Cost Function Estimation Under Risk

Rulon D. Pope and Richard E. Just

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Applied Economics - U of M
1594 Buford Ave 232 ClaOff
St Paul MN 55108-6040 USA

Brigham Young University
and
University of Maryland at College Park

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Table 1. Effect of Production Risk on Errors in Ex Post Cost Function Estimates

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Biases</th>
<th>Mean Squared Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_e = 0$</td>
<td>$\sigma_e = .15$</td>
</tr>
<tr>
<td>$B_{11}$</td>
<td>-.010</td>
<td>-.252</td>
</tr>
<tr>
<td>$B_{12}$</td>
<td>.007</td>
<td>-.090</td>
</tr>
<tr>
<td>$B_{22}$</td>
<td>-.005</td>
<td>-.112</td>
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<tr>
<td>$\sigma_{11}$</td>
<td>.16</td>
<td>7.11</td>
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<tr>
<td>$\sigma_{12}$</td>
<td>.10</td>
<td>3.70</td>
</tr>
<tr>
<td>$\sigma_{22}$</td>
<td>.09</td>
<td>2.09</td>
</tr>
</tbody>
</table>

* True parameter values are $B_{11} = 10$, $B_{12} = B_{21} = 6$, $B_{22} = 4$, $\sigma_{11} = 4.083$, $\sigma_{12} = 1.633$, and $\sigma_{22} = 1.987$. 
Dual applications of cost functions have become a common means of generating theoretically plausible systems of factor demands and of making inferences about economies of scale, technical change, and factor substitution (e.g., Binswanger). Some studies suggest that cost function approaches are superior for investigating properties of production technologies such as nonjointness (e.g., Leathers). For empirical purposes, however, cost function methodologies have had a serious and, all too often, unrecognized limitation in that they do not admit production risk (Pope and Chavas). Risk is a fundamental characteristic of most agricultural production. This paper shows that common dual cost function methods are not appropriate with stochastic production even under risk neutrality. An alternative dual cost function approach is illustrated and applied that admits production risk. The approach extends to the case of risk averse producers under a restricted albeit common stochastic specification of technology.

The reason that typical cost function methodologies do not admit production risk is that they are conditioned on actual output. However, actual output is unobservable at the time of input decision making if production is stochastic. Hence, a cost function so conditioned cannot serve as a first-stage problem for the relevant expected profit maximization or expected utility maximization problem (Pope and Chavas). We show that the decision-relevant cost function must be based on expected output. When production is stochastic, the decision-relevant expected output is not observable so the cost function cannot be estimated directly as in the nonstochastic production case.

This paper considers estimation of ex ante cost functions which are conditioned on the decision-relevant expected output rather than actual output. Unobservability of expected output is addressed by using the distance function and the duality that exists between the distance and cost functions. Ex ante methodology is generalized for a major class of stochastic production functions in the case of risk averse decision makers. Results show that conventional ex post methods result in substantial estimation bias. An application to aggregate U.S. agricultural data also demonstrates improved plausibility of results.
Ex Post and Ex Ante Cost Functions

The ex post and ex ante cost function approaches are most usefully contrasted by comparison of two decision problems, each of which can be broken into two stages. Consider

Problem 1:

\[ \text{Max}_x \{ \pi = pf(x,\varepsilon) - rx \} \]

Stage 1: \( \text{Min}_x \{ rx \mid f(x,\varepsilon) \geq y \} = c(r,y,\varepsilon) \)

Stage 2.: \( \text{Max}_y \{ py - c(r,y,\varepsilon) \} \)

Problem 2:

\[ \text{Max}_x \{ E[\pi] = E[pf(x,\varepsilon) - rx] \} \]

Stage 1: \( \text{Min}_x \{ rx \mid E[f(x,\varepsilon)] \geq \bar{y} \} = \bar{c}(r,\bar{y}) \)

Stage 2.: \( \text{Max}_y \{ py - \bar{c}(r,\bar{y}) \} \)

where \( \pi \) is profit, \( p \) is output price, \( y \) is output quantity, \( x \) is a vector of input quantities, \( r \) is a corresponding vector of input prices, \( \varepsilon \) is a stochastic production disturbance, and \( \bar{y} \) is expected output (before production disturbances are realized). In Problem 1, the decision maker fully realizes stochastic disturbances before making input decisions. In Problem 2, the decision maker must make input decisions before stochastic disturbances are realized. At best, expected profits rather than actual profits can be maximized.

Problem 1 corresponds to conventional dual cost function approaches. The associated cost function generated in Stage 1 is an ex post cost function. It is conditioned on actual output. Under standard regularity assumptions (McFadden, 1978), this cost function generates the system of input demand equations,

\[ c_i(r,y,\varepsilon) = x(r,y,\varepsilon), \]

where subscripts represent differentiation. These demand equations assume that the stochastic production disturbance is observed before decision making. Clearly, this is an absurd assumption for major agricultural input decisions such as land allocation and seed use that must be decided in advance of weather
and pest realizations.

Alternatively, we contend that actual output in most agricultural production problems is unknown at the time of major input decision choices so that the basic optimization problem solved by the firm cannot be conditioned on actual output.\(^1\) If the production disturbance is unknown at the time of decision making, then the producer can at best maximize expected profit rather than actual profit as in Problem 2. The associated cost function in Stage 1 is an ex ante cost function. It is conditioned on expected output. Under similar regularity conditions, the ex ante cost function generates the relevant system of input demand equations,

\[ (2) \quad \tilde{c}(r, \tilde{y}) = \tilde{x}(r, \tilde{y}), \]

which is conditioned on expected output.

To date, cost function applications in agriculture have simply ignored uncertainty and represented the cost function as \(c(r,y)\). Accordingly, input demand specifications derived from cost functions have been conditioned on actual output as in (1) rather than on expected output as in (2). Whether the production disturbance is represented explicitly or not, this is the ex post approach of Problem 1.

**Bias with Ex Post Methods**

Use of the ex post specification in Problem 1 when the underlying decision problem follows the ex ante case in Problem 2 raises fundamental concerns regarding estimation because of the bias in estimation that follows from incorrectly interpreting actual output observations as observations on decision quantities. We submit that these concerns are serious and apply to essentially all agricultural cost function studies to date.

To demonstrate the problem conceptually, suppose production is homothetic in which case \(c(r,\tilde{y}) = \phi(\tilde{y})\theta(r)\) and further assume constant returns to scale in which case \(\phi(\tilde{y}) = \tilde{y}\). Then consider estimation of the demand system in (2). For simplicity of exposition, suppose that \(\theta(r)\) is linear in parameters (as in the generalized Leontief case) in which case \(\theta(r) = W\beta\) where \(W\) is a function of \(r\) and \(\beta\) is an unknown parameter vector. The demand system in (2) thus becomes
(3) \[ x = \bar{y}W_1\beta + \delta \]

where \( \delta \) is a vector of disturbances added for econometric purposes (assumed to be stochastically independent of prices).\(^2\)

To contrast the ex ante and ex post cases, assume that actual output differs from expected output by a random disturbance, \( y = \bar{y} + \varepsilon, E(\varepsilon) = 0 \), and that expected output is replaced by actual output for estimation purposes. Substituting \( \bar{y} = y - \varepsilon \) in (3), the demand system becomes

(4) \[ x = yW_1\beta + \tilde{\delta} \]

where \( \tilde{\delta} = \delta + \varepsilon W_1\beta \). Clearly, the disturbance vector \( \tilde{\delta} \) in the actual production case of (4) is correlated with the regressors even though the disturbance vector \( \delta \) in the expected-production specification of (3) is not. Furthermore, the correlation is necessarily strong when randomness of production is important because \( \varepsilon W_1\beta \) is perfectly correlated with \( yW_1\beta \). Both biased and inconsistent estimates necessarily result whenever production is stochastic.

To demonstrate the severity of this problem explicitly, we report a brief Monte Carlo comparison of estimators for the case of risk neutrality. Suppose \( c \) follows Diewert's (1971) homothetic generalized Leontief cost function with constant returns to scale, i.e., \( \theta(r) = r^{n/2} \beta r^{1/2} \), for which the associated system of input demands is,

(5) \[ x_t = \bar{y}_t r_t^{n/2} \beta_t \tilde{r}_t + \delta_t, \]

where \( \beta \) is a symmetric matrix of positive parameters and \( \tilde{r}_t \) is a diagonal matrix with the inverse square roots of respective \( r_t \) elements on the diagonal. If \( n = 2 \), only three parameters must be estimated (\( \beta_{11}, \beta_{12} = \beta_{21}, \) and \( \beta_{22} \)).

Table 1 presents Monte Carlo results for estimation of the input demand system in (5) with \( n = 2 \) when actual observations \( y_t \) are substituted for expected output \( \bar{y}_t \) following (4). Both bias and mean squared error of parameter estimates are presented for both the cost function parameters and the covariance matrix of the demand system. The experiment is based on 45 observations per repetition with 5000
With stochastic production, the expected production function $g(x) = \mathbb{E}[f(x,e)]$ can be recovered similarly from $\bar{c}(r,\bar{y})$. That is, under stochastic production the distance function is $\Psi(\bar{y},x) = \min \{1 - \bar{c}(r,\bar{y}) + rx\}$ in which case expected production function follows $g(x) = \max_y \{\bar{y} \mid \Psi(\bar{y},x) \geq 1\}$. Thus, expected production can be specified in terms of the parameters of the cost function.

To illustrate the associated ex ante estimation procedure, let $\bar{c}(r,\bar{y};\beta)$ denote the ex ante cost function where $\beta$ is a vector of parameters. If $\beta$ is known, then $\bar{y}$ is obtained as

$$
\bar{y} = \max_{\bar{y}} \{\bar{y} \mid \min_r [1 - \bar{c}(r,\bar{y};\beta) + rx] \geq 1\},
$$

(6) $$
= \max_{\bar{y},\lambda} \{\bar{y} + \lambda(\min_r [1 - \bar{c}(r,\bar{y};\beta) + rx] - 1)\}
= \max_{\bar{y},\lambda} \{\bar{y} - \lambda[\bar{c}(r,\bar{y};\beta) - rx]\}.
$$

where $\lambda$ is a Lagrangian multiplier. This specification can be substituted for $\bar{y}$ obtaining a regression relationship fully specified in terms of the cost function. For example, in the case of estimating the cost function alone rather than the input demand system, the least squares estimator for the model $c_t = \bar{c}(r_t,\bar{y}_t;\beta) + u_t$, $t = 1,\ldots,T$, is obtained by

$$
\min_\beta S^2 = \sum_{t=1}^T \left[ c_t - \bar{c}(r_t,\max_{\bar{y},\lambda,t} \{\bar{y}_t - \lambda[\bar{c}(r_t,\bar{y}_t;\beta) - r_t x_t]\};\beta) \right]^2.
$$

Note also that this least squares optimization can be simplified by eliminating maximization with respect to $r_t$ because, by common efficiency assumptions, the observed $r_t$'s are the distance function minimizing $r_t$'s. Similarly, substituting (6) for $\bar{y}$ in (3) obtains a regression relationship for the input demand system fully specified by the cost function. Under risk neutrality, this approach has been utilized only once to date on a data set now 15 years old despite its obvious advantages (Pope and Just).

**Ex Ante Cost Functions for Risk Averse Firms**

In the remainder of this paper, we explore extension of the concept of ex ante cost functions to the case of risk aversion. However, this is done only under a restrictive stochastic specification of technology. Specifically, assume the production function follows the multiplicative form, $y = g(x)h(e)$, where $\mathbb{E}[h(e)]$.

This form has been criticized previously by Just and Pope as not sufficiently general to handle some
interesting risk problems. However, the multiplicative form continues to be widely used. When data are highly aggregated, distinct measurement of both marginal mean and variance effects is difficult. This section formulates a general representation of the cost function for the case of expected utility maximization with multiplicative production risk. The ex post cost function approach is shown to be incompatible with expected utility maximization. Then the ex ante approach is developed and demonstrated.

Formally, compatibility of a cost function with expected utility maximization can be demonstrated by defining a two-stage optimization problem in which cost minimization serves as a first stage of expected utility maximization. Specifically, consider the two-stage choice problem,

\begin{equation}
\text{(7) Stage 1: } \min_x \{rx \mid g(x) \geq \bar{y}\} = \bar{c}(r,\bar{y}).
\end{equation}

\begin{equation}
\text{(8) Stage 2: } \max_{\bar{y}} E\{u[w_0 + p g(\bar{x},\bar{y})]h(\varepsilon) - \bar{c}(r,\bar{y})\},
\end{equation}

with optimal Stage 1 choice vector \( \bar{x}(r,\bar{y}) \) and optimal Stage 2 choice \( \bar{y}(w_0,r) \). The corresponding single-stage expected utility problem is,

\begin{equation}
\text{(9) } \max_x E\{u[w_0 + p g(x)h(\varepsilon) - rx]\},
\end{equation}

with optimal choice vector \( x^*(w_0,r) \) where \( u \) is utility and \( w_0 \) is initial wealth. Compatibility of (7)-(8) and (9) requires

\begin{equation}
\text{(10) } \bar{x}(r,\bar{y}) = x^*(w_0,r).
\end{equation}

Obviously, for the special case of risk neutrality, this cost function is applicable in the Stage 2 problem which reduces to \( \max_\varepsilon p\bar{y} - \bar{c}(r,\bar{y}) \) because \( \bar{y} = g(x) = E[g(x)h(\varepsilon)] \).

Clearly, as in the risk neutral case, any choice of \( x \) in Stage 1 cannot be random (depending on \( \varepsilon \)) and satisfy (10) because random phenomena are unknown at the time of decision making when \( x^* \) is chosen. This rules out ex post minimization where \( \bar{c} \), depends on \( \varepsilon \). Thus, any cost function compatible with expected utility maximization must be an ex ante cost function just as in the case of expected profit maximization.

The interesting simplification that occurs in (7)-(9) when production is multiplicative is that fixing
expected production in Stage 1 also fixes risk. Thus, all necessary information is carried to Stage 2 for purposes of making decisions compatible with (9).

The first-order conditions for (7) assuming an interior solution are

\[(11) \quad r - \lambda g_x = 0, \]
\[\bar{y} - g(x) = 0,\]

where \(\lambda\) is a vector of Lagrangian multipliers (shadow prices). Solving these equations obtains input demands \(x(r, \bar{y})\), cost function \(\bar{c}(r, \bar{y}) = r\bar{x}(r, \bar{y})\), and shadow prices \(\lambda(r, \bar{y}) = \bar{c}_\bar{y}\). The first-order condition for (8) is

\[(12) \quad E[u'(p h(e) - \bar{c}_\bar{y})] = 0.\]

Economically, these conditions imply that price is equated to the marginal cost of expected output subject to the expected output relationship inherent in the production technology.

To establish compatibility, the combination of (7) and (8) must be equivalent to directly maximizing the certainty equivalent in (9) which has first-order condition

\[(13) \quad E[u'(p g_x h(e) - r)] = 0.\]

To see that compatibility is achieved, multiply (12) by \(g_x\) which is nonstochastic and can be taken inside the expectation,

\[(14) \quad E[u'(p g_x h(e) - \bar{c}_\bar{y} g_x)] = 0.\]

Then substitute \(\lambda = \bar{c}_\bar{y}\) in (11) to obtain \(r = \bar{c}_\bar{y} g_x\) which when substituted into (14) obtains (13).

The results of this section demonstrate the ex ante cost function need not depend on risk or risk preferences. In this case, both cost function specification and estimation for the case of non-neutral risk preferences can parallel the risk neutral case. We emphasize, however, that this convenience is obtained only with multiplicative production disturbances.
Ex Ante Cost Function Specification and Estimation Under Risk

To this point, the concept of ex ante cost functions has been generalized to the case of all forms of von Neumann-Morgenstern risk preferences under multiplicative risk. This approach can be readily generalized to include price uncertainty and multiple outputs. Thus, these results substantially extend the set of problems under which cost function methodology is applicable. For empirical purposes, however, it remains to consider appropriate functional forms to impose in specification of ex ante cost functions under risk. Clearly, derivation of cost specifications from primal specifications of production technology is not tractable except under the simplest of forms. Alternatively, direct specification of ex ante cost functions as in conventional flexible dual approaches is desirable if the appropriate properties to impose can be determined.

To determine the properties of the ex ante cost function, the results of standard duality theory apply in a straightforward manner. That is, so long as the input requirement set, \( \{x \mid g(x) \geq \bar{y}\} \), is input conventional and convex, the properties for \( \tilde{c}(r,\bar{y}) \) with respect to both \( r \) and \( \bar{y} \) are the same as in standard duality theory. Specifically, \( \tilde{c} \) is nondecreasing, concave, and positively linearly homogeneous in \( r \) and \( \tilde{c}_r = x \). Also, \( \tilde{c} \) is nondecreasing in \( \bar{y} \), nonnegative, and \( \tilde{c}(r,0) = 0 \). Under homotheticity, \( \tilde{c} \) takes the form \( \tilde{c}(r,\bar{y}) = \phi(\bar{y})\theta(r) \) where under constant returns to scale \( \phi(\bar{y}) = \bar{y} \).

Because these properties exactly parallel the nonstochastic production case, the same functional specifications in common use for cost functions under certainty can also be used for this risk case, e.g., Cobb-Douglas, translog, generalized quadratic, and generalized Leontief forms. For estimation purposes, however, the production risk case is necessarily burdened by the problem of unobservability of expected production. Thus, the approach to estimation must take account of that unobservability just as in the stochastic production case under risk neutrality discussed above. Specifically, actual production cannot be substituted for expected production to estimate the cost function without bias and inconsistency in estimation. Alternatively, the distance function approach developed by Pope and Just for the stochastic production case applies. Thus, the input demand system \( x = \tilde{c}(r,\bar{y};\beta) + \delta \) with disturbance vector \( \delta \) and
parameter vector $\beta$ can be estimated by replacing $\tilde{y}$ with $\tilde{y} = \max_{x, \lambda} \{\tilde{y} - \lambda[(r, \tilde{y}; \beta) - rx]\}$ following (6) and then following standard system maximum likelihood methods.

An Analysis of U.S. Agriculture

Likely stochastic production is a more crucial assumption in agriculture than in any other sector of the economy. Production decisions are made and then weather and other natural phenomena cause production to deviate from expected levels. Furthermore, substantial literature has been amassed demonstrating the presence of risk averse behavior in agriculture. Thus, estimation of cost functions for agriculture, either directly or through input demand system estimation, without the risk and risk preference generalizations of this paper appears to be inappropriate. In this section, we present estimates of a cost function for U.S. agriculture using these generalizations with a generalized Leontief ex ante cost function specification.

Before application, two considerations are necessary in the description of technology. First, the technology must be specified to account for the well documented problem of technical change (Kislev and Peterson, 1982). Second, the technology must be specified to recognize the fixity of land in agriculture. A useful yet simple extension of (5) that accomplishes both these ends under is

$$\tilde{y}(r, \tilde{y}) = \phi(\tilde{y}) (r_t^{1/2} \beta r_t^{1/2} + L_t \gamma r_t + t \alpha r_t) + \delta_t r_t,$$

where $t$ is time, $L_t$ is land availability at time $t$, $\delta_t$ is a vector of serially independent random disturbances, $\alpha$ and $\gamma$ are parameter vectors compatible for multiplication with $r_t$, and $\beta$ is a symmetric positive parameter matrix as in Diewert’s generalized Leontief cost model. The last term in parentheses represents technical change following Binswanger and Parks.

For the case of generalized Leontief technology, a somewhat more general method of obtaining $\phi(\tilde{y})$ than suggested by (6) is possible. Diewert has shown for the nonstochastic production case that $y_t = \phi^{-1}(1/\nu_t)$ where $\nu_t$ is the maximal root of $\tilde{y}_t^{1/2} \beta \tilde{x}_t^{1/2}$ and $\tilde{x}_t$ is a diagonal matrix with $x_t$ on the diagonal. A parallel result for the stochastic production case yields $\tilde{y}_t = \phi^{-1}(1/\nu_t)$ in this case. Therefore, $\phi(\tilde{y})$ can be replaced by $1/\nu_t$ where calculation of $\nu_t$ replaces the optimization in (6) in the estimation process.
Thus, the associated cost-function/input-demand system is

\[ c_i = \frac{1}{\nu_t} \left( r_i^{1/2} \beta r_i^{1/2} + L_t \gamma r_i + \alpha r_i \right) + \delta r_i, \]
\[ x_i = \frac{1}{\nu_t} \left( r_i^{1/2} \beta x_i^{1/2} + L_t \gamma + \alpha \right) + \delta, \]

where \( \tilde{r}_i \) is a diagonal matrix with the inverse square roots of respective \( r_i \) elements on the diagonal. Note that the disturbance term in the cost specification follows the McElroy additive general error model which generates a systematically consistent specification of disturbances in the cost and demand system specifications. Because \( r_i \tilde{x}_i (r_i, \tilde{y}_i) = \tilde{c}_i (r_i, \tilde{y}_i), t = 1, \ldots, T, \) one equation in this system is redundant and must be deleted for estimation. Maximum likelihood estimates are invariant to the equation deleted (Berndt and Savin, 1975). For simplicity, we delete the cost equation and estimate the demand system.

The data used here are the annual aggregate data on U.S. agriculture developed by Eldon Ball at the Economic Research Service from 1948 through 1989 which include a single aggregate output with six inputs: fertilizer, capital, labor, materials, pesticides, and land. All data were mean scaled for estimation. Equation (15) was estimated by feasible generalized least squares for both the ex post and ex ante cases. In both cases, estimates were obtained by imposing positive symmetry on \( \beta \) which implies global concavity of the cost function estimates. Because the ex post case requires explicit specification of \( \phi(\tilde{y}_i) \), the ex post estimates are based on the assumption of \( \phi(\tilde{y}_i) = \tilde{y}_i \), and are developed using the conventional approach of replacing \( \tilde{y}_i \) by \( y_i \) for estimation. The results are presented in Table 2.

Note that the fit obtained with both models is exceptional (see the footnote of Table 2). Correlation of predicted and dependent variables ranges from .942 to .993 for the ex ante estimates and from .906 to .987 for the ex post estimates. The fit is somewhat better for the ex ante case although this may be due to the somewhat more restrictive assumption of \( \phi(\tilde{y}_i) = \tilde{y}_i \) in the ex post case. Table 2 also reveals major differences in the ex ante and ex post estimates. A comparison of these results is not conclusive. However, some of the signs in the ex post results are more difficult to accept than for the ex ante results while the implications of others are quite different. Perhaps intuition has the strongest implications for the technical change coefficients. For example, the most overwhelming characteristic of technical change one
would expect to find in American agriculture over the last 50 years is a replacement of labor by capital. These expectations have been verified in numerous studies. According to the ex ante results, technical change has indeed been capital using \( \alpha_2 > 0 \) and labor saving \( \alpha_3 < 0 \) and both results are significant at the one-percent level. According to the ex post results, however, technical change has been both capital and labor saving although only the labor result is significant.

Note further, however, that the two sets of estimates imply a very different role for additional land at the margin. For example, the ex post results imply that land substitutes with all other inputs while the ex ante results imply a complementary relationship with fertilizer, capital, and pesticides. The latter relationships correspond better with an agriculture where, say, fertilizer and pesticide inputs tend to be used in fixed proportions with land, and increases in output holding land fixed are accomplished by switching to more labor- and materials-intensive crops. Note that a fixed proportions use of pesticides tends to result from label requirements for use of pesticides under EPA regulations while agronomic practices and recommendations tend to limit the amount of fertilizer that can be usefully applied to a given piece of land. Similarly, for a given crop/technology, the amount of cultivation per unit of land tends to be determined by the size of machinery whereas changes in crop/technology choice tend to cause a change in labor and materials. Thus, the ex ante results tend to appear somewhat more plausible.

Own price elasticities of demand at sample means are presented in Table 3 for both the ex ante and ex post estimates. All input demands exhibit inelasticity as expected. However, the more inelastic results for labor and fertilizer obtained by the ex ante approach are more consistent with estimates, for example, by Antle (1984) whose aggregate input demand elasticities obtained assuming profit maximization are -.01 for labor and -.25 for chemicals (which includes fertilizer). On the other hand, the capital demand elasticity is the same as Antle's for the ex post case and somewhat higher for the ex ante approach. The smaller ex ante estimates for labor and fertilizer are also more in line with earlier estimates of input demand elasticities by non-dual methods (e.g., see Griliches' references). On balance, the comparison of elasticities with previous studies by non-cost-function methods tends to favor the ex ante approach.
Conclusions

This paper demonstrates that conventional cost function approaches are not applicable for problems with stochastic production. Conventional ex post cost function specifications cannot serve as a first stage of a two stage expected profit or expected utility maximization problem. Monte Carlo results demonstrate that substantial bias and inconsistency in cost function estimates occurs as a result. These results invalidate conventional cost function analysis for most agricultural problems.

Alternatively, the cost function consistent with expected profit and expected utility maximization is demonstrated to be an ex ante cost function. Appropriate ex ante cost function specifications are explored for the case of risk neutral preferences and for general risk preferences under multiplicative production risk. It is shown that conventional specifications are appropriate if conditioned on expected rather than actual output. The use of a distance function approach is shown to facilitate estimation of such ex ante cost functions. An application to the case of aggregate U.S. agriculture demonstrates that different and more plausible results are obtained with ex ante estimation than with ex post estimation.
Footnotes

1 In reality, some decisions must be made before unforeseen circumstances arise while other
decisions are made in response to circumstances as they arise. For example, major input
decisions such as land allocation, seed use, and labor and machinery use in field preparation and
planting must be made before weather and crop disease conditions are observed. Alternatively,
pesticide applications may be made as crop disease or weed conditions are observed. Ideally,
production studies should consider both types of inputs distinctly. However, the primary focus
of production studies to date is on inputs of the former type.

2 Of course, one could also consider joint estimation of the cost equation, \( c = \bar{y}W\beta + e \),
with the system of demand equations (where \( e \) is the associated random error). However, typical
specifications lead to linear dependence of such a system so that one equation must be discarded
for estimation (see the application section below). Typical practice involves discarding the cost
equation so for ease of exposition we discuss estimation of the demand system.

3 The data for \( r \) and \( \bar{y} \) were generated randomly and independently according to a uniform
distribution with mean 3 and range \( \pm 1 \) for \( r_1 \), mean 6 and range \( \pm 2 \) for \( r_2 \), and mean 100 and
range \( \pm 33 \) for \( \bar{y} \).

4 Disturbances in input demand equations were generated by \( \delta_1 = (u_1 - .5)\sigma_1, \delta_2 = \rho\delta_1 +
(u_2 - .5)\sigma_2, \sigma_1 = 7, \sigma_2 = 4, \) and \( \rho = .4 \) where \( u_1 \) and \( u_2 \) are independent uniform random
variables in the unit interval. Thus, \( \sigma_{11} = 4.083, \sigma_{12} = 1.633, \) and \( \sigma_{22} = 1.987 \).

5 The results are presented here in the context of conventional expected utility maximization
although the same principles also apply to modern treatments of generalized expected utility (e.g.,
Machina, 1982).

6 The input requirement set, \( \{ x \mid g(x) \geq z \} \), is assumed nonempty, compact, and strictly
convex.
When the input requirement set is not convex, reasonable results may still be obtained as in the standard theory but they are not as straightforward (McFadden, 1978).

Note that Diewert's leading case with $\beta \gg 0$ is sufficient but not necessary for concavity of the cost function in observed input prices. Positivity of $\beta$ was imposed by estimating parameters corresponding to the square root of elements of the $\beta$ matrix. Standard errors for corresponding estimates of $\beta$ were then derived by using a first-order Taylor series approximation based on estimated standard errors for the estimated square roots.
References


repetitions. The actual production levels, \( y_t \), needed to apply conventional ex post methods were generated by \( y_t = \bar{y}_t e^{\varepsilon_t} \) where the production disturbance \( \varepsilon_t \) is assumed to be independently and identically distributed with a normal distribution, \( \varepsilon_t \sim N(0, \sigma^2) \), \( t = 1, \ldots, T \). The true parameters used to generate costs and input demands are \( \beta_{11} = 10, \beta_{12} = \beta_{21} = 6, \) and \( \beta_{22} = 4 \). The estimates are developed by feasible generalized least squares. That is, the covariance matrix for \( \delta \) is estimated on the basis of least squares residuals and then used in generalized least squares estimation following Zellner's seemingly unrelated regression method.

The results in Table 1 are first presented for the case of nonstochastic production, \( \sigma^2 = 0 \), as a benchmark. The biases and mean squared errors for cost function parameters are negligible as expected according to asymptotic econometric theory (first and fifth columns of Table 1). Biases in cost function parameters do not exceed .125\% and biases in estimates of the covariance matrix do not exceed 6.1\%. When \( \sigma^2 \) increases to .15, .3, and .6, the biases in cost function parameter estimates increase almost exponentially to as much as 2.8\%, 10.5\%, and 35.9\% of parameter values, respectively (upper left quadrant of Table 1). Biases in estimated covariance terms increase much more dramatically to as much as 227\%, 854\%, and 2874\%, respectively (lower left quadrant of Table 1). Mean squared errors also increase similarly. When \( \sigma^2 = 0 \), percentage root mean squared errors do not exceed 11\% for cost function parameters and 27\% of covariance matrix parameters. As \( \sigma^2 \) increases to .15, .3, and .6, percentage root mean squared errors are as high as 13\%, 19\%, and 44\%, respectively, for cost function parameters, and as high as 240\%, 884\%, and 2966\%, respectively, for covariance parameters.

In summary, ex post cost function estimation under stochastic production causes serious bias and imprecision in cost function parameter estimates, while covariance parameter estimates may bear little resemblance to the true parameters. The seriousness of covariance estimation problems suggests that hypothesis tests about cost function parameters could be particularly misleading. The results in Table 1 thus demonstrate that production uncertainty has fundamental and adverse implications for ex post cost function estimation even when decision makers are risk neutral. While these results are developed here
for the more complex case of factor demand system estimation, similar difficulties occur in estimation of
the cost equation (see Pope and Just for an example of cost equation estimation bias).

**Alternative Approaches to Cost Function Estimation With Stochastic Production**

One approach to dealing with the endogeneity of \( y \) in (4) is to consider instrumental variable estimation.

From the underlying problem, one can note \( y \) depends in part on \( r \) and \( p \) which are the exogenous variables
in the problem. An instrumental variables approach would use \( r \) and \( p \) as instruments and proceed with
estimation of (4). While this approach overcomes the problem of inconsistent estimation, efficiency is lost
by using a specification that likely has substantially larger variation of disturbances (the covariance of \( \delta \)
is likely much larger than the covariance of \( \delta \)). Furthermore, except in extremely simple cases where self-
dual functions are applicable, efficiency is thus lost by using an instrumental variable expression
representing \( y \) as a function of \( r \) and \( p \) that is incompatible with the cost function specification.

Another approach is to substitute the expected production relationship, \( g(x) = \mathbb{E}[f(x,e)] \), for \( y \) in
(3). This approach also encounters two problems. First, endogenous variables are introduced on the right-
hand side for which instrumental variables or other appropriate procedures must be used. Second, the
specification chosen for the cost function has implications for the specification of \( g(x) \) which are only
adequately reflected by the associated duality between cost and production. Any ad hoc specification of
\( g(x) \) may be incompatible with the specification chosen for the cost function.

This section briefly reviews a third approach to estimation for the case of risk neutrality that
substitutes a distance function specification for \( y \) in the cost function thus representing expected production
in a manner fully compatible with the cost function specification. This approach has been shown elsewhere
to yield consistent and asymptotically efficient estimators and to yield small sample efficiency
approximating the hypothetical case where \( y \) is observable (Pope and Just). To understand this approach,
recall that the distance function rather than the production function is the direct dual of the cost function
(Shephard, p. 160) and that the distance function under certainty is defined by \( \Psi(y,x) = \min \{1 - c(r,y) + rx \} \) from which the production function is found as \( f(x) = \max_y \{y \mid \Psi(y,x) \geq 1\} \) (Shephard, p. 171).
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Ex Ante Estimates</th>
<th>Ex Post Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>Standard Error&lt;sup&gt;a&lt;/sup&gt;</td>
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<tr>
<td>$\beta_{11}$</td>
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<td>.88061</td>
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<td>-.43089 **</td>
</tr>
<tr>
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<td>$\gamma_2$</td>
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</tr>
<tr>
<td>$\alpha_5$</td>
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<td>.36462 **</td>
</tr>
</tbody>
</table>

<sup>a</sup> Single asterisks indicate significance at the .05 level and double asterisks indicate significance at the .01 level. Note that $T = 42$ with $\ln L = 1.38$ for the ex ante case and $\ln L = -3.47$ for the ex post case. Correlations (not squared correlations as would be comparable to the usual $R^2$ statistic) between actual and predicted inputs are .942, .973, .956, .944, and .993 for the ex ante case and .906, .922, .960, .922, and .987 for the ex post case for fertilizer, capital, labor, materials, and pesticides, respectively.
### Table 3. Estimated Price Elasticities of Demand

<table>
<thead>
<tr>
<th>Input</th>
<th>Ex Ante</th>
<th>Ex Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Fertilizer</td>
<td>-.543</td>
<td>-.610</td>
</tr>
<tr>
<td>2. Capital</td>
<td>-.334</td>
<td>-.250</td>
</tr>
<tr>
<td>3. Labor</td>
<td>-.434</td>
<td>-.634</td>
</tr>
<tr>
<td>4. Materials</td>
<td>-.504</td>
<td>-.355</td>
</tr>
<tr>
<td>5. Pesticides</td>
<td>-.432</td>
<td>-.433</td>
</tr>
</tbody>
</table>


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