32), the price of storage is unambiguously negative while, in a period of surplus, $\tilde{p}_{t-1} < 0$ and the price of storage <u>may</u> become positive.

The terms that multiply $\tilde{p}_{t-1}$ in expression (30) and (32) for the bias and the price of storage, respectively, have opposite signs. From this, it follows that the bias will behave in an <u>opposite</u> way to that of the price of storage. This result makes intuitive sense. Consider, for instance, the case of wheat. When supplies are abundant and stocks are above normal, the stockholders would require a positive return to carry the surplus stocks into subsequent periods. Thus, the price of storage should become positive or nearly positive. However, at the same time, risk-averse stockholders would hedge in futures most of the carry-over stocks for fear of further price declines. Hence, hedging in the market would most probably be net short, and this would cause the futures price to be <u>below</u> the expected spot price as the model predicts [equation (30)].

By using expressions (30) and (32), we can obtain the following expression:

(33) $$p^{f}_{t,t-1} - p^{e}_{t,t-1} = (1 + \eta)\ (\bar{p}_f - \bar{p}_s) - \eta\left(p^{f}_{t,t-1} - p_{t-1}\right)$$

where $\eta = r\gamma\phi\sigma_1^2$, and it is a unitless positive number. From (33), it is clear that, when the price of storage is negative (a situation which is likely before the harvest of some agricultural commodity), then the futures-price bias may be positive, or, if negative, it will be smaller than the amount of normal backwardation. When the price of storage is positive (a situation common in the postharvest periods for agricultural commodities), then the bias would be negative and larger than the amount of the normal backwardation.

V. REFLECTIONS ON EMPIRICAL ESTIMATES OF THE BIAS

With the help of the theory exhibited in the previous section, several observations on empirical tests for futures price bias can be made. The theory that was developed is consistent with Cootner's [1960a] argument that futures prices would tend to fall on average from the period prior to the harvest of a storable agricultural commodity until the end of harvest and would tend to rise from the period after the peak of harvest until just before the next harvest. From equation (28), we can derive an expression for the futures price as follows:

$$(34) \qquad \tilde{p}^{f}_{t,t-1} = \frac{r + \gamma\phi\sigma_1^2}{1 + \delta\phi\sigma_1^2}\tilde{p}_{t-1}.$$

In other words, the movements of the futures price are in the same direction as the movements in the spot price. Cootner, indeed, found this to be the case despite the fact that he used only 10 years of data.

However, this observation, while consistent with the existence of a futures price bias, is not sufficient proof. Notice that equation (34) would hold even if $\overline{p}_f - \overline{p}_s = 0$, i.e., no long-run bias.

Since it has been shown in earlier sections that the bias is variable over time and it alternates between positive and negative values, it is clear that, in order to empirically estimate the underlying long-run or normal bias, $\overline{p}_f - \overline{p}_s$, one should use data for a period in which the spot-price distribution is symmetric about its mean. However, because the government has been such an important factor in preventing agricultural commodity spot prices from falling below specified floor levels, the observed spot-price distributions for such commodities are more likely to be skewed with more weight on positive price deviations from the mean. This was especially true in the 1950s, 1960s, and early 1970s. Under these circumstances, all empirical tests that used data from such price periods (e.g., the test of Dusak) are a priori biased toward giving a less-negative (or even positive) value for the futures price bias.

Another observation that is derived from the theory is the following. Remember that the amount of the underlying long-run futures price bias, $\overline{p}_f - \overline{p}_s$, was found to be a direct function of c--the amount of normal carry-overs during the period t - 1 to t. What consists of "normal carry-overs," however, is quite different in various phases of the production and marketing cycle of a commodity. For instance, for an agricultural product, normal carry-overs would be highest during the period after harvest and would then decline continuously until the next harvest. According to the above theory, this would imply that the normal backwardation, $\overline{p}_f - \overline{p}_s$, would be highest in the periods of peak stocks and lowest in the periods of low stocks.

This observation is quite consistent with the known behavior of the price of storage for storable agricultural commodities (cf. Working [1948]). The price of storage is known to be small and even positive in the early marketing season of an agricultural crop and then progressively turns negative (inverse carrying charge) toward the end of the marketing season before the next harvest. Using equation (32), which expresses the price of storage as a function of the underlying normal bias and a term that depends on the current spot-price deviation from its mean, this behavior can be easily rationalized.

Consider the crop-marketing year as composed of two half periods, the earlier one of ample stocks and the later one of low stocks. Let the spot price be x below the average yearly price in the first period and x above it in the second period. By the argument above, the normal bias would be large in the first half of the year and small in the latter half. Assume, for simplicity, that the bias is close to -x in the first half of the year and close to zero in the second half. Then, according to (32), the price of storage would be close to zero in the first half of the year and close to -x in the second half--a situation that resembles the aforementioned observed behavior of the price of storage.

The above argument also points out a theoretically reasonable way to go about empirically identifying the size of the long-run normal backwardation, $\bar{p}_f - \bar{p}_s$. Notice from (30) and (32) that, when $\tilde{p}_{t-1} = 0$, then $p^f_{t,t-1} - p^e_{t,t-1} = \bar{p}_f - \bar{p}_s = p^f_{t,t-1} - p_{t-1}$. In other words, when $\tilde{p}_{t-1} = 0$, the price of storage is exactly equal to the futures price bias. It seems, therefore, that a good empirical way to test for a futures market bias would be to consider the observed price of storage in the middle of the crop year when spot prices are near their average for the year and then average this price of storage over many crop years to filter out the effects of inflation and other exogenous factors that could affect the market in any year.

VI. SUMMARY AND CONCLUSIONS

A dynamic model of the simultaneous determination of spot and futures prices has been constructed by piecing together theoretical behavior models of producers, inventory stockholders, processors, and futures speculators in the market of a storable commodity. The model produced results consistent with Keynes' theory of normal backwardation and Working's theory of the price of storage. Furthermore, it identified the determinants of both the long-run or normal futures price bias as well as the determinants of the bias that exists in every period.

The major result of the theoretical analysis was the demonstration that the futures price bias is composed of two terms: One term is constant and always negative and is a function of the amount of normal carry-overs during the period of expiration of the futures contract. The other term depends on the deviation of the current spot price from its long-run average value and is, hence, highly variable. The futures price bias was found to behave in a way opposite to that of the observable price of storage. However, in periods where the spot price is near its long-run average value, the bias is exactly equal to the observable price of storage.

The theory was used to illustrate some possible pitfalls of earlier empirical attempts to identify the futures price bias. Furthermore, the theory can be used as a guide in choosing the correct time period and futures contract for the empirical identification of futures price bias in subsequent research.

FOOTNOTES

*This paper has benefited from comments and suggestions by Gordon C. Rausser and Holbrook Working. However, the responsibility is mine for errors and omissions.

[1]See, Commodity Futures Trading Commission [1977].

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