Production Insurance and Input Use: An Analytical Framework

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Duality applies under uncertainty. In particular, Chambers and Quiggin (1998) have shown that dual cost structures exist for the continuous, stochastic technologies most familiar to agricultural economists. Beyond merely demonstrating existence, however, this finding has important implications for the analysis of stochastic decisionmaking. For years now, agricultural economists have intensively studied decisionmaking by producers facing stochastic technologies. And yet, no commonly accepted body of 'stylized facts' exists for most truly interesting formulations of this problem. Some have even questioned the relevance of the cost minimization hypothesis for risk-averse decisionmakers (Pope and Chavas). More generally, apart from a number of results that have been established for trivially stochastic situations, e.g., price but not production uncertainty, there is no common agreement as to what one can expect from a risk-averse producer facing a stochastic world.

A particular case in point is the literature on the economic implications of the public and private provision of crop insurance. The basic theory of crop insurance has been studied by several authors (Nelson and Loehman, Chambers). However, few really new results have emerged. For example, under the expected-utility hypothesis, a primary result is that optimality requires that a risk-neutral insurer offer full insurance to risk-averse farmers producing a single stochastic output so long as no informational asymmetries exist between the insurer and the farmer (Nelson and Loehman). But it is easy to recognize this result as a direct consequence of basic re-insurance results established much earlier by Borch.

Other authors have studied the related and potentially more important question of how input utilization is affected by the provision of crop insurance or income support (Quiggin 1992, Ramaswami, Hennessey). Intuitively, one expects that providing insurance encourages producers to undertake risky activities that carry with them the promise of higher expected returns. Reasoning thus, one also expects that inputs which might be perceived as enhancing the riskiness of the production outcome would be used more intensively in the presence of insurance than in its absence. Conversely, inputs which do little to enhance productivity, but which act as damage-control agents, would be used less intensively in the presence of insurance than in its absence. When stated in this fashion, these facts seem to be self-evident. However, the existing literature suggests that this is not generally the case even if attention is restricted to single-output, single-input technologies (Ramaswami; Horowitz and Lichtenberg). Because such technologies are highly restrictive, the natural implication seems to be that little, if anything, can be said for more realistic technologies.

The goal of this paper is to demonstrate, by way of theoretical example, the importance of the duality between cost and stochastic technologies by studying the impact of crop insurance upon input utilization. The basic model is a state-contingent formulation of the problem, which encompasses both production and price uncertainty, and that allows full exploitation of the duality between the technology and the cost structure in comparative-static analyses. In particular, we can rely on this duality and a stochastic version of Shephard's lemma to allow us to examine input responsiveness to the provision of crop insurance in a new and informative manner that does not rely on the single-input, single-output stochastic production function model that has dominated most previous studies. Moreover, this formulation carries it with the additional benefit that it also allows us to consider preference structures that are far more general than the expected-utility preference structures that have been used in the existing literature.

Using this formulation of the problem, we show that it is straightforward to develop a complete, analytical framework for analyzing the impact of crop insurance (and more generally any other comparative static problem regarding input utilization) that can be usefully illustrated with graphical techniques that should be familiar to virtually all economists. In particular, the analytical framework presents a decomposition of the problem reminiscent of the classic Hicks-Allen decomposition of the Slutsky effect familiar from rudimentary consumer theory.

1. Model and Assumptions

1.1. A State-Contingent Technology

Following Chambers and Quiggin (1996, 1997, 1999), the stochastic technology is represented by a multi-product, state-contingent input correspondence. To make this explicit, suppose that the states of nature are given by the set $\Omega = \{1, 2, ..., S\}$, let $\mathbf{x} \in \Re_{+}^{N}$ be a vector of inputs committed prior to the resolution of uncertainty, and let $\mathbf{z} \in \Re_{+}^{M \times S}$ be a vector of state-contingent outputs. So, if state $s \in \Omega$ is realized (picked by 'Nature'), and the producer has chosen the *ex ante* input-output combination $(\mathbf{x}, \mathbf{z})$, then the realized or *ex post* output vector is $\mathbf{z}^s$ corresponding to the sth column of $\mathbf{z}$. In other words, the observed output is an M-dimensional vector $\mathbf{z}^s$ where z_m^s corresponds to the mth output that would be produced in state s.

The input correspondence, $X : \Re_+^M \rightarrow \Re_+^N$, maps matrices of state-contingent outputs into input sets that are capable of producing that state-contingent output matrix. Formally, it is defined by

$$X(\mathbf{z}) = \{\mathbf{x} \in \Re_+^N : \mathbf{x} \text{ can produce } \mathbf{z} \in \Re_+^{MxS}\}.$$

We impose the following axioms on $X(\mathbf{z})$:

X.1 $X(\mathbf{0}_{MxS}) = \Re_+^N$ (no fixed costs), and $\mathbf{0}_N \in X(\mathbf{z})$ for $\mathbf{z} \geq \mathbf{0}_{MxS}$ and $\mathbf{z} \neq \mathbf{0}_{MxS}$ (no free lunch).

X.2 $\mathbf{z}' \leq \mathbf{z} \Rightarrow X(\mathbf{z}) \subset X(\mathbf{z}')$.

X.3 $\mathbf{x}' \geq \mathbf{x} \in X(\mathbf{z}) \Rightarrow \mathbf{x}' \in X(\mathbf{z})$.

X.4 $\lambda X(\mathbf{z}) + (1-\lambda)X(\mathbf{z}') \subset X(\lambda\mathbf{z} + (1-\lambda)\mathbf{z}') \qquad 0 \leq \lambda \leq 1.$

X.5 $X(\mathbf{z})$ is closed for all $\mathbf{z} \in \Re_+^{MxS}$.

The first part of X.1 says that doing nothing is always feasible, while the second part of X.1 says that realizing a positive output in any state of nature requires the committal of some inputs. X.2, free disposability of state-contingent outputs, says that if an input combination can produce a particular matrix of state-contingent outputs then it can always be used to produce a smaller matrix of state-contingent outputs. X.3 implies that inputs have non-negative marginal productivity. X.4 tells us that the state-contingent technology is convex, and intuitively it leads to diminishing marginal productivity of inputs. X.5 is a technical assumption that ensures the existence of the revenue-cost function that we develop next.

1.2. The revenue-cost function

Denote by $\mathbf{p} \in \Re_{++}^{M \times S}$ the matrix of state-contingent output prices corresponding to the matrix of state-contingent outputs. The interpretation of $\mathbf{p}$ is basically the same as $\mathbf{z}$. If 'Nature' picks $s \in \Omega$, then the vector of realized spot prices is $\mathbf{p}^s \in \Re_{++}^M$. We assume that the producers in question are competitive in the sense that they take these state-contingent output prices and the prices of all inputs as given. The state-contingent revenue vector $\mathbf{r} = \mathbf{pz} \in \Re_+^S$ has typical elements of the form $r_{s=}\mathbf{p}^s \bullet \mathbf{z}^s$.

In all cases we consider, producers will be concerned with state-contingent revenue rather than output *per se*, and it is useful to consider the *revenue-cost function* defined as

$$C(\mathbf{w}, \mathbf{r}, \mathbf{p}) = \min \left\{ \mathbf{w} \cdot \mathbf{x} : \mathbf{x} \in X(\mathbf{z}), \sum_m p_{ms} z_{ms} \geq r_s, s \in \Omega \right\}$$

if there exists a feasible state-contingent output array capable of producing $\mathbf{r}$ and ∞ otherwise. Here $\mathbf{w} \in \Re_{++}^N$ represents a strictly positive vector of input prices that the producer takes as given. The properties of $C(\mathbf{w}, \mathbf{r}, \mathbf{p})$ that follow from X.1-X.5 (Chambers and Quiggin, 1999):

Properties of the Revenue-Cost Function (CR):

CR.1$C(\mathbf{w}, \mathbf{r}, \mathbf{p})$ is positively linearly homogeneous, non-decreasing, concave, and continuous in $\mathbf{w} \in \Re_{++}^N$.

CR.2 Shephard's Lemma.

CR.3 $C(\mathbf{w}, \mathbf{r}, \mathbf{p}) \geq \mathbf{0}$ with equality if and only if $\mathbf{r} = 0$.

CR.4 $\mathbf{r}' \geq \mathbf{r} \Rightarrow C(\mathbf{w}, \mathbf{r}', \mathbf{p}) \geq C(\mathbf{w}, \mathbf{r}, \mathbf{p})$.

CR.5 $\mathbf{p}' \geq \mathbf{p} \Rightarrow C(\mathbf{w}, \mathbf{r}, \mathbf{p}') \leq C(\mathbf{w}, \mathbf{r}, \mathbf{p})$.

CR.6 $C(\mathbf{w}, \mathbf{r}_{-s}, \theta r_{s,}, \mathbf{p}_{-s}, \theta \mathbf{p}_s) = C(\mathbf{w}, \mathbf{r}_{-s}, \theta r_{s,}, \mathbf{p}_{-s}, \theta \mathbf{p}_s), \quad \theta > 0$.

CR.7$C(\mathbf{w}, \mathbf{r}, \mathbf{p}) = C(\mathbf{w}, \mathbf{r}/k, \mathbf{p}/k), \quad k > 0$.

CR.8 $C(\mathbf{w}, \mathbf{r}, \mathbf{p})$ is convex in $\mathbf{r}$.

For analytic simplicity, we shall typically assume that $C(\mathbf{w}, \mathbf{r}, \mathbf{p})$ is smoothly differentiable in all state-contingent revenues and input prices. By assuming a differentiable in revenues cost structure, we, therefore, rule out the stochastic-revenue function approach and the non-stochastic production approach of Sandmo (Chambers and Quiggin, 1998, 1999).[1]

[1] If production is non-stochastic, then it follows immediately that

$$C(\mathbf{w}, \mathbf{r}, \mathbf{p}) = \mathbf{Max}_{1,2,\ldots S} \left\{ C^f(\mathbf{w}, r_s, p_s) \right\}$$

where

$$C^f(\mathbf{w}, r_s, p_s) = Min \left\{ \mathbf{wx} : p_s f(\mathbf{x}) \geq r_s \right\}.$$

Generally, neither this function or the one corresponding to the stochastic-revenue function will be everywhere

1.3. Preferences

The producer's preferences are represented by an increasing function of his vector of state contingent net returns.

$$\mathbf{y} = \mathbf{r} - (\mathbf{w} \cdot \mathbf{x})\, \mathbf{1}_s,$$

where $\mathbf{1}_S$ is the S-dimensional unit vector. As Chambers and Quiggin (1998) demonstrate, without loss of generality, the producer's preferences can thus be expressed in terms of the revenue-cost function as

$$\mathbf{y} = \mathbf{r} - \mathbf{C}(\mathbf{w}, \mathbf{r}, \mathbf{p})\mathbf{1}_s.$$

Following Quiggin and Chambers (1998) and Chambers and Quiggin (1999), these preferences over state-contingent net returns will be represented in terms of a continuous, strictly increasing preference function $W : \Re^s \to \Re$. A producer is said to be *risk-averse with respect to the probability vector* $\boldsymbol{\pi} \in \Pi$ *if*

$$W(\bar{y}\mathbf{1}^S) \geq W(\mathbf{y}), \forall \mathbf{y}$$

where $\bar{y}\mathbf{1}^S$ is the state-contingent outcome vector with $\bar{y} = \sum_{s \in \Omega} \pi_s y_s$ occurring in every state of nature (Yaari, 1969; Quiggin and Chambers, 1998).

If preferences are smoothly differentiable, the vector of probabilities is unique and proportional to the marginal rate of substitution between state-contingent incomes along the equal-incomes vector. More concretely, without loss of generality, if preferences are smoothly differentiable

$$\pi_s = \frac{W_s(c\mathbf{1}^S)}{\sum_{t \in \Omega} W_t(c\mathbf{1}^S)}, \quad s \in \Omega, \quad c \in \Re.$$

Pictorially, therefore, the *fair-odds line*, which gives the locus of points having the same expected value and whose slope is given by minus the relative probabilities is given by the slope of the tangent to the producer's indifference curve at the bisector. Figure 1 illustrates.

In order to impose some structure upon preferences other than simple aversion to risk, consider the partial ordering $\preceq_\pi$ of risky outcomes which possess a common mean for the probability vector $\boldsymbol{\pi}$. This partial ordering is defined by

$$\mathbf{y} \preceq_\pi \mathbf{y}'$$

smoothly differentiable in revenues or outputs respectively.

if and only if $\mathbf{y}$ and $\mathbf{y}'$ have the same mean and $\mathbf{y}$ is less risky than $\mathbf{y}'$ in the sense of Rothschild and Stiglitz. Chambers and Quiggin (1997) define a function $W : \Re^S \to \Re$ to be *generalized Schur-concave* for π if $\mathbf{y} \preceq_\pi \mathbf{y}' \Rightarrow W(\mathbf{y}) \geq W(\mathbf{y}')$.

A comment about generalized Schur concavity is worthwhile. Unlike the assumption of expected-utility maximization, generalized Schur concavity doesn't impose additive separability across states of nature. Consequently, it does not rely upon the independence axiom which has proved vulnerable to a variety of criticisms. Even so, the expected-utility functional with concave u is generalized Schur-concave as can be recognized from the result due to Rothschild and Stiglitz that if $\mathbf{y} \preceq_\pi \mathbf{y}'$ then $\mathbf{y}$ would be preferred to $\mathbf{y}'$ by all individuals with risk-averse expected-utility preferences. More generally, generalized Schur concavity characterizes a number of preference classes, which are consistent with risk-aversion in our sense, but which are not consistent with expected utility. An obvious example is given by individuals with maximin preferences

$$W(\mathbf{y}) = \min\{y_1, ..., y_S\}.$$

This class of preferences is risk-averse in our sense for all possible probability vectors (note it is not differentiable), and it is also generalized Schur concave. Another obvious class of generalized Schur concave preferences is the mean-variance class. More generally, virtually all preference functions currently in use, including the rank-dependent models (Quiggin 1982, Yaari 1987) and weighted-utility models (Chew 1983) are consistent with generalized Schur concavity. The main result of Machina (1982) may be restated in our terminology as saying that preferences are generalized Schur concave if and only if the local utility function is everywhere concave.

In what follows, we shall frequently restrict attention to the case where W is smoothly differentiable. In that case, a basic result due to Chambers and Quiggin (1997), which we state in lemma form for future use, will prove useful:

Lemma 1 If $W : \Re^S \to \Re$ is generalized Schur-concave and once continuously differentiable everywhere on its domain, then :

$$\left(\frac{W_s(\mathbf{y})}{\pi_s} - \frac{W_r(\mathbf{y})}{\pi_r}\right)(y_s - y_r) \leq 0,$$

for all s and r.

2. Production Equilibrium in the Absence of Insurance

As a point of comparison, we first present some basic results on the production choices of risk-neutral and risk-averse producers in the absence of insurance. Suppose the risk-neutral producer's subjective probabilities are given by the vector $\boldsymbol{\pi}$. Then her first-order conditions on $\mathbf{r}$ may be written in the notation of complementary slackness as

$$\pi_s - C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) \leq 0, \quad r_s \geq 0, \quad s \in \Omega$$

where

$$C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) = \frac{\partial C(\mathbf{w}, \mathbf{r}, \mathbf{p})}{\partial r_s}.$$

That is, the marginal cost of increasing revenue in any state is at least equal to the subjective probability of that state. Pictorially, therefore, we represent the producer equilibrium by a hyperplane being tangent to an isocost curve of the producer. Figure 2 illustrates. Here the slope of the hyperplane is determined by the ratio of the producer's subjective probabilities, *the fair-odds line*, and the isocost curve is determined by the equilibrium level of revenue-cost. This is exactly analogous to the representation of production equilibrium in the non-stochastic, multi-product case. Instead of determining an optimal mix of outputs as in the non-stochastic multi-product case, however, the producer equilibrium now determines the optimal mix of state-contingent revenues. This analogy naturally suggests interpreting the producer's subjective probabilities as the producer's subjective prices of the state-contingent revenues.

Summing the first-order conditions on $\mathbf{r}$ yields an *arbitrage condition*

$$\sum_{s \in \Omega} C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) \geq \sum_{s \in \Omega} \pi_s = 1. \tag{2.1}$$

Intuitively speaking, $\sum_{s \in \Omega} C_s(\mathbf{w}, \mathbf{r}, \mathbf{p})$ is the marginal cost of increasing all state-contingent revenues by the same small amount in each state of nature, i.e., it is the marginal cost of a sure increase in revenue of one unit. Hence, (2.1) simply requires that this cost be at least as large as the associated sure increase in returns. If it were not, the decisionmaker could increase profit with probability 1, and she would thus have an incentive to continue expanding all revenues equally. For an interior solution, (2.1) must hold as an equality.

We shall refer to the set of revenue vectors $\mathbf{r}$ satisfying (2.1) for given $\mathbf{w}$,$\mathbf{p}$ as the *efficient set*, denoted $\Xi(\mathbf{w},\mathbf{p})$,

$$\Xi(\mathbf{w},\mathbf{p}) = \left\{ \mathbf{r} : \sum_{s\in\Omega} C_s(\mathbf{w},\mathbf{r},\mathbf{p}) \geq 1 \right\}.$$

We call the boundary of $\Xi(\mathbf{w},\mathbf{p})$ the *efficient frontier* and note that its elements are given by:

$$\bar{\Xi}(\mathbf{w},\mathbf{p}) = \left\{ \mathbf{r} : \sum_{s\in\Omega} C_s(\mathbf{w},\mathbf{r},\mathbf{p}) = 1 \right\}.$$

By the homogeneity properties of the revenue-cost function, we can conclude that: $\Xi(\theta\mathbf{w},\theta\mathbf{p}) = \theta\Xi(\mathbf{w},\mathbf{p})$ and $\bar{\Xi}(\theta\mathbf{w},\theta\mathbf{p}) = \theta\bar{\Xi}(\mathbf{w},\mathbf{p})$, $\theta > 0$ (Chambers and Quiggin, 1999). That is, the efficient set and the efficient frontier are positively linearly homogeneous in input and output prices.

Different risk-neutral producers may hold different subjective probabilities. Regardless of the individual's subjective probabilities, however, a revenue vector $\mathbf{r}$ is potentially optimal for some risk-neutral decision-maker only if (2.1) holds. Hence, $\bar{\Xi}(\mathbf{w},\mathbf{p})$ can be interpreted naturally as the collection of state-contingent revenues that are potentially expected-profit maximizing. To see why, suppose that (2.1) holds for an arbitrary revenue vector, call it $\hat{\mathbf{r}}$. Now construct a set of probabilities by setting $\hat{\pi}_s = C_s(\mathbf{w},\hat{\mathbf{r}},\mathbf{p})$ for all s. Because they belong to the efficient set and are derived from a non-decreasing revenue-cost function, these probabilities are positive and sum to one. Moreover, a risk-neutral individual having such probabilities would choose $\hat{\mathbf{r}}$ as the expected-profit maximizing vector of state-contingent revenues. The correspondence of the producer's subjective probabilities with these state-contingent marginal costs then determines the optimal point on the efficient set.

Now let us turn to the case where the producer is not risk-neutral but risk-averse with generalized Schur-concave preferences[2]. The producer chooses state-contingent revenues to maximize:

$$W(\mathbf{y}) = W(\mathbf{r} - C(\mathbf{w},\mathbf{r},\mathbf{p})\mathbf{1}_S).$$

So long as the preference function is smoothly differentiable in state-contingent revenues,

[2]Risk-neutral preferences are trivially generalized Schur-concave.

then the first-order condition on r_s is:

$$W_s(\mathbf{y}) - C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) \sum_{t \in \Omega} W_t(\mathbf{y}) \leq 0, \quad r_s \geq 0,$$

with complementary slackness.

The arbitrage condition (2.1) can be derived as a consequence of summing these first-order conditions

$$\sum_{s \in \Omega} C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) \geq 1. \tag{2.2}$$

We conclude from (2.2) that a producer with generalized Schur-concave preferences chooses a revenue vector that is in the efficient set. Hence, as observed by Chambers and Quiggin (1997), there always exists a vector of probabilities that will induce a risk-neutral individual to choose the same production pattern as that chosen by one with generalized Schur-concave preferences. In general, however, these probabilities derived from the efficient frontier will not correspond to the producer's subjective probabilities unless she is herself risk-neutral.

Observe, that condition (2.2) holds with equality for an interior solution even in the absence of differentiability of W. Suppose that

$$\sum_{s \in \Omega} C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) > 1,$$

and that $\mathbf{r}$ is strictly positive. Then, as before, revenue can be reduced by one unit in every state of the world generating a cost reduction of more than one unit leading to an increase in net returns with probability 1. Pictorially, this production equilibrium is illustrated by a tangency between the producer's indifference curve and one of her isocost curves as illustrated in Figure 3.

More generally, the production equilibrium will be characterized by a level curve of the producer's indifference curve just sitting on a level curve of his revenue-cost function. This implies, for example, that when preferences are of the maximin form, producers completely stabilize revenues and produce where the efficient frontier intersects the equal-revenue curve (the bisector).

Several points should be made here to facilitate comparison of the risk-neutral production pattern with that associated with generalized Schur-concave preferences. For an interior solution, it is obvious that a risk-neutral producer chooses his state-contingent revenues so that for all $t, s \in \Omega$

$$\frac{C_s(\mathbf{w}, \mathbf{r}, \mathbf{p})}{\pi_s} = \frac{C_t(\mathbf{w}, \mathbf{r}, \mathbf{p})}{\pi_t}.$$

Moreover, summing the first-order conditions for a risk-neutral producer, it follows immediately by complementary slackness that

$$\sum_{s\in\Omega} \pi_s r_s - \sum_{s\in\Omega} C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) r_s = 0. \tag{2.3}$$

Expression (2.3) requires that the marginal profitability of increasing the optimal state-contingent revenue vector radially is zero for a risk-neutral producer[3].

However, assuming an interior solution and differentiable preferences, it follows from the risk averter's first-order condition and Lemma 1 that:

$$\left(\frac{C_s(\mathbf{w}, \mathbf{r}, \mathbf{p})}{\pi_s} - \frac{C_t(\mathbf{w}, \mathbf{r}, \mathbf{p})}{\pi_t}\right)(r_s - r_t) \leq 0. \tag{2.4}$$

Expression (2.4) implies an inverse covariance between the elements of the state-contingent revenue vector $\mathbf{r}$ and the vector with typical element, $\frac{C_s(\mathbf{w},\mathbf{r},\mathbf{p})}{\pi_s}$. Hence, we conclude:

$$\sum_{s\in\Omega} C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) \left(r_s - \sum_{t\in\Omega} \pi_t r_t\right) \leq 0. \tag{2.5}$$

Expression (2.5) and the arbitrage condition imply that a risk averter with generalized Schur-concave preferences will choose an optimal state-contingent revenue vector that is characterized by the fact that a small radial expansion of it will lead to an increase in expected profit[4].

[3]More formally, the left-hand side of (2.3) is the directional derivative of expected profit in the direction of the state-contingent revenue vector, i.e.,

$$\frac{\partial}{\partial \lambda}\left[\sum_{s\in\Omega} \pi_s \lambda r_s - C(\mathbf{w}, \lambda\mathbf{r}, \mathbf{p})\right]$$

evaluated at $\lambda = 1$. Expression (2.3) requires that this directional derivative be set to zero. Hence, a radial change in revenue has no effect on expected profit.

[4]The finding that a radial expansion of the optimal state-contingent revenue vector for an individual with generalized Schur-concave preferences increases expected profit generalizes to the case of multiple-output price and production uncertainty and general preferences Sandmo's result that in the absence of production

Generally speaking, therefore, the risk-averter does not equate his marginal rate of transformation between state-contingent revenues to the ratio of probabilities as a risk-neutral individual would. Furthermore, the risk averter operates on a smaller scale than a risk-neutral producer in the sense that the former can radially expand his optimal state-contingent revenue vector and increase profit while the latter cannot. In a word, the risk averter trades off expected return in an effort to provide self insurance against the price and revenue risk that he faces. And because the preference function is generalized Schur concave, then, in the neighborhood of the equilibrium, the revenue-cost function must behave as though it, too, were generalized Schur concave. Accordingly, in that neighborhood, there must a negative correlation between marginal cost and the level of the state-contingent revenues.

Expression (2.4) corresponds to Peleg and Yaari's notion of *risk-averse efficiency*. Risk-averse efficiency intuitively means that any state-contingent revenue vector satisfying this property would be optimal for some risk-averse individual who incurred the same level of revenue-cost. Pictorially, one can visualize the result by returning to Figure 3 and noting that the producer equilibrium is characterized by a tangency between her indifference curve and her isocost curve. Also recall from Figure 1 that the ratio of probabilities (the slope of the fair-odds line) is given by the slope of the producer's indifference curve at the bisector. As drawn in Figure 3, it follows immediately that, in absolute value, the slope of the fair-odds line is flatter than the slope of the isocost curve at the optimum. Then coupling this fact with the fact that $r_2 > r_1$ in the illustration gives (2.4) in the two-state case. It also follows from (2.4) that any suitably small mean preserving multiplicative spread of the equilibrium revenue vector leads to a fall in revenue cost. In Figure 3, this is illustrated by the fair-odds line cutting the isocost frontier from below.[5]

uncertainty and in the presence of expected-utility preferences, a risk averter always produces less than a risk-neutral producer. When there is a single non-stochastic output and price uncertainty, then even for generalized Schur concave preferences, the current result implies that a risk-averter produces less than a risk-neutral individual.

[5]A mean-preserving multiplicative spread is represented by a movement from the equilibrium point to the northwest along the fair-odds line.

3. Decomposing Input Adjustments

Having provided a brief survey of productive decisionmaking under uncertainty in a state-contingent framework, our next task is to specify an algorithm for examining how input utilization responds to the provision of crop insurance. Our starting point is the recognition via CR. 2 (Shephard's Lemma) that optimal input demands can be recaptured directly from the revenue-cost function as

$$\mathbf{x}(\mathbf{w},\mathbf{r},\mathbf{p}) = \nabla_{\mathbf{w}} C(\mathbf{w}, \mathbf{r}^*(\mathbf{w},\mathbf{p}), \mathbf{p})$$

where $\mathbf{r}^*(\mathbf{w},\mathbf{p})$ denotes the producer's optimal state-contingent revenue vector. So, for example, if input and output prices remain constant, comparing input demands for a risk-neutral individual with those of an individual with strictly generalized Schur-concave preferences, assuming both share the same technology, is simply a matter of comparing the same input demand function evaluated at two different optimal state-contingent revenue vectors. More generally, comparing different input demands arising from the same technology requires the ability to compare different state-contingent revenue vectors.

In an uncertain world, different state-contingent revenue vectors may usefully be compared in two dimensions. The first compares their relative expected returns, while the latter contains some measure of their riskiness. A risk-averse individual is, by definition, willing to trade off some increase in expected returns in return for an (appropriately defined) reduction in riskiness. Hence, it is imperative that any decomposition that we might suggest should clearly recognize these two dimensions of the decisionmaker's problem. Therefore, in what follows we decompose all comparisons of different revenue vectors, and hence their associated input demands, into two effects. The first is a *pure risk effect* which keeps means constant but allows riskiness to vary, and the latter is an *expansion* effect which measures the difference in expected returns.[6]

[6] In actuality, there are an infinity of ways to compute the pure risk effect and expansion effect depending upon how one makes the adjustment in mean values. Here we always restrict attention to the case where the mean adjustment is consistent with a radial expansion because we think it would be most familiar to most economists. More generally, the expansion can be measured in any direction. Which direction is chosen will be typically determined by a number of factors. For example, if it is assumed that individuals have

Figure 4 illustrates our proposed decomposition for revenue changes. Let point A in that figure be the risk-neutral individual's optimum and point B be the risk-averter's optimal point. Now suppose that we want to compare these two optima and their associated input demands. For the purpose of discussion, we will make the comparison in terms of moving from B to A. However, it is also perfectly plausible to consider the move from A to B, but we defer that analysis to the reader's initiative.

The decomposition we employ breaks that move down into two effects. The first is the movement from B to the point C which is on the same fair-odds line as B. Point C has the same expected revenue as at B, but the same revenue mix as at A (is on the same ray as A). Because comparing points B and C involves comparing outcomes with the same mean, then in some sense (which we define precisely in a minute) the difference between B and C must be solely a difference in the riskiness of the two prospects. We shall call that comparison the pure risk effect.

The second effect, which measures the difference in the means of the two prospects, is associated with the movement (in this case) outward along the ray from C to point A. (A is arrived at by deflating point C by the ratio of C's mean to B's mean.) We shall refer to this movement in the revenue vector as the (radial) expansion effect. Combining these two effects allows us to arrive at a mean-compensated decomposition of revenue and input adjustments.

To make the mean-compensated decomposition meaningful, we need a clear definition of what it means for one state-contingent revenue vector to be riskier than another which possesses the same mean. Following Chambers and Quiggin (1999), we define a risk ordering, denoted $\preceq_W$, directly in terms of the preference function W. Hence, if $\mathbf{y}$ and $\mathbf{y}'$ share a common mean, then $\mathbf{y} \preceq_W \mathbf{y}'$ if $W(\mathbf{y}) \geq W(\mathbf{y}')$. (Strictly speaking, $\mathbf{y} \preceq_W \mathbf{y}'$ should be read as $\mathbf{y}$ is less risky than $\mathbf{y}'$ for preferences W. However, we shall simply say that $\mathbf{y}$ is less risky than $\mathbf{y}'$.) So, in terms of Figure 4, C is riskier than B if it lies on a lower indifference

preferences exhibiting constant relative risk aversion then the radial decomposition we suggest is usually the most appropriate because the expansion effect involves no change in the riskiness of the revenue bundle. However, if preferences exhibit constant absolute risk aversion, then an expansion effect measured in the direction of the equal-incomes vector would be more appropriate as it would involve no change in the inherent riskiness of the revenue bundle.

curve than B.

Now that we have defined a risk-ordering, the final piece that we need is a way to relate that risk-ordering to input utilization. In the past, considerable attention has been devoted to the notions of *risk-reducing* and *risk-increasing inputs* (Pope and Kramer). Intuitively, these notions seem clear: risk-reducing inputs reduce the riskiness of output, and risk-increasing inputs increase the riskiness of state-contingent outputs. Clear as this intuition seems, writers on production under uncertainty have struggled with formalizing a definition of risk-increasing and risk-reducing inputs that matches this simple intuition and which accords with general notions of increases and decreases in risk[7].

The state-contingent approach adopted in this paper, however, leads to a rather different perspective. Rather than thinking of input choices, in combination with random variation, determining a stochastic output, we consider inputs and state-contingent outputs to be chosen jointly, in a preference maximizing fashion subject to a state-contingent input correspondence. Hence, it is natural to think in terms of complementarity between input choices and more or less risky state-contingent output patterns rather than in terms of simple causal relationships between input choices and risk.

Therefore, following Chambers and Quiggin (1996, 1999)[8], we define input n as a risk complement (risk substitute) at $\mathbf{r}$ if

$$\mathbf{r} \preccurlyeq_W \mathbf{r}' \Rightarrow x_n(\mathbf{w}, \mathbf{r}', \mathbf{p}) \geq x_n(\mathbf{w}, \mathbf{r}, \mathbf{p})(x_n(\mathbf{w}, \mathbf{r}', \mathbf{p}) \leq x_n(\mathbf{w}, \mathbf{r}, \mathbf{p})).$$

Here the intuition is clear. Something is a risk complement if more of it is used with more risky state-contingent revenue vectors than with less risky state-contingent revenue vectors. Just the reverse logic applies for a risk substitute. Because we have been able to make our notion of 'more risky' precise and to compensate for mean differences in state-contingent revenue vectors, this intuition accords closely with the more commonly popular notion of a risk increasing (risk reducing) input. However, we prefer our terminology because it emphasizes the simultaneity between the input choice and the state-contingent revenue

[7] An input is typically called risk-increasing or risk-reducing depending upon the sign of a second partial derivative of stochastic production function (e.g., Quiggin 1992, Ramaswami).

[8] In Chambers and Quiggin (1996), the terms risk increasing and risk decreasing were used in place of risk complements and risk substitutes.

choice. and it is a proper risk comparison as a mean-compensation is involved.

Several comments are in order. First, it's not a purely technological definition. It depends upon both the technology and the producer's objective function W. (Also recall that probabilities, which are required for the mean compensation, in our framework are determined in a subjective manner from the producer's preferences over state-contingent outcomes.) Second, it's a local notion as it's expressly given at a point in state-contingent output space. And third,

$$\mathbf{r} \preccurlyeq_{\pi} \mathbf{r}' \Rightarrow \mathbf{r} \preccurlyeq_{W} \mathbf{r}'$$

if W is generalized Schur concave. Thus, via Lemma 1 this definition leads to a natural characterization of risk complementarity and risk substitutes in terms of partial derivatives of input demand functions

Lemma 2 Suppose that the revenue-cost function is twice differentiable in all its arguments.

Input n is a risk complement for a generalized Schur-concave preference structure at $\mathbf{r}$ only if for all $r, s \in \Omega$:

$$\left(\frac{\frac{\partial x_n(\mathbf{w},\mathbf{r},\mathbf{p})}{\partial r_r}}{\pi_r} - \frac{\frac{\partial x_n(\mathbf{w},\mathbf{r},\mathbf{p})}{\partial r_s}}{\pi_s} \right) (r_r - r_s) \geq 0.$$

Input n is a risk substitute for a generalized Schur-concave preference structure at $\mathbf{r}$ only if for all $r, s \in \Omega$:

$$\left(\frac{\frac{\partial x_n(\mathbf{w},\mathbf{r},\mathbf{p})}{\partial r_r}}{\pi_r} - \frac{\frac{\partial x_n(\mathbf{w},\mathbf{r},\mathbf{p})}{\partial r_s}}{\pi_s} \right) (r_r - r_s) \leq 0.$$

From Lemma 2, it follows immediately that $\frac{\frac{\partial x_n(\mathbf{w},\mathbf{r},\mathbf{p})}{\partial z_r}}{\pi_r}$ is inversely (positively) correlated with r_r if x_n is a risk substitute (complement), whence for risk substitutes

$$\sum_{s\in\Omega} \frac{\partial x_n(\mathbf{w},\mathbf{r},\mathbf{p})}{\partial r_s} \left(r_s - \sum_{r\in\Omega} \pi_r r_r \right) \leq 0,$$

with the reversed inequality for risk complements. In words, an input is a risk substitute if its responsiveness to state-contingent revenue variation is large and positive for the lowest revenue states and small, and possibly negative, for the highest income states. Here the intuition is relatively straightforward. If an input is a risk substitute, it will tend to be used the most in producing the least risky revenue distributions. It is, therefore, natural to expect it to be most positively responsive to the lowest revenue states and the least responsive to the highest revenue states because this type of flexibility will be associated with smoother (less risky) revenue distributions.

4. Input Use and Insurance

4.1. Producer Equilibrium

We assume that the insurer is risk-neutral and competitive. For simplicity, we assume that the insurer has the same information set as the producer, and the producer is risk-averse for the insurer's subjective probabilities, which we continue to denote as π. Because the insurer can observe 'Nature's' draw from Ω, he can write state-contingent insurance contracts. An actuary employed by the insurer would regard as fair any contract for which

$$\sum_s \pi_s I_s = 0.$$

where I_s denotes the net indemnity paid by the insurance company in state s. Any equilibrium insurance contract offered by a competitive, risk-neutral insurance company must be *actuarially fair* in this sense. To be actuarially fair, therefore, the net indemnity schedule must involve positive payouts in some states and negative payouts in other states of nature.

We now consider how the farmer would optimally exploit the presence of a competitive crop insurance market. Given the freedom to choose any actuarially fair contract, the representative farmer's optimal production *cum* insurance scheme solves:

$$\max_{\mathbf{I},\mathbf{r}} \left\{ W\left(\mathbf{r} + \mathbf{I} - C\left(\mathbf{w}, \mathbf{r}, \mathbf{p}\right)\right) : \sum_s \pi_s I_s = 0 \right\}. \tag{4.1}$$

Recognizing that now

$$y_s = r_s + I_s - C\left(\mathbf{w}, \mathbf{r}, \mathbf{p}\right),$$

shows that (4.1) can be rewritten after a simple change of variables as

$$\max_{\mathbf{y},\mathbf{r}} \left\{ W\left(\mathbf{y}\right) : \sum_s \pi_s y_s = \sum_s \pi_s r_s - C\left(\mathbf{w}, \mathbf{r}, \mathbf{p}\right) \right\}.$$

Therefore, it follows immediately that, regardless of whether her preferences are smoothly differentiable or not, the farmer chooses her state-contingent revenue vector to

$$\max_{\mathbf{r}} \left\{ \sum_s \pi_s r_s - C(\mathbf{w}, \mathbf{r}, \mathbf{p}) \right\}.$$

Verifying this fact is easy. Suppose that the farmer has chosen a particular state-contingent revenue vector, an indemnity vector, and thus a net-returns vector which is not consistent with this strategy. The farmer then can obviously hold her net-return vector constant while rearranging her production choices to generate a larger amount of income than before. This extra income could then be used to raise all state-contingent net returns thus making the farmer better off with certainty. Because her objective function is non-decreasing in these state-contingent net incomes, she'll choose her production vector to maximize expected profit.

Presuming she chooses the expected profit maximizing state-contingent revenue vector, notice that by her risk aversion[9] the indemnity schedule (evaluated at the expected profit maximizing state-contingent revenue)

$$I_s = \sum_t \pi_t r_t - r_s, \ \ s \in \Omega.$$

dominates all others because it guarantees her a certain income of

$$\max_{\mathbf{r}} \left\{ \sum_s \pi_s r_s - C(\mathbf{w}, \mathbf{r}, \mathbf{p}) \right\},$$

which is the best that she could possibly hope for. Even a risk-neutral individual would at least weakly prefer this contract to all others. Moreover, this indemnity schedule breaks even for the insurer.

So we've established that: *Risk-averse farmers who face an actuarially fair insurance contract will produce in the same fashion as a risk-neutral farmer.* In the presence of an actuarially fair insurance market, a risk-averse farmer's production pattern is independent of her risk preferences. An immediate implication is that a farmer's optimal revenue choice in the presence of actuarially fair crop insurance contract belongs to the efficient set. These

[9] Notice risk aversion with respect to $\boldsymbol{\pi}$ is sufficient here. We need not invoke generalized Schur concavity of W.

results confirm, for our more general preference and production structure, the full-insurance result originally obtained by Nelson and Loehman.

These arguments can be illustrated by using calculus based arguments. Note that if preferences are smoothly differentiable, the farmer's first-order conditions are:

$$\begin{aligned} \frac{\partial W(\mathbf{y})}{\partial y_s} - \lambda \pi_s &= 0, \quad s \in \Omega, \\ \frac{\partial W(\mathbf{y})}{\partial y_s} - \frac{\partial C(\mathbf{w}, \mathbf{r}, \mathbf{p})}{\partial r_s} \sum_{s \in \Omega} \frac{\partial W(\mathbf{y})}{\partial y_s} &\leq 0, \quad r_s \geq 0, s \in \Omega, \end{aligned} \tag{4.2}$$

in the notation of complementary slackness. Here, λ is the non-negative Lagrangian multiplier associated with the zero profit constraint for the insurance company. Because W is increasing in all state-contingent net revenues, $\lambda > 0$. This Lagrangian multiplier is interpretable in several ways: Most familiarly, it is the shadow value of the zero-profit constraint in the farmer's maximization problem. However, an alternative interpretation is also possible. Sum the first set of S conditions in (2.4) to obtain:

$$\lambda = \sum_{s \in \Omega} \frac{\partial W(\mathbf{y})}{\partial y_s}. \tag{4.3}$$

The Lagrangian multiplier can also be interpreted as the directional derivative of the farmer's preference function in the direction of the equal income ray (the bisector), which is economically interpretable as the marginal utility associated with raising all state-contingent incomes by one (small) unit. Expression (4.3), therefore, represents an arbitrage condition between the insurer and the farmer. The opportunity cost of an another dollar available for indemnities must equal the farmer's marginal gain from receiving such a payment with certainty.

Hence, it follows immediately from these conditions that for an interior equilibrium the farmer produces where for all $r, s \in \Omega$

$$\frac{C_s(\mathbf{w}, \mathbf{r}, \mathbf{p})}{\pi_s} = \frac{C_t(\mathbf{w}, \mathbf{r}, \mathbf{p})}{\pi_t},$$

just as would a risk-neutral individual. Now by Lemma 1 on generalized Schur concave preferences it follows immediately that the producer completely stabilizes his income stream through the use of net indemnities.

Figure 5 illustrates pictorially for the case of smooth preferences[10]. The isocost curve in that figure represents the level curve of $C(\mathbf{w}, \mathbf{r}, \mathbf{p})$ as evaluated at the optimal level of state-contingent production. It is drawn as tangent to the fair-odds line at the optimal state-contingent production point (r_1^*, r_2^*) reflecting the fact that the farmer picks her revenue vector to maximize expected net income. The farmer will now trade with the insurance company along the fair-odds line until her marginal rate of substitution between state-contingent incomes is the same as the insurer's. And since this equalization occurs at the bisector for smooth generalized Schur-concave preferences, the producer ultimately locates there.

4.2. The effect of insurance on input use

Assessing the impact that actuarially fair crop insurance has on input utilization, thus, reduces to comparing the input decisions made by a risk-neutral producer and a risk-averse producer. Generally speaking, there are four possible outcomes when expressed in terms of the expansion effect and the pure-risk effect. Both the expansion effect and the risk effect can be positive, in which case the overall effect on an input's use is positive. The expansion effect can be positive and the risk effect can be negative, in which case the overall effect is ambiguous. The expansion effect can be negative and the risk effect negative, in which case the overall effect is negative. And finally, the expansion effect can be negative and the risk effect positive, in which case the overall effect is ambiguous.

More finely, however, there exists an even larger number of possibilities. For example, the expansion effect on input utilization could be positive because the expansion effect on revenues is positive and the input is non-regressive to radial expansions in revenues. Alternatively, the expansion effect on input utilization could be positive because the expansion effect on revenues is negative, but the input is regressive in radial expansions of revenues. Similarly, a negative effect could emerge from a positive (negative) expansion effect on revenues

[10]More generally, since a dominance argument was used to establish these results, it follows immediately that they apply regardless of whether preferences or the technology are smooth. Moreover, in the case of non-smooth preferences, Chambers and Quiggin (1999) following Segal and Spivak show that farmers can fully insure even in the absence of an actuarially fair insurance contract.

and non-regressivity (regressivity) to radial expansions of revenues.

Similar ambiguities arise from the risk effect as well. For example, an input could be a risk substitute and the risk effect in terms of revenues could be associated with an increase in risk. The input risk effect would then be negative. The other obvious possibilities can be enumerated by the reader.

Our strategy for sorting through the possible results is somewhat different than the strategy typically pursued in previous studies. There the common strategy is to impose some type of structure upon the producer's preference structure, for example, constant absolute risk aversion or decreasing absolute risk aversion. The results, thus obtained, are limiting for at least two reasons. First, compared to the preference structure utilized in the current paper, the preference structure utilized in other studies (expected utility) is quite restrictive in that it imposes additive separability across states of nature. Moreover, it is widely recognized to rely on a weak conceptual basis because empirical evidence routinely refutes the crucial independence axiom underlying expected-utility theory. Thus, these studies are in the position of imposing additional structure on a model that has already been demonstrated to be empirically flawed.

Second, the production structure that underlies all these studies is even more restrictive because it imposes an extreme form of non-substitutability between state-contingent outputs (Chambers and Quiggin, 1998). And as Chambers and Quiggin (1999) demonstrate, the differences between a risk-neutral producer's production pattern and a risk-averter's production pattern in that framework ultimately reduce to determining whether the risk-averter produces more or less of a single reference state-contingent revenue which automatically determines all other revenue levels and the level of input utilization. In effect, the stochastic production function model can always be reduced to a trivial single-input problem. Given the extreme restrictiveness of the production model and the fact that input committal really plays no role in determining the inherent riskiness of the state-contingent revenue vector, it's not surprising, therefore, that one is forced to place even more stringent restrictions on preferences to obtain results.

So in what follows, we follow an alternative strategy and place no restrictions on preferences other than that they be consistent with risk aversion and generalized Schur concavity.

Instead, we examine restrictions on the shape of the isocost frontiers for the state-contingent technology.

The first restriction that we consider is what Chambers and Quiggin (1999) have referred to as *constant relative riskiness* of the revenue-cost function[11]. Constant relative riskiness, in the current context, is equivalent to requiring that the revenue-cost function be homothetic in state-contingent revenues. The main economic consequence of this fact is that the expansion path in state-contingent revenue space that is defined by the locus of points which are expected-revenue maximizing for fixed levels of revenue cost is linear. This emerges from the fact that isocost frontiers for this technology are radial blow-ups of a reference isocost curve[12].

Let the farmer's revenue vector in the absence of insurance be denoted by $\mathbf{r}^A$ and the farmer's revenue vector in the presence of actuarially fair insurance be denoted by $\mathbf{r}^F$. Then, as discussed earlier, notice that the effect of providing insurance can be broken down into two parts, the mean-compensated move from $\mathbf{r}^A$ to $\frac{\sum_{s\in\Omega}\pi_s r_s^A}{\sum_{s\in\Omega}\pi_s r_s^F}\mathbf{r}^F$ and the radial movement from this mean-compensated term to $\mathbf{r}^F$. The expression, $\frac{\sum_{s\in\Omega}\pi_s r_s^A}{\sum_{s\in\Omega}\pi_s r_s^F}\mathbf{r}^F$, corresponds to point C in Figure 4.

Now notice that because $\frac{\sum_{s\in\Omega}\pi_s r_s^A}{\sum_{s\in\Omega}\pi_s r_s^F}\mathbf{r}^F$ is either a radial expansion or a radial contraction of $\mathbf{r}^F$, it lies on the producer's expansion path and hence must be the most profitable state-contingent revenue combination for the revenue-cost level

$$C\left(\mathbf{w}, \frac{\sum_{s\in\Omega}\pi_s \mathbf{r}_s^{\mathbf{A}}}{\sum_{s\in\Omega}\pi_s \mathbf{r}_s^{\mathbf{F}}}\mathbf{r}^{\mathbf{F}}, \mathbf{p}\right).$$

Also notice, however, that $\frac{\sum_{s\in\Omega}\pi_s r_s^A}{\sum_{s\in\Omega}\pi_s r_s^F}\mathbf{r}^F$ has the same expected revenue as $\mathbf{r}^A$. Thus, the cost associated with $\mathbf{r}^A$ must be at least as large as that associated with $\frac{\sum_{s\in\Omega}\pi_s r_s^A}{\sum_{s\in\Omega}\pi_s r_s^F}\mathbf{r}^F$. If it were not, that would mean that the same expected revenue could be obtained from $\mathbf{r}^A$ at a cost level lower than $C\left(\mathbf{w}, \frac{\sum_{s\in\Omega}\pi_s \mathbf{r}_s^{\mathbf{A}}}{\sum_{s\in\Omega}\pi_s \mathbf{r}_s^{\mathbf{F}}}\mathbf{r}^{\mathbf{F}}, \mathbf{p}\right)$. But this contradicts the fact that $\frac{\sum_{s\in\Omega}\pi_s \mathbf{r}_s^{\mathbf{A}}}{\sum_{s\in\Omega}\pi_s \mathbf{r}_s^{\mathbf{F}}}\mathbf{r}^{\mathbf{F}}$ lies on

[11] Constant relative riskiness and constant absolute riskiness as defined below are defined analogously to constant absolute risk aversion and constant relative risk aversion for the general state-contingent preference function W as in Quiggin and Chambers (1999). A straightforward extension of those arguments leads to the maintained relationship between homotheticity and translation homotheticity.

[12] This result is completely analogous to the result that maximum revenue expansion paths for non-stochastic, multi-output technologies which exhibit output homotheticity are straight lines.

the firm's expansion path. Figure 6 illustrates this fact by having the point of intersection between the fair-odds line through $\mathbf{r}^A$ and the risk-neutral expansion path lie below the firm's isocost curve for $\mathbf{r}^A$.

Revealed-preference arguments, therefore, lead us to conclude that

$$\mathbf{r}^{\mathbf{A}} - C\left(\mathbf{w}, \mathbf{r}^{\mathbf{A}}, \mathbf{p}\right) \preceq_W \frac{\sum_{s\in\Omega} \pi_s \mathbf{r}_s^{\mathbf{A}}}{\sum_{s\in\Omega} \pi_s \mathbf{r}_s^{\mathbf{F}}} \mathbf{r}^{\mathbf{F}} - C\left(\mathbf{w}, \mathbf{r}^{\mathbf{A}}, \mathbf{p}\right).$$

If this ordering of the outcomes did not hold, then $\mathbf{r}^A$ would not be the optimal choice for a risk averter. Notice that the preceding arguments have established that $\frac{\sum_{s\in\Omega} \pi_s \mathbf{r}_s^{\mathbf{A}}}{\sum_{s\in\Omega} \pi_s \mathbf{r}_s^{\mathbf{F}}} \mathbf{r}^{\mathbf{F}}$ is less costly than $\mathbf{r}^{\mathbf{A}}$, thus it represents a feasible choice for this level of revenue cost. Hence, we conclude that

$$\mathbf{r}^{\mathbf{A}} \preceq_W \frac{\sum_{s\in\Omega} \pi_s \mathbf{r}_s^{\mathbf{A}}}{\sum_{s\in\Omega} \pi_s \mathbf{r}_s^{\mathbf{F}}} \mathbf{r}^{\mathbf{F}}.$$

From this observation we can state the following result.

Result 1 If the producer's revenue-cost function exhibits constant relative riskiness, the pure-risk effect of the provision of actuarially fair insurance on an input is positive if the input is a risk complement and negative if the input is a risk substitute.

Generally speaking the expansion effect for a technology exhibiting constant relative riskiness can require either a shrinking or an expansion of the risk-neutral optimum depending upon the rate at which marginal costs of the state-contingent outputs rise. So, as a general matter, we cannot make a clear pronouncement as to what will be the effect of the provision of crop insurance for a producer facing such a technology without placing further structure upon the problem.

There does exist a class of technologies for which one can obtain clear results about both the expansion and the pure risk effects. That technology is the member of the class of translation-homothetic[13] technologies (Chambers and Färe), which Chambers and Quiggin (1999) refer to as exhibiting *constant absolute riskiness.* The technology exhibits constant absolute riskiness if

$$C(\mathbf{w}, \mathbf{r}, \mathbf{p}) = \hat{C}\left(\mathbf{w}, \mathbf{T}\left(\mathbf{r}, \mathbf{p}, \mathbf{w}\right), \mathbf{p}\right)$$

[13] Translation homothetic technologies are the class of quasi-homothetic technologies which can be represented by non-decreasing translations of a reference isoquant in a given direction (Chambers and Färe).

where

$$T\left(\mathbf{r}+\delta\mathbf{1}^{S},\mathbf{p},\mathbf{w}\right)=T\left(\mathbf{r},\mathbf{p},\mathbf{w}\right)+\delta,\quad \delta\in\Re,$$

$$T\left(\lambda\mathbf{r},\lambda\mathbf{p},\mathbf{w}\right)=\lambda T\left(\mathbf{r},\mathbf{p},\mathbf{w}\right),\quad T\left(\mathbf{r},\mathbf{p},\lambda\mathbf{w}\right)=T\left(\mathbf{r},\mathbf{p},\mathbf{w}\right)\quad \lambda>0,$$

and $\hat{C}\left(\mathbf{w},\mathbf{T}\left(\mathbf{r},\mathbf{p},\mathbf{w}\right),\mathbf{p}\right)$ is positively linearly homogeneous in input prices, homogeneous of degree zero in $T\left(\mathbf{r},\mathbf{p},\mathbf{w}\right)$ and $\mathbf{p}$, non-decreasing and convex in $T\left(\mathbf{r},\mathbf{p},\mathbf{w}\right)$, non-increasing in $\mathbf{p}$. $T\left(\mathbf{r},\mathbf{p},\mathbf{w}\right)$ is non-decreasing and convex in $\mathbf{r}$.

Intuitively, technologies which exhibit constant absolute riskiness have isocost curves which are parallel to one another as one moves in a direction parallel to the bi-sector. Therefore, increasing revenue by the same amount in all states of nature has no effect on the rate at which state-contingent revenues substitute for one another in the technology. In that sense, constant absolute riskiness is the natural production analogue of general risk-averse preferences which exhibit constant absolute risk aversion.[14]

The most important property that technologies exhibiting constant absolute riskiness have for state-contingent technologies is that the cost level corresponding to the efficient set is unique for such technologies. Hence, in this special case, the efficient set corresponds exactly to a unique isocost contour. The easiest way to discern this property is to differentiate both sides of the expression

$$T\left(\mathbf{r}+\delta\mathbf{1}^{S},\mathbf{p},\mathbf{w}\right)=T\left(\mathbf{r},\mathbf{p},\mathbf{w}\right)+\delta$$

with respect to δ and evaluate the resulting directional derivative at $\delta=0$ to obtain

$$\sum_{s\in\Omega}T_s\left(\mathbf{r},\mathbf{p},\mathbf{w}\right)=1.$$

Using this fact and our definition of constant absolute riskiness, it follows immediately that in this case, the arbitrage condition (2.1) can be written as

$$\hat{C}_T\left(\mathbf{w},\mathbf{T}\left(\mathbf{r},\mathbf{p},\mathbf{w}\right),\mathbf{p}\right)\geq 1.$$

Thus, assuming an interior solution the arbitrage condition determines a unique level of T and thus of revenue cost. In this context, notice that T may naturally be thought of as a

[14]Blackorby and Donaldson refer to this class of functions as unit translatable. Chambers and Färe call functions which are translation homothetic in the direction of the equal revenue ray BD-translation homothetic.

revenue aggregate which has the property that increasing all state-contingent revenues by one unit increases it by one unit. For technologies exhibiting constant absolute riskiness, (2.1) simply reduces to equating the marginal cost of that revenue aggregate to one.

Because both risk averters and risk-neutral individuals produce in the efficient set, it follows immediately that:

Result 2 If the technology exhibits constant absolute riskiness, the introduction of production insurance does not affect the level of revenue cost incurred by a risk-averse entrepreneur.

Accordingly, the only effect that production insurance has on the risk-averse entrepreneur is to change his optimal revenue mix to that associated with a risk-neutral individual. This brings with it an increase in expected revenue, at no additional cost, that can be used along with the production insurance to enhance the producer's overall welfare.

The production decisions for a risk-averse producer in the presence of insurance and in its absence can be illustrated graphically as in Figure 7 when the technology exhibits constant absolute riskiness. There the producer produces at $\mathbf{r}^F$ when insurance is provided and at $\mathbf{r}^A$ in its absence. It is pictorially obvious and generally true that

$$\frac{\sum_{s\in\Omega}\pi_s r_s^A}{\sum_{s\in\Omega}\pi_s r_s^F} < 1. \tag{4.4}$$

The inequality follows from the fact that $\mathbf{r}^F$ must be associated with the highest expected revenue consistent with the constant level of cost.

Result 3 If the technology exhibits constant absolute riskiness, the introduction of production insurance increases the level of expected revenue produced by a risk-averse producer.

Moreover, a revealed preference argument exactly parallel to the one used in the discussion of constant relative riskiness reveals that

$$\mathbf{r^A} \preceq_W \frac{\sum_{\mathbf{s}\in\boldsymbol{\Omega}}\boldsymbol{\pi}_\mathbf{s}\mathbf{r}_\mathbf{s}^\mathbf{A}}{\sum_{\mathbf{s}\in\boldsymbol{\Omega}}\boldsymbol{\pi}_\mathbf{s}\mathbf{r}_\mathbf{s}^\mathbf{F}}\mathbf{r^F}. \tag{4.5}$$

Straightforward consequences of (4.5) and (4.4) are

Result 4 If the technology exhibits constant absolute riskiness, then the pure risk effect on an input is positive (negative) if the input is a risk complement (risk substitute).

Result 5 If the technology exhibits constant absolute riskiness, the expansion effect on an input is positive (negative) if the input is non-regressive (regressive) in radial expansions of revenue.

Because there exist unambiguous results for both the pure-risk and expansion effects on input utilization, it is now an easy matter to obtain some clear-cut overall results. We have:

Corollary 1 If the technology exhibits constant absolute riskiness, an input's utilization increases as a result of the introduction of insurance if the input is a risk complement and it is non-regressive in radial expansions of the state-contingent revenue vector.

Corollary 2 If the technology exhibits constant absolute riskiness, an input's utilization decreases as a result of the introduction of insurance if the input is a risk substitute and it is regressive in radial expansions of the state-contingent revenue vector.

5. Discussion of Results

The results that we have presented show that regardless of the preference structure, there are a number of things which can be said about the input response to the provision of (actuarially fair) insurance. For example, consider the case of a technology that exhibits constant absolute riskiness. Then it follows from our discussion that any input which is both a risk complement and which is not radially regressive in revenues will be used more heavily in the presence of insurance than in its absence.

So intuitively, one might think in terms of an input like chemical fertilizer which would seem to be a natural risk complement and which empirical evidence would also suggest is not a regressive input. Then, one could immediately conclude that an individual using a technology characterized by constant absolute riskiness would use more chemical fertilizer in the presence of insurance. This coincides nicely with popular wisdom on such inputs. Conversely, one sees that the pure-risk effect will lead the producer to utilize less risk-substitute inputs, such as pesticides. But more generally, the introduction of insurance might ultimately force even these inputs' utilization to rise as a result of the expansion effect if pesticides are not regressive to radial expansions of the state-contingent revenues.

For the class of technologies exhibiting constant relative riskiness, we see that the pure risk effect is always distinguishable and unambiguous. Thus the pure risk effect would push a farmer to use more risk-complementary inputs in the presence of insurance and fewer risk substitutes.

Perhaps the most important aspect of our results is that they establish that neither risk complementarity nor risk substitutability is sufficient to determine whether an input's utilization increases or decreases as a result of the provision of insurance. While this may seem counterintuitive, it is quite reasonable once one recognizes that provision of insurance evokes at least two responses on the part of producers. The first, which we have called the pure-risk effect, is the change in the mix of state-contingent revenues which changes the riskiness (from the producer's perspective) of the optimal state-contingent revenue bundle. Generally, we expect the producer to move to a more risky revenue bundle as market provision of insurance substitutes for the insurer's need to self insure. It is for this effect where the notions of risk complementarity and risk substitutability are most relevant. But providing insurance also influences the producer's scale of operation, and these scale adjustments can either reinforce or modulate the pure-risk adjustment depending upon the input's responsiveness to radial changes in the revenue vector.

Our results are most directly comparable with the results of Horowitz and Lichtenberg and Ramaswami who study input and supply responsiveness to the provision of insurance in the presence of moral hazard. Both of those papers report sufficient conditions for providing insurance to increase the use of a single scalar input. For example, Ramaswami shows that if that input is risk-reducing, in his sense, and preferences are expected-utility preferences exhibiting non-increasing absolute risk aversion, its use will fall as a result of an introduction of crop insurance. Notice, in particular, that this finding implies that under these circumstances that output or revenue will fall in every state of nature.

In our study, to concentrate our focus on the construction of an analytical framework we have abstracted from the moral-hazard problem by assuming that the insurer can write state-contingent contracts. However, it is an easy consequence of results reported in Chambers and Quiggin (1999, Chapter 7) that provision of the production insurance of the type considered by Ramaswami moves the producer out of the efficient set. This is the natural extension of the Ramaswami result to the multiple-output, multiple-input technology that we consider. Moreover, it is straightforward to show that for technologies exhibiting constant absolute riskiness that revenue cost falls after such insurance is provided[15]. Given these results, it is then straightforward to sort out the effects on individual inputs by using the methodology developed above. Of course, if one is willing to impose even more structure upon preferences (for example, constant absolute risk aversion) while still not requiring maximisation of expected utility, one can obtain even sharper results.

6. Concluding Remarks

This paper studies the impact of crop insurance upon input utilization by risk-averse decisionmakers using the state-contingent formulation of Chambers and Quiggin (1996, 1997, 1999). This framework allows us to rely on a version of Shephard's lemma for stochastic technologies to examine input responsiveness to the provision of crop insurance that does not rely on the single-output stochastic production function model that has dominated most previous studies. The only restriction placed upon preferences is that they be consistent with a very mild form of risk aversion. Moreover, the methodology developed in this paper can be used in any situation where one seeks to determine the comparative-static effect on input use of some change in the producer's economic environment.

We show that it is straightforward to develop a framework for analyzing the impact of crop insurance on input use that can be usefully illustrated with graphical techniques that should be familiar to virtually all economists. Using this framework, we isolate a number of results including sufficient conditions on the technology for the provision of insurance to lead to either an increase or a decrease in the use of an input.

[15] As noted earlier, under constant absolute riskiness, the efficient set determines a unique level of cost. If the producer doesn't operate in the efficient set, it turns out that his cost must fall as a straightforward consequence of the properties of constant absolute riskiness.

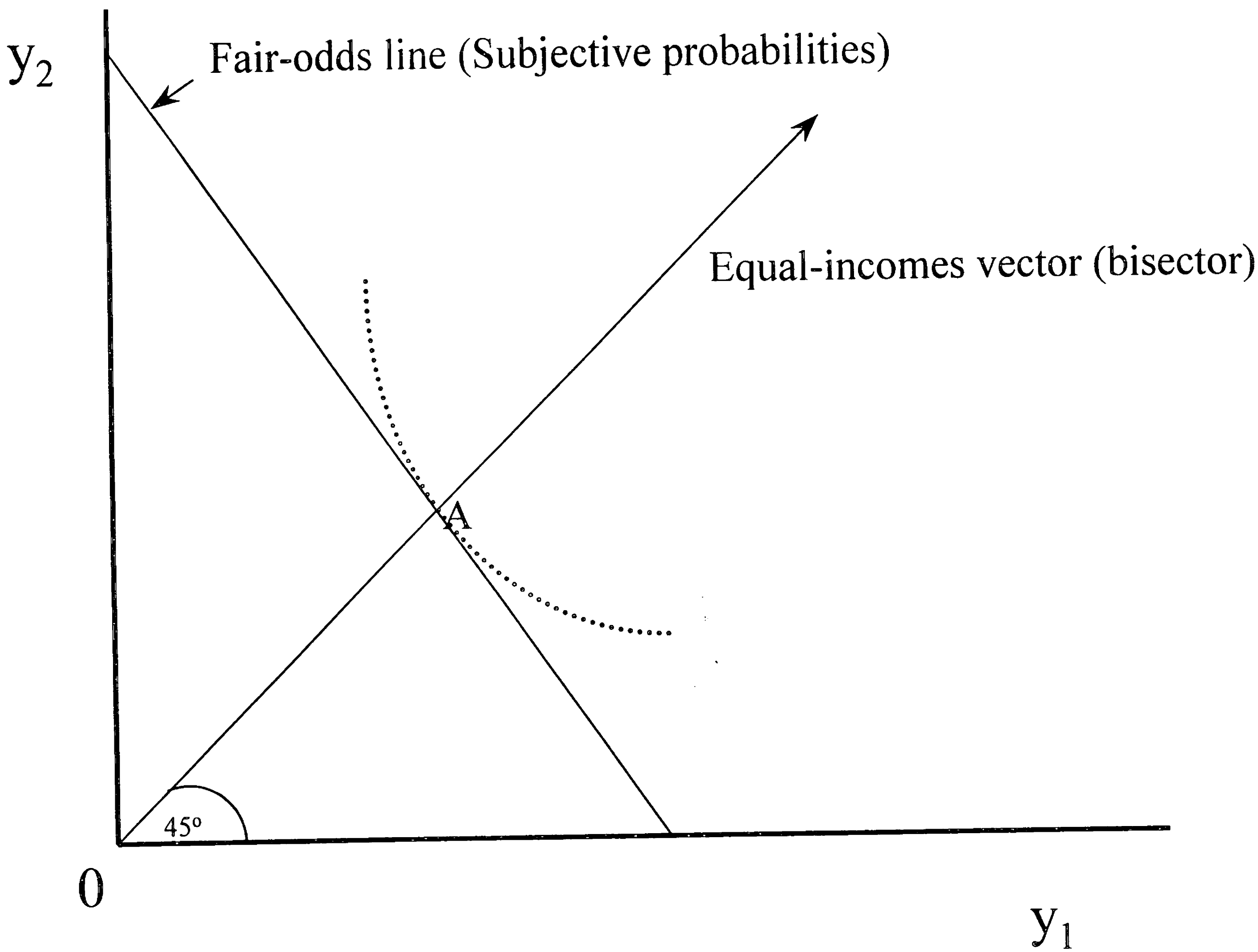

Figure 1: Risk-averse preferences

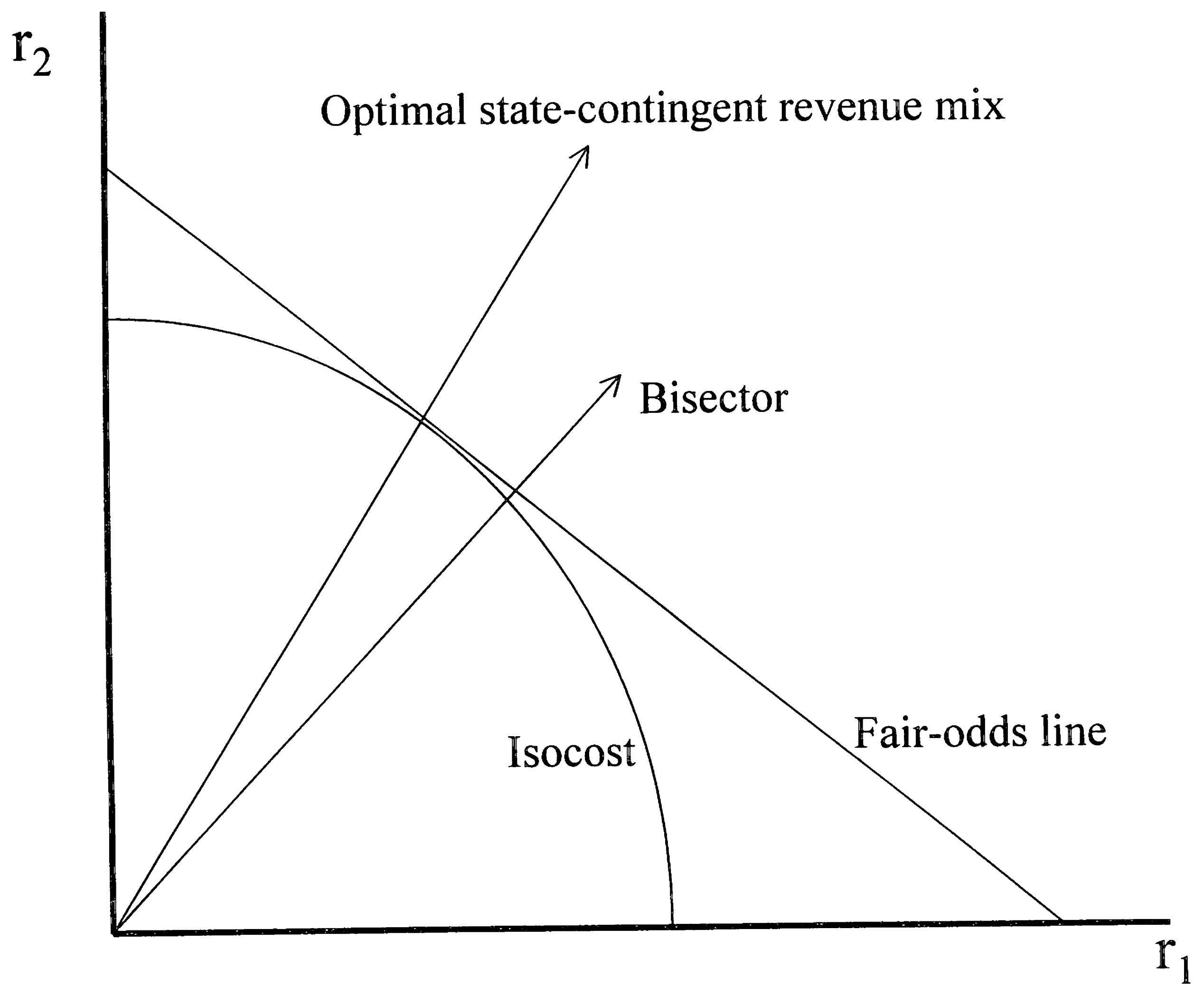

Figure 2: Risk-neutral production equilibrium

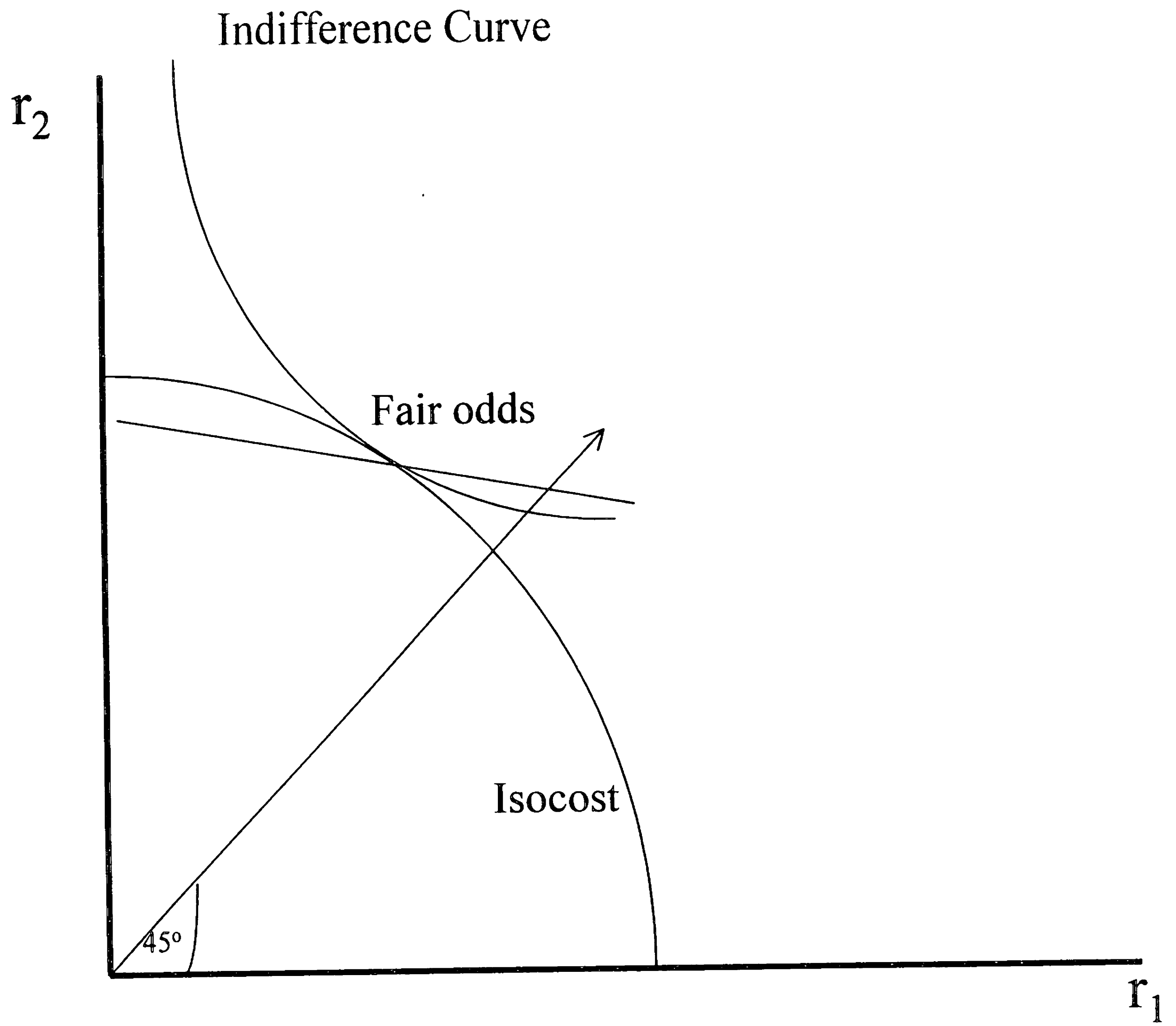

Figure 3: Risk-averse production equilibrium

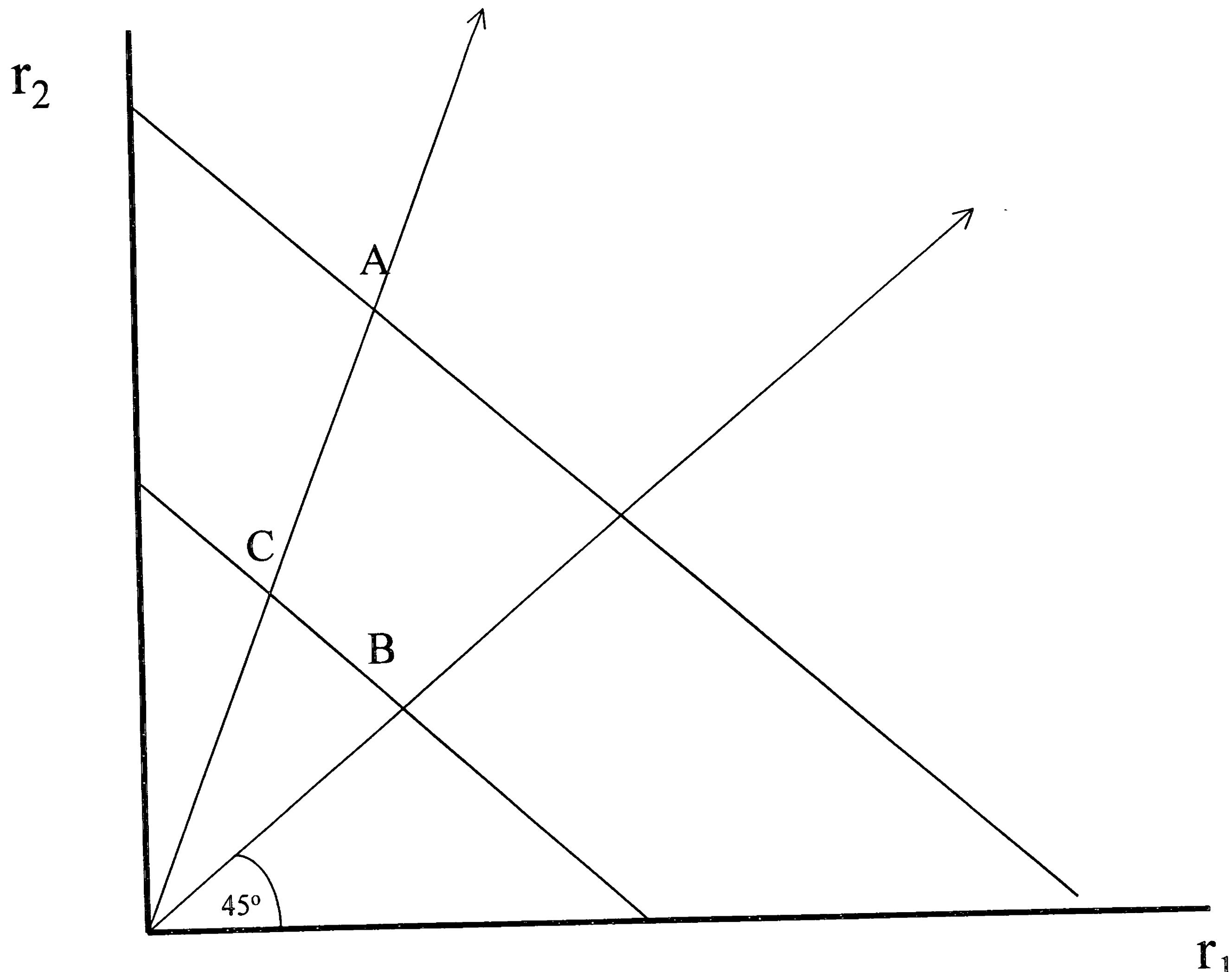

Figure 4: Decomposing Revenue Adjustments

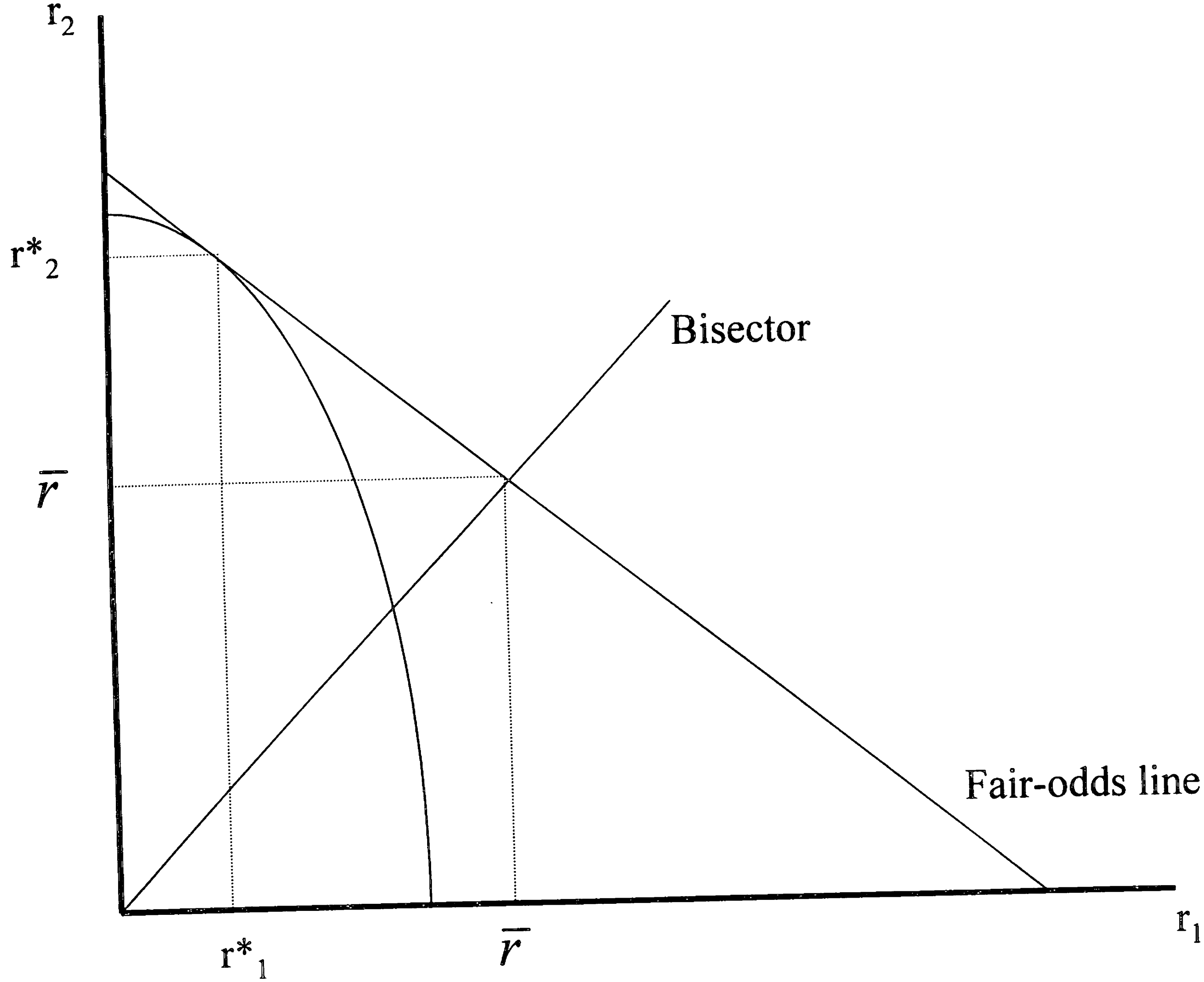

Figure 5: Equilibrium with Fair Insurance

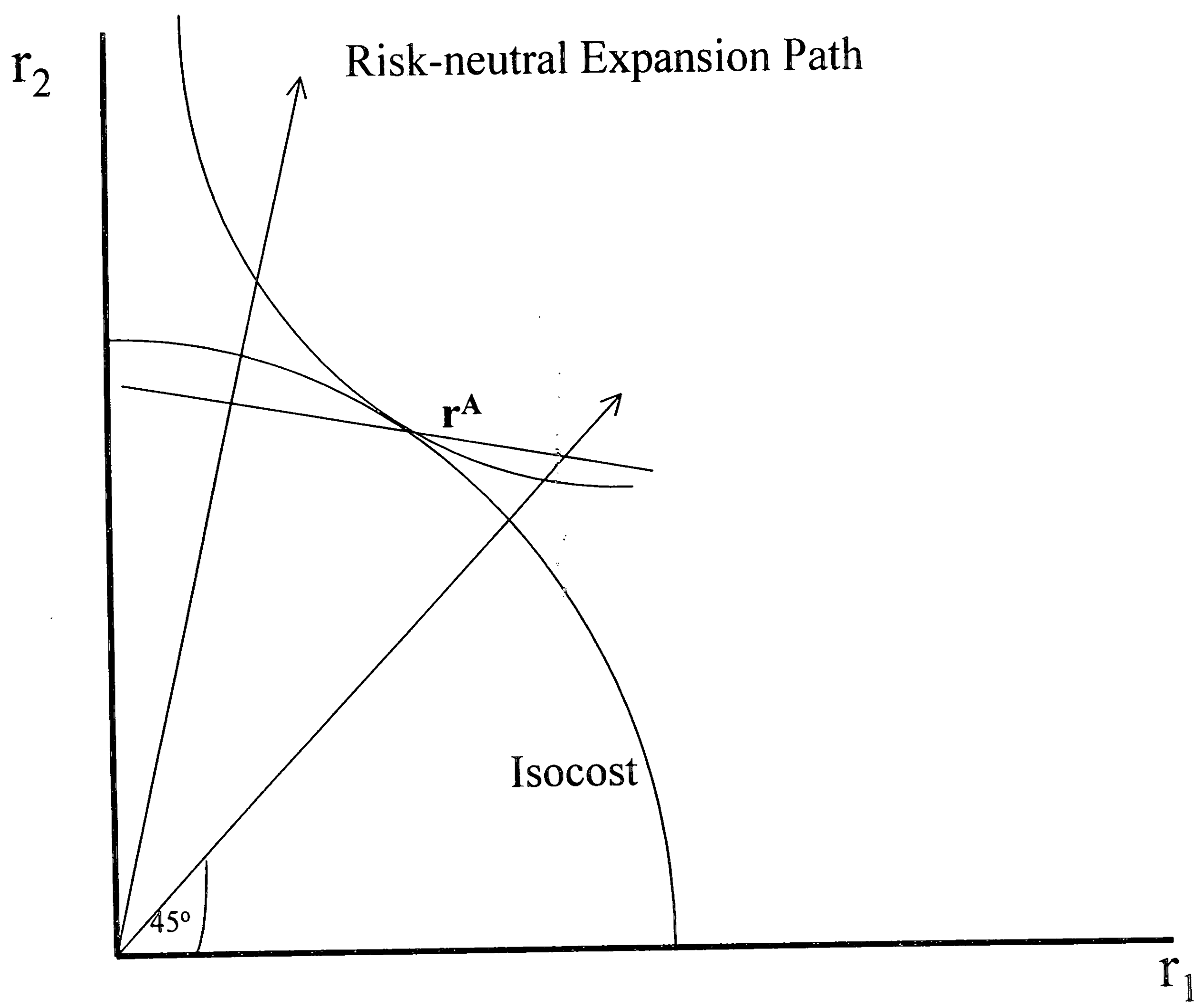

Figure 6: Constant relative riskiness and insurance

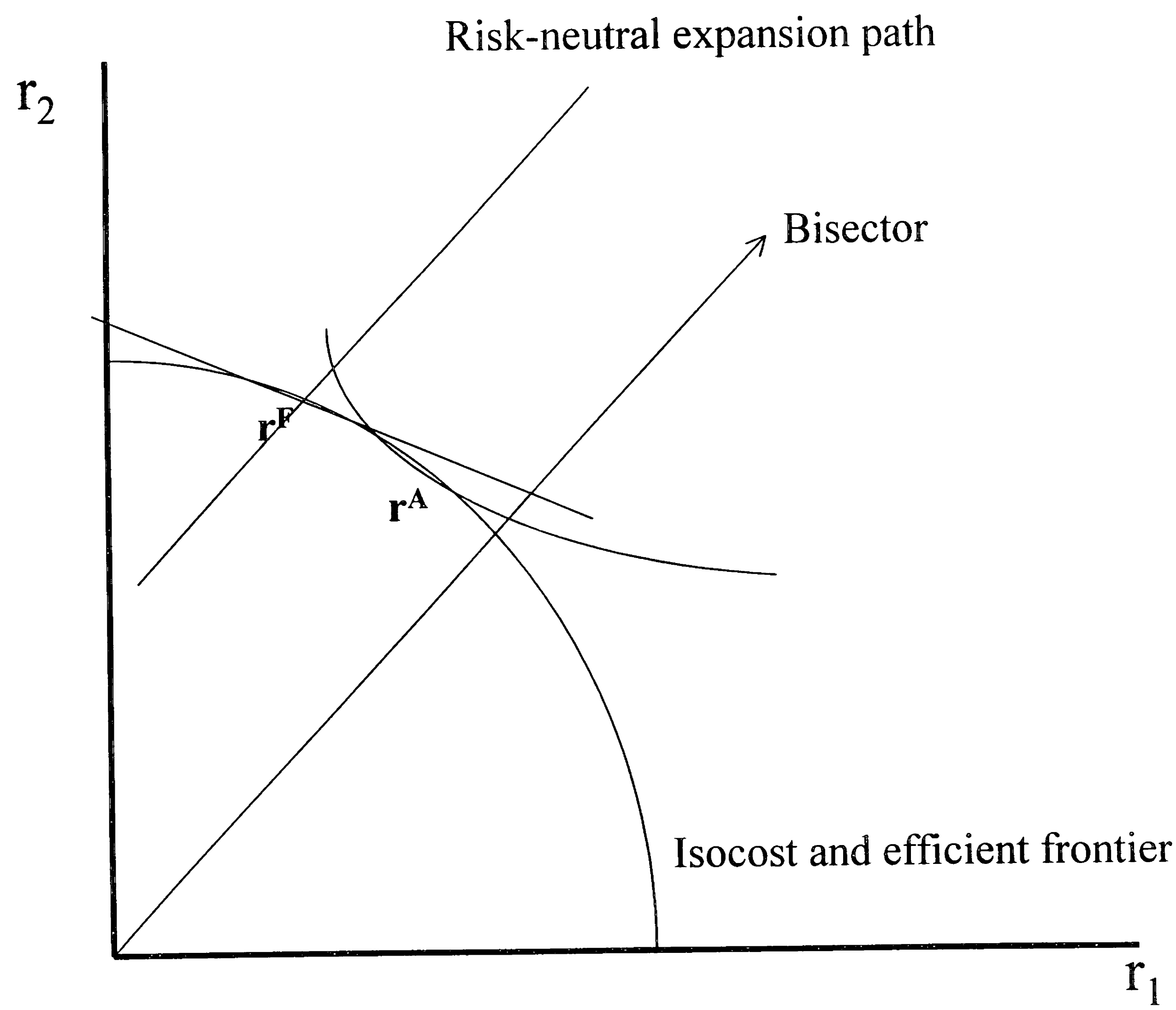

Figure 7: Constant absolute riskiness and insurance

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