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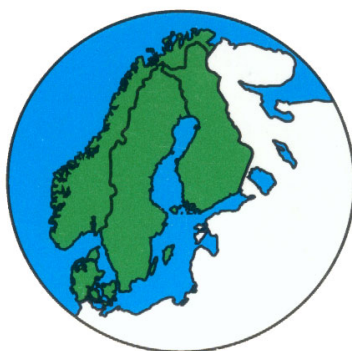
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Risk Aversion and Optimal Rotation: a Stochastic Efficiency Approach

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Abstract

A new stochastic efficiency analysis approach, called stochastic efficiency with respect to a function (SERF), that partitions a set of risky alternatives in terms of certainty equivalents (CEs) for a specified range of attitudes to risk, is applied to analyse average optimal rotation strategies at different levels of forest owner's risk aversion. Using Norwegian forest data with stochastic timber price and volume growth, the empirical results show that the optimal rotation length increases with increasing degree of risk aversion. It is also found that the effect of risk aversion is lower with higher interest rates, while the size of the investment cost affects only the level of the CE, with the forest owner's risk aversion being relatively unimportant.

Keywords: Dynamic risk analysis, stochastic budgeting, stochastic dominance with respect to a function.

1. Introduction

Risk management has received increased attention in the forest economics literature (Braze and Newman 1999). However, with some exceptions (e.g., Caulfield 1988, Gong and Löfgren 2003), the relationship between risks involved and the forest owner's degree of risk aversion has not been subject to attention in the analyses. This is despite the fact that forest owners have to make investment decisions in the face of increased price uncertainty as well as uncertainty about future growth and quality of retained stands. Under assumed certainty, guidelines are available for the optimal forest rotation length for different investment costs, interest rates and growth conditions (Johansson and Löfgren 1985, for a rigorous treatment. In addition there exist also in most countries handbooks for practical forest management). But, as far as we know, no general guidelines about optimal rotation length exist when both the entailed risk and the forest owner's risk aversion are taken into account. Guidelines that cover these cases would be useful to investors who are deciding whether investing is worthwhile, as well as for policy makers.

Forest investment and management decisions have a long time horizon with significant risk. The variations in consequences can be large relative to the decision maker's (DM's) wealth and hence the cost of ignoring risk aversion might be too high to be ignored (Anderson and Hardaker 2003).

The returns from investing in a forest naturally depend on how that forest is managed. In particular, the choice of the rotation length must be resolved in advance since it determines the

flows of costs and benefits over time. Most analyses of optimal rotation in a stochastic setting are formulated as a multistage decision process, and solved using dynamic programming.

In this paper we use an alternative approach using conventional budgeting methods. In particular, we apply a stochastic budgeting model within a stochastic dominance framework. The forest owners' temporary income for which risk aversion is likely to be slight is implicitly ignored in our model, and the focus is on the risk in permanent income, i.e., the general level of income over a long-time horizon. Using stochastic budgeting, we investigate the optimal rotation length under various assumptions and so determine the profitability to the DM of investing in a forest.

Caulfield (1988) and Gong (1998) used stochastic dominance analysis to explore the economic rotation of a forest with given assumptions about risk aversion. Caulfield investigated effects of stochastic volume growth, and especially changes in optimal rotation age followed by risk of fire. By using second-degree stochastic dominance (SSD) he found that the risk-efficient rotations might be shorter than would be the case in the absence of risk aversion. Gong investigated the effects of price risk using SSD analysis to show that risk-averse forest owners typically will harvest earlier than risk-neutral forest owners. Caulfield and Newman (1999) stated that the impact on rotation ages of incorporating price risk is uncertain.

In this paper we analyse how risk aversion affects the choice of an optimal forest rotation strategy and hence the initial investment decision using stochastic efficiency with respect to a function (SERF) (Hardaker et al. in press). SERF is a relatively new variant of the widely used stochastic dominance with respect to a function (SDRF) (Meyer 1977). SDRF has stronger discriminatory power (with respect to partially ranked risky alternatives) than first- and second-degree stochastic dominance. The greater discrimination is achieved through the introduction of bounds on the absolute risk-aversion coefficient within a SSD analysis. SERF, which partitions a set of risky alternatives in terms of certainty equivalents (CEs) for a specified range of attitudes to risk, is more transparent, easier to implement, and has even stronger discriminating power than conventional SDRF. The method is illustrated with a simple example comparing the merits of different average rotation lengths for a hypothetical forest investment in Norway. The empirical results show that, in cases with stochastic timber prices and volume growth, optimal rotation length on average increases with increasing degree of risk aversion.

2. Modelling framework and methods

Resolving the investment decision requires the optimal rotation strategy of the forest to be decided, given stochastic timber price and volume growth and a risk-averse decision maker. However the effects of risk aversion on the optimal rotation length are ambiguous and are therefore an empirical question, as also deduced by Caulfield and Newman (1999).

In our computed model, we do not know the forest owner's utility function and some efficiency criteria that allow some ranking of risky alternatives when the exact degree of risk aversion is not known must be used. A much used efficiency criterion given risk aversion is SSD. SSD assumes that the DM prefers more income to less and is not risk preferring, i.e., that absolute risk aversion bounds are $0 \leq r_a(w) < +\infty$. In empirical work it is often found that SSD are not discriminating enough to yield useful results.

An alternative to SSD is SDRF, which was introduced by Meyer (1977). In SDRF absolute risk aversion bounds are reduced to $r_L(w) \leq r_a(w) \leq r_U(w)$, and ranking of risky

scenarios is defined for all DMs whose absolute risk aversion coefficients lie anywhere between lower and upper bounds $r_L(w)$ and $r_U(w)$, respectively.

In this paper we apply a more straightforward and potentially more discriminating method called stochastic efficiency with respect to a function (SERF) (Hardaker et al. in press). SERF partitions alternatives in terms of CEs as a selected measure of risk aversion is varied over a defined range. Conventional SDRF picks only the pairwise dominated alternatives, thus one can expect that pairwise SDRF may not isolate the smallest possible efficient set. By contrast, SERF will potentially identify a smaller efficient set than SDRF because it picks only the utility efficient alternatives, comparing each with all the other alternatives simultaneously.

The maximand used to determine the optimal rotation length is the equivalent annuity of the whole forest. It is necessary to work in annuity terms because we are comparing investments of different lengths. The equivalent annuity can be interpreted as the forest owner's permanent income, meaning the average income over several years. Risk aversion is likely to be important in assessing stochastic permanent income levels. Given stochastic timber price and volume growth and assuming risk aversion, we base the choice of the optimal rotation (and hence the assessment of the profitability of the investment decision) on the expected utility of permanent income.

Under an assumption of stationarity, the distribution of permanent income (i.e. distribution of equivalent annuity) is measured by the distribution of the NPV of one stand of area A/T where A is total forest area and T is rotation length. In other words, the distribution of the NPV to an infinite horizon of one stand in our model is also the distribution of the equivalent annuity to an infinite horizon for the whole forest area.

If we return to our decision problem of the forest owner, we ignore any probability of natural disaster problems and exclude the possibility of partial harvesting before clearcutting each stand. Costs of silviculture, thinnings etc. are taken into account in the cost for the replanting year. Thus, the cash flow in the NPV formula for each stand consists only of the year with planting and the year with clearcut.

Given stochastic timber prices and volume growth, the one stand expected NPV of an infinite series of rotations of length T is calculated as

$$E[NPV] = \frac{\bar{p}(T) \cdot \bar{f}(T) \cdot e^{-iT} - I_0}{1 - e^{-iT}} \quad (1)$$

where I_0 are investment costs in the start of each rotation, $\bar{p}(T)$ is the stochastic stationary timber price less harvesting costs in year T , $\bar{f}(T)$ is the stochastic stationary timber volume in year T , and i is the discount rate.

For each risky alternative and for a chosen form of the utility function, the subjective expected utility hypothesis means that utility of permanent income (equivalent annuity) can be calculated depending on the degree of risk aversion, r , and the distribution of the NPV from one stand as:

$$E[U(NPV, r)] = U(NPV, r) = \int U(NPV, r) f(NPV) dNPV \quad (2)$$

Then U is calculated for selected values of r in the range r_1 to r_2 . The CE s for each of these values of U are found by:

$$CE(NPV, r) = U^{-1}(NPV, r) \quad (3)$$

The general rule for SERF analysis for the given assumptions is that the efficient set contains only those alternatives that have the highest (or equal highest) CE for some value of r in the relevant range.

The CDF distribution of one stand NPV in our model depends on many risky input variables, so we constructed a Monte Carlo stochastic simulation model to represent the forest owner's risky decision environment. We implemented the stochastic Monte Carlo simulation model within the SERF approach for our forest decision problem through the following steps:

1. Select a rotation length, T ;
2. Run one iteration and record the cash flows;
3. Select a coefficient of absolute risk aversion, r_a , within the range r_1 to r_2 ;
4. Convert cash flows to utilities, by using a negative exponential utility function, and find utility of the one stand net present value, $U(NPV)$ (as a measure for utility of permanent income and equivalent annuity);
5. Loop back to 3;
6. Loop back to 2;
7. When enough iterations, expected utility of NPV is computed for each r , and the inverse function is used to get CE s for each r ;
8. Loop back to 1 and do it all again for a different rotation length;
9. For each value of r_a , select as optimal the rotation length with the highest CE ; investment is worth-while only if the optimal CE s are positive.

3. Application

In this section, the approach outlined above is applied for a hypothetical forest property in Norway. For simplicity, we assume that the forest will be the forest owner's sole source of wealth.

The size of the chosen forest property is 3000 ha which is planted with Norway spruce with an even site quality index (H40) of G14. The uncertainty concerning future growth and quality has several different sources. Forestry is exposed to physical, climatic, and biological risks (e.g. insect attacks and rot). And from time to time extreme weather (for example gales) can cause natural disasters such as windthrows. In addition, the forest owner regards the price and market risk as important. We assume that the forest owner is risk-averse, but that we have not quantified exactly how risk-averse he is.

The presence of risk and risk aversion aspects makes it difficult to decide the optimal

rotational strategy, but some general guidelines are needed. In this illustration we construct an optimal rotation model taking volume and price risk into account. The results from such a model can give general guidelines about optimal rotation cycles for forest owners or prospective forest investors.

3.1. Variable Specification

The net stumpage value is a function of price, timber quality and volume, and harvesting costs. The timber quality and expected volume changes with the age of the stand. The volume function is estimated from a database of forest production and development of stand plots (site index G14) in Eastern Norway collected by the Norwegian Institute of Land Inventory for the years 1994 to 2002. Since we observed heteroskedasticity in the data, we estimated a cubic function with multiplicative heteroskedasticity (Harvey 1976):

$$f(t) = 0.7053 * t + 0.0787 * t^2 - 0.000528 * t^3 + e_t \quad (5)$$

$$Var(e_t) = 21.39^2 \exp(0.0488 * t - 0.000172 * t^2)$$

where $f(t)$ is volume per ha at forest age t .

The correlation of stand volume levels between years was calculated to be 0.95, implying that the expected volume curve from any stage onwards is conditional to the volume in the current year. This stochastic dependency is included in the analysis by defining the conditional expected yield in any year, given the volume in the previous year (where we assume that volume is normal distributed and the change in volume per ha between years is bivariate normal).

During a rotation there are changes in the portion of various log grades that can be produced from the harvested stems. Generally, increased age means a higher portion of high valued product (sawlogs) and lower portion of low valued grades (pulpwood). Thus, the expected timber price increases with age of stand. Moreover, timber quality becomes less even with age so that the price variation becomes greater. In order to capture these effects we estimated a price function where mean price and price variation depend on stand age. We first applied the relative price function for Norway spruce, which has height, diameter and the relative price difference between sawlogs and pulpwood ((sawlog price-pulpwood price)/pulpwood price) as input parameters. Given volume and increment functions for Norway spruce, and the actual relative timber prices in Eastern Norway for 2001, we got figures for relative prices for various stand ages. Normalising to a stand age of 79 years (i.e. we assumed that this age yielded the mean price), we got the following expressions for expected gross value per m³, $p(t)$ and the variance at a given time, t :

$$p(t) = E[p(t)] = 113.02 + 2.709t + e_t \quad (6)$$

$$Var(e_t) = (28.24 + 06769t)^2$$

We further assumed that the expected price is constant after a stand reaches 100 years of age, because this logs at this age is supposed to give the maximum sawlog share.

Investment costs were set to NOK (Norwegian kroner) 7000 ha⁻¹. This figure was chosen to represent the investment of the average forest owner. The real interest rate was assumed to be 2 per cent per annum.

3.2. Results

Stochastic simulation means that we can evaluate a given rotation strategy with any required degree of precision simply by setting the appropriate simulation sample size. We used a sample size of 5000 to get good estimates of the distribution of the chosen objective variable for any specified strategy.

We ran the simulation for rotation lengths from 60 to 110 years, with 10-years intervals and for four different degrees of risk aversion. A rough and ready classification of degrees of risk aversion, based on the relative risk aversion with respect of wealth, $r_r(w)$, is in the range 0.5 (hardly risk-averse at all) to about 4 (very risk-averse). In this paper we are not considering utility and risk aversion in terms of wealth, but in terms of permanent income (where the uncertainty relates to the long-run level of income). The relation between absolute risk aversion with respect of permanent income and relative risk aversion with respect to wealth is $r_a(x) \approx r_r(w)/x$ where x is permanent income [5]. We assume that the typical level of a forest owner's permanent income is NOK 150 000 per annum. Then a value of $r_a(x)$ in the range 0.00000667 to 0.00002667 corresponds to $r_r(w)$ in the range 0.5 to 4. This range was used as the risk aversion bounds in this analysis.

The SERF approach using a negative exponential utility function resulted in the CE-graph shown in Fig. 1.

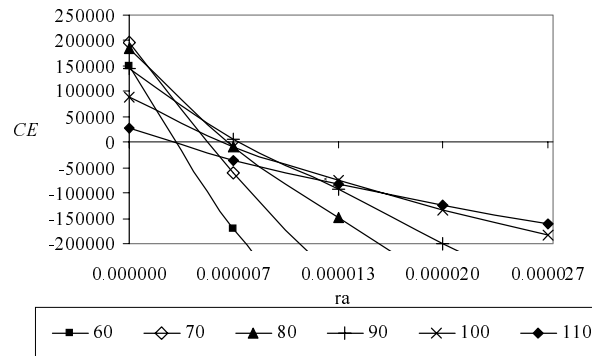


Figure 1. CE-graph of optimal forest strategies (60 – 110 years) for different degree of risk aversion. CE is expected equivalent annuity in NOK per year.

From the CE-graph we can observe:

- The forest owner's degree of risk aversion has a strong influence on the optimal rotation strategy. For a rotation strategy of, e.g., 80 years the CE annuity per year is reduced by 296% from risk neutrality to the most risk-averse case;
- A rotation length 70 or 80 years gives the highest CE for a risk indifferent forest owner ($r_a(x) = 0$);

- A moderately risk-averse forest owner (i.e. where $r_a(x) = 0.000013$ and $r_r(w) \approx 2$) should be almost indifferent between rotation lengths of 90, 100 and 110 years;
- A very risk-averse forest owner ($r_a(x) = 0.0000267$), ($r_r(w) \approx 4$) should prefer a rotation length of 110 years;
- At absolute risk aversion levels less than about 0.000008, there exists at least one rotation length that yields a positive CE, i.e. investment is worthwhile (subject to comparison with other non-forest alternatives);
- For very risk averse forest owners (with coefficient of absolute risk aversion higher than 0.000008) the investment is unprofitable, since all rotation strategies yields a negative CE.

3.3. Sensitivity Analysis

The results might be sensitive to variable specification. We therefore conducted sensitivity analyses by changing the interest rate from 2 to 3 per cent per annum; the investment cost from NOK 7000 to NOK 3500 ha⁻¹; and a combination of these two. The results from this analysis are shown in Fig. 2.

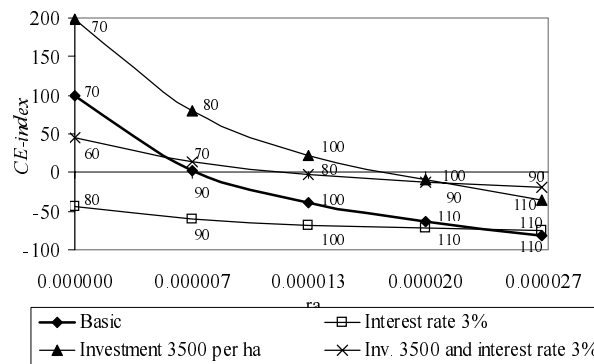


Figure 2. Sensitivity analysis of choice of interest rate, investment cost and the combination of both. For further details see the explanation in the text.

The frontiers (optimal choices) for each case are drawn in Fig. 2. The frontier curves are represented by index values, where the CE for a risk-neutral forest owner in the “basic” alternative is indexed to 100. In the basic alternative the interest rate is 2 per cent per annum and the investment cost is NOK 7000 ha⁻¹. The numbers inside the figure show the optimal rotation age for each case at the plotted degree of risk aversion.

Our empirical results show that a risk-neutral forest owner would be best advised to choose a longer rotation length (70 to 80 years) when the interest rates increase and approximately the same (though slightly shorter) rotation length when the investment cost decreases, as is expected from theory (Johansson and Löfgren 1985). From these results we thus also conclude that:

- the degree of risk aversion becomes less important in the choice of the optimal rotation strategy with the increase in the interest rate;
- the size of the investment cost affects only the level of the CE, and only slightly the importance of the degree of risk aversion.

4. Conclusions

Earlier studies, which have focused on either price risk or volume growth risk, have found that risk-averse forest owners typically will harvest earlier than risk-neutral forest owners. Our empirical results, that include both price and volume growth risk, show that the optimal length of the rotation strategy increases with increasing degree of risk aversion. Some reasons for this somewhat result that is contrary to earlier findings can be suggested. First, mathematically there is no a priori reason to expect that risk aversion lead to shorter rotations. Second, our model is an infinite horizon stationary model, which implies that the same uncertainty applies for a rotation length of, e.g., 80 years today as for a rotation length of 80 years that starts, e.g., 500 years in the future. The limitation of the stationarity property is that the uncertainty is likely to increase over time (from rotation to rotation) for a real forest owner, yet this aspect is not accounted for in the stationary model. This limitation may disfavour shorter rotations.

We have assumed the same investment strategy for all degrees of risk aversion. The results show that, for very risk-averse forest owners, the CEs are negative. The policy implication of this is that such types of forest owners will choose to invest less in forestry, or not to invest at all.

In the literature an often-used way to analyse optimal rotation decisions for a forest is to use of stochastic dynamic programming. It may be argued that stochastic simulation is more flexible and easier to implement and understand than stochastic dynamic programming. Our SERF-approach does not account for non-stationarity (as is also normally the case for infinite horizon dynamic programming). We have used our model to illustrate how general guidelines can be determined for investment based on a rotational strategy that matches each case, including the DM's degree of aversion to permanent income risk. Of course, in practice each forest owners does not have to decide at the start what rotation length to adopt. He can look at each section and decide when to harvest. This aspect is probably better analysed using dynamic programming.

We have assumed that it is sufficient to use a utility function defined in terms of the average annual income (equivalent annuity). At best this is an approximation of an intertemporal utility function. In the real risky world with a less than perfect capital market, the discounted cash flow method has several limitations. For future research, alternative utility functions should be considered, especially if the analysis accounts for the reality that income flows through time may be irregular.

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