M.B. OBWANA, K.P. KALIRAJAN AND R.T. SHAND*

On Measuring Farmer-Specific and Input-Specific Allocative Efficiency

Abstract: The objective in this paper is to suggest an alternative method of estimating farmer-specific and input-specific allocative efficiency, taking into account the influence of the methods of application of inputs on output. Further, we compare the proposed measures with those calculated based on the existing conventional method using the same data set.

INTRODUCTION

The economic viability of a farmer depends on two important questions. First, how efficiently are the inputs used, given the technology, and secondly, how much of the inputs are utilized in the production process, given the technology and market prices. Examination of the above two questions is vital to the survival of farmers in the long run. There are several ways to answer these questions. The method most frequently used in the literature to answer the first question is to estimate the stochastic frontier output and to compare with the actual realized output popularized by Aigner *et al.* (1977) and Meeusen and van den Broeck (1977). The second question is usually answered by equating the marginal value product (MVP) calculated from the farm's realized production function with marginal cost (MC) of the variable inputs under the behavioural assumption of profit maximization.

While calculating the MVP, it is always assumed that the input-specific production response coefficients do not vary among farmers. However, as the magnitudes of coefficients reflect the 'contributions' of inputs which, in turn, are determined by the methods of their application, it is very unlikely that these coefficients are in fact constant across farmers. This can only occur when all the farmers are using the same methods of application of inputs. Empirical studies show that methods of application of inputs—and therefore the input-specific response coefficients—do vary among farmers (see, for example, IRRI (1979), Smith and Umali (1985), Kalirajan and Obwona (1994), among others). This means that testing of allocative efficiencies following the constant response coefficients approach may not produce reliable results.

The objective in this paper is to suggest a method of estimating farmer-specific and input-specific allocative efficiency, taking into account the influence of the methods of application of inputs on output. Further, we compare these measures with efficiency measures calculated based on the existing conventional method using the same data set.

MODELLING FARMER-SPECIFIC PRODUCTION BEHAVIOUR

Empirical evidence shows that with the same levels of inputs, different levels of outputs are obtained by following different methods of input application. This implies that the

* Australian National University.

different methods of applying various inputs will influence the output differently. Therefore, the existing stochastic frontier production function approach estimating constant slope but varying intercept coefficient does not appear to be meaningful. The random coefficient regression models popularized by Swamy (1971) and Hsiao (1986) facilitates modelling the differences in the methods of application of inputs by farmers by introducing variations not only in intercepts but also in slope coefficients of farmer-specific production functions. Furthermore, whether such modelling is valid with data can also be tested statistically.

The random coefficients regression model can be specified as:

(1) $y_i = x_i'\beta_i + \varepsilon_i; \quad i = 1,\dots,N$

where y_i is an observed dependent variable and x_i is a $K\times 1$ vector of known non-random values of independent variables. ε_i is a random error term with mean zero and constant variance, σ_ε^2. We assume that each $K\times 1$ coefficients vector β_i varies from the mean response coefficient vector, $\bar{\beta}$, by a vector of random error terms, u_i, that is,

(2) $\beta_i = \bar{\beta} + u_i$

When $x_{1i} = 1$, the additive equation error term cannot be distinguished from the randomly varying intercept. Consequently, ε_i is usually not explicitly included in Equation (1) (Hildreth and Houck, 1968).

For all the N observations, Equations (1) and (2) can be written more compactly as:[1]

(3) $y = X\bar{\beta} + w$

where $w = D_x u$ and y is an $N\times 1$ vector, X is an $N\times K$ matrix of stacked X_i', D_x is an $N\times NK$ diagonal matrix of X_i', u is an $NK\times 1$ vector of u_i's. We assume that

(4) $E(u_i) = 0; \quad i = 1,\dots,N$

$$E(u_{ki}u_{lj}') = \begin{cases} \Lambda_u \text{ if } k = l \text{ and } i = j \\ 0 \quad \text{otherwise} \end{cases}$$

An estimator which is best, linear and unbiased (BLU) of $\bar{\beta}$ in (3) is

(5) $\hat{\beta} = (X'\Sigma_w^{-1}X)^{-1}X'\Sigma_w^{-1}y$

where

(6) $\Sigma_w = D_x(I_N \otimes \Lambda_u)D_x'$

and

$$\Lambda_u = \operatorname{diag}(\sigma_{u11},\dots,\sigma_{uKK})$$

I_N in (6) is an indentity matrix of order N, $\otimes$ represents the Kronecker product and σ_{ukk}; ($k = 1,\ldots,K$) are elements of Λ_u. The ith diagonal element of $\sum_w$ in Equation (6) is

$$\sum_{wi} = \sum_{k=1}^{K} \sigma_{ukk} x_{ki}^2$$

Since Λ_u elements are not known, they have to be estimated. Hildreth and Houck (op. cit.) suggests several methods of estimating the elements of Λ_u. In this study, we follow the version of Hildreth and Houck procedure modified by Singh *et al.* (1976).[2]

After estimating Λ_u and obtaining the estimates of the mean response coefficients, $\bar{\beta}$ the individual response coefficient estimates of the β_is are given by

$$(7) \quad \hat{\beta}_i = \hat{\bar{\beta}} + \Lambda_u X_i' \left[X_i \Lambda_u X_i'\right]^{-1} \left(y_i - X_i \hat{\bar{\beta}} \right); \quad i = 1,\ldots,N$$

and are best linear and unbiased (Griffiths, 1972).

The validity of the application of the random coefficients specification to the data can be examined by following the test method suggested by Swamy (1971). If the production response coefficients in Equation (1) are random, then Λ_u will contain non-zero elements. Thus, the appropriate test for randomness is $H_o : \Lambda_u = 0$ against $H_a : \Lambda \neq 0$. Swamy (1971) developed a likelihood ratio test in the context of panel data models. The test statistic used here is a straightforward application of Swamy's test with only one period, that is, $T = 1$.

FARMER-SPECIFIC AND INPUT-SPECIFIC ALLOCATIVE EFFICIENCY

The criterion for determining the optimal levels of inputs used is to locate the point on the farmer-specific production function that has the highest associated isoprofit line. At this point, profits will be maximized. This point is characterized by a tangency condition: the slope of the farmer-specific production function should be equal to the slope of the farmer-specific isoprofit line. Since the slope of the production function is the marginal physical product, and the slope of the isoprofit line is the ratio of the price of the factor input to the price of the output, this condition can be written as:

$$(8) \quad MVP_k = MC_k; \qquad k = 2,\ldots,M$$

Now, considering a Cobb–Douglas production function with $M - 1$ variable inputs and $(K - M + 1)$ fixed, denoted by x and z, respectively. The farmer-specific production function can be written as:

$$(9) \quad \ln y_i = \bar{\beta}_1 + \sum_{k=2}^{M} \beta_{ki} \ln x_{ki} + \sum_{j=M+1}^{K} \alpha_{ji} \ln z_{ji} + \varepsilon_i; \qquad i = 1,\ldots,N$$

The marginal productivity conditions in Equation (8) for profit maximization are

$$\begin{aligned}
&\beta_{2i}\frac{y_i}{x_{2i}} = \frac{p_{2i}}{p_{yi}} \\
(10)\quad &\beta_{3i}\frac{y_i}{x_{3i}} = \frac{p_{3i}}{p_{yi}} \\
&\vdots = \vdots \\
&\beta_{Mi}\frac{y_i}{x_{Mi}} = \frac{p_{Mi}}{p_{yi}}; \qquad i = 1,...,N
\end{aligned}$$

Equations (9) and (10) yield the following system of equations:

$$\begin{aligned}
&\beta_{2i}\ln x_{2i} + \beta_{3i}\ln x_{3i} + ... + \beta_{Mi}\ln x_{Mi} - \ln y_i = -\sum_{j=M+1}^{K}\alpha_{ji}\ln z_{ji} - \ln\beta_{1i} \\
(11)\quad &\ln x_{2i} \qquad - \ln y_i = \ln\beta_{2i} - \ln p_{2i} + \ln p_{yi} \\
&\qquad = \vdots \\
&\ln x_{Mi} - \ln y_i = \ln\beta_{Mi} + \ln p_{yi} \\
&i = 1,...,N
\end{aligned}$$

These are NM equations in NM unknowns consisting of $x_2, x_3,..., x_M$ and y. The parameters (βs) are estimates from Equation (9). The solutions to Equation (11) are the optimal output levels denoted by say, y^0, along with the optimal input levels $x_2^0, x_3^0,..., x_M^0$.

Now a measure of farmer-specific and input-specific allocative efficiency (ISAE) to examine whether there is any under- or over-utilization of variable inputs can be defined as

$$(12)\quad ISAE_{ki} = \frac{x_{ki}}{x_{ki}^0};\ k = 2,...,M \text{ and } i = 1,...,N$$

where x_{ki} is the observed level of the kth input used by the ith farmer, and x_{ki}^0 is the optimal level of the kth input of the ith farmer obtained as solutions to the profit maximizing system, Equation (11). The above measure of *ISAE* can be equal to, greater or less than 1. When *ISAE* is equal to 1, it means that the farmer is efficiently allocating the particular input. On the hand, when *ISAE* is either greater or less than 1, this implies that the farmer is not efficient in choosing the level of the concerned input. More specifically, *ISAE* greater than 1 means that that particular variable input is being over-utilized, and *ISAE* less than 1 implies that the variable input in question is being under-utilized.

DATA AND EMPIRICAL ANALYSIS

Data for the present study came from a cost of cultivation project conducted by the Tamil Nadu Agricultural University in 1986. A random sample of 64 farmers growing the modern cotton variety MCU-5 in Madurai district, Tamil Nadu State in India was chosen for empirical analysis[3]. Sample farmers were operating between 5 and 10 acres of land. They may be named medium-sized farmers according to Indian standard.

The following Cobb–Douglas type of production function was estimated:[4]

$$(13)\quad \ln y_i = \bar{\beta}_1 + \sum_{k=2}^{3} \beta_{ki} \ln x_{ki} + \alpha_{4i} \ln z_{4i} + \varepsilon_i;\ i = 1,2,\ldots,64$$

where

y = amount of cotton in tonnes
x_2 = labour in man days
x_3 = fertilizer in kilograms
z_4 = area operated in acres

βs are farmer-specific and variable input-specific response coefficients, α is the farmer-specific and fixed input-specific response coefficient, ε is the random disturbance term.

Table 1 *Mean Response Coefficients and the Range of Estimates of the Actual Response Coefficients*

Inputs	Coefficients	Mean response coefficients	Range of actual response coefficients
Constant	β_1	4.3072 (0.8765)	4.2813 – 4.3216
Labour	β_2	0.2518 (0.1204)	0.2316 – 0.2648
Fertilizer	β_3	0.2094 (0.0919)	0.1913 – 0.2216
Land	α_4	0.5206 (0.2582)	0.5004 – 0.5324

Note: Figures in parentheses are standard errors of estimates.

The mean response coefficients estimated as in Equation (5) are given in Table 1. These coefficients have theoretically acceptable signs and magnitudes and are significant at the 5 percent level. These coefficients can be considered as the production coefficients calculated based on conventional approach of using the weighted least squares estimation of production functions.

Table 2 *Farmer-specific and Input-specific Allocative Efficiencies of Variables Inputs*

	Number of farmers			
	Conventional method		Suggested method	
ISAE	Labour	Fertilizer	Labour	Fertilizer
<1	2	10	7	33
=1	43	30	11	14
>1	19	24	46	17
Total	64	64	64	64

Now, to arrive at the farmer-specific optimal levels based on the conventional approach, the above coefficients are substituted along with farmer-specific and input-specific prices in Equation (11). Then the farmer-specific and input-specific allocative efficiency measures are calculated as given in Equation (12) and the results are presented in Table 2.

Next, the hypothesis that the response coefficients are fixed across observations is tested by using the likelihood ratio test developed by Swamy (1971). The asymptotic $\chi^2_{\frac{1}{2}K(K+1)}$ chi-square test statistic was calculated to be 28.96 (with 10 degrees of freedom) which is significant at the 1 percent level. This means that the null hypothesis may be rejected and that the use of the fixed coefficients model is rejected in favour of the varying coefficients model of the present data set.

Actual response coefficients for individual observations are calculated using Equation (7) and the range of actual response coefficients is given in Table 1. The results show that there are variations in the farmer-specific and input-specific actual response coefficients. Thus, the results indicate that the conventional approach of modelling the production behaviour of sample farmers where the same response coefficients are assigned to each observation without first testing statistically is not appropriate.

Finally, using the above actual farmer-specific and input-specific production response coefficients along with farmer-specific and input-specific prices in Equation (11), the optimal input levels are calculated. The farmer-specific and input-specific allocative efficiencies are then calculated and the results are presented also in Table 2.

These results show that only about 17 percent and 22 percent of farmers appear to have efficiently allocated their labour and fertilizer inputs respectively. The corresponding figures from the conventional method are much higher, that is, 67 percent and 47 percent, respectively. In other words, when examined using our suggested approach, about 83 percent of sample farmers appear to be allocating the labour input inefficiently, while the corresponding figure for fertilizer is 78 percent. However, in the case of conventional approach which is based on the assumption of fixed input-specific production response coefficients for all sample farmers, only 33 percent of farmers seem to be allocating the labour input inefficiently and for the fertilizer, it is 53 percent. As the hypothesis of fixed production response coefficients for all the sample farmers has earlier on been rejected, in the light of the above results, it may be concluded that the conventional method of calculating allocative efficiencies is misleading.

CONCLUSION

Drawing on the principles of the random coefficient regression model, this paper suggests an approach to model the impact on allocative efficiency of different methods of application of a given technology at the farm level. The effect of different methods of applying a given technology on output manifests in the form of yielding different magnitudes of production coefficient across observations. The ratio of the actual level of input used to the optimal level of input calculated using the actual response coefficients and farmer-specific prices, provides a measure of farmer-specific and input-specific allocative efficiency.

The empirical results show that measuring allocative efficiency using the conventional method which does not take into account the possibility of variation in production response coefficients among farmers, may provide misleading results. This implies that any measurement of the allocative efficiencies should be preceded by testing whether the production response coefficients do vary among farmers.

NOTES

1 Without the random error term ε_i since $u_{1i} = \varepsilon_i$ by specification.

2 Hsiao (1975) shows that the Hildreth–Houck estimator is equivalent to the minimum-norm quadratic unbiased estimator (MINQUE) of Rao (1970).

3 Cotton is an important commercial crop in India, and Tamil Nadu is one of the major cotton producing states in India (Hitchings, 1983).

4 A translog functional form was estimated using the data set. But, the test based on translog estimates for a Cobb–Douglas functional form could not be rejected. Furthermore, a Cobb–Douglas production function has also been proved to be suitable in earlier empirical studies on cotton production in Tamil Nadu State (see, for example, Subramanian, 1986).

REFERENCES

Aigner, D.J., Lovell, C.A.K and Schmidt, P., 1977, 'Formulation and Estimation of Stochastic Frontier Production Function Models', *Journal of Econometrics*, Vol. 6, No. 1, pp.21–37.

Griffiths, W. E., 1972, 'Estimation of Actual Response Coefficients in the Hildreth–Houck Random Coefficient Model', *Journal of American Statistical Association*, Vol. 67, No. 339, pp.633–635.

Hildreth, C. and Houck, J.P., 1968, 'Some Estimates for Linear Model with Random Coefficients', *Journal of American Statistical Association*, Vol. 63, No. 322, pp.584–595.

Hitchings, J.A., 1983, The Economics of Cotton Cultivation in India — Supply and Demand for 1980–90, World Bank Staff Working Paper No. 68, Washington, D.C.

Hsiao, C., 1975, 'Some Estimation Methods for a Random Coefficients Model', *Econometrica*, Vol. 43, No. 2, pp.305–325.

Hsiao, C., 1986, *Analysis of Panel Data*, Econometric Society Monographs No. 11, Cambridge University Press, New York.

IRRI, 1979, 'Farm-level Constraints to High Rice Yields in Asia 1974–77', International Rice Research Institute, Los Banõs, Philippines.

Kalirajan, K.P. and Obwona, M.B., 1994, 'Frontier Production Function: The Stochastic Coefficients Approach', *Oxford Bulletin of Economics and Statistics*, Vol. 56, No. 1, pp. 87–96.

Meeusen, W. and van den Broeck, 1977, 'Efficiency Estimation from Cobb–Douglas Production Function with Composed Error', *International Economic Review*, Vol. 18, pp.435-444.

Rao, C.R., 1970, 'Estimation of Heteroscedastic Variances in Linear Models', *Journal of the American Statistical Association*, Vol. 65, No. 329, pp.161–72.

Singh, B., Nager, A.L., Chouldhry, N.K. and Raj. B., 1976, 'On the Estimation of Structural Changes: A Generalization of the Random Coefficients Regression Model', *International Economic Review*, Vol. 17, No. 2, pp.340–361.

Smith, J. and Umali, G., 1985, 'Production Risk and Optimal Fertilizer Rates: A Random Coefficient Model', *American Journal of Agricultural Economics*, Vol. 67, No. 3, pp.654–659.

Subramanian, G., 1986, 'Labour Demand and Supply Responsiveness of Cotton in Madurai District, Tamil Nadu', *Indian Journal of Agricultural Economics*, Vol. 41, No.2, pp.155–163.

Swamy, P.A.V.B., 1971, *Statistical Inference in Random Coefficient Regression Models*, Springer-Verlag, New York.

DISCUSSION OPENING — Steven A. Neff *(Economic Research Service, USDA)*

The paper uses a random coefficients technique to evaluate a 1986 study by G. Subramanian of farmer-specific and input-specific allocative efficiency for cotton farmers in Tamil Nadu, India. The present paper, following Subramanian, uses a Cobb–Douglas production function. I have no criticism of the application of the method. I do not find technical flaws.

Having said said, I will argue a bit with the premise, I will question the conclusion, and I will raise doubts about its applicability. The paper begins by asserting that a farmer's economic viability depends on the quantity of inputs and how effectively they are used, given technology and market prices. Of course technology and market prices are not given to farmers in many parts of the world, including Zimbabwe. Yesterday I heard a farmer in a communal area name the problems his group had encountered in the current growing season, including the breakdown of an irrigation pump. This intra-seasonal technology change was not a choice, but the technology was, for his farm, not a given. A large scale commercial farmer said that the Cotton Marketing Board is not guaranteeing the price of cotton in the coming year, so his viability depends crucially on his marketing results. I could give other examples for the United States, but I think I have made the point that the farmers' viability depends on many things, among which technology and market prices are not given. The paper is a bit oversold at the outset.

I promised to question the interpretation of the results. Applying the random coefficients technique, the authors find that only 17 percent and 22 percent of farmers are using labour and fertilizer inefficiently. Subramanian had found that 67 percent and 47 percent of farmers were using labour and fertilizer efficiently. Now I don't question that individual farmers' response coefficients are different, but I would question whether the difference can all be attributed to variations in the effectiveness of application of the inputs. I suspect that some of what is being called inefficiency is actually due to variations in labour and fertilizer quality. I might excuse the authors from responsibility for data quality because they are using an existing data set, but some of the 'inefficiency' may actually be due to data quality.

If I were the authors' student, I would see very dramatically the effect of taking into account the fact that response coefficients are not fixed. If I were the professor, I would have demonstrated very effectively that this point should be considered. If I were not a professor or student, but the state minister for agriculture in Tamil Nadu, my perspective would be quite different. If had seen the original study that concluded that my farmers are using their inputs efficiently, I might have concluded that there was no need to increase funding for my extension service. Now I have another study on exactly the same topic using exactly the same data, and it comes to an exactly opposite conclusion. What should I do? Should I rely on the one that favours my bias? Should I ask for a new study that looks at each assumption in a different way or uses another technique or a different functional form (after all, why use Cobb–Douglas?) that neither of these studies have considered? I am afraid that I am likely to ignore the studies and make my policy decisions based on other criteria. In short, it would have been helpful if the authors had offered some guidance on the applicability of the results.

GENERAL DISCUSSION — Claude Mehier, Rapporteur *(France)*

In the discussion of Fantino and Veeman's paper, the authors were asked several questions about how inputs were measured. The interest centred on capital components. Measures of multi-factor productivity are generally influenced by how the land input is measured. The authors were asked whether the results would change if account were taken of resource stocks and flows. There was further discussion of the importance the influence of changes in technology and the problem of measuring capital input as quality changed. A similar question was raised with regard to changes in labour quality over time. Other discussion concerned the influence of economies of scale and the tendency for total factor productivity measures to follow a cyclical pattern in the United States.

The authors of the other two papers were not present. A brief summary of Kalirajan and Shand's paper was presented by Oeivind Hoveid (Norway). Steve Neff (ERS, US Department of Agriculture) presented a brief outline of Obwana, Kalirajan and Shand's paper.

Participants in the discussion included Simeon Ehui (ILCA, Ethiopia), Franco Rosa (University of Udine, Italy) W. Huffman (Iowa State University) and Heinrich Hockmann (University of Gottingen, Germany).