



**AgEcon** SEARCH  
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

*The World's Largest Open Access Agricultural & Applied Economics Digital Library*

**This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.**

**Help ensure our sustainability.**

Give to AgEcon Search

AgEcon Search  
<http://ageconsearch.umn.edu>  
[aesearch@umn.edu](mailto:aesearch@umn.edu)

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

## K.P. KALIRAJAN AND R.T. SHAND\*

### *Modelling and Measuring Technical Efficiency: An Alternative Approach*

**Abstract:** In the literature, technical efficiency is measured as the ratio of observed output to potential output. Although there is no a priori theoretical reasoning, in the stochastic framework of measuring technical efficiency, the potential output is defined as a neutral shift from the observed output. The objective in this paper is to suggest a method to measure technical efficiency without having to consider the potential output as a neutral shift from the observed output.

## INTRODUCTION

Technical efficiency, one of the two components of economic efficiency, is defined as the ability and willingness of any producing unit to obtain the maximum possible potential output from a given set of inputs and technology. In the literature, technical efficiency is measured as a ratio of actual output to the potential output (Aigner, Lovell and Schmidt, 1977; and Meeusen and van den Broeck, 1977). Based on techniques of estimating the potential output, the approaches to measuring technical efficiency generally vary from programming to statistical estimation<sup>1</sup>. In the latter approach, a firm-specific stochastic production frontier involving outputs and inputs is defined as follows:

$$(1) \quad y_i^* = f(x_i) \exp(v_i)$$

where,  $x_i$  is a vector of  $m$  inputs,  $v_i$ s are statistical random errors with  $N(0, \sigma_v^2)$ , and  $y_i^*$  is the maximum possible stochastic potential output for the  $i$ th firm, which varies over time for the same firm and across firms in the same period.

It is rational to assume that firms may not know the parameters of their own frontier production function exactly for various reasons, and that this lack of knowledge is manifest principally as technical inefficiency. Therefore, the realized production function of the  $i$ th firm may be modelled as follows:

$$(2) \quad y_i = y_i^* \exp(u_i)$$

where  $\exp(u_i)$  is defined as a measure of observed technical efficiency of the  $i$ th firm. It is further assumed that  $u_i \leq 0$ . When  $u_i$  takes the value zero, it means that the  $i$ th firm is technically fully efficient and realises its maximum possible potential output. On the other hand, when  $u_i$  assumes values less than zero, it means that the  $i$ th firm is not fully technically efficient and so produces output which is less than its potential output. Now, a measure of technical efficiency for the  $i$ th firm can be defined as:

$$(3) \quad \exp(u_i) = \frac{y_i, \text{given } u_i}{y_i^*, \text{given } u_i = 0}$$

\* Australian National University.

To obtain the above measure the denominator has to be estimated, as the numerator is the observed output level. Assuming a functional form to represent the technology in Equation (1), and a density function for  $u$  in Equation (2), the denominator can be estimated by using the maximum likelihood methods.

There are three apparent limitations to this approach. First, the technology is parametrized by some *ad hoc* functional forms involving outputs and inputs, which is restrictive. Second, assuming a density function for  $u_i$  is not based on any theoretical reasoning. Finally, and most importantly, the frontier production function defined in Equation (1) is assumed to be a neutral shift from the observed production function, Equation (2), which is questionable. Statistical tests are available and have been carried out to validate the selection of functional forms and the distributional assumption for  $u_i$ , but, the question as to why the frontier should be a neutral shift from the observed production function has not received much attention in the literature.

The objective in this paper is to suggest a method to estimate the frontier production function using cross-sectional data and to measure firm-specific technical efficiency for individual observations, when the frontier shifts non-neutrally from the observed production function. The following section explains the methodology which is followed by the estimation procedures.

## FRONTIER WITH NON-NEUTRAL SHIFT

When technical efficiency is measured by using Equation (3), the underlying assumption is that the frontier is a neutral shift from the realized production function. This constant-slope, variable-intercept approach raises a basic question about the concept of technical efficiency. Where does technical efficiency come from? How does a firm achieve its technical efficiency? The literature indicates that a firm obtains its full technical efficiency by following the best practice techniques, given the technology. In other words, technical efficiency is determined by the method of application of inputs regardless of the levels of inputs. This implies that the different methods of applying various inputs will influence the output differently. That is, the slope coefficients will vary from firm to firm. Therefore, the constant-slope approach of measuring technical efficiency is not consistent with the definition of technical efficiency. The following specification of the production process which is consistent with the concept of technical efficiency, facilitates estimation of firm-specific technical efficiency for individual observations.

Assuming a Cobb–Douglas technology, the production relationship can be written as follows:<sup>2</sup>

$$(4) \quad \ln y_i = \beta_{i1} + \sum_{j=2}^k \beta_{ij} \ln x_{ij} + u_i \text{ for } i = 1, 2, \dots, n$$

where  $y_i$  refers to the level of the output of the  $i$ th firm;  $x_{ij}$  refers to the level of the  $j$ th input used by the  $i$ th firm;  $\beta_{i1}$  is the intercept term for the  $i$ th firm;  $\beta_{ij}$ ,  $j > 1$ , is the actual response of the output to the method of application of the  $j$ th input by the  $i$ th firm, and  $u_i$  refers to the random disturbance term which is  $N(0, \sigma_u^2)$ .

Further, let,  $\beta_{ij} = \beta_j + v_{ij}$ ,  $\beta_{i1} = \beta_1 + v_{i1}$

$$E(v_{ij}) = 0, \quad E(v_{i1}) = 0, \quad E(\beta_{ij}) = \beta_j$$

$$\text{var}(\beta_{ij}) = E(\beta_{ij} - \beta_i)(\beta_{ik} - \beta_i)$$

$$= \sigma_{jj} \text{ for } j = k$$

$$= 0 \text{ for } j \neq k$$

$$= V$$

Now, with the above assumptions, following Swamy (1971), Equation (4) can be rewritten as:

$$(5) \quad \ln y_i = \beta_1 + \sum_{j=2}^k \beta_j \ln x_{ij} + w_i \quad \text{for } i = 1, 2, \dots, n$$

$$\text{where } w_i = \sum_{j=2}^k v_{ij} \ln x_{ij} + v_{i1} + u_i$$

$$E(w_i) = 0$$

$$\text{var}(w_i) = \sigma^2 + \sum_j \sigma_{jj} \ln x_{ij}^2 + \sigma_{11}$$

$$\text{cov}(w_i, w_j) = 0 \text{ for } i \neq j$$

The assumptions underlying Equation (5) are as follows. (a) Technical efficiency is achieved by adopting the best practice techniques which involve the efficient use of inputs without having to increase their levels. Technical efficiency stems from two sources. First, the efficient use of each input which contributes individually to technical efficiency, and can be measured by the magnitudes of the varying random slope coefficients ( $\beta_{ij}$ s, excluding the intercepts). Second, when all the inputs are used efficiently, then it may produce a combined contribution over and above the individual contributions. This latter 'lump sum' contribution, if any, can be measured by the varying random intercept term. (b) The highest magnitude of each response coefficient and the intercept form the production coefficients of the potential frontier production function. Let  $\beta_1^*, \beta_2^*, \dots, \beta_k^*$  be the estimates of the parameters of the frontier production function:

$$\beta_j^* = \max\{\beta_{ij}\} \text{ for } i = 1, 2, \dots, n \text{ and } j = 1, 2, \dots, k$$

Now, the firm-specific potential frontier output for each observation can be worked out as:

$$(6) \quad \ln y_i^* = \beta_1^* + \sum_{j=2}^k \beta_j^* \ln x_{ij} \text{ for } i = 1, 2, \dots, n$$

where  $x_{ij}$  is the actual level of the  $j$ th input used by the  $i$ th firm. The frontier output given by Equation (6) necessarily indicates the non-neutral shift of the frontier from the actual production function. Now, a measure of technical efficiency can be defined as follows:

$$(7) \quad E_i = \frac{y_i}{\exp(\ln y_i^*)}$$

where  $E_i$  = realized output/potential output and varies between 0 and 1.

## ESTIMATION

Harville (1977) proposed a number of alternative estimators. Some of these, such as the maximum likelihood (ML) and the restricted ML, are only defined for legitimate values of  $\nu$ . Although procedures exist for guaranteeing the estimated  $\nu$  to be always non-negative at each iteration, the maximum may be at the boundary. Further, as the likelihood function is not globally concave, it allows for multiple local maxima (Maddala, 1971). The above problems can be eliminated by following the iterated GLS-likelihood maximization methods suggested by Breusch (1987) with some modifications.

The principles underlying Breusch's suggestions are as follows: the parameters are classified into two groups, of which, one represents the mean response coefficients and the other shows the variance parameters. The estimates are modified by maximising the likelihood over one classification with the other fixed at its estimated value from the previous step. The response coefficients are modified by GLS using  $\nu$  estimated from the previous step, and  $\nu$  is usually modified by using the residuals from the previous GLS estimates. The iterated GLS algorithm, which is computationally simple, usually provides feasible solutions (Breusch, 1987).

Now, following Griffiths (1972), the actual firm-specific and input-specific response coefficient predictor for the  $i$ th observation  $\hat{\beta}_{ij}$ , which is the best linear unbiased predictor (BLUP) can be obtained as follows:

$$(8) \quad \hat{\beta}_{ij} = \beta + \frac{\nu x_i}{x_i' \nu x_i} \hat{w}_i$$

The response coefficients representing the potential frontier production function can be identified as follows from the above estimates:

$$(9) \quad \hat{\beta}_j^* = \max \{ \hat{\beta}_{ij} \} \text{ for } i = 1, 2, \dots, n \text{ and } j = 1, 2, \dots, k$$

Now, the firm-specific potential frontier output for each observation can be worked out as:

$$(10) \quad \ln y_i^* = \beta_1^* + \sum_{j=2}^k \hat{\beta}_j^* \ln x_{ij}$$

where  $x_{ij}$ s refer to actual levels of inputs used by the  $i$ th firm. Calculation of firm-specific technical efficiency for individual observation can then be calculated as follows:

$$TE = \frac{(y_i)}{\exp(\ln y_i^*)}$$

## DATA AND RESULTS

Data for the present study were drawn from a random sample of 68 farmers growing high-yielding paddy variety IR 36 in the village of Solavanthan in Madurai district in Tamil Nadu State of India. The sample farms are well-irrigated and they are of medium size (between 5 and 10 acres), The selected village is visited frequently by the extension officials.

The following Cobb–Douglas type of production function has been assumed for the present study

$$(11) \ln y_i = \beta_{i1} \chi_{i1} + \sum_{j=1}^k \beta_{ij} \ln \chi_{ij} + u_i \text{ for } i = 1, 2, \dots, 68.$$

where,

$y$  = high yielding (IR 36) paddy output in tonnes

$\chi_1$  = 1,...a constant term

$\chi_2$  = pre-harvest labour man days

$\chi_3$  = fertiliser in kgs.

$\chi_4$  = animal labour days

$\chi_5$  = area in acres multiplied by a relevant soil fertility index

$u$  = statistical 'white noise', which is  $N(0, \sigma^2)$

The mean response coefficients of inputs are given in Table 1. All the coefficients are significant at the 5 per cent level and they all have theoretically acceptable signs and magnitudes. At the outset, the validity of the modelling of Equation (11) as a random coefficient specification for the present data set has been examined by following the testing procedures suggested by Breusch and Pagan (1979). They pointed out that the random coefficient specification, as described above, fits the class of heteroscedastic error models and have proposed a Lagrange multiplier test, which has the same asymptotic properties as the likelihood ratio tests in standard situations. The test results indicate that the random coefficient specification in Equation (11) could not be rejected for the present data set. This means that our modelling of the production process in the study area is appropriate. The Ramsey's (1969) RESET test for functional form was used and the calculated test Statistic  $F_{(1,63)} = 2.72$ . The tabulated  $F_{(1,63)}$  value at the 5 per cent level was 3.65. Therefore, the Cobb–Douglas functional form could not be rejected for the data set.

Table 2 shows the range of actual response coefficients of inputs for individual observations. The variation in the farm-specific and input-specific elasticity coefficients is substantial. This means that the methods of application of different inputs vary among

sample farms and, consequently, individual contributions of inputs to output differ from farm to farm. The estimates of the production coefficients of the frontier are derived using (9) and the results are given in Table 2. These estimates indicate the maximum possible contribution of each input to output when the inputs are applied efficiently following the best practices techniques. Further, these estimates are derived relaxing the conventional assumption that the frontier output is a neutral shift from the realised output.

**Table 1** *Iterated GLS Estimates of the Mean Response Coefficients and the Variance Coefficients*

Inputs	Unit of measurement	Iterated GLS estimates	
		Variance coefficient	Mean response coefficient
Constant	-	0.1216	0.3916 (0.1428)
Labour	Man days	0.1318	0.2012 (0.1065)
Fertilizer	kgs	0.1176	0.2584 (0.1265)
Animal labour	days	0.0820	0.0616 (0.0303)
Area	acres	0.1372	0.4763 (0.1329)

Notes: Figures in parentheses denote standard errors. Number of observations: 68.  
Log likelihood -118.34.

**Table 2** *Range of Estimates of Actual Response Coefficients and Estimates of Frontier Production Function*

Inputs	Range of actual response coefficients	Estimates of the frontier production function
Constant	0.3896–0.3982	0.3982
Labour	0.1923–0.2106	0.2106
Fertilizer	0.2485–0.2615	0.2615
Animal power	0.0592–0.0678	0.0678
Area	0.4318–0.4682	0.4682

**Table 3** *Frequency Distribution of Farm-Specific Technical Efficiency Measures*

Efficiency measures(%)	Number of firms	Percentage
71 – 75	0	29.4
76 – 80	16	23.5
81 – 85	14	20.6
86 – 90	13	19.1
91 – 95	5	7.4
Total	68	100

Following Equation (10), the potential frontier outputs for individual observations have been estimated and the calculated farm-specific technical efficiency measures for each sample farmer are shown in Table 3 in a frequency form. The efficiency measures range from 0.71 to 0.94.

## CONCLUSIONS

The fixed coefficient frontier production function methodology hitherto used restricts measurement of efficiency to an overall measure. But, it is rational to argue that, depending on which farm uses which best practice technique with which input, production coefficients would vary from farm to farm. This provides the rationale and the necessity for the use of the variable coefficient frontier production function to measure firm-specific technical efficiencies. The results reveal substantial variations in the actual farm-specific and input-specific response coefficients, which means that methods of application of different inputs vary among farms, and consequently individual contributions of inputs to output differ from farm to farm.

In the light of the above findings, this method of measuring technical efficiency, which relaxes the conventional assumption of a neutral shift of the frontier function from the actual production function, provides valuable additional information to policy-makers. Not only can the analysis distinguish which farmers are more or less efficient, but also with respect to which inputs. This should, for example, give greater guidance as to the most appropriate direction for extension advice and calls for research on the reasons for variations in individual efficiencies.

## NOTES

<sup>1</sup> For a detailed discussion, see Bauer, 1990.

<sup>2</sup> Equation (4) is a modification of the Hildreth-Houck (1968) random coefficient regression model. Unlike the Hildreth-Houck model, an additive disturbance term has been included in the proposed model, in addition to the random intercept term.

## REFERENCES

- Aigner, D.J., Lovell, C.A.K. and Schmidt, P., 1977, 'Formulation and Estimation of Stochastic Frontier Production Function Models', *Journal of Econometrics*, Vol. 6, No. 1, pp.21-37.
- Bauer, P.W., 1990, 'Recent Developments in the Econometric Estimation of Frontier', *Journal of Econometrics*, Vol. 46, No. 1, pp.39-56.
- Breusch, T.S., 1987, 'Maximum Likelihood Estimation of Random Effects Models', *Journal of Econometrics*, Vol. 36, No. 3, pp.383-389.
- Breusch, T.S. and Pagan, A.R., 1979, 'The Lagrange Multiplier Test and its Application to Model Specification in Econometrics', *Review of Economic Studies*, Vol. 47, pp. 239-253.
- Griffiths, W.E., 1972, 'Estimation of Actual Response Coefficients in the Hildreth-Houck Random Coefficient Model', *Journal of the American Statistical Association*, Vol. 67, No. 339, pp.633-635.
- Harville, D.A., 1977, 'Maximum Likelihood Approaches to Variance Component Estimation and to Related Problems', *Journal of the American Statistical Association*, Vol. 72, No. 358, pp.584-595.
- Hildreth, C. and Houck, J.P., 1968, 'Some Estimators for a Linear Model with Random Coefficients', *Journal of the American Statistical Association*, Vol. 63, No. 322, pp.584-595.



- Maddala, G.S., 1971, 'The Use of Variance Models in Pooling Cross Section and Time Series Data', *Econometrica*, Vol. 39, No. 2, pp.341–358.
- Meeusen, W. and van den Broeck, J., 1977, 'Efficiency Estimation from Cobb Douglas Production Functions with Composed Error', *International Economic Review*, Vol. 18, pp.435–444.
- Ramsey, J., 1969, 'Tests of Specification Errors in Classical Linear Least-Squares Regression Analysis', *Journal of the Royal Statistical Society, Series B*, Vol. 31, No. 2, pp.350–371.
- Swamy, P.A.V.B., 1971, *Statistical Inference in a Random Coefficient Regression Model*, Berlin, Springer, Verlag.