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A DYNAMIC PUTTY-CLAY MODEL OF POLLUTION CONTROL

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A DYNAMIC PUTTY-CLAY MODEL OF POLLUTION CONTROL[1]

1. INTRODUCTION

THE PUTTY-CLAY APPROACH was introduced by Johansen [8] and Salter [11] in the late 1950s and came into common use in macroeconomic growth models such as those of Solow [12], Massell [9], Phelps [10], and Bliss [2] as well as in some microeconomic models such as that of Hjalmarsson [5]. These developments offer important alternatives to neoclassical production theory that take account of the Cambridge School's objections to assumptions of malleability of capital. The putty-clay approach specifically permits the existence of different capital goods that are not interchangeable.

More recently, Hochman and Zilberman [6] have demonstrated some important short-run implications of the putty-clay approach in the context of pollution control. The imposition of new pollution controls or revision of existing ones may cause some firms to fail in the short run because of short-run production rigidities attached to nonmalleable capital. In this context, Hochman and Zilberman [6] examine policy trade-offs between output, pollution, and employment in a competitive industry. They specifically focus on the comparative effects of the two most popular pollution controls--Pigouvian taxes and the standards and pricing approach of Baumol and Oates [1]. Their results show that, in the short run, taxes provide a first-best policy which both minimizes cost and maximizes the sum of producer and consumer surpluses in attaining a given environmental goal. However, taxation may, in fact, increase the amount of pollution per unit of output, and it generally results in lower output and higher prices than standards which attain the same pollution

goal. Hochman and Zilberman thus conclude that one explanation for the prevalent use of standards in reality is due to inflation and unemployment considerations.

The purpose of this paper is to extend these considerations to the longer run case where the regulated polluting industry has the opportunity to adjust to regulation by installing new capital goods and using new techniques. The questions of comparative effects of taxes and standards on output growth and, specifically, on inflation and unemployment raised by Hochman and Zilberman's work are basically dynamic questions and, thus, cannot be answered in a short-run framework. By considering the time path of investment induced by pollution regulation, the analysis here yields simple expressions for the industry's production, output price, pollution, and employment over time which enable comparison of the effects of taxes and standards over time.

While the analysis in Hochman and Zilberman relies heavily on Johansen's approach, the analysis in this paper is inspired by the intuitive approach of Salter. Here, however, Salter's model is both quantified and modified to include pollution and environmental policies. Two key assumptions are made to bring the problem into a manageable framework: (1) continuous market clearing and (2) instantaneous introduction of capital. These assumptions are necessary to attain continuous Salter equilibrium.

Through explicit treatment of future price expectations, two distinguished types of Salter equilibrium result. The industry may be in Salter equilibrium with an expected rate of price growth different from the actual rate; it can also be in Salter equilibrium with fulfilled price expectations. The latter case (fulfilled price expectations) defines a more extensive concept of equilibrium which will be called <u>expectations equilibrium</u>. It can be proved

that, during a period in which policy parameters are not changed, the regulated industry attains an expectations equilibrium path from any initial point (Zilberman [13]). Thus, investigation of the behavior in expectations equilibrium is important for understanding the dynamic behavior of an industry.

While the expectations equilibrium behavior of output price and quantity can be expressed by manageable and meaningful mathematical expressions, the expressions for the behavior of pollution and labor use are complicated and difficult to interpret. To understand the behavior of these variables, yet a further equilibrium concept is developed--long-run expectations equilibrium (LREE). An industry which operates as if in expectations equilibrium for all time under the same policy parameters is said to be in LREE. Output price and production are the same for an industry in expectations equilibrium and a hypothetical industry with the same parameters which is in LREE; and, furthermore, Zilberman [13] shows that pollution and aggregate labor-use functions do not deviate substantially from the expectations equilibrium functions. Therefore, the LREE functions are instructive for policy analysis.

In this context the results of this paper show that imposition of a pollution tax which achieves a constant rate of pollution is socially optimal and that a constant pollution rate can be attained by increasing the pollution tax over time at a rate which depends on production function elasticities, the rate of technological change, the rate of growth of wages, the demand elasticity, and the rate of growth in demand. In addition, results show that the same constant pollution rate can be attained by imposing a pollution/production standard if the pollution/production standard is tightened over time at a rate depending on the same factors. By way of comparison of tax and standard policies which attain the same such pollution rates, it is found that the rates

of growth in price and output are the same but that the length of productive life of capital is higher under standards where the relative cost of pollution imposed on aging capital is lower; as a result, the absolute level of output might be higher under standards than under taxes (and, of course, prices are lower). It thus appears that the usual anti-inflation argument in favor of standards only has merit on a one-time basis in the context of changing from taxes to standard; after adjustment, the rate of inflation is the same as under tax regulation while social welfare is reduced.

To develop these results, output is assumed to be produced with pollution as well as capital and labor. Pollution is considered as an input to simplify the analysis. Considering pollution as an input is not unreasonable and, in some cases, is even natural. For example, in agricultural production processes using pesticides, the pesticides are inputs and pollutants at the same time. In many other cases, models which consider pollution as an input can be transformed into models with pollution as an output (and vice versa) without affecting the outcome. A possible rationalization for considering pollution as an input is to view environmental quality as a good of which there is a given endowment for each period. Pollution is then the quantity of environmental quality used in production.

Specifically, the ex ante production function which determines the technology of new production capacity at any given time follows a linear homogeneous Cobb-Douglas production function;[2] ex post production follows Leontief functions according to the putty-clay approach. The state of technology at any given time is reflected by the multiplicative factor in the Cobb-Douglas function. Thus, technical changes are neutral and result in an

isoelastic shift of the production function. To enable comparison of investments in different periods, capital in the ex ante production function is measured by the initial investment at constant prices. This definition thus requires the use of some price index of capital goods.

Within this framework, the industry is assumed to be competitive and to maximize profits. Finally, to bring the dynamic problem into tractable proportions which can be meaningfully discussed, exogenous factors are assumed to follow exponential growth paths. For example, the wage rate relative to the price of capital is assumed to grow exponentially at a rate which does not exceed the interest rate;[3] the multiplicative factors reflecting demand growth and the state of technology grow exponentially. When imposed, the tax on pollution grows exponentially and the standard on the pollution-output ratio declines exponentially. The interest rate, however, is constant. In addition, entrepreneurs expect output price to grow exponentially at a rate which is a convex combination of the previous price growth rate expectation and the observed rate.[4] For simplicity, however, all firms are assumed to correctly forecast wage rates.

2. THE MODEL

Suppose production at time t (time subscripts are omitted for convenience) follows constant returns technology with unit isoquant

$$(1) \qquad 1 = e^{nt} A k^{\alpha_1} \ell^{\alpha_2} \gamma^{1-\alpha_1-\alpha_2}$$

where k is the capital-output ratio, ℓ is the labor-output ratio, γ is the pollution-output ratio, ($n > 0$, $A > 0$), and $0 \leq \alpha_1 \leq 1$, $0 \leq \alpha_2 \leq 1$, $0 \leq \alpha_1 + \alpha_2 \leq 1$. To simplify the analysis, let capital be used as a numeraire so that the capital price index is constant; and so, without loss of generality, the price of capital input is identically 1. To formalize other assumptions regarding exogenous factors, assume

$$(2) \qquad w(\tau) = w(t)e^{g(\tau-t)}$$

$$(3) \qquad v(\tau) = v(t)e^{s(\tau-t)}$$

$$(4) \qquad P^{e}_{\tau}(\tau) = P(t)e^{\lambda(t)(\tau-t)}$$

where g is the rate of wage growth ($g > 0$), s is the rate of emission tax growth, and $\lambda(t)$ is the perceived rate of price growth at time t. The perceived rate of output-price growth follows the adaptive expectations mechanism

$$(5) \qquad \dot{\lambda}(t) = \beta \left[\frac{\dot{P}(t)}{P(t)} - \lambda(t)\right], \qquad 0 \leq \beta \leq 1.$$

The constant return-to-scale assumption implies that the technologies chosen by the industry are the ones that maximize the perceived discounted profit per unit of output capacity when they are introduced. Since output price is assumed to grow slower than labor cost, capacity introduced at time t will have finite production life, denoted by $\bar{t}(t)$. Assuming that new capacity is introduced at every point in time, firms will utilize all the capacity introduced at time t during the period $[t, t + \bar{t}(t)]$ and cease using it thereafter.

Following the above arguments, using (2), (3), and (4) and assuming that the price of capital is 1 for all t, the optimal technology for time t is determined by solving

$$(6) \qquad \max_{\bar{t},\, k,\, \ell,\, \gamma} \quad \frac{P}{r_1(\lambda, \bar{t})} - \frac{w\ell}{r_1(g, \bar{t})} - \frac{v\gamma}{r_1(s, \bar{t})} - k$$

subject to (1) where

$$r_1(a, \bar{t}) = (r - a)\left[1 - e^{-(r-a)\bar{t}}\right]^{-1}$$

is the principal and interest payment rate associated with amortization over a life of $\bar{t}$ at interest rate $r - a$. (Note that P, w, ℓ, v, γ, k, and $\bar{t}$ are functions of time; for simplicity, the argument is omitted from these functions.)

Solving the optimization problem yields the following Lemma.

LEMMA 1: The optimal technology of machines introduced at time t with pollution taxes follows.

$$(7)\qquad k^*(t) = \frac{e^{-\eta t}}{A}\left[\frac{\alpha_1}{2^{\alpha}r_1(g,\bar{t})}\frac{w(t)}{}\right]^{\alpha_2}\left[\frac{\alpha_1}{1-\alpha_1-\alpha_2}\frac{v(t)}{r_1(s,\bar{t})}\right]^{1-\alpha_1-\alpha_2}$$

$$(8)\qquad \ell^*(t) = \frac{e^{-\eta t}}{A}\left[\frac{\alpha_2}{\alpha_1}\frac{r_1(g,\bar{t})}{w(t)}\right]^{\alpha_1}\left[\frac{\alpha_2}{1-\alpha_1-\alpha_2}\frac{v(t)}{r_1(s,\bar{t})}\frac{r_1(g,\bar{t})}{w(t)}\right]^{1-\alpha_1-\alpha_2}$$

$$(9)\qquad \gamma^*(t) = \frac{e^{-\eta t}}{A}\left[\frac{1-\alpha_1-\alpha_2}{\alpha_2}\frac{r_1(s,\bar{t})}{v(t)}\right]^{\alpha_1}\left[\frac{1-\alpha_1-\alpha_2}{\alpha_2}\frac{w(t)}{r_1(g,\bar{t})}\frac{r_1(s,\bar{t})}{v(t)}\right]^{\alpha_2}$$

$$(10)\qquad P(t)\,e^{\lambda(t)\bar{t}(t)} = w(t)\,e^{g\bar{t}(t)}\,\ell(t) + v(t)\,e^{s\bar{t}(t)}\,\gamma(t).$$

A second-order condition for the optimal choice is

$$(11)\qquad (r-\lambda)\,Pe^{-(r-\lambda)\bar{t}} + \frac{k}{\alpha_1}\left\{\alpha_2\,r_2(g,\bar{t})\,[1-r_1(g,\bar{t})] + (1-\alpha_1-\alpha_2)\,r_2(s,\bar{t})\,[1-r_1(s,\bar{t})]\right\} > 0$$

where

$$r_2(a,\bar{t}) = (r-a)\,e^{-(r-a)\bar{t}}\left[1-e^{-(r-a)\bar{t}}\right]^{-1}.$$

PROOF: To simplify the analysis, the optimization is performed in two stages. First, optimal $\gamma(t)$, $\ell(t)$, and $k(t)$ are computed for a given $\bar{t}(t)$; second, profit as a function of $\bar{t}(t)$ is maximized, and the optimal $\bar{t}(t)$ is derived. In the first stage the optimization problem is to find

$$(12)\qquad \pi(\bar{t}) = \max_{\gamma,\ell,\,k}\ \frac{P}{r_1(\lambda,\bar{t})} - \frac{w\ell}{r_1(g,\bar{t})} - \frac{v\gamma}{r_1(s,\bar{t})} - k$$

subject to (1), where $\pi(\bar{t})$ is perceived profit for an optimal policy at a given time $\bar{t}$. Since $P/r_1(\lambda, \bar{t})$ is constant for a given $\bar{t}$, the optimization problem in (12) collapses into a standard cost minimization choice along an isoquant of a Cobb-Douglass production function where the input prices are $w/r_1(g, \bar{t})$, $v/r_1(s, \bar{t})$, and 1. The first-order conditions for such a problem consist of the isoquant equation [equation (1) in this case] and the conditions which equate marginal rates of substitution to input price ratios;

$$(13) \qquad \frac{w}{r_1(g, \bar{t})} = \frac{\alpha_2}{\alpha_1} \frac{k}{\ell}$$

$$(14) \qquad \frac{v}{r_1(s, \bar{t})} = \frac{1 - \alpha_1 - \alpha_2}{\alpha_1} \frac{k}{\gamma}.$$

By introducing (13) and (14) into (1), one can derive (7); substituting the explicit expression for k* into (13) and (14) yields (8) and (9).

To find the optimality condition determining the perceived length of capital introduced at time t, introduce (13) and (14) into (12) to yield

$$(15) \qquad \pi(\bar{t}) = \frac{P(t)}{r_1(\lambda, \bar{t})} - \frac{k}{\alpha}.$$

Differentiating (15) with respect to $\bar{t}$ yields

$$(16) \qquad \frac{\partial \pi}{\partial \bar{t}} = Pe^{-(r-\lambda)\bar{t}} - \frac{k}{\alpha_1} [\alpha_2 \, r_2(g, \bar{t}) + (1 - \alpha_1 - \alpha_2) \, r_2(s, \bar{t})].$$

By introducing (12) and (13) into (15), equating to zero, and dividing both sides by e^{-rt}, (10) is derived. The second-order condition for choosing the optimal perceived life of capital is $\partial^2\pi/\partial t^2 < 0$. Thus, differentiating (16) with respect to $\bar{t}$ yields the second-order condition in (11).

3. POLLUTION CONTROL OVER TIME IN AN INDUSTRY REGULATED BY TAXES

Suppose that the industry is in Salter equilibrium. In this case, prices are determined so as to clear the market of the output good instantaneously and continuously. Assuming that new capital goods can be introduced instantaneously and that new capacity is added continuously, the equilibrium price at each point in time is determined such that perceived profit is zero. Using (12) and (15) obtains

$$(17) \qquad P = r_1(\lambda, \bar{t}) \left[\frac{w\ell}{r_1(g, \bar{t})} + \frac{\gamma v}{r_1(s, \bar{t})} + k \right] = r_1(\lambda, \bar{t}) \frac{k}{\alpha_1} .$$

Further, introducing the definition of k(t) from (7) obtains

$$(18) \qquad P(t) = r_1(\gamma, \bar{t}) \frac{e^{-\eta t}}{A\alpha_1} \left[\frac{\alpha_1}{\alpha_2} \frac{w(t)}{r_1(g, \bar{t})} \right]^{\alpha_2} \left[\frac{\alpha_1}{1 - \alpha_1 - \alpha_2} \frac{v(t)}{r_1(s, \bar{t})} \right]^{1-\alpha_1-\alpha_2} .$$

Hence, the price level at time t is a function of given parameters and exogenous factors (technological parameters, wages, taxes, and interest rate) and two endogenous variables (the perceived rate of price growth and the perceived length of productive life of capital of vintage t). Further manipulations also obtain:

PROPOSITION 1: The perceived length of operation of capital goods of vintage t is increasing in the perceived rate of output price growth and declining in the rates of wage growth and pollution tax growth; in addition, the perceived length of operation of capital goods of vintage t is increasing, constant, or decreasing depending on whether the rate of wage growth is greater than, equal to, or less than the rate of growth of pollution tax, i.e.,

$$(19) \qquad \frac{\partial \bar{t}(t)}{\partial \alpha_2} \gtrless 0 \qquad \Leftrightarrow \qquad g \gtrless s.$$

PROOF: By introducing the price equation (17) into (16), equating it to zero, and writing $r_1(\cdot, \cdot)$ explicitly whenever it appears, $\bar{t}(t)$ can be determined as the solution of

$$(20) \quad \frac{[r - \lambda(t)]\, e^{\lambda(t)\bar{t}(t)}}{1 - e^{-[r-\lambda(t)]\,\bar{t}(t)}} = \frac{\alpha_2(r - g)\, e^{g\bar{t}(t)}}{1 - e^{-(r-g)\bar{t}(t)}} + \frac{(1 - \alpha_1 - \alpha_2)\,(r - s)\, e^{s\bar{t}(t)}}{1 - e^{-(r-s)\bar{t}(t)}}.$$

By introducing (17), the second-order condition in (11) further implies

$$(21) \quad J = r_1(\lambda, \bar{t}) \cdot r_2(\gamma, \bar{t}) - \alpha_2\, r_1(g, \bar{t})\, r_2(g, \bar{t}) - (1 - \alpha_1 - \alpha_2)\, r_1(s, \bar{t})\, r_2(s, \bar{t}) > 0.$$

Differentiation of (20) yields

$$(22) \quad \begin{bmatrix} \frac{\partial \bar{t}(t)}{\partial \lambda(t)} \\ \frac{\partial \bar{t}(t)}{\partial g} \\ \frac{\partial \bar{t}(t)}{\partial s} \\ \frac{\partial \bar{t}(t)}{\partial \alpha_1} \\ \frac{\partial \bar{t}(t)}{\partial \alpha_2} \end{bmatrix} = \frac{1}{J} \begin{bmatrix} m_1(\lambda) \\ -\alpha_2 m_1(g) \\ -(1 - \alpha_1 - \alpha_2)\, m_1(s) \\ m_2(s) \\ -[m_2(g) - m_2(s)] \end{bmatrix}$$

where

$$m_1(a) = \frac{e^{-(r-a)\bar{t}}}{1 - e^{-(r-a)\bar{t}}} \left[\frac{\bar{t}(r - a)}{1 - e^{-(r-a)\bar{t}}} - 1 \right] \qquad \text{for } a < r$$

and

$$m_2(a) = r_2(a, \bar{t}).$$

By (21), $J > 0$ and $m_2(\cdot) > 0$ since it is a product of positive terms. The first part of Proposition 2 follows if

$$(23) \qquad \frac{\bar{t}(r-a)}{1-e^{-(r-a)\bar{t}}} - 1 > 0 \qquad \text{for } a < r.$$

But condition (23) is true since

$$\frac{\bar{t}(r-a)}{1-e^{-(r-a)\bar{t}}} - 1 = \int_0^{\bar{t}} \frac{(r-a)\,[1-e^{-(r-a)\tau}]}{1-e^{-(r-a)\bar{t}}}\,d\tau \qquad a \leq r$$

and

$$1 > e^{-(r-a)\bar{t}} \qquad \text{for } 0 \leq \tau \leq \bar{t}, \quad a < r.$$

To prove (19), note that $\partial m_2(a)/\partial a = m_1(a) > 0$. Hence, $g \gtrless s \iff m_2(g) \gtrless m_2(s)$, and condition (19) follows.

3.1 Expectations Equilibrium

Turn now to the consideration of industry behavior in expectations equilibrium. An industry is said to be in expectations equilibrium if the perceived rate of price growth is equal to the actual rate of price growth, i.e., $\lambda(t) = \dot{P}(t)/P(t)$. It can be proved using the adaptive expectations scheme in (6) that the industry under consideration converges to a dynamic path of prices with expectations equilibrium from any initial rate of perceived price growth [13]. Thus, investigation of the behavior of the industry in expectations equilibrium is interesting since it is the long-run behavior.

From the assumption that price expectations grow exponentially, it is clear that, when price expectations are fulfilled, prices grow exponentially at a constant rate, say, $\bar{\lambda}$. Also, from (20), $\bar{t}(t)$ becomes a function of variables which are independent of time when $\lambda(t) = \bar{\lambda}$; thus, one obtains:

COROLLARY 1: For an industry in expectations equilibrium, the length of the productive life of capital is equal for all vintages, i.e.,

$$(24) \qquad \dot{\lambda}(t) = 0 \Rightarrow \dot{\bar{t}}(t) = 0.$$

The length of the productive life of capital can thus be denoted by $\bar{t}$. The value of $\bar{t}$ is determined by solving (20) with $\bar{\lambda} = \lambda$. (Bliss [2] and Hjalmarsson [5] also found that capital goods of different vintages have the same length of productive life in equilibrium.) The result in Corollary 1 allows considerably simpler characterization of output prices and capital, labor, and pollution inputs in expectations equilibrium.

LEMMA 2: In expectations equilibrium with pollution controlled by pollution taxes, output price and capital, labor, and pollution inputs follow:

$$(25) \qquad \frac{\dot{P}(t)}{P(t)} = \frac{\dot{k}(t)}{k(t)} = g\alpha_2 + (1 - \alpha_1 - \alpha_2)\, s - n = \bar{\lambda},$$

$$(26) \qquad \frac{\dot{\ell}(t)}{\ell(t)} = \bar{\lambda} - g, \qquad \frac{\dot{\gamma}(t)}{\gamma(t)} = \bar{\lambda} - s,$$

$$(27) \qquad \frac{k(t)/\dot{\ell}(t)}{k(t)/\ell(t)} = g, \qquad \frac{k(t)/\dot{\gamma}(t)}{k(t)/\gamma(t)} = s.$$

PROOF: Equations (25) and (26) are verified by differentiating (7), (8), (9), and (17) with respect to time; introducing (24); and additionally noting that $\dot{P}(t)/P(t) = \bar{\lambda}$ in expectations equilibrium. Then introducing (25), (26), and (19) into $(k/\dot{\ell})/(k/\ell) = \dot{k}/k - \ell/\ell$ and $(k/\dot{\gamma})/(k/\gamma) = \dot{k}/k - \dot{\gamma}/\gamma$ obtains (27).

Equation (27) indicates that more modern vintages have decreasing pollution ratios when the pollution tax growth rate (relative to the capital price index) is positive. Using (26), one can deduce that, in order to be sure that more modern vintages will have decreasing pollution-output ratios, the pollution tax growth rate must be higher than the output price growth rate. Also, equation (26) verifies that more modern vintages have decreasing labor-pollution ratios if the pollution tax grows faster than wages.[5]

Since price expectations are fulfilled in expectations equilibrium, the expected length of operation of capital is equal to the actual length.

Proposition 1 analyzes the behavior of the perceived length of operation generally. But these results can be further modified for cases of expectations equilibrium where a well-defined dependency exists between the rate of the industry's price growth, the rates of growth of labor and pollution prices, and labor and pollution share coefficients.

PROPOSITION 2: For an industry having a dynamic price path with expectations equilibrium, the length of the period of operation of capital goods is independent of the rates of growth of pollution tax and labor wage and declines with a rise in the neutral rate of technical change.

PROOF: Total differentiation of (20) for $\dot{\bar{t}}(t) = 0$ and $\dot{\bar{\lambda}}(t) = 0$ using the notation introduced in (22) yields

$$d\bar{t} = \frac{1}{J}\left\{m_1(\bar{\lambda})\, d\bar{\lambda} - \alpha_2\, m_1(g)\, dg - (1 - \alpha_1 - \alpha_2)\, m_1(s)\, ds + m_2(s)\, d\alpha_1 - [m_2(g) - m_2(s)]\, d\alpha_2\right\}. \tag{28}$$

Also from (25)

$$d\bar{\lambda} = \alpha_2\, dg + (1 - \alpha_1 - \alpha_2)\, ds - d\eta - s\, d\alpha_1 + (g - s)\, d\alpha_2. \tag{29}$$

Introducing (29) into (28), yields

$$\begin{bmatrix} \frac{d\bar{t}}{dg} \\ \frac{d\bar{t}}{ds} \\ \frac{d\bar{t}}{d\eta} \\ \frac{d\bar{t}}{d\alpha_2} \\ \frac{d\bar{t}}{d\alpha_2} \end{bmatrix} = \frac{1}{J} \begin{bmatrix} -\alpha_2[m_1(g) - m_1(\bar{\lambda})] \\ -(1 - \alpha_1 - \alpha_2)\,[m_1(s) - m_1(\bar{\lambda})] \\ -m_1(\bar{\lambda}) \\ m_2(s) - s m_1(\bar{\lambda}) \\ -[m_2(g) - m_2(s)] + [(g - s) - m_1(\bar{\lambda})] \end{bmatrix}. \tag{30}$$

Differentiation of $m_1(a)$ then obtains

$$\frac{\partial m_1(a)}{\partial a} = \frac{\bar{t}e^{-(r-a)\bar{t}}}{\left[1 - e^{-(r-a)\bar{t}}\right]^2} \left[\frac{\bar{t}(r-a)}{1 - e^{-(r-a)\bar{t}}} - 1\right]$$

(31)

$$- \frac{\bar{t}e^{-(r-a)\bar{t}}}{1 - e^{-(r-a)\bar{t}}} \left[\frac{\bar{t}(r-a)}{1 - e^{-(r-a)\bar{t}}}\right] - 1 = 0 \text{ for } a < r.$$

Thus, $m_1(g) = m_1(s) = m_1(\bar{\lambda})$. Introducing these into (30) and using the knowledge that $J > 0$ and $m_1(\bar{\lambda}) > 0$ [by (21) and (23)] proves the proposition.

REMARK 1: Capital goods operate as long as output price is not smaller than average variable costs. Initially, price is higher than average variable costs, but costs grow faster and eventually overtake price. Increases in the rate of wage growth or pollution tax growth cause an increase in the rate of growth of variable costs, thus reducing the length of operation period. Increases in the rates of growth of pollution taxes and wages also increase the rate of price growth in expectations equilibrium, thus extending the operation period. These conflicting effects cancel one another; as a result, rates of wage and pollution tax growth do not affect the length of operation period in expectations equilibrium.

Corollary 1, Lemma 2, and Proposition 2 examine the behavior of prices and productive capacity under expectations equilibrium in an industry regulated by taxes. The welfare effects on the industry, however, also depend critically on actual production. Assuming the market clears at all times, quantities are determined from the demand relation for each given output price. Suppose demand is given explicitly by

(32) $\quad X(t) = e^{\delta t} DP(t)^{-\mu}$

where $X(t)$ is quantity demanded per unit of time at time t, δ is rate of

growth of demand, D is scale coefficient of demand, and μ is elasticity of demand. By introducing the price function (18) into (32), the equilibrium output rate function is given by

$$X(t) = A^{\mu} De^{(\delta-\mu\eta)\bar{t}(t)} r_1(\lambda, \bar{t})^{-\mu} \left[\frac{\alpha_1}{\alpha_2} \frac{w(t)}{r_1(g, \bar{t})}\right]^{-\mu\alpha_2}$$

(33)

$$\left[\frac{\alpha_1}{1 - \alpha_1 - \alpha_2} \frac{v(t)}{r_1(s, \bar{t})}\right]^{-\mu(1-\alpha_1-\alpha_2)}.$$

PROPOSITION 3: For an industry in expectations equilibrium, the dynamic path of output grows at a constant exponential rate, say, ρ, which is equal to the rate of growth of demand minus the rate of growth in output price multiplied by the elasticity of demand, i.e., in expectations equilibrium,

$$(34) \quad \frac{\dot{X}(t)}{X(t)} = (\delta - \mu\bar{\lambda}) = \delta - \mu [g \alpha_2 + (1 - \alpha_1 - \alpha_2) s - \eta] = \rho.$$

PROOF: Differentiating (33) and introducing (24) yields (34).

REMARK 2: By Proposition 3, the dynamic path of output of an industry in expectations equilibrium is described by

$$(35) \quad X(t) = X_0 e^{\rho(t-t_0)}$$

where X_0 is the rate of output at time t_0. It is evident from (34) that the rate of output growth is affected negatively by the rates of growth of wages and emission taxes. The wage and pollution tax levels at time t_0 affect only the rate of output production but not the rate of output growth.

3.2 Long-Run Expectations Equilibrium

At each point in time, output is produced by capital goods of a variety of vintages. Even for an industry in expectations equilibrium, dynamic paths of pollution and employment can be considerably different depending on the age

distribution of productive capacity. So as to analyze only a single time path of behavior, consider the concept of LREE. An industry is said to be in LREE if it operates as if in expectations equilibrium for all time, i.e., $\dot{P}/P = \bar{\lambda}$ for $-\infty < t < \infty$. This assumption guarantees a continuous and well-behaved distribution of productive capacity. The behavior of an industry in LREE is of interest since it is a limit toward which an industry tends to verge after long periods of operation under the same policy parameters.

By Corollary 1, an expectations equilibrium price path results in a constant length of operation period, $\bar{t}$, for all vintages of capital. Therefore, output at time t is produced only by capital goods with age no greater than $\bar{t}$, i.e., the active vintages at t are defined by the segment $[t - \bar{t}, t]$. Thus,

$$(36) \qquad X(t) = \int_{t-\bar{t}}^{t} c(\tau)\, d$$

where $c(\tau)$ is the rate of output capacity introduced at time τ. Similarly, the rates of labor use and pollution generation at time t by an industry in LREE are

$$(37) \qquad L(t) = \int_{t-\bar{t}}^{t} \ell(\tau)\, c(\tau)\, d\tau$$

and

$$(38) \qquad Z(t) = \int_{t-\bar{t}}^{t} \gamma(\tau)\, c(\tau)\, d\tau,$$

respectively. The function $c(t)$ is computed by differentiating $X(t)$ with respect to time,

$$(39) \qquad \dot{X}(t) = c(t) - c(t - \bar{t});$$

hence,

$$(40) \qquad c(t) = \dot{X}(t) + c(t - \bar{t}).$$

The assumption of LREE ensures that $X(\tau)$ is differentiable for every $\tau \leq t$. By differentiation of $X(\tau)$ with respect to time at $\tau = t - n\bar{t}$, one finds

(41) $\quad c(t - n\bar{t}) = \dot{X}(t - n\bar{t}) + c[t - (n + 1)\bar{t}] \qquad$ for $n = 0, 1, \ldots, \infty$.

By applying (41) recursively, c(t) can be represented as

$$(42) \qquad c(t) = \sum_{j=0}^{\infty} \dot{X}(t - \bar{t}j) = \rho X_0 \, e^{\rho \bar{t}} \sum_{j=0}^{\infty} e^{-\rho \bar{t} j} = \frac{\rho X_0 \, e^{\rho \bar{t}}}{1 - e^{-\rho \bar{t}}}$$

where $X_0 = X(0)$. Inserting $\gamma(t) = \gamma_0 \, e^{(\gamma - s)t}$ and (42) into (38) also yields

$$(43) \qquad \begin{aligned} X(t) &= \int_{t-\bar{t}}^{t} \frac{\rho X_0 \, \gamma_0 \, e^{(\rho + \bar{\lambda} - s)\tau} \, d\tau}{1 - e^{-\rho \bar{t}}}; \\ &= \frac{\rho \gamma_0 \, X_0 \, e^{(\rho + \bar{\lambda} - s)t} \, [1 - e^{-(\rho + \bar{\lambda} - s)t}]}{(\rho + \bar{\lambda} - s)(1 - e^{-\rho \bar{t}})} \end{aligned}$$

where $\gamma_0 = \gamma(0)$. This proves:

PROPOSITION 4: For an industry in LREE, the rate of growth of new output capacity is equal to the rate of growth in production, i.e.,

$$(44) \qquad \frac{\dot{c}(t)}{c(t)} = \rho,$$

and the rate of growth in pollution is equal to the rate of growth in output plus the rate of growth of the pollution-output ratio, i.e.,

$$(45) \qquad \frac{\dot{Z}(t)}{Z(t)} = \rho + \bar{\lambda} - s.$$

REMARK 3: The Pigouvian tax, equating the marginal social costs of pollution and the marginal cost of pollution abatement, has been shown to yield Pareto optimality and is widely considered the best antipollution policy in theory [1]. However, in practicality, estimation of the social costs of pollution is difficult. Therefore, planning agencies have tended to adopt the

standards and pricing approach which is to set an overall regional standard of pollution and then impose taxes designed to attain it. This approach is suggested in Baumol and Oates, and its properties are analyzed there in a static framework. The foregoing analysis provides a much-needed generalization for the dynamic case. The results, in fact, show that an exponentially growing tax can limit the rate of growth of pollution over time or sustain a certain pollution level for an industry in LREE. The rate of growth of taxes that ensures a constant pollution level, i.e., $\bar{Z}(t) = Z$ for all t, is derived from (45) by setting $\dot{Z}(t) = 0$ and using (25) and (34),

$$(46) \qquad s_0 = \frac{(\alpha_2 g - \eta)(1 - \mu) + \delta}{\mu(1 - \alpha_1 - \alpha_2) + (\alpha_1 + \alpha_2)} .$$

By introducing $s = s_0$ into (34), the rate of growth of output when the pollution level is constant can be determined as a function of δ, μ, g, η, α_1, and α_2. Also using (26), the rate of growth in the pollution-output ratio corresponding to $s = s_0$ is $\dot{\gamma}(t)/\gamma(t) = -\rho_0$. From (46), one finds that the antipollution tax which insures a constant pollution rate may decline with time, e.g., with a low rate of demand growth, δ, low elasticity of demand ($\mu < 1$), and a high rate of technical change, η. In these cases, however, output is stagnating; and the pollution-output ratio is declining significantly.

A tax policy which ensures that an industry is in LREE generates a constant level of pollution consists of two parameters. The first parameter is the rate of growth of the pollution tax given by equation (46) which ensures that the rate of pollution growth is zero. The second parameter (v_0), the tax rate for time zero which ensures $Z(0) = Z$, is given in the following lemma.

LEMMA 3: An industry in LREE generates a constant level of pollution $\bar{Z}$ if the pollution tax rate grows exponentially with rate S_0; and the tax rate at

time 0 is a decreasing homogeneous function of the pollution goal of the form,

$$(47) \qquad v_0 = b_0 \, \bar{Z}^{-b_1}$$

where

$$b_0 = \left\{ \frac{\bar{t}_0 \, \rho_0 \, A^{\mu} \, D}{1 - \ell^{-\rho_0 t_0}} \, r_1(\bar{\lambda}_0, \bar{t}_0)^{-\mu} \left[\frac{r_1(g, \bar{t}_0) \, \alpha_2}{w(0)} \right]^{\alpha_2(\mu-1)} \right.$$

$$\left. \alpha_1^{-[\alpha_1 + \mu(1-\alpha_1)]} \right\}^{b_1} (1 - \alpha_1 - \alpha_2) \, r_1(s_0, \bar{t}_0)$$

$$b_1 = - \left[(\alpha_1 + \alpha_2) + (1 - \alpha_1 - \alpha_2) \, \mu \right]^{-1}.$$

PROOF: Introducing $Z(t) = \bar{Z}$ and $\rho + \bar{\lambda} - s = 0$ into (43) and using L'Hospital's rule yields

$$(48) \qquad \bar{Z} = X_0 \, \gamma_0 \, \bar{t}_0 \, \rho_0 \left[1 - e^{-\rho_0 \bar{t}_0} \right]^{-1},$$

and $\bar{t}_0$ is the length of productive life of capital in LREE with a constant pollution level. By introducing s_0 and the rate of price growth associated with s_0--i.e., $\bar{\lambda}_0 = g \, \alpha_2 + s_0(1 - \alpha_1 - \alpha_2) - n$--into (20), the value of $\bar{t}_0$ is determined. To find v_0, write X_0 and γ_0 as functions of v_0.

By (19),

$$(49) \qquad \gamma_0 = b_2 \, v_0^{-(\alpha_1+\alpha_2)}$$

where

$$b_2 = \left[(1 - \alpha_1 - \alpha_2) \, r_1(s_0, \bar{t}_0) \right]^{(\alpha_1+\alpha_2)} \alpha_1^{-\alpha_1} \left[\frac{w(0)}{r_1(g, \bar{t}_0) \, \alpha_2} \right]^{\alpha_2}.$$

By (33),

$$(50) \qquad X_0 = b_3 \, v_0^{-\mu(1-\alpha_1-\alpha_2)}$$

where

$$b_3 = A^{\mu}\, D\, r_1(\bar{\lambda}_0,\ \bar{\tau}_0)^{-\mu} \left[\frac{\alpha_2\, r_1(g,\ \bar{\tau}_0)}{\alpha_1 \quad w(0)}\right]^{\mu\alpha_2}$$

$$\left[\frac{1 - \alpha_1 - \alpha_2}{\alpha_1}\, r_1(s_0,\ \bar{\tau}_0)\right]^{\mu(1-\alpha_1-\alpha_2)} .$$

Introducing (49) and (50) into (48) yields (47).

3.3 Dynamic Optimality of Effluent Taxes

Previous work in a static context has shown that the standards and pricing approach to pollution regulations (1) minimizes production cost of a given level of output when total pollution is constrained [1], (2) maximizes the sum of producer and consumers surpluses resulting from the activities of a polluting industry when the total pollution generated by the industry is restricted by a given upper bound [6], and (3) maximizes social welfare subject to a pollution constraint in a general equilibrium setting [3]. Here the opportunity taxation is extended to a dynamic context. The welfare criterion is maximization of the discounted sum of consumption benefits minus production cost (including new investment) caused by the output of the polluting industry (note that this measure includes tax revenues gained by government) subject to an environmental constraint that does not permit aggregate pollution to exceed a threshold level at any time. Consumption benefits of the output are measured by the consumer surplus area under the demand curve. Thus, the social optimization problem is

$$\max_{\substack{k(t),\ell(t),\gamma(t)\ 0 \\ X(t),q(t,\tau),c(t)}} \int_0^{\infty} e^{-rt} \left[\int_0^{x(t)} D^{-1}(z)dz - w(t) \right.$$

(51)

$$\left. \int_{-\infty}^{t} \ell(t) q(\tau, t)\, d\tau - k(t)\, c(t) \right] dt$$

subject to

(52) $$1 = e^{-\eta t} A k^{\alpha_1} \ell^{\alpha_2} \gamma^{1-\alpha_1-\alpha_2},$$

(53) $$\bar{Z} \geq \int_{-\infty}^{t} q(\tau, t)\, \gamma(\tau) d\tau \qquad \text{for all } t,$$

(54) $$X(t) = \int_{-\infty}^{t} q(\tau, t) d\tau \qquad \text{for all } t,$$

(55) $$0 \leq q(\tau, t) \leq c(\tau) \qquad \text{for all } t, \leq t$$

where $q(\tau, t)$ is the output capacity of machines of vintage τ which are employed at t. Constraint (55) indicates that $q(\tau, t)$ cannot exceed the actual output capacity of vintage τ, while equation (54) states that total output capacity of the machines employed at time t is equal to the rate of output production at time t. In equation (53), $\bar{Z}$ is the maximal permitted pollution rate.

The optimization problem in (51) can be expressed as a Lagrangian problem,

$$\begin{aligned} L = \max \int_0^{\infty} e^{-rt} \Bigg\{ & \int_0^{x(t)} D^{-1}(z)dz - w(t) \int_0^{t} \ell(\tau)\, q(\tau, t)\, d\tau - k(t)\, c(t) \\ & + \psi(t) \left[1 - e^{-\eta t} k(t)^{\alpha_1} \ell(t)^{\alpha_2} \gamma(t)^{1-\alpha_1-\alpha_2} \right] \\ & + \theta(t) \left[\bar{Z} - \int_{-\infty}^{t} q(\tau, t)\, \gamma(\tau) d\tau \right] \\ & + \zeta(t) \left[\int_{-\infty}^{t} q(\tau, t) d\tau - X(t) \right] + \int_{-\infty}^{t} \phi(\tau, t) [c(\tau) - q(\tau, t)] \\ & + \chi(\tau, t)\, q(\tau, t) d\tau \Bigg\} dt \end{aligned} \tag{56}$$

where $\psi(t)$, $\theta(t)$, $\zeta(t)$, $\phi(\tau, t)$, and $\chi(\tau, t)$ are functions that denote the shadow prices of the constraints at each point in time. The first-order conditions, in addition to (52)-(55), include (among others)

$$(57)\quad \frac{\partial L}{\partial X(t)} = e^{-rt} D^{-1}[X(t)] - \zeta(t) = 0 \qquad \text{for } t = 0, \infty;$$

$$(58)\quad \frac{\partial L}{\partial \ell(t)} = \int_t^{\infty} w(\tau)\, e^{-r(\zeta - t)}\, q(t,\tau)\, d\tau + \psi(t)\, \frac{\alpha_2}{\ell(t)} = 0 \qquad \text{for } t = 0, \infty;$$

$$(59)\quad \frac{\partial L}{\partial k(t)} = -e^{-rt}\, c(t) + \psi(t)\, \frac{\alpha_1}{k(t)} = 0 \qquad \text{for } t = 0, \infty;$$

$$(60)\quad \frac{\partial L}{\partial \gamma(t)} = -\psi(t)\, \frac{1 - \alpha_1 - \alpha_2}{\gamma(t)} - \int_t^{\infty} \theta(\tau)\, q(t,\tau)\, d\tau = 0 \qquad \text{for } t = 0, \infty;$$

$$(61)\quad \frac{\partial L}{\partial c(t)} = -e^{-rt}\, k(t) + \int_t^{\infty} \phi(t,\tau)\, d\tau \geq 0 \qquad \text{for } t = 0, \infty;$$

$$(62)\quad \frac{\partial L}{\partial q(\tau, t)} = \zeta(t) - e^{-rt}\, w(t)\ell(\tau) - \phi(t)\, \gamma(\tau) + \chi(\tau, t) - \phi(\tau, t) = 0$$

$$\text{for } t = 0, \quad ; \infty = -\infty, t;$$

$$(63)\quad \frac{\partial L}{\partial \phi(\tau, t)}\, \phi(\tau, t) = [c(t) - q(\tau, t)]\, \phi(\tau, t) = 0 \qquad \text{for } t = 0, \quad ; \infty = -\infty, t;$$

$$(64)\quad \frac{\partial L}{\partial \chi(\tau, t)}\, \chi(\tau, t) = \chi(\tau, t)\, q(\tau, t) = 0 \qquad \text{for } t = 0, \quad ; \infty = -\infty, t.$$

At the optimal solution, $\zeta(t)$ is the discounted shadow price of output and $\phi(t)$ is the discounted shadow price of pollution. Equation (57) indicates that the determination of output price and quantity by the intersection of supply and demand is a condition for optimality. Conditions (62)-(64) imply that all the output capacity introduced in time t will be employed at $\tau \geq t$ if output price at τ is greater than average variable costs at τ. The output capacity of vintage τ will not be employed at all at $\tau > t$ if the average variable cost for this vintage at t is above output price. By (59) and (62)-(64), new capital goods are introduced at time t as long as their price does not exceed the discounted value of the gains they generate.

Introducing the results in (62)-(64) into (58) and (59), one finds at the optimal solution that the ex ante marginal rate of substitution between labor and capital goods of vintage t is equal to the discounted cost of employing

one unit of labor during the productive lives of these capital goods. Similarly, by (58) and (60), the ex ante marginal rate of substitution between pollution and capital for capital goods of vintage t is equal to the discounted sum of the shadow prices of the pollution constraint during the period of operation of this vintage.

The first-order conditions of the social optimization problem are satisfied by a competitive industry, with full information about future taxes, that is regulated by a pollution tax equal to the shadow price of the pollution constraint, $\theta(t)$. Note that the implied tax is not necessarily growing at a constant rate. When the tax is imposed on an industry with partial future price information (price expectations are determined by an adaptive scheme), it does not necessarily result in socially optimal behavior. Since this work analyzes industries with adaptive future price expectations regulated by pollution taxes with a constant rate of growth, it is of interest to find under what circumstances the behavior of such industries is socially optimal.

PROPOSITION 5: For an industry in LREE, a pollution tax policy with a constant rate of growth, which results in the generation of a constant level of pollution, is the optimal policy for maximization of the discounted sum of producer and consumer surpluses when pollution generation is bounded by a constant for an infinite time period that starts at any point in time.

PROOF: Consider an industry in LREE which is restricted to generate a constant level of pollution Z by a tax scheme given by

$$v(t) = v_0 e^{s_0 t} \tag{65}$$

where s_0 is given in (46) and v_0 is given in (47). For such an industry, new technology at each point in time is determined according to (1), (11), (13) and (14). Capital goods are employed as long as output price is not lower than average variable costs. Output price is equal to the average discounted

cost of the newest technology at each point in time, and output is determined by the intersection of supply and demand. This behavior satisfies the first-order conditions for the optimization problem in (51) given in part by (52)-(55) and (57)-(64).

REMARK 4: The tax scheme in (65) results in the generation of a constant level of pollution Z by a regulated industry in LREE. However, when this tax scheme is imposed on an industry in short-run expectations equilibrium, pollution generation will not necessarily be constant at all times. For some periods, the industry will generate more pollution than the upper bound Z and in other periods less. Following the same line of analysis as in Proposition 5, however, it can be proved that a pollution tax policy with a constant rate of growth s_0 and an initial rate v_0 maximizes the discounted sum of economic surpluses for an industry whose permitted pollution level is bounded from above by the pollution function of an industry in expectations equilibrium which starts operation at a given point in time and is regulated by the pollution tax mentioned above (see Zilberman [13] for further discussion).

REMARK 5: When a constant level of pollution is generated in LREE equilibrium, the sum of the rates of growth of output and the pollution-output ratio is zero by (45). Therefore, for a growing industry, the pollutionoutput ratio is decreasing with time.

Generally, however, the threshold level $\bar{Z}$ is determined arbitrarily, and in most cases policymakers are interested in restricting pollution to an interval rather than to a specific value. Thus, policymakers will tolerate some deviations from the threshold level $\bar{Z}$ in various times, at least when the deviations are in both directions. Thus, such policymakers should favorably

consider regulating an industry which starts operation at a time zero with a tax rate of the form

$$v(t) = v_0 e^{s_0 t}$$

because this policy attains the desired pollution level most efficiently, and its pollution function oscillates around $\bar{Z}$ in the short run and converges to it in the long run (Zilberman [13] demonstrates this point).

It is plausible that this tax policy is useful for regulating industries with many different distributions of capital goods at the start of the regulation period. Since these industries have some productive capacity at time zero, the scale of their discrete process of capital accumulation will not be as large as in the case of an industry which starts operation at time zero. Thus, in the period of taxation, their age distributions of capital will be smoother; and they may converge to LREE faster than an industry that starts operation at time zero. Therefore, it is reasonable that their pollution functions after taxes also oscillate around the threshold level in the short run, with perhaps smaller deviations, while converging to it in the long run. Such a pattern of behavior is probably tolerable for policymakers in many cases.

4. POLLUTION CONTROL OVER TIME IN AN INDUSTRY REGULATED BY STANDARDS

In many cases pollution is not controlled by taxation but by direct regulations (standards). This section analyzes a particular kind of standard—an upper bound on the pollution-output ratio of new machines. This standard may become more stringent with time; i.e., the upper bound on the pollution-output ratio may be reduced for more modern technologies. For example, such an evolution of standards seems to characterize regulation of automobile emission policies. For a discussion of other pollution problems where this type of standard is common see [13].

Let $\bar{\gamma}(t)$ denote the upper bound on the pollution-output ratio of machines of vintage t and let this upper bound decrease with time at a constant rate following

$$\bar{\gamma}(t) = \bar{\gamma}_0 e^{-\phi t}, \quad \phi \geq 0$$

where γ_0 is the upper bound on the pollution-output ratio of machines introduced at time zero and ϕ is the rate of decline of the pollution-output ratio.[6] When pollution is not priced, profit maximizers will operate with the highest feasible pollution output ratio

$$\gamma(t) = \bar{\gamma}(t). \tag{66}$$

Introducing (66) into (1), the ex ante production function at time t becomes

$$Q(t) = Ae^{\eta t} K(t)^{\alpha_1} L(t)^{\alpha_2} \left[\bar{\gamma}(t)Q(t)\right]^{1-\alpha_1-\alpha_2}. \tag{67}$$

This relationship can be rearranged as

$$\begin{aligned} Q(t) &= \left[Ae^{\eta t} \bar{\gamma}(t)^{(1-\alpha_1-\alpha_2)t}\right]^{1/(\alpha_1+\alpha_2)} K(t)^{\alpha_1/(\alpha_1+\alpha_2)} L(t)^{\alpha_2/(\alpha_1+\alpha_2)} \\ &= \left[A\gamma_0^{(1-\alpha_1-\alpha_2)}\right]^{1/(\alpha_1+\alpha_2)} e^{\left\{[\eta-\phi(1-\alpha_1-\alpha_2)]/(\alpha_1+\alpha_2)\right\}t} \\ &\quad K(t)^{\alpha_1/(\alpha_1+\alpha_2)} L(t)^{\alpha_2/(\alpha_1+\alpha_2)} \\ &= \tilde{A}e^{\tilde{\eta}t} K(t)^{\tilde{\alpha}_1} L(t)^{\tilde{\alpha}_2}. \end{aligned} \tag{68}$$

Thus, the ex ante production function of an industry with a constrained pollution-output ratio $\bar{\gamma}(t)$ is treated by the entrepreneurs as a Cobb-Douglas production function of capital and labor with capital share coefficient $\alpha_1 = \alpha_1/(\alpha_1 + \alpha_2)$, scale parameter

$$\tilde{A} = \left[A\bar{\gamma}_0^{(1-\alpha_1-\alpha_2)}\right]^{(1-\alpha_1-\alpha_2)},$$

and neutral rate of technical change $\tilde{\eta} = [\eta - \phi(1 - \alpha_1 - \alpha_2)]/(\alpha_1+\alpha_2)$. This formulation of the production function will be called the "relevant" production function under standards. Since, under standards, pollution is costless, the share coefficients of labor and capital for the relevant production function are larger than the actual share coefficients. Furthermore, the deviations of the relevant share coefficient from the actual ones are increased with the actual coefficient of pollution.

The relevant production function with capital and labor as inputs will be used for planning purposes by unregulated industries with bounded pollution-output ratios. For these cases, there is no contradiction between having an ex ante production function of degree one with capital, labor, and pollution as inputs and behavior that can be explained by a Cobb-Douglas function of degree one with only capital and labor as inputs.

The production function formulation in (68) permits the use of some of the key results obtained in the previous sections for the case analyzed in this section. That is, introducing the parameters of the relevant production function into the equations derived for industries regulated by taxes yields the formulations describing the behavior of untaxed industries with a bounded pollution-output ratio. Thus, one can derive the following directly from (7)-(9) and (18).

LEMMA 4: For an industry with a bounded pollution-output ratio, the optimal technology of machinery introduced at time t is given by

$$(69) \qquad k(t) = \left[\frac{e^{-[\eta-\phi(1-\alpha_1-\alpha_2)]t}}{A\bar{\gamma}_0}\right]^{1/(\alpha_1+\alpha_2)} \left[\frac{\alpha_1 w(t)}{\alpha_2 r_1(g, t)}\right]^{\alpha_2/(\alpha_1+\alpha_2)}$$

$$(70)\quad \ell(t) = \left[\frac{e^{-[\eta-\phi(1-\alpha_1-\alpha_2)]t}}{A\bar{\gamma}_0}\right]^{1/(\alpha_1+\alpha_2)} \left[\frac{\alpha_2 r_1(g, \bar{t})}{\alpha_1 w(t)}\right]^{\alpha_1/(\alpha_1+\alpha_2)}$$

$$\gamma(t) = \bar{\gamma}_0 e^{-\phi t},$$

and

$$(71)\quad P(t) = r_1(\lambda, \bar{t})\ \frac{k(t)\ (\alpha_1 + \alpha_2)}{\alpha_1}.$$

Similarly, from (20), the perceived length of the productive life of a capital good introduced at time t is $\bar{t}(t)$ and is implicitly given by

$$(72)\quad \frac{[r - \lambda(t)e]^{\gamma(t)\bar{t}(t)}}{1 - e^{-[r-\lambda(t)]\bar{t}(t)}} = \frac{(r - g)\ e^{g\bar{t}(t)}}{\left[1 - e^{-(r-g)\bar{t}(t)}\right]}\ \frac{\alpha_2}{(\alpha_1 + \alpha_2)}.$$

4.1 Expectations Equilibrium

The notion of expectations equilibrium introduced earlier can also be applied to cases where pollution is regulated by standards. Since behavior under standards can be treated as a special case of the behavior under taxes (using the relevant production function), it follows from results mentioned earlier that an industry will tend toward a price path of expectations equilibrium from any initial point under standards. Thus, the expectations equilibrium price path for standards is of interest in explaining tendencies in nonequilibrium (expectations).

By Corollary 1, the length of the economic life of capital in expectations equilibrium is constant for all vintages and is equal to, say, $\bar{t}_\phi$. The rate of output price growth in expectations equilibrium is constant and equal to, say, $\bar{\lambda}_\phi$. By applying Lemma 2, one can thus show directly that

$$(73)\quad \frac{\dot{P}(t)}{P(t)} = g\ \frac{\alpha_2}{\alpha_1 + \alpha_2} - \frac{\eta - \phi(1 - \alpha_1 - \alpha_2)}{\alpha_1 + \alpha_2} = \bar{\lambda}_\phi,$$

$$(74)\quad \frac{\dot{k}(t)}{k(t)} = \bar{\lambda}_\phi,\ \frac{\dot{\ell}(t)}{\ell(t)} = \bar{\lambda}_\phi - g,\ \frac{\dot{\gamma}(t)}{\gamma(t)} = -\phi$$

for an industry regulated by standards in expectations equilibrium.

REMARK 6: The results in (69) through (74) enable some comparison of the behavior of prices of industries in expectations equilibrium under taxes and under standards. Since the relevant share coefficient of labor is larger than the actual share coefficient, it follows from (73) that the contribution of the rate of wage growth to the rates of price increase and capital intensification is larger under standards than under taxes. Under standards, this contribution is increasing with the share coefficient of pollution.

Following the same methodology as in the previous section, the behavior of production under standards can be derived by introducing the parameters of the relevant production function into the equations describing production behavior under taxes. Thus, from Proposition 3, one can derive:

PROPOSITION 6: For an industry regulated by standards, the rate of output growth in expectations equilibrium is equal to the rate of growth of demand minus the rate of growth in output price multiplied by the elasticity of demand

$$(75) \qquad \frac{\dot{X}(t)}{X(t)} = (\delta - \mu\bar{\gamma}_{\phi}) = \left\{\delta - \mu\,\frac{g\alpha_2 - [\eta - \phi(1 - \alpha_1 - \alpha_2)]}{\alpha_1 + \alpha_2}\right\} = \rho_{\phi}.$$

4.2 Long-Run Expectations Equilibrium

Because the behavior of the industry of this paper tends toward LREE under taxes, the same result must also hold for an industry regulated by standards since the behavior under standards can be analyzed as a special case of the behavior under taxes. Again, by analogy, (42) implies

$$(76) \qquad c(t) = \frac{\rho_{\phi}\, X_0 \cdot e^{\rho_{\phi} t}}{1 - e^{-\rho_{\phi} \bar{t}_{\phi}}}.$$

Also note that using $\gamma(t) = \gamma_0 e^{-\phi t}$ and (76) in

$$Z(t) = \int_{t-\bar{t}_\phi}^{t} c(\tau)\ \gamma(\tau)\ d\tau$$

obtains

$$(77) \qquad Z(t) = \frac{\rho_\phi\ \bar{\gamma}_0\ X_0\ e^{(\rho_\phi - \phi)t}\left[1 - e^{-(\rho_\phi - \phi)\bar{t}_\phi}\right]}{(\rho_\phi - \phi)\left(1 - e^{-\rho_\phi \bar{t}_\phi}\right)}$$

This proves:

PROPOSITION 7: For an industry regulated by standards in LREE, the rate of growth of new output capacity is equal to the rate of growth of output, i.e.,

$$(78) \qquad \frac{\dot{c}(t)}{c(t)} = \rho_\phi,$$

and the rate of growth of pollution is equal to the rate of growth of output minus the rate of growth of the pollution-output ratio of new technologies, i.e.,

$$(79) \qquad \frac{\dot{Z}(t)}{Z(t)} = \rho_\phi - \phi.$$

REMARK 7: Condition (78) indicates that, for an industry regulated by standards which generates a constant level of pollution in LREE, the rate of decline in the pollution-output ratio of new technologies is equal to the rate of growth in output, i.e,

$$(80) \qquad \rho_{\phi_0} = \phi_0,$$

where ϕ_0 is the rate of decline in the pollution-output ratio of new technologies that ensures $Z(t) = \bar{Z}$.

To develop further results in the following section comparing taxes and standards, the following lemma is also useful.

LEMMA 5: An industry in LREE generates a constant level of pollution $\bar{Z}$ if it is regulated by a standard restricting the pollution-output ratio of new machines which is decreasing exponentially with the rate ϕ_0 given by

$$(81) \qquad \phi_0 = \frac{\delta\,(\alpha_1 + \alpha_2) - \mu\,(g\alpha_2 - \eta)}{(\alpha_1 + \alpha_2) + \mu\,(1 - \alpha_1 - \alpha_2)},$$

and the upper bound on the pollution-output ratio of machines of vintage zero is given by

$$(82) \qquad \bar{\gamma}_0 = d\bar{Z}^{\alpha_1+\alpha_2}$$

where

$$(83) \qquad d = \frac{1}{A}\left[\frac{\bar{\tau}_\phi\,\rho_{\phi_0}}{1 - e^{-\rho_{\phi_0}\bar{t}_{\phi_0}}}\,\frac{1}{D}\,r_1(\lambda_\phi, {}_0\bar{\tau}_\phi)_0^{\mu}\right]^{-(\alpha_1+\alpha_2)}\left[\frac{\alpha_1}{\alpha_2}\,\frac{w(0)}{\alpha_1(g, \bar{\tau}_{\phi_0})}\right]^{\alpha_2}.$$

PROOF: Analogous to Lemma 3.

5. COMPARISON OF TAXES AND STANDARDS IN LONG-RUN EXPECTATIONS EQUILIBRIUM

It is possible to attain a constant level of pollution with either taxes or standards as Lemmas 3 and 5. Given the optimality of a tax which generates a constant level of pollution (Proposition 5), it is interesting to compare the dynamic behavior of other variables when both taxes and standards attain the same constant level of pollution in LREE. From Proposition 5, taxation is the more efficient of the two in terms of producer and consumer surpluses. However, other considerations may also be important from a political point of view. The following propositions compare the effects of these policies on output price, output, pollution, and capital age.

PROPOSITION 9: For an industry in LREE which generates a constant pollution level, the rates of growth of price and output and the rate of decline in

the pollution-output ratio of new technologies are the same whether or not pollution is regulated by taxes or by standards, i.e.,

$$(84) \qquad \left(-\frac{\dot{\gamma}}{\gamma}\right)_{s_0} = \phi_0 = \rho_{\phi_0} = \frac{\delta\,(\alpha_1 + \alpha_2) - \mu\,(g\alpha_2 - n)}{(\alpha_1 + \alpha_2) + \mu\,(1 - \alpha_1 - \alpha_2)} = \rho_0.$$

PROOF: The first equality of (84) follows (74), the second from (80), the third from (81), and the fourth by substituting (46) into (34). The remaining assertions follow trivially.

Even though the rates of growth of prices, output, and pollution are equal under taxes and standards, their levels are not necessarily indentical when both policies are directed at attaining the same constant pollution level. One reason for the possible difference is that the length of life of capital is not necessarily identical under both policies.

PROPOSITION 9: For an industry regulated to attain a constant pollution level in LREE, the length of the productive life of capital under standards is longer than under taxes if the output of the industry is growing, and the length of the productive life of capital is equal under both policies if the output of the industry is constant.

PROOF: By (20), the productive life under taxes can be derived from

$$(85) \qquad \frac{(r - \bar{\lambda}_0)\, e^{\bar{\lambda}_0 \bar{t}_0}}{1 - e^{-(r-\bar{\lambda}_0)\bar{t}_0}} = \frac{\alpha_2\,(r - g)\, e^{g\bar{t}_0}}{1 - e^{-(r-g)\bar{t}_0}} + \frac{(1 - \alpha_1 - \alpha_2)\,(r - s_0)\, e^{s_0 \bar{t}_0}}{1 - e^{-(r-s_0)\bar{t}_0}}.$$

By (71), the productive life of capital under standards is derived from

$$(86)\qquad (\alpha_1 + \alpha_2)\,\frac{\left(r - \bar{\lambda}_{\phi_0}\right) e^{\lambda_{\phi_0} \bar{t}_{\phi_0}}}{1 - e^{-\left(r-\bar{\lambda}_{\phi_0}\right)\bar{t}_{\phi_0}}} = \alpha_2\,\frac{(r-g)e^{g\bar{t}_{\phi_0}}}{1 - e^{-(r-g)\bar{t}_{\phi_0}}}.$$

Since $\bar{\lambda}_0 = \bar{\lambda}_{\phi_0}$, subtracting (86) from (85) yields

$$(87)\qquad a_1 = a_2$$

where

$$a_1 = [r_2(\bar{\lambda}_0, \bar{t}_0) - r_2(s_0, \bar{t}_0)]\,(1 - \alpha_1 - \alpha_2)$$

and

$$a_2 = \alpha_2[r_2(g, \bar{t}_{\phi_0}) - r_2(g, \bar{t}_0].$$

By (45), $\rho_0 = s_0 - \lambda_0$; thus, $s_0 - \lambda_0 > 0$ when $\rho_0 > 0$. Since $r_2(a, \bar{t})$ is increasing in a and the expression a_1 is negative when $s_0 > \lambda_0$, a_2 is also negative. Thus, $r_2(g, \bar{t}_{P_0}) - r_2(g, \bar{t}_0) > 0$, and $\bar{t}_{\phi_0} > \bar{t}_0$ since $e^{gt}/[1\ e^{-(r-g)t}]$ is increasing in $\bar{t}$. When $\rho_0 = 0$, a_1 in (87) is zero; hence, $a_2 = 0$ and $\bar{t}_{\phi_0} = \bar{t}_0$.

REMARK 8: One explanation of the shorter productive life of capital under taxes is that the burden of pollution taxation becomes heavier as a capital good ages, while under standards a firm is not burdened by the pollution it generates.

LEMMA 6: For an industry restricted to a constant pollution level in LREE, standards yield higher output than taxes if

$$(88)\qquad G = (\alpha_1 + \alpha_2)\,\frac{1 - e^{-(r-\bar{\lambda}_0)\bar{t}_0}}{1 - e^{-(r-\bar{\lambda}_0)\bar{t}_{\phi_0}}}\left(\frac{\bar{t}_{\phi_0}}{\bar{t}_0}\,\frac{1 - e^{-\rho_0\bar{t}_0}}{1 - e^{-\rho_0\bar{t}_{\phi_0}}}\right)^{1/(\alpha_1+\alpha_2)}$$

$$\cdot\left[\frac{1 - e^{-(r-g)\bar{t}_{\phi_0}}}{1 - e^{-(r-g)\bar{t}_0}}\right]^{\alpha_2/(\alpha_1+\alpha_2)} < 1.$$

PROOF: Since the rate of output growth is equal under both policies ($\rho_0 = \rho_{\phi_0}$), it is sufficient to find under what conditions the rate of output at time zero under standards $\left(X_{\phi_0}\right)$ is greater than the rate of output at time zero under taxes, X_0. From (48) and an equation analogous to (48) associated with the relevant production function, use of $\rho_{\phi_0} = \rho_0$ implies that

$$(89)\qquad \frac{X_0}{X_{\phi_0}} = \frac{\bar{t}_{\phi_0}}{\bar{t}_0}\,\frac{1 - e^{-\rho_0\bar{t}_0}}{1 - e^{-\rho_0\bar{t}_{\phi_0}}}\,\frac{\gamma_{\phi_0}}{\gamma_0}\,.$$

Substituting (70) into (71) , (7) into (18), and dividing one result by the other obtains

$$\frac{P_{0_\phi}}{P_0} = (\alpha_1 + \alpha_2)\,\frac{1 - e^{-(r-\bar{\lambda}_0)\bar{t}_0}}{1 - e^{-(r-\bar{\lambda}_0)\bar{t}_\phi}}\left(\frac{\gamma_0}{\bar{\gamma}_{\phi_0}}\right)^{1/(\alpha_1+\alpha_2)}$$

(90)

$$\cdot\left[\frac{1 - e^{-(r-g)\bar{t}_{\phi_0}}}{1 - e^{-(r-g)\bar{t}_0}}\right]^{\alpha_2/(\alpha_1+\alpha_2)}.$$

Using the demand equation

$$(91) \qquad \frac{P_{0_\phi}}{P_0} = \left(\frac{X_0}{X_{\phi_0}}\right)^{1/\mu}$$

and substituting (89) and (91) into (90) then yields

$$(92) \qquad \left(\frac{X_0}{X_{\phi_0}}\right)^{-(\alpha_1+\alpha_2)-\mu} = G,$$

from which Lemma 6 follows.

PROPOSITION 10: For an industry restricted to a constant pollution level in LREE, which produces constant levels of output under both policies, more output is produced under standards than under taxes, i.e.,

$$(93) \qquad X_{0_\phi} > X_0 \qquad \text{if} \qquad \rho_{\phi_0} = 0.$$

PROOF: By Lemma 6, $\bar{t}_0 = \bar{t}_{\phi_0}$ since $\rho_0 = 0$. Introducing this result into (92) yields $G = \alpha_1 + \alpha_2 < 1$ so by Lemma 6, $X_{0_\phi} > X_0$.

When both policies attain the same constant level of pollution, by generating constant output levels, Proposition 9 states that the length of the productive life of capital is equal under both policies. Suppose that output, prices, and the pollution-output ratio are also the same under both policies. Since, at each point in time, price is equal to the average variable costs of the new technology, the average discounted costs are equal under both technologies. But, under standards, pollution is not priced; hence, the average costs under standards are lower. Thus, the outcomes under different policies are different. Under taxes, prices must be higher; hence, output must be larger under standards. Since both policies generate the same pollution levels, the pollution-output ratio under standards must be lower.

When output is growing with time, the different impact that taxes and standards have on average costs results in a difference in the length of the

productive life of capital. Proposition 9 states that, under standards, the productive life of capital is longer. Thus, from (89), the product of output with the pollution-output coefficient of the newest machines at time zero is larger under taxes than under standards, i.e.,

(94) $X_0\gamma_0 > X_{\phi_0}\bar{\gamma}_{\phi_0}$ when $\rho_0 > 0$.

Condition (94) indicates that behavior under taxes differs from behavior under standards in more aspects than the one mentioned above; in addition to differences in the length of the productive life of capital, there are differences in output and/or pollution-output ratios. However, condition (94) excludes the possibility that standards result in both higher output and pollution-output ratios.

It is also unreasonable to have both higher pollution-output ratios and lower output under standards. The reason is that higher pollution intensity under standards leads to less intensive use of other inputs. Thus, discounted average costs and, hence, prices under standards must be lower, and output must be higher. In addition, the pollution output ratio under standards is lower than under taxes even though less-polluting machines are introduced under standards, machines are utilized for a longer period of time in meeting the same aggregate pollution target.

6. CONCLUSIONS

This paper derives, analyzes, and compares the dynamic implications of exponentially increasing antipollution taxes and exponentially decreasing antipollution standards.

The results are basically as follows.

Taxes are the most efficient policy to attain a constant target level of pollution over time, thus confirming and extending the results obtained in the literature for the static case. Pollution taxation results in factor obsolescence of capital goods, and new capital goods are more polluting under taxes than under standards.

Unlike the static case, neither policy clearly generates more output in lower prices over time. Only in the case where the output level is constant over time do standards clearly result in more production and lower output price. However, the rates of output and price growth are the same under both policies. Thus, even in cases where standards generate more production and result in lower output prices, standards may hold some advantage over taxes in terms of macropolicy concerned with inflation but only on a one-time basis.

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FOOTNOTES

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[2]Since without regulations or taxes on pollution firms would use infinite quantities of pollution, feasible pollution-output ratios are assumed to be bounded from above.

[3]This assumption, which postulates that wages grow exponentially and more rapidly than other prices, follows Salter's arguments and leads to the tendency toward capital-intensive technologies even with neutral technological changes. Salter used empirical evidence from the United States to justify this assumption and explained the relative increase in labor prices by technical progress in capital goods industries.

[4]Following Harcourt [4], the expected rate of price growth is lower than the rate of wage growth. Harcourt's explanation is that wages are expected to reflect increases in prices and overall productivity; therefore, when productivity is improving at all, businessmen expect wages to rise faster than prices.

[5]Assuming that the output price growth rate is lower than the rate of growth of wages, $\lambda < g$, introduction of the expression for λ from (26) into the last inequality yields a constraint on the feasible values of the policy variable s. This constraint is

$$s < g\left(1 + \frac{\alpha_1}{1 - \alpha_1 - \alpha_2}\right) + \frac{n}{1 - \alpha_1 - \alpha_2}.$$

Since in most cases the capital share is higher than the pollution share and the pollution share is very small, the upper bound on the rate of growth of

the pollution tax is significantly higher than the wage growth rate. Policies considered in the rest of this work are assumed to comply with this constraint.

[6]While the pollution-output ratio bound is assumed to decline over time, for purposes of discussion, the analysis also holds if the bound increases, in which case $\phi < 0$.

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