Working Paper Series

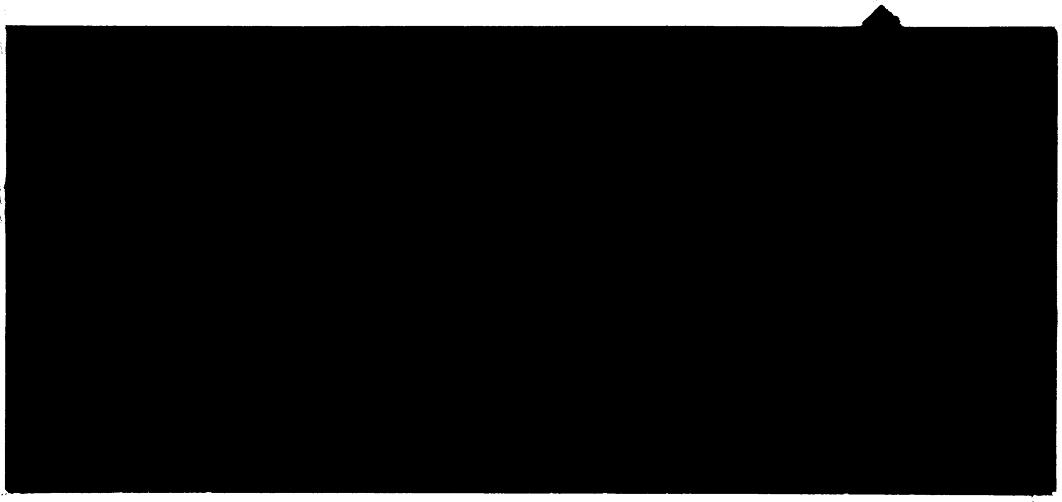
[redacted]

DEPARTMENT OF AGRICULTURAL AND
RESOURCE ECONOMICS

BERKELEY

CALIFORNIA AGRICULTURAL EXPERIMENT STATION

University of California



Division of Agricultural Sciences
UNIVERSITY OF CALIFORNIA

Working Paper No. 174 Rev.

EXPORT TAXES VERSUS BUFFER STOCKS AS OPTIMAL EXPORT POLICIES UNDER UNCERTAINTY

by

Alexander H. Sarris

California Agricultural Experiment Station
Giannini Foundation of Agricultural Economics

January, 1982

EXPORT TAXES VERSUS BUFFER STOCKS AS OPTIMAL
EXPORT POLICIES UNDER UNCERTAINTY*

by

Alexander H. SARRIS

Department of Agricultural and Resource Economics
University of California
Berkeley, California 94720

Optimal state-dependent export taxes and costly-to-store buffer stocks are compared in their welfare implications for an exporter possessing monopoly power in the international trade of a volatile commodity. Optimal stochastic control is used to derive the optimal buffer stock rules. It is shown that, if the internal and external fluctuations facing the exporter are large, if the storage costs are low, and if the price elasticity of export supply is small relative to that of export demand, the exporter would gain more from a buffer stock than from an optimal export tax. World welfare is always increased by buffer stocks, as opposed to tariffs; and, under some conditions, the foreign country might also benefit and, hence, not retaliate.

Correspondence to: Alexander H. Sarris
Department of Agricultural and Resource Economics
University of California
Berkeley, CA 94720

EXPORT TAXES VERSUS BUFFER STOCKS AS OPTIMAL EXPORT POLICIES UNDER UNCERTAINTY

1. Introduction

There has been much discussion in recent years about international commodity price instability and, in particular, the detrimental effects that it has on the economies of the developing countries that export volatile primary products. This concern had led to efforts to find ways to stabilize international commodity markets by multilateral agreements such as the United Nations Conference on Trade and Development (UNCTAD) Integrated Programme for Commodities. Most of these efforts, however, have produced little in the way of viable arrangements. This is not surprising since it is well known from the theoretical literature on commodity price stabilization [e.g., Hueth and Schmitz (1972) and Turnovsky (1978)] that any international price stabilization scheme will benefit some countries and hurt others. Obviously, the governments of the countries that are expected to lose from an agreement are reluctant at best to join any such scheme, thus making broad international cooperation on such issues virtually impossible. The recent failure of the attempts to renew the International Wheat Agreement attests to the difficulty of convincing losers to cooperate.

Given the failure of multilateral efforts, the only remaining route for Less Developed Country (LDC) exporters of volatile primary materials is to explore narrower schemes to enhance their position. It is to this problem that this paper is addressed.

Monopoly power, which is a prerequisite for any kind of action, can be obtained in the international market of any commodity by cooperation among many exporters if it is not possessed by any single country--as the

Organization of Petroleum Exporting Countries (OPEC) has so clearly demonstrated. The question, however, arises as to which export policy is best for the welfare of the group of the cooperating exporters treated as one unit under conditions of world market uncertainty. Under certainty, for example, it is well known that an optimal export tax is the policy that maximizes net welfare gains from an export monopoly. Under uncertainty, however, such a policy might be inferior on welfare grounds to an appropriately designed buffer stock scheme. It might then be optimal for the exporters to operate their own buffer stock.

In this paper, the optimal export tax policy for a monopolistic exporter is compared with an optimal buffer stock policy also operated by the exporter. The same information about domestic and foreign markets is assumed to be available to the exporter in the design of both optimal policies. Assuming a quadratic storage cost function, the optimal buffer stock policy is derived as the solution to an optimal control problem. The average expected welfare gains for the exporter under the optimal buffer stock scheme and under the optimal export tax scheme are then compared.

The results show that, when the magnitudes of storage costs are low, the coefficients of variation of the domestic and foreign excess demand fluctuations are high and, when the price elasticity of export supply is much lower than the price elasticity of foreign demand, the optimal buffer stock policy is preferred to an optimal export tax policy on welfare grounds.

In an examination of international welfare effects, it is shown that a unilaterally operated buffer stock will yield positive--albeit smaller--gains for the world as a whole than will an optimal world buffer stock. Furthermore, there are cases in which the importing country would benefit, as well, from the buffer stock policy of an exporting country. By contrast, an optimal export tax policy will necessarily reduce world welfare and hurt the importing country. This implies that a nationalistic buffer stock policy is far less likely to lead to retaliatory actions from importers.

Section 2 presents the model that is used for the analysis. Section 3 derives the optimal buffer stock rules. In section 4, a comparison of the two policies with respect to their expected welfare gains and domestic price variances is made. Section 5 analyzes the distributional implications of the two policies for the international market, and section 6 summarizes the arguments and conclusions of the paper.

2. The model

The exporting country or group of countries will be treated as one unit. The commodity under consideration is assumed to be produced and consumed competitively in the domestic market. The domestic supply and/or demand curves are subject to random shocks that translate into a random, upward-sloping excess supply for the exporting country. Linearity in the supply and demand curves and additivity of the random shocks will be assumed for simplicity of exposition. The demand for a country's exports will be represented by a downward-sloping linear excess demand curve of the rest of the world that is also subject to random shocks that are independent of domestic shocks. This last assumption is also one of convenience with results easily extendable to more complicated situations.

The excess supply curve of the exporting country (which can be thought of as the difference between domestic supply and demand) in period t is

$$ES_t = -e + fp_t + v_t, \tag{1}$$

where e, f > 0, v_t is a zero mean random variable uncorrelated over time with variance equal to σ_v^2, and p_t denotes the border price.

The world demand for the exportable commodity in period t is given by the expression

$$WD_t = g - hp_t + u_t, \tag{2}$$

where g, h > 0 and u_t is a zero mean random variable with variance σ_u^2, uncorrelated over time and independent of v_t for all t.

Under no interference from the exporting country, the unrestricted world price would be equal to

$$p_{0t} = \frac{e + g}{f + h} + \frac{u_t - v_t}{f + h} \tag{3}$$

with a variance equal to

$$\sigma_0^2 = \frac{\sigma_u^2 + \sigma_v^2}{(f + h)^2} . \tag{4}$$

The quantity exported annually can be found by substituting eq. (3) in eq. (1); it is equal to

$$q_{0t} = \frac{fg - eh}{f + h} + \frac{fu_t + hv_t}{f + h} . \tag{5}$$

To ensure positive average exports for the exporter, the following innocuous condition must be imposed:

$$fg - eh > 0 \tag{6}$$

It will be assumed that, in the implementation of the optimal policy in year t, the exporting country has perfect knowledge of the current world market disturbances, v_t and u_t, in addition to the constant values of the other parameters. This can be justified if, for example, in the beginning of a trading year, very accurate forecasts of the world excess demand and the domestic excess supply situations can be made. Although this assumption is not very realistic, it provides a very useful benchmark for further analysis. Innacurate forecasts would reduce welfare gains from any policy but would not alter the ordering of policies analyzed later.

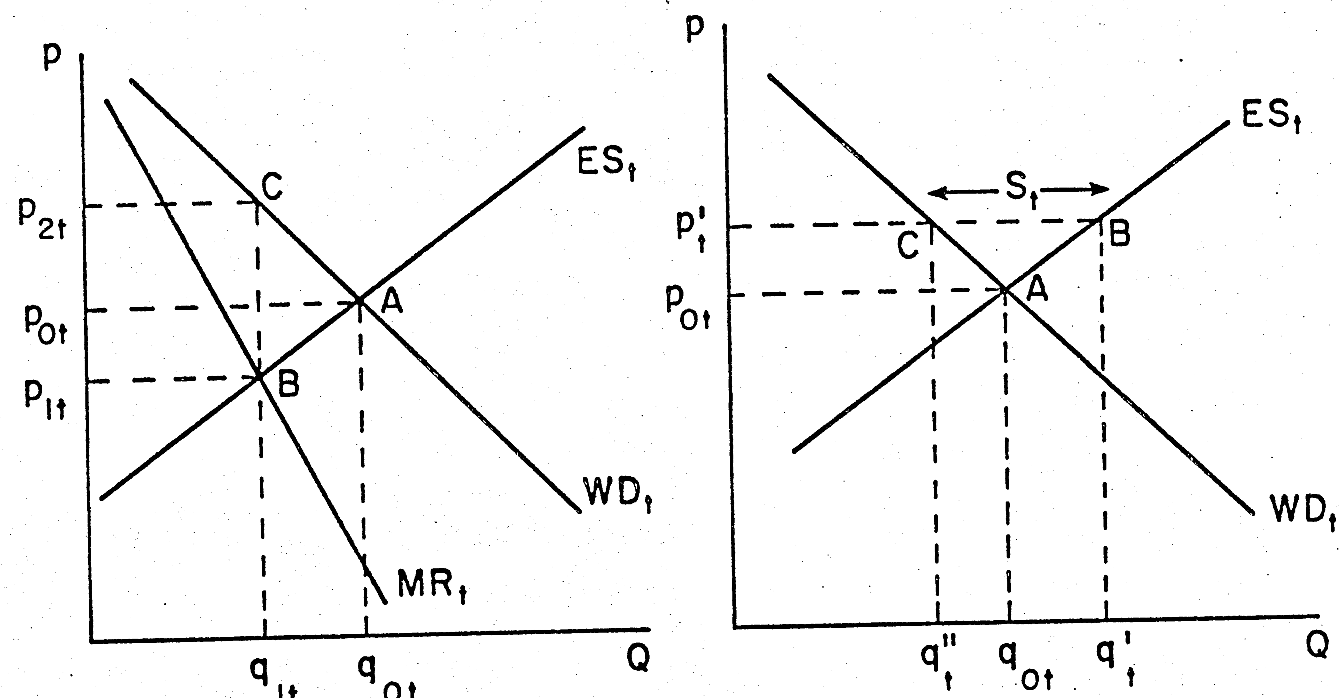
In the absence of buffer stocks, the best policy of the exporting country would be to impose an optimal export tax in every period. Fig. 1a illustrates this policy. By equating MR_t, the curve that represents the marginal revenue from exports, with the excess supply curve, ES_t, the optimal export tax in period t, $p_{1t}p_{2t}$, is obtained. The net welfare benefit to the exporting country in period t is equal to

$$W_t^T = q_{1t}(p_{2t} - p_{0t}) - \frac{1}{2}(q_{0t} - q_{1t})(p_{0t} - p_{1t}). \tag{7}$$

The optimal export tax varies from period to period as the excess supply and demand curves shift. Consequently, the expression for W_t^T in eq. (7) is also a function of the state of nature.

By using the model in eqs. (1) and (2), the following expressions can be obtained for the quantities and prices of the optimal export tax policy (notation corresponds to that of fig. 1a)

$$q_{1t} = \frac{fg - eh}{2f + h} + \frac{fu_t + hv_t}{2f + h} \tag{8}$$



a. Export Tax

b. Buffer Stock

Fig. 1. Welfare effects of an optimum export tax and a buffer stock.

$$p_{1t} = \frac{g + 2e}{2f + h} + \frac{u_t - 2v_t}{2f + h} \tag{9}$$

$$p_{2t} = \frac{g(f + h) + eh}{h(2f + h)} + \frac{u_t(f + h) - v_t h}{h(2f + h)} . \tag{10}$$

Substituting these expressions along with eqs. (3) and (5) in the expression (7) for the yearly net welfare gain and taking the expected value of the resulting function, it can be shown[1] that the average net welfare gain to the exporting country from imposing in every period the optimal export tax is equal to

$$W^T \equiv E\left(W_t^T\right) = \frac{f(fg - eh)^2}{2h(2f + h)\,(f + h)^2} + \frac{f\left(f^2\,\sigma_u^2 + h^2\,\sigma_v^2\right)}{2h(2f + h)\,(f + h)^2} \tag{11}$$

or, if we define by $\bar{q}_0$ the average exports under uninterfered trade,

$$\bar{q}_0 \equiv E(q_0) = \frac{fg - eh}{f + h} ,$$

$$W^T = \frac{f\bar{q}_0^{\,2}}{2h(2f + h)} + \frac{f\left(f^2\,\sigma_u^2 + h^2\,\sigma_v^2\right)}{2h(2f + h)\,(f + h)^2} . \tag{12}$$

Fig. 1b illustrates a situation in which the exporting country interferes in the market by operating a buffer stock. If in a particular period the parastatal agency that is empowered by the exporter to carry the buffer stock policy purchases an amount, S_t (by convention, $S_t > 0$ when there are net additions to the stock), equal to the distance, BC, in fig. 1b, then the international and domestic price is raised to p_t'. The net benefit to the domestic economy is equal to p_{0t} AB p_t'. Via the operations of the stock agency, the taxpayers in that period, however, lose an amount equal to the area, q_t' BC q_t'', which is the cost of purchasing the stocks to maintain the price at p_t'. Furthermore, the storage agency must pay for the yearly storage costs.

Denote the accumulated public stocks after the purchases during this period as I_t, where

$$I_t = \sum_{i=1}^{t} S_i = I_{t-1} + S_t. \tag{13}$$

These public stocks are assumed to be distinct from and additional to the inventories that are already privately held.[2]

Then it will be assumed that yearly storage costs are given by the quadratic function, $c\, I_t^2$, where c is some positive constant. Note that, with such a specification, "storage" costs will be incurred not only when $I_t > 0$ but also when $I_t < 0$, denoting a drawdown of total commercial plus public inventories. The positive "storage" cost when $I_t < 0$ is meant, then, to capture the loss of convenience yield[3] to the country as a whole as a result of holding a smaller amount of total stocks. The quadratic specification is a rather gross approximation to a realistic public storage cost function which is more likely to be as depicted in fig. 2. The storage cost function in fig. 2 reflects a sharply rising convenience yield of additional inventories when there is a scarcity of total stocks ($I_t < 0$); furthermore, for $I_t > 0$, it illustrates a gently rising marginal storage cost that reflects the slowly growing scarcity of aggregate storage capacity. The

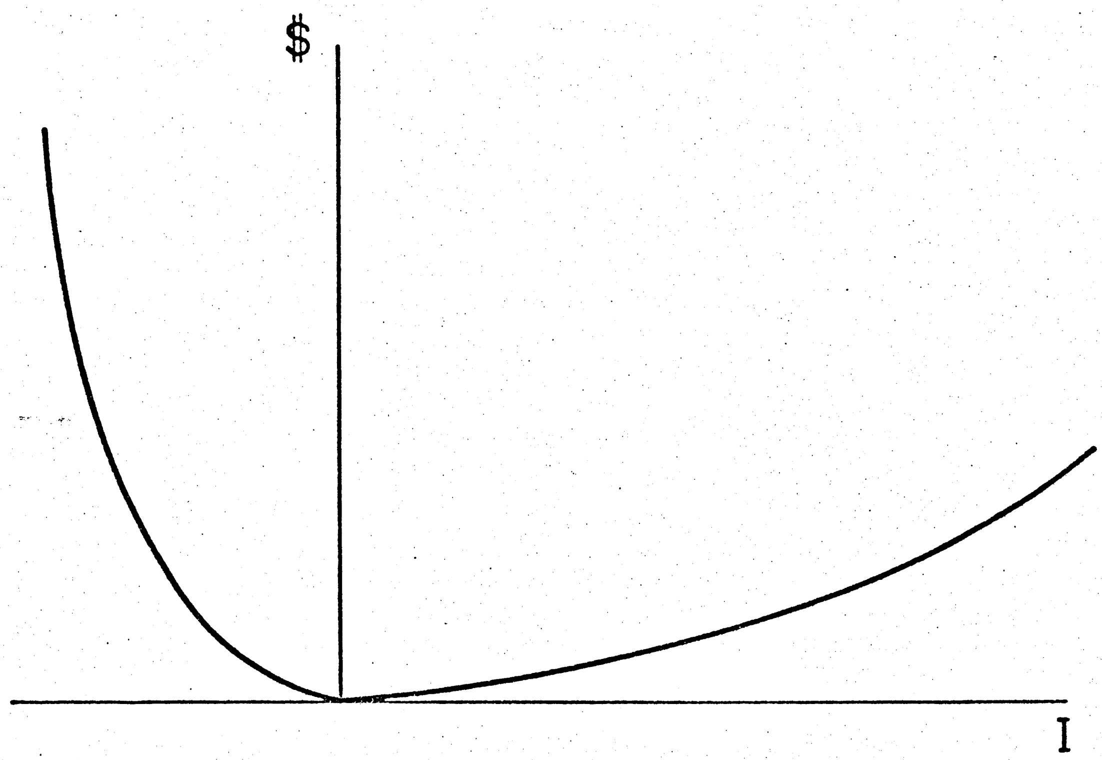


Fig. 2. A realistic public storage cost function

quadratic specification is adopted for analytical convenience only and could be easily changed in a numerical exercise.[4]

Notice, finally, that the assumption of positive publicly held stocks is not appropriate since our concern is with public and not total (private plus public) stocks which, of course, must always be positive.

The net benefit to the exporter (see fig. 1b) is then equal to

$$W_t^B = \frac{1}{2} (q_{ot} + q_t') (p_t' - p_{0t}) - S_t p_t' - c I_t^2 . \tag{14}$$

This expression holds for positive and negative S_t.

3. The optimal buffer stock policy

To find the optimal buffer stock policy, we first evaluate expression (14). The market equilibrium condition under a buffer stock policy in year t is

$$ES_t = WD_t + S_t . \tag{15}$$

Substituting eqs. (1) and (2) in eq. (15) and recalling eq. (3), the international price in year t is found to be

$$p_t' = p_{ot} + \frac{S_t}{f + h} . \tag{16}$$

The quantity, q_t', of exports can be found by substituting the expression for p_t' from eq. (16) in the equation for the excess supply curve [eq. (1)]. Substituting all of the resulting expressions in eq. (14), we obtain

$$W_t^B = -x\ S_t^2 + (y + zv_t + wu_t)\ S_t - c\ I_t^2$$
$$= -\ (x + c)\ S_t^2 + (y + zv_t + wu_t - 2c\ I_{t-1})\ S_t - c\ I_{t-1}^2\ , \tag{17}$$

where the constants, x, y, z, and w, are defined as follows:

$$x = \frac{f + 2h}{2(f + h)^2} \tag{18}$$

$$y = -\ \frac{e(f + 2h) + gh}{(f + h)^2} \tag{19}$$

$$z = \frac{f + 2h}{(f + h)^2} \tag{20}$$

$$w = -\ \frac{h}{(f + h)^2}\ . \tag{21}$$

Suppose that the buffer stock is operated for T periods. Consider the following stochastic control problem

$$\max_{\{S_1, S_2, \ldots, S_T\}} E \left\{ \sum_{t=1}^{T} \left[W_t^B\ (S_t,\ v_t,\ u_t,\ I_{t-1}) \right] \right\} \tag{22}$$

subject to

$$I_t = I_{t-1} + S_t \tag{23}$$

and the initial condition, $I_o = 0$.

The optimization allows all rules of the form

$$S_t = S_t(v_1, u_1, I_o; v_2, u_2, I_1; \ldots; v_t, u_t, I_{t-1}). \tag{24}$$

Given the quadratic nature of W_t^B, this problem can be solved analytically by applying the technique of discrete stochastic dynamic programming.[5] The backward recursive equation of dynamic programming for period t can be written as follows:

$$G_{t,T} = W_t^B(S_t, v_t, u_t, I_{t-1}) + E\left\{G^o_{t+1,T}(I_t, v_{t+1}, u_{t+1})\right\} \tag{25}$$

where $G^o_{t+1,T}$ denotes the optimal "gain to go" starting in period t + 1. Bellman's principle of optimality allows us to solve the problem in each period by maximizing $G_{t,T}$ with respect to all rules of the type $S_t(v_t, u_t, I_{t-1})$.

Starting in period T, we can solve for the optimal S_T as a linear function of v_T, u_T, and I_{T-1} and obtain $G^o_{T,T}$ as a quadratic function of I_{T-1}, v_T, and u_T. Continuing the process backward, we find that, at every step j, $G_{j,T}$ is a quadratic function of I_{j-1}, v_j, and u_j.

Assume that, in general,

$$\begin{aligned} G^o_{t+1,T} = {} & \alpha_{t+1} v_{t+1} + \beta_{t+1} u_{t+1} + \gamma_{t+1} v_{t+1} u_{t+1} + \delta_{t+1} v^2_{t+1} \\ & + \varepsilon_{t+1} u^2_{t+1} + \eta_{t+1} I_t + \pi_{t+1} I^2_t + \theta_{t+1} I_t v_{t+1} \\ & + \lambda_{t+1} I_t u_{t+1} + \mu_{t+1} . \end{aligned} \tag{26}$$

Since $I_t = S_t + I_{t-1}$ and S_t is a function of random variables that have subscripts less than or equal to t, I_t is independent of v_{t+1} and u_{t+1}. Taking the expectation of eq. (26) and substituting the resulting expression and the expression from eq. (17) into eq. (25), we find that $G_{t,T}$ is a quadratic function of S_t. Maximizing that function with respect to S_t, we obtain the following expression for the optimal public storage policy in year t:

$$S_t^o = \frac{y + \eta_{t+1} + zv_t + wu_t}{2(x + c - \pi_{t+1})} - \frac{(c - \pi_{t+1})\, I_{t-1}}{(x + c - \pi_{t+1})} . \tag{27}$$

Substituting this equation for S_t^o into the expression for $G_{t,T}$, we obtain the following expression for $G_{t,T}^o$:

$$\begin{aligned} G_{t,T}^o = {} & \alpha_t\, v_t + \beta_t\, u_t + \gamma_t\, v_t\, u_t + \delta_t\, v_t^2 + \varepsilon_t\, u_t^2 + \eta_t\, I_{t-1} \\ & + \pi_t\, I_{t-1}^2 + \theta_t\, I_{t-1}\, v_t + \lambda_t\, I_{t-1}\, u_t + \mu_t \end{aligned} \tag{28}$$

where

$$\alpha_t = \frac{(y + \eta_{t+1})\, z}{2(x + c - \pi_{t+1})} \tag{29a}$$

$$\beta_t = \frac{(y + \eta_{t+1})\, w}{2(x + c - \pi_{t+1})} \tag{29b}$$

$$\gamma_t = \frac{zw}{2(x + c - \pi_{t+1})} \tag{29c}$$

$$\delta_t = \frac{z^2}{4(x + c - \pi_{t+1})} \tag{29d}$$

$$\epsilon_t = \frac{w^2}{4(x + c - \pi_{t+1})} \tag{29e}$$

$$\eta_t = \eta_{t+1} - \frac{(c - \pi_{t+1})(y + \eta_{t+1})}{(x + c - \pi_{t+1})} \tag{27f}$$

$$\pi_t = \frac{(c - \pi_{t+1})^2}{(x + c - \pi_{t+1})} - (c - \pi_{t+1}) \tag{29g}$$

$$\theta_t = - \frac{(c - \pi_{t+1})\, z}{(x + c - \pi_{t+1})} \tag{29h}$$

$$\lambda_t = - \frac{(c - \pi_{t+1})\, w}{(x + c - \pi_{t+1})} \tag{29i}$$

$$\mu_t = \mu_{t+1} + \delta_{t+1}\,\sigma_v^2 + \epsilon_{t+1}\,\sigma_u^2 + \frac{(y + \eta_{t+1})^2}{4(x + c - \pi_{t+1})} \,. \tag{29j}$$

These recursive equations have the following boundary conditions:

$$\begin{aligned} \alpha_{T+1} &= \beta_{T+1} = \gamma_{T+1} = \delta_{T+1} = \epsilon_{T+1} = \eta_{T+1} = \pi_{T+1} = \theta_{T+1} \\ &= \lambda_{T+1} = \mu_{T+1} = 0. \end{aligned} \tag{30}$$

The crucial equations among the 10 recursive equations in eq. (29) are eqs. (29g) and (29f). Eq. (29g) can be readily shown to have two real stationary solutions, one positive and one negative. Of these, only the negative one can yield a stable stationary solution for η_t from eq. (29f). These considerations then lead to the following limiting stationary values for the first nine parameters in eq. (29).

$$\alpha^* = 0 \tag{30a}$$

$$\beta^* = 0 \tag{30b}$$

$$\gamma^* = \frac{2\ zw}{4(x + c - \pi^*)} \tag{30c}$$

$$\delta^* = \frac{z^2}{4(x + c - \pi^*)} \tag{30d}$$

$$\epsilon^* = \frac{w^2}{4(x + c - \pi^*)} \tag{30e}$$

$$\eta^* = -y \tag{30f}$$

$$\pi^* = \frac{1}{2}\left(c - \sqrt{c^2 + 4\ xc}\right) \tag{30g}$$

$$\theta^* = -\frac{(c - \pi^*)\ z}{x + c - \pi^*} \tag{30h}$$

$$\lambda^* = -\frac{(c - \pi^*)\ w}{x + c - \pi^*}\ . \tag{30i}$$

Notice that eq. (29j) does not have a limiting value. Given, however, eq. (30f) and the fact that δ_t and ϵ_t have limiting values, μ_1 can be closely approximated for large T by the expression

$$\mu_1 \approx (T - 1)\ (\delta^*\ \sigma_v^2 + \epsilon^*\ \sigma_u^2)\ . \tag{31}$$

It is clear now that the optimal value of the criterion (22) is equal to the expected value of $G^o_{1,T}$, which in turn is equal to

$$E\left(G^o_{1,T}\right) = \delta_1\ \sigma_v^2 + \epsilon_1\ \sigma_u^2 + \mu_1\ . \tag{32}$$

The average single-period gain from the optimal buffer stock policy in this T period case is equal to

$$\frac{1}{T} E\, G^{o}_{1,T} \ .$$

Letting T go to infinity and considering eq. (31) and the fact that δ_1 and ε_1 tend toward their limiting values, δ^* and E^*, we find that the average yearly net benefit to the exporting country from operating an optimal buffer stock policy is

$$W^B \equiv \lim_{T\to\infty} \frac{1}{T} E(G^{o}_{1,T}) = \delta^* \sigma_v^2 + \varepsilon^* \sigma_u^2 \ . \tag{33}$$

The optimal "steady state" buffer stock policy can be found if we substitute the stationary values of eq. (30) into eq. (27):

$$S^{o}_{T} = \frac{zv_t + wu_t}{2(x + c - \pi^*)} - \frac{(c - \pi^*)\, I_{t-1}}{(x + c - \pi^*)} \ . \tag{34}$$

For zero storage costs (namely, $c = \pi^* = 0$), W^B achieves its maximum value, which is equal to

$$W^{B}_{max} = \frac{\sigma_v^2 (f + 2h)^2 + \sigma_u^2\, h^2}{2(f + h)^2\, (f + 2h)} \ . \tag{35}$$

Notice that, if the storage cost is negligible, then the optimal buffer stock policy depends on the values of the current and not the earlier disturbances (embodied in I_{t-1}).

An interesting thing to notice from eq. (33) is that, since $\delta^*, \varepsilon^* > 0$, the exporting country always gains by operating an optimal buffer stock policy despite the fact that it pays for the stocks and the storage costs. In other words, an exporting country with monopoly power does not have to rely on international sharing of the buffer stocks or their costs in order to enhance its position. The gains to the exporter are larger for larger domestic and foreign excess demand fluctuations, lower storage costs, and higher values of the slope parameters, f and h (namely, lower values of the parameter x).

4. Optimal export taxes versus optimal buffer stocks

We will now compare the average yearly welfare gain from an optimal buffer stock in expression (33) to the average yearly welfare gain from the optimal tariff given in eq. (12). Before we do this, we revise the expression, $x + c - \pi^*$, which appears in the denominators of the expressions for both δ^* and ε^* in eq. 33 (cf. eqs. 30d and 30e).

Recall that π^* is the (negative) solution of the algebraic equation (cf. eq. 29g):

$$\pi = \frac{(c - \pi)^2}{x + c - \pi} - (c - \pi) = - \frac{x(c - \pi)}{x + c - \pi} . \tag{36}$$

From eq. (36) we obtain

$$x + c - \pi^* = - \frac{x(c - \pi^*)}{\pi^*} = \frac{x}{\rho} , \tag{37}$$

where

$$\rho = - \frac{\pi^*}{c - \pi^*} . \tag{38}$$

Substituting the expression in eq. (37) for the denominators of δ^* and ε^* in eqs. (30d) and (30e) and then using eqs. (18), (20), and (21), we obtain

$$\delta^* = \frac{(f + 2h)\,\rho}{2(f + h)^2} \tag{39}$$

$$\varepsilon^* = \frac{h^2\,\rho}{2(f + 2h)\,(f + h)^2} \quad .$$

The parameter, ρ, is a function of c as can be seen from eq. (38). Using the value of π^* from eq. (30g), we find

$$\rho = \frac{\sqrt{1 + \frac{4x}{c}} - 1}{\sqrt{1 + \frac{4x}{c}} + 1} \quad . \tag{41}$$

From eq. (41) the behavior of ρ as a function of c is apparent. The variable, ρ, is a monotonically decreasing function of c in the interval $(0, \infty)$. For $c \to 0$, $\rho \to 1$, and for $c \to \infty$, $\rho \to 0$. The last-mentioned tendency for ρ to approach zero as c tends to infinity captures the fact that welfare gains from buffer stocks become negligible as the costs of storage rise appreciably.

Given these preliminaries, we can write a general expression for the difference between W^T and W^B from eqs. (12) and (33) as follows:

$$W^T - W^B = \frac{f\bar{q}_o^2}{2h(2f + h)} + \frac{\sigma_u^2}{2(f + h)^2}\left[\frac{f^3}{h(2f + h)} - \frac{h^2\,\rho}{f + 2h}\right] + \frac{\sigma_v^2}{2(f + h)^2}\left[\frac{fh}{2f + h} - (f + 2h)\,\rho\right] \quad . \tag{42}$$

We shall examine various special cases of eq. (42).

It is clear from eq. (42) that, if σ_u^2 and σ_v^2 are small, then $W^T - W^B > 0$. So, buffer stocks should not even be considered unless the magnitudes of the excess demand fluctuations are substantial. Noting from the analysis that $0 \leq \rho \leq 1$ and that the closer ρ is to one (namely, the lower the storage costs) the more favorable the case for buffer stocks becomes, let us examine the difference, $W^T - W^B$, for the limiting case, $\rho = 1$ (the case of zero storage costs). Under this assumption, we have, after some manipulation of eq. (41),

$$W^T - W^B = \frac{f\bar{q}_o^{\,2}}{2h(2f + h)} + \frac{\sigma_u^2(f^2 - h^2)}{2h(2f + h)\,(f + 2h)} - \frac{\sigma_v^2}{2f + h}\,. \tag{43}$$

The negativity of the expression in eq. (43) provides a <u>necessary</u> but by no means sufficient condition for the preference of buffer stocks over the optimal tariffs.

Consider the case in which all of the market fluctuations are caused by domestic excess supply variability. Then a simple necessary condition for the preference by the exporter of buffer stocks over optimal yearly tariffs is

$$\frac{\bar{q}_o^{\,2}}{2(2f + h)}\left(\frac{e_s}{e_d} - \frac{2\sigma_v^2}{\bar{q}_o^{\,2}}\right) < 0, \tag{44}$$

where $e_s/e_d = f/h$ and e_s and e_d are the absolute values of the price elasticities of export supply and import demand, respectively, evaluated at $(\bar{p}_o, \bar{q}_o)$, where $\bar{p}_o = E(p_{ot})$.[6]

If the price elasticity of export supply is low relative to the price elasticity of foreign import demand, and, in addition, if the coefficient of variation of export supply (the quantity $\sigma_v/\bar{q}_o$) is high, then condition (44) is likely to be satisfied and buffer stocks might indeed be a better policy than tariffs.

If market fluctuations are caused totally by fluctuations in the import market, then a necessary condition for the preference of buffer stocks over optimal tariffs is

$$\frac{f\,\sigma_u^2}{2h(2f+h)}\left(\frac{\bar{q}_o^{\,2}}{\sigma_u^2}+\frac{e_s^2-e_d^2}{e_s(e_s+2e_d)}\right)<0\,. \tag{45}$$

If the price elasticity of export supply is low relative to the price elasticity of foreign demand and if, furthermore, the coefficient of variation of foreign import demand ($\sigma_u/\bar{q}_o$) is high, then condition (45) is likely to be fulfilled and buffer stocks would be preferred to optimal tariffs. These general considerations do not change when we include storage costs.

In summary, it appears that a country should consider implementing a buffer stock instead of an optimal export tax as a means of exploiting international monopoly power if all of the following conditions hold:

1. The coefficients of variation of its export supply and the foreign import demand are high.
2. The price elasticity of export supply is small compared to the price elasticity of the foreign import demand.
3. The costs of storage are low.

5. International repercussions of national buffer stocks

One of the considerations that makes exporters with monopoly power think twice about imposing export taxes in an effort to exploit their international monopoly power is the threat of retaliation by importers. Ever since the classic study by Johnson (1953-54), it has been known that--if the foreigners retaliate through imposition of tariffs--it is possible that everybody will be be worse off than they were before the trade taxes were enacted. Furthermore, even if there is no retaliation, there is always a net loss in world welfare represented in fig. 1a by the triangle, ABC. These arguments, needless to say, carry over to the uncertainty case. Average world welfare loss in this case would be measured by the expected value of the area inside the triangle, ABC (cf. fig. 1a).

The implementation of a buffer stock, however, engenders quite different worldwide effects. It will be shown in this section that expected world welfare always increases from the implementation of an optimal buffer stock policy by one country. Furthermore, it will be shown that, under some conditions, the threat of retaliation is nullified because the foreign country benefits, as well, although it does not participate in the design of the buffer stock policy.

From fig. 1b it can be seen that the welfare gain (loss if negative) of the importer in period t from implementation of an optimal buffer stock policy by the exporter is equal to

$$W_t^F = \frac{1}{2}\left(p_{ot} - p_t'\right)\left(q_{ot} + q_t''\right) . \tag{46}$$

Using the expressions derived earlier, this can be written as

$$W_t^F = \frac{1}{(f+h)^2}\left[\frac{h}{2}\left(S_t^o{}^2\right) - (fg - eh + fv_t + hu_t)\, S_t^o\right], \tag{47}$$

where S_t^o is the exporter's optimal policy [eq. (34)].

The quantity we are interested in is the following:

$$W^F = \lim_{T \to \infty} \frac{1}{T}\, E\left(\sum_{t=1}^{T} W_t^F\right). \tag{48}$$

In eq. (48), W^F represents the average yearly welfare gain (or loss) to the importer when the exporter implements an optimal buffer stock policy for a long period of time.

In the Appendix it is shown that

$$W^F = \frac{1}{(f+h)^2}\left[\frac{hz\,\sigma_v^2}{2\omega}\left(\frac{z}{2\omega(2-\phi)} - 1\right) + \frac{w\,\sigma_u^2}{2\omega}\left(\frac{hw}{2\omega(2-\phi)} - f\right)\right], \tag{49}$$

where

$$\omega = x + c - \pi^* \tag{50}$$

$$\phi = \frac{c - \pi^*}{x + c - \pi^*}, \qquad 0 \le \phi \le 1\,. \tag{51}$$

Expression (49) is a monotonically decreasing function of c.[7] The maximum of W^F is attained for c = 0 and (after appropriate substitutions and some algebraic rearranging) is equal to

$$W^F_{max} = \frac{h}{2(f+h)^2}\left[\frac{\sigma_u^2[h^2 + 2f(f+2h)]}{(f+2h)^2} - \sigma_v^2\right] . \tag{52}$$

Notice that eq. (52) is likely to become positive when the fluctuations of import demand are much larger than the fluctuations of export supply. This conclusion still holds when storage costs are nonzero.

World net welfare gains can be found by adding W^F from eq. (49) and W^B from eq. (33). The resulting, rather cumbersome expression is always positive.[8]

The maximum value of $W^F + W^B$ resulting from zero storage costs can be readily evaluated by adding the expressions in eqs. (52) and (35), and it results in the simple formula

$$W^B_{max} + W^F_{max} = \frac{\sigma_u^2\ h(2f+3h)}{2(f+h)\ (f+2h)^2} + \frac{\sigma_v^2}{2(f+h)} . \tag{53}$$

As the storage cost, parameter c, increases, world welfare declines; but it never becomes negative.

The implications of the analysis above are the following: Whenever an exporting country finds it advantageous from its national viewpoint to implement an optimal buffer stock policy, it also contributes toward an improvement in world welfare. Furthermore, such a nationalistic policy might not evoke retaliatory policies from importers because the importers themselves might receive a windfall welfare gain.

The methodology and procedures developed earlier can also be used to answer the question, What is an optimal buffer stock policy for the world as a whole? Without presenting the analysis, the answer is the following: The optimal buffer stock policy for the world as a whole is given by

$$S^o_{Wt} = \frac{z'v_t + w'u_t}{2(x' + c - \pi')} - \frac{c - \pi'}{x' + c - \pi'} I_{t-1}. \tag{54}$$

where

$$x' = \frac{1}{2(f + h)}, \quad z' = \frac{1}{f + h}, \quad w' = -\frac{1}{f + h} \tag{55}$$

$$\pi' = \frac{1}{2}\left(c - \sqrt{c^2 + 4x'c}\right). \tag{56}$$

The average yearly world welfare gain from applying the optimal buffer stock policy is equal to

$$W^W = \frac{(z')^2 \sigma_v^2 + (w')^2 \sigma_u^2}{4(x' + c - \pi')}. \tag{57}$$

Interestingly enough, when c = 0, eq. (54) reduces to $S^o_{Wt} = v_t - u_t$. This is a policy that implies complete price stabilization at a price equal to $E(p_{ot})$. This then shows that price stabilization via buffer stocks is a welfare-optimal policy for the world as a whole if storage costs are negligible. If storage costs are nonzero, complete price stabilization is never optimal, but some degree of stabilization is optimal. This is true because the variance of world price under a buffer stock policy [such as eq., (54)] can be readily shown to be lower than the variance of the free-trade price.

The value of W^W can also be shown to be larger than the value of $W^F + W^B$. This highlights the fact that a nationalistic optimal buffer stock policy, although it improves world welfare, does not achieve the highest world welfare possible.

6. Concluding remarks

It has been shown that the condition of low storage costs, high domestic and/or foreign fluctuations, and a low price elasticity of excess supply relative to the price elasticity of foreign demand should make it advantageous for an exporter (or group of exporters) with international monopoly power to implement a buffer stock policy, as opposed to the best alternative policy, namely, a yearly computed optimum export tax. Such a policy would tend to stabilize the world price and might even benefit the importer thus lessening the chance of retaliatory action. This is quite different from the situation with an optimal export tax which--by necessarily hurting the importer--might evoke retaliation.

In the real world, tariffs and export taxes are fixed and not adjusted from year to year, mainly for practical reasons. The optimal value for a constant <u>ad valorem</u> export tax under uncertainty [the formula is rather complicated and can be found in Sarris (1979b)] necessarily yields a net welfare gain to the exporting country lower than what was computed earlier [cf. eq. (12)] thus making the case for a buffer stock even stronger.

The implementation of a buffer stock policy, however, will also yield gains somewhat lower than the ones computed earlier [cf. eq. (33)]. This is because practical implementation will necessitate forecasts of the values of the yearly domestic and foreign fluctuations, v_t and u_t, and the errors of these forecasts are likely to be far from negligible. However, the agency that is in charge of the buffer stock could update and correct its rule as more accurate information about v_t and u_t becomes available in the course of a marketing period thus minimizing potential losses.

A policy of combining export taxes with buffer stocks could also be considered. Such a policy will necessarily be better than either one alone. This is true because choosing two variables optimally in every period (rather than choosing one) necessarily leads to higher flexibility and welfare.

In an era of increasing uncertainty and unwillingness of developed countries to enter internationally cooperative market management arrangements for volatile commodities of importance to LDCs and in a period during which threats of retaliation to export tax or subsidy policies are more likely to be carried out, the organization of national or regional buffer stock schemes might be the only way for primary exporting nations to exploit whatever monopoly power they possess.

Appendix

Derivation of the average importer welfare gain

From eq. (34) it is easy to see that $E(S_t^o) = 0$. This follows from the initial condition, $I_o = 0$, and the fact that all v_t and u_t are random variables with zero means. Taking the expected value of W_t^F in eq. (46) and using eq. (34), we obtain

$$E\left(W_t^F\right) = \frac{1}{(f+h)^2}\left[\frac{h}{2}\, E\left(S_t^o\right)^2 - \left(\frac{fw\ \sigma_u^2}{2\omega} + \frac{hz\ \sigma_v^2}{2\omega}\right)\right] \tag{A1}$$

where we have defined $\omega \equiv x + c - \pi^*$.

Squaring S_t^o in eq. (34) and taking its expected value, we obtain

$$E\left(S_t^o\right)^2 = \frac{z^2\ \sigma_v^2}{4\omega^2} + \frac{w^2\sigma_u^2}{4\omega^2} + \phi^2\ E(I_{t-1})^2 \tag{A2}$$

where

$$\phi \equiv \frac{c - \pi^*}{x + c - \pi^*} < 1 \ .$$

Since

$$I_{t-1} = S_{t-1}^o + I_{t-2} = \frac{zv_{t-1} + wu_{t-1}}{2\omega} + (1 - \phi)\ I_{t-2}, \tag{A3}$$

we obtain that

$$E(I_{t-1})^2 = \frac{z^2\ \sigma_v^2}{4\omega^2} + \frac{w^2\ \sigma_u^2}{4\omega^2} + (1 - \phi)^2\ E(I_{t-2})^2. \tag{A4}$$

Using eq. (A4) repeatedly, we have

$$E(I_{t-1})^2 = \frac{z^2\sigma_v^2 + w^2\ \sigma_u^2}{4\omega^2} \cdot \frac{1-(1-\phi)^{2t}}{1-(1-\phi)^2}. \tag{A5}$$

Substituting eq. (A5) in eq. (A2), we obtain

$$E\left(S_t^o\right)^2 = \frac{z^2\ \sigma_v^2 + w^2\ \sigma_u^2}{4\omega^2}\left(\frac{2-\phi(1-\phi)^{2t}}{2-\phi}\right). \tag{A6}$$

Substituting now eq. (A6) into eq. (A2), we derive the following expression:

$$\frac{1}{T}\sum_{t=1}^{T} E\left(W_t^F\right) = \frac{1}{(f+h)^2}\left\{\frac{h}{2}\left(\frac{z^2\ \sigma_v^2 + w^2\ \sigma_u^2}{4\omega^2}\right)\left[\frac{2}{2-\phi} - \frac{\phi\left[1-(1-\phi)^{2T+2}\right]}{T(2-\phi)\ [1-(1-\phi)^2]}\right] - \left(\frac{hz\ \sigma_v^2 + fw\ \sigma_u^2}{2}\right)\right\}. \tag{A7}$$

We can now easily obtain

$$W^F = \lim_{T\to\infty}\sum_{t=1}^{T} E\left(W_t^F\right) = \frac{1}{(f+h)^2}\left(\frac{h}{2-\phi}\ \frac{z^2\ \sigma_v^2 + w^2\ \sigma_u^2}{4\omega^2} - \frac{hz\ \sigma_v^2 + fw\ \sigma_u^2}{2\omega}\right). \tag{A8}$$

Footnotes

*Giannini Foundation Paper No. 640 (reprint identification only). My special thanks to an unknown referee whose perceptive comments have led to substantial improvements of the original analysis.

[1]E(·) will denote the expected value of the variable or expression inside of the parentheses.

[2]As Newberry and Stiglitz (1981, Chapter 14) and Sarris (1979a) have shown, the public stocks will generally not be additional to the privately held ones because private stockholders will change their inventories in response to the government policy. In general, the public stocks will have to increase by more than, say, α to achieve a net addition to total stocks of α. Additionality of public stocks will occur only if private stockholders are infinitely risk averse. This consideration implies that the public profits from purchases and sales of stocks, as well as the public storage costs, probably are underestimates of the true ones. Since, however, total public agency gains are equal to the difference between these two quantities, the net effect is unclear.

[3]For expositions of the theory of convenience yield, see Brennan (1958) and Working (1949).

[4]For a recent empirical analysis using a different storage cost function, see Burt, Koo, and Dudley (1980).

[5]For a complete description and some applications of this elegant and powerful technique for solving stochastic control problems, see Chow (1975).

[6]Because it is the ratio of the elasticities that matters for the signing of expression (44), the particular point at which the elasticities are evaluated does not matter as long as it is an international equilibrium point.

[7]This is seen clearly by noting that $\omega(2 - \phi) = 2x + c - \pi^*$, which is an increasing function of c.

[8]To show this, notice that the only negative term of $W^F + W^B$ is the one multiplying σ_v^2 in the expression for W^F in eq. (49). Considering now only the coefficient of σ_v^2 in the total expression, $W^F + W^B$, and taking into account eqs. (30d) and (50), we obtain that the coefficient is equal to

$$\frac{hz^2}{(f + h)^2\, 4\omega^2(2 - \phi)} + \frac{zf}{4\omega(f + h)^2} > 0 \; .$$

References

Brennan, M. J., 1958, The supply of storage, American Economic Review, 47, 50-72.

Burt, O. R., W. W. Koo and N. J. Dudley, 1980, Optimal stochastic control of U. S. wheat stocks and exports, American Journal of Agricultural Economics, 62, 172-187.

Chow, G. C., 1975, Analysis and Control of Dynamic Economic Systems (John Wiley and Sons, Inc., New York).

Hueth, D. and A. Schmitz, 1972, International trade in intermediate and final goods: Some welfare implications of destabilized prices, Quarterly Journal of Economics 86, 351-365.

Johnson, H. G., 1953-54, Optimum Tariffs and Retaliation, Review of Economic Studies XXI, 142-153; also in Johnson, H. G., 1967, International Trade and Economic Growth (Harvard University Press, Cambridge, MA).

Newberry, D. M. and J. E. Stiglitz, The Theory of Commodity Price Stabilization (forthcoming).

Sarris, A. H., 1979a, Public intervention in speculative commodity markets, Working Paper No. 91 (Giannini Foundation of Agricultural Economics, University of California, Berkeley, CA).

Sarris, A. H., 1979b, Optimal international commercial policies under uncertainty. Paper presented at the North American Winter Meeting of the Econometric Society, Atlanta, GA.

Turnovsky, S. J., 1978, The distribution of welfare gains from price stabilization: A survey of some theoretical issues, in F. G. Adams and S. Klein (eds.), Stabilizing World Commodity Markets (Lexington Book, MA).

Working, H., 1949, The theory of the price of storage, American Economic Review 31, 1254-1262.