Dynamic Economic Models with Uncertainty and Irreversibility: Methods and Applications

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ABSTRACT

This paper reviews results for dynamic economic models subject to uncertainty and/or irreversibility constraints. The primary emphasis is on the characteristics of optimal solutions, the long run dynamics of the optimal state process, and nonlinearities or deviations from certainty equivalence introduced by the combination of irreversibility and uncertainty. The paper briefly reviews several recent applications in intertemporal economics including the theory of optimal growth with irreversible investment, consumption and savings behavior under liquidity constraints, harvesting single-species fisheries, conjunctive management of surface and ground water, and the economics of environmental preservation.

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I. INTRODUCTION

Two fundamental aspects of economic decision making are: (1) the fact that many actions are at least partially irreversible, and (2) the fact that many choices must be made in an environment of uncertainty over future states of nature. This paper surveys methods in the theory and application of discrete time Markov decision models to dynamic economic allocation problems with these characteristics. First, the paper examines the characteristics of optimal solutions, the long run dynamics of the optimal state process, and nonlinearities or deviations from certainty equivalence due to the combination of irreversibility and uncertainty. Then, the paper reviews specific applications in intertemporal economics including the theory of optimal growth with irreversible investment, consumption and savings behavior under liquidity constraints, natural resource allocation, and the economics of environmental preservation.

II. THE MARKOV DECISION MODEL.

This paper is concerned with economic optimization problems that can be modeled as a Markov decision process specified by S, $\{A(s):S \rightarrow A\}$, q, U, and δ; where S and A are nonempty convex subsets of $\mathbb{R}$, the sets of states and actions of the system; for each s, A(s) is a nonempty, continuous and compact-valued correspondence that specifies the admissible actions from s; q is a conditional probability measure on the Borel subsets of S governing the transition of the state given (s,a); U is a continuous, bounded function on $S \times A$ that denotes the one-stage reward or utility; and $0 \leq \delta < 1$ is the discount factor.

The state evolves according to a transition equation $s_{t+1} = g(s_t, a_t, \omega_{t+1})$ where $g(\cdot,\cdot,\omega) \in C(S \times A)$, and $\{\omega_t\}$ is a sequence of independent, identically distributed random variables with common distribution ϕ on Ω, a compact subset of $\mathbb{R}^m$. This determines the transition kernel on

S, $q(B|s,a) = \int_\Omega 1_B[(g(s,a,\omega))]\phi(d\omega)$ for all B, the Borel subsets of S. At the beginning of each period the agent observes the state s_t and then takes an action a_t. As a consequence the agent receives utility $U(s_t,a_t)$, and the system moves to a new state $s_{t+1} = g(s_t,a_t,\omega_{t+1})$.

The partial history at date t is given by $h_t = (s_0,a_0,\ldots,a_{t-1},s_t)$. A <u>policy</u> π is a sequence $\{\pi_0,\pi_1,\ldots\}$, where π_t is a conditional probability measure on A such that $\pi_t(A(s_t)|h_t) = 1$. A policy is <u>Markovian</u> if for each t, π_t depends only on s_t. In this case π can be represented by a sequence $\{a_0,a_1,\ldots\}$, where each a_t is a Borel measurable map from S to A such that $a_t(s)\in A(s)$ for all $s\in S$. A Markovian policy is <u>stationary</u> if $a_t(s) \equiv a(s)$ for all t, where $a(\cdot)$ is some Borel measurable function from S to A. $a(\cdot)$ is called a <u>stationary policy function</u> and the policy defined by $a(\cdot)$ is denoted $a^{(\infty)}$.

Associated with a policy π and an initial state s is an expected discounted sum of rewards over time $V_\pi(s) = E\Sigma_{t=1}^{\infty}\delta^t U(s_t,a_t)$, where $\{s_t,a_t\}$ are generated by π and q in the obvious manner. A policy, π^*, is <u>optimal</u> if $V_{\pi^*}(s) \geq V_\pi(s)$ for all policies π and all $s\in S$, and V_{π^*} is referred to as the <u>value function</u>. By a well-known contraction mapping argument it can be shown that the value function satisfies the functional equation

$$V_{\pi^*}(s) = \max_{a\in A(s)} U(s,a) + \delta \int V_{\pi^*}(g(s,a,\omega))d\phi(\omega).$$

Under the assumptions above, V is continuous. Further, if $a^*(s)$ is the set of maximizers of the above equation at $s\in S$, then a^* is an upper-semicontinuous correspondence from S to A which admits a measurable selection and the policy, $a^{*(\infty)}$, defined through $a^*(s)$ is a stationary optimal policy.

In dynamic economic models one is often interested in how the stationary optimal policy function $a^*(s)$ varies with s. For $x,y\in\mathbb{R}$ let $x\wedge y = \min[x,y]$ and $x\vee y = \max[x,y]$. The function $U(s,a)$ is said to be <u>supermodular</u> on $S\times A$ if $U(s\wedge s',a\wedge a') + U(s\vee s',a\vee a') \geq U(s,a) + U(s',a')$ for all (s,a) and (s',a') in $S\times A$. In economic maximization problems supermodularity is interpreted as a form of complementarity among arguments in U. The reason being that for C^2 utility functions supermodularity is equivalent to $d^2U/dsda \geq 0$, so an increase in one argument raises the marginal utility of the other. A correspondence $\Gamma(s)$ is defined to be <u>expanding</u> on S if $s \leq s'$ implies $\Gamma(s) \subseteq \Gamma(s')$. $\Gamma(s)$ is <u>ascending</u> on S if $s \leq s'$, $a \in \Gamma(s)$, $b \in \Gamma(s')$ implies $a \wedge b \in \Gamma(s)$ and $a \vee b \in \Gamma(s')$.

PROPOSITION 1. *Assume (i) U is supermodular on $S \times A$, (ii) g is independent of s, and (iii) A(s) is ascending on S. Then $a^*(s)$ is ascending in s.*

<u>Proof of Proposition 1.</u> Let $a \in a(s)$ and $a' \in a(s')$ for $s \geq s'$. Since A(s) is ascending, $a \vee a' \in A(s)$ and $a \wedge a' \in A(s')$. Note that $\delta EV(g(a \vee a',\omega)) + \delta EV(g(a \wedge a',\omega)) = \delta EV(g(a,\omega)) + \delta EV(g(a',\omega))$. Hence, $0 \geq U(s,a \vee a') + \delta EV(g(a \vee a',\omega)) - U(s,a) + \delta EV(g(a,\omega)) \geq U(s',a') + \delta EV(g(a',\omega)) - U(s',a \wedge a') + \delta EV(g(a \wedge a',\omega)) \geq 0$, where the first and last inequalities follow from the principle of optimality and the middle inequality is due to the supermodularity of U. Since $U(s,a \vee a') + \delta EV(g(a \vee a',\omega)) - U(s,a) + \delta EV(g(a,\omega)) \geq 0$ it follows that $a \vee a' \in a(s)$. Similarly since $0 \geq U(s',a') + \delta EV(g(a',\omega)) - U(s',a \wedge a') + \delta EV(g(a \wedge a',\omega))$, it follows that $a \wedge a' \in a(s')$. This implies that a(s) is ascending. //

PROPOSITION 2. *Assume (i) U is nondecreasing on S, concave on $S \times A$, strictly concave on A, and supermodular on $S \times A$, (ii) g is nondecreasing on S, nonincreasing on A, concave on $S \times A$, and supermodular on $S \times A$, and (iii) A(s) has a convex graph, and is expanding and ascending on S. Then $a^*(s)$ is single-valued and nondecreasing in s.*

<u>Proof of Proposition 2.</u> V is concave since U and g are concave and A(s) has convex graph. V is nondecreasing since U and g are nondecreasing in s and A(s) is expanding. a(s) is single-valued and continuous by the strict concavity of U in a. Let $a \in a(s)$ and $a' \in a(s')$, where $s \geq s'$. If $a \geq a'$, then $V(g(s \vee s',a \vee a',\omega)) + V(g(s \wedge s',a \wedge a',\omega)) = V(g(s,a,\omega)) + V(g(s',a',\omega))$. If $a < a'$, then $V(g(s \vee s',a \vee a',\omega)) = V(g(s,a',\omega))$ and $V(g(s \wedge s',a \wedge a',\omega)) = V(g(s',a,\omega))$. g is nondecreasing in s and nonincreasing in a so that $g(s,a,\omega) \geq g(s,a',\omega) \geq g(s',a',\omega)$. Thus, there exists $0 \leq \theta \leq 1$ such

that $g(s,a',\omega) = \theta g(s,a,\omega) + (1-\theta)g(s',a',\omega)$. Supermodularity of g implies $g(s',a,\omega) \geq g(s,a,\omega) + g(s',a',\omega) - g(s,a',\omega)$. Combined with the previous equality this gives $g(s',a,\omega) \geq (1-\theta)g(s,a,\omega) + \theta g(s',a',\omega)$. This and V nondecreasing implies $V(g(s \vee s',a \vee a',\omega)) + V(g(s \wedge s',a \wedge a',\omega)) = V(g(s,a',\omega)) + V(g(s',a,\omega)) \geq V(\theta g(s,a,\omega)+(1-\theta)g(s',a',\omega)) + V((1-\theta)g(s,a,\omega)+\theta g(s',a',\omega))$. The concavity of V then yields $V(\theta g(s,a,\omega)+(1-\theta)g(s',a',\omega)) + V((1-\theta)g(s,a,\omega)+\theta g(s',a',\omega)) \geq V(g(s,a,\omega)) + V(g(s',a',\omega))$ which establishes that $V(g(s, a,\omega))$ is supermodular in (s,a). The remainder of the proof follows similar arguments to the proof of Proposition 1. //

The main advantage of Proposition 1 is that it does not require concavity of U and/or g. This permits a characterization of optimal policies in nonconvex problems. The main strength of Proposition 2 is that it allows the future state to depend on both the current state and action.

Dynamics of optimal state processes

The monotonicity of optimal policy functions is useful in characterizing the dynamics of optimal processes in infinite horizon models. The following 2 questions are of interest. First, do optimal processes converge to a limiting stationary distribution? Second, when is the stationary distribution unique so that the limiting behavior of optimal processes is independent of initial conditions? Propositions 1 and 2 provide an avenue for addressing these questions since they can be used to show when the optimal state evolves according to a monotone Markov transition equation $s_{t+1} = H(s_t,\omega_{t+1})$.

Define $\omega^t = (\omega_1,\ldots,\omega_t)$, let ϕ^t be the joint distribution of ω^t on Ω^t, and define $H^t(s,\omega^t) = H(\ldots(H(H(s,\omega_1),\omega_2,\ldots,\omega_t)$. Let μ be any probability distribution for s. The probability distribution of the optimal process s_t from $s_0 = s$ is defined by $\phi^t\mu(B) = \int_S \phi^t(\{\omega^t \in \Omega^t | H^t(s,\omega^t) \in B\})\mu(ds)$, where B is any (Borel) subset of $\mathbb{R}$. μ is an invariant probability if $\phi^1\mu = \mu$. A subset S' of $\mathbb{R}$ is said to be ϕ-invariant if it is closed and if $\phi(\{\omega \in \Omega | H(s,\omega) \in S' \text{ for all } s \in S'\}) = 1$. A subset S" is a minimal ϕ-invariant set if it is ϕ-invariant and if any strict subset of S" is not ϕ-invariant. Define $H_m(s) = \inf_{\omega\in\Omega}H(s,\omega)$ and $H_M(s) = \sup_{\omega\in\Omega}H(s,\omega)$. Assume that the transition function H satisfies:

A.H.1. $H(s,\omega)$ is nondecreasing in s for each $\omega\in\Omega$,

A.H.2. $H(s,\omega)$ is jointly continuous in s and ω,

A.H.3. $H_M(s) > H_m(s)$ for all $s\in S$.

The following recurrence result is used to characterize the convergence of the optimal state process to an invariant distribution.

PROPOSITION 3 (Dubins and Freedman [1966]). *Suppose A.H.1-A.H.3 hold and let S' be a ϕ-invariant closed interval. If there exists a unique minimal ϕ-invariant closed interval S" in S' then there is one and only one invariant probability μ^* in S'. Furthermore, for each probability μ on S', the distribution function of $\phi^t\mu$ converges uniformly to the distribution function of μ^*.*

Let the space S be defined by the closed interval $[\underline{s},\overline{s}]$. Define the sequence $\{a_n,\hat{a}_n,b_n\}^{\infty}{}_{n=1}$ inductively by $a_n = \mathrm{Inf}\{s\geq b_{n-1}|H_m(s)=s\}$, $b_n = \mathrm{Inf}\{s>a_n|H_M(s)=s\}$, $\hat{a}_n = \mathrm{Sup}\{s\in[a_n,b_n]|H_m(s)=s\}$, where $b_0 = \underline{s}$. H_m and H_M have at least one fixed point by Brouwer's fixed point theorem. Let N^* be the maximum n such that the fixed points a_n, $\hat{a}_n$, and b_n are well-defined. The space S can now be expressed as

$$S = [b_0,\hat{a}_1) \cup \bigcup_{n=1}^{N^*} [\hat{a}_n,b_n] \cup \bigcup_{n=2}^{N^*} (b_{n-1},\hat{a}_n) \cup (b_{N^*},\overline{s}] .$$

The next results characterize the limiting behavior of optimal processes.

PROPOSITION 4. *Suppose that A.H.1-A.H.3 hold. (a) Fix any integer $n\in[1,N^*]$. Then the set $[\hat{a}_n,b_n]$ is a ϕ-invariant interval and $[\hat{a}_n,b_n]$ is a unique ϕ-invariant interval in itself. There exists a unique ϕ-invariant distribution on $[\hat{a}_n,b_n]$. If $s_0\in[\hat{a}_n,b_n]$ then the distribution function of s_t converges* *uniformly to the distribution function of the invariant distribution on* $[\hat{a}_n, b_n]$. *(b) The set* $\tau = \cup^{N}_{n=2} (b_{n-1}, \hat{a}_n) \cup [b_0, \hat{a}_1) \cup (b_N{*}, \overline{s}]$ *is transient. If* $s_0 \in \tau$, *then, with probability one,* s_t *will in finite time leave* τ *and never return.*

COROLLARY TO PROPOSITION 4. *Fix any distribution function* F_0 *for* s_0, *let* s_t *be generated by the transition function* $H(s,\omega)$, *and define* N^* *as above. Let* F_t *be the resulting distribution function of* s_t. *Then (a)* F_t *converges uniformly as* $t \to \infty$ *to the distribution function,* F^*, *of an invariant probability, and (b) there is a unique invariant probability on S if and only if there is only one interval of the type* $[\hat{a}_n, b_n]$, *i.e., if and only if* $N^* = 1$.

Nonlinearity and deviations from certainty equivalence

An important subset of Markov decision problems are those characterized by a quadratic reward function, a linear transition equation with additive stochastic terms, and an unconstrained choice set. In this class of problems the optimal decision rule is linear and equivalent to the solution obtained when the disturbance is replaced with its expected value. This property is known as certainty equivalence.

In constrained problems certainly equivalence breaks down. Generally, optimal decision rules are nonlinear and it is no longer valid to replace random variables by their conditional mean in the solution. These facts have important implications for optimizing models of economic agents, some of which will be discussed below.

Bibliographic notes to Section II.

The primary focus of this paper is on discrete time models. The literature on uncertainty and irreversibility in continuous time dynamic economic models is surveyed by Pindyck [1991]. Bhattacharya and Majumdar [1981] provide an excellent review of some additional results for stochastic economic models. There is a large literature on the theory and applications of Markov decision processes in economics. The preeminent reference is Stokey and Lucas (with Prescott) [1989]. The formal dynamic programming framework developed here is based on Furukawa [1972]. The structure of the model is similar to that of Blume, Easley, and O'Hara [1982]. On the analysis of monotone optimal policy functions see Topkis [1978] and Heyman and Sobel [1984, ch. 8]. Serfozo [1976] also examines this issue. Hopenhayn and Prescott [1992] provide the most complete treatment for dynamic optimization problems. Milgrom and Shannon [1994] extend Topkis' results. Proposition 1 is similar to Heyman and Sobel [1982, Corollary 8-5b]. Mendelssohn and Sobel [1980] and Dechert and Nishimura [1983] were the first to apply these techniques to economic growth problems. Recent applications are Amir, Mirman, and Perkins [1991] and Olson and Roy [1994]. Most results on the monotonicity of optimal investment and consumption policies in economic models can be obtained from Propositions 1 and 2. Economists have also focussed on models with non-monotone policy functions that are capable of generating periodic or chaotic behavior (Brock and Dechert [1991] provide a comprehensive survey).

The discussion of the convergence of optimal processes with monotone Markov transition equations is based on Majumdar, Mitra, and Nyarko [1989] and Nyarko and Olson [1991]. Bhattacharya and Lee [1988] extend the work of Dubins and Freedman [1966]. Hernández-Lerma, Montes-De-Oca, and Cavazos-Cadena [1991] provide a good discussion of additional recurrence relations. Related developments in economic modeling can be found in Razin and Yahav [1979], Futia [1982], Stokey and Lucas (with Prescott) [1989], and Hopenhayn and Prescott [1992]. In models of economic growth, the convergence of optimal processes to a unique invariant distribution is examined by Brock and Mirman [1972, 1973], Mirman and Zilcha [1975, 1977], Majumdar, Mitra, and Nyarko [1989], Olson [1989], and Nyarko and Olson [1994], among others. Donaldson and Mehra [1983] consider a problem with correlated shocks.

A formal treatment of the certainty equivalence principle can be traced to Theil [1954, 1957] and Simon [1956]. The loss of certainty equivalence under choice constraints is noted in Malinvaud [1969]. Henry [1974] and Arrow and Fisher [1974] established conditions under which the combination of irreversibility and Bayesian learning cause systematic deviations from certainty equivalence, a phenomenon Henry referred to as the "irreversibility effect." Further analysis and extensions are in Jones and Ostroy [1984] and Freixas and Laffont [1984].

3. APPLICATIONS IN ECONOMICS

Economic growth and irreversible investment

In the one-sector model of economic growth with random technology shocks, an economic agent allocates an aggregate production good between per capita consumption, c_t, and per capita capital, x_t, in each period so as to maximize the expected discounted sum of utility. Utility is denoted by U(c), where U is an increasing, strictly concave function of consumption. The production technology is given by a net output function $F(x_t,\omega_{t+1})$ and a constant rate of depreciation, d, which together yield a gross output function, $f(x_t,\omega_{t+1}) = F(x_t,\omega_{t+1})+(1-d)x_t$. It is assumed that F is increasing and concave in x. Investment is defined by $i_t = x_t - (1-d)x_{t-1}$.

With positive consumption, and in the absence of a depreciation constraint on capital or a nonnegativity constraint on investment the main results are: (1) optimal solutions are characterized by an intertemporal arbitrage condition $U'(c_t) = \delta EU'(c_{t+1})f'(x_t,\omega_{t+1})$, (2) optimal consumption and investment are increasing functions of output, (3) optimal processes converge to a unique invariant distribution, (4) optimal solutions are also the solution to a decentralized, competitive economy in which all forms of the consumption/capital good have the same price, (5) the price of existing capital relative to its replacement stock (known as Tobin's q) is always 1, and (6) the model is capable of producing fluctuations in output and investment that generally mimic those observed in aggregate economic data.

When investment is constrained by depreciation, many of these conclusions must be modified. Sargent [1980] shows that the price of existing capital may be less than that of new capital (Tobin's q is less than 1), and that current investment decisions depend on agent's expectations about the extended future. He also proves that optimal processes converge to a unique limiting distribution. These results are extended in Olson [1989]. In the deterministic case Majumdar and Nermuth [1982] prove that monotonicity and convergence results hold, even if the technology is non-convex, and that investment is constrained for a finite number of periods along an optimal path. Mitra and Ray [1983] develop a duality theory for optimal solutions under irreversible investment and show that optimal programs need not be competitive in an economy where all forms of the aggregate good are priced the same. Optimal programs are competitive in an economy with separate prices for both the output and the capital good. Other studies include Mitra [1978, 1983]. Dow and Olson [1992] investigate the implications of irreversible investment for real business cycle models of the economy. They show that irreversibility influences how capital and other economic variables respond to technology shocks and that the degree of randomness is significant in determining the extent to which there are deviations from certainty equivalence. When random shocks are small, such as those as measured at the aggregate level, then irreversibility has very little influence. At larger variations consistent with those found in durable good sectors of the economy, investment constraints have a noticeable effect on output, investment, and consumption. Constraints on the transferability of capital across sectors of the economy appear to be more important than depreciation constraints. Bertola and Caballero [1994] provide strong evidence that the smooth, persistent behavior of observed aggregate investment in the U.S. can be rationalized by unit-level irreversibility constraints. In a somewhat different setting Demers [1991] demonstrates that firms will invest more cautiously under irreversible investment when they anticipate better information in the future. Further, there is a gradual adjustment in the capital stock to its desired level, rather than full adjustment in a single period.

Consumption and savings with liquidity constraints

The response of individual consumption/saving behavior to uncertainty about income has long been a central issue in economics. The following model of individual life cycle consumption is typically used to analyze this problem: max $E\Sigma\delta^t U(c_t)$ subject to $x_t = (1+r)x_{t-1} + \omega_t - c_t$, where x_t = saving in period t, c_t = consumption in period t, ω_t = stochastic income received at the beginning of period t, r = interest rate on saving, and δ = subjective discount factor. When there are no constraints on consumption or saving the optimal solution satisfies what is known as the permanent income hypothesis. The primary implications are (Hall [1978]): (1) optimal consumption is based solely on expected lifetime wealth or permanent income and is insensitive to anticipated changes in income, (2) no information at time t aside from c_t helps to predict c_{t+1}, (3) aside from a trend, the behavior of marginal utility resembles a random walk, (4) consumption is characterized by $U'(c_t) = \delta(1+r)EU'(c_{t+1})$, and (5) when utility is quadratic, consumption is a linear function of current wealth and the expected value of future income. A considerable amount of empirical research testing this model has produced somewhat mixed results (see Deaton [1992] for a summary). In particular, there is evidence of precautionary saving and excess sensitivity of consumption to anticipated changes in income when compared to the predictions of the basic model.

A potential short-coming of the basic model is that it does not account for liquidity constraints or the inability to borrow against future income. Such restrictions have been suggested as a likely explanation for many of the disparities between observed consumption and the permanent income hypothesis. For consumers subject to liquidity constraints, optimal consumption is characterized by 3 patterns of behavior (e.g., Dow and Olson [1991]). At low wealth levels, consumers are unable to borrow to increase consumption so they consume their entire wealth. At intermediate wealth levels, consumption is not liquidity constrained yet it is lower than certainty equivalent consumption due to the prospect of being constrained in the future. At sufficiently high wealth levels, liquidity constraints are never binding at any point in the future and consumption is the same as in the unconstrained, certainty equivalent case. The combined consumption function is nonlinear and consistent with both precautionary saving and excess sensitivity.

There have been a number of investigations of the importance of liquidity constraints for different aspects of economic behavior. The degree to which liquidity constraints are responsible for deviations from certainty equivalence in consumption data is studied by Flavin [1985], Zeldes [1989], and Alessie, Kapteyn, and Melenberg [1989], among others. Deaton [1991] shows that liquidity constraints are capable of explaining a number of disparate phenomena in micro and macroeconomic data. Liquidity constraints increase the effectiveness of fiscal policy even if the future tax implications of government spending are perfectly foreseen (Heller and Starr [1979], Hubbard and Judd [1986]). Scheinkman and Weiss [1986] show that adding liquidity constraints to a general equilibrium model can produce price and output fluctuations similar to those observed in actual business cycles. Brock and LeBaron [1990] examine a production-based asset pricing model in which firms are borrowing constrained. The effect of liquidity constraints is to transform production shocks of short memory into return shocks of longer memory, offering a possible explanation of the observed phenomena of mean reversion in firm valuation time series data.

Natural resource management

The management of natural resources is a problem that extends across a number of disciplines including economics. Fisheries management and water allocation are two examples.

A. Fisheries management

In each period an agent harvests c_t from the available resource biomass y_t. The resource stock grows according to the stock-harvest-recruitment relationship $y_{t+1} = f(x_t,\omega_{t+1})$, where $x_t = y_t - c_t$ represents resource escapement and ω_t reflects the influence of random environmental factors on growth in the resource biomass. To obtain a harvest c_t, the agent expends fishing effort, e_t, where the effort needed to attain a given harvest depends on the size of the resource stock. Migration of the resource implies a stochastic yield-effort relation given by $e_t = e(c_t,y_t,\epsilon_t)$, where ϵ_t is an i.i.d. random variable. The expected utility (profit) of harvests is given by $U(c_t,y_t) = E_\epsilon u(e(c_t,y_t,\epsilon_t),c_t)$. The problem facing the fishery manager is to select a sequence of harvests that maximizes the expected discounted sum of utility from harvests over time subject to $0 \leq c_t \leq f(x_{t-1},\omega_t)$.

This problem fits easily within the framework developed in the preceding section. There exist optimal harvest and escapement policies $c_t = C(x_{t-1},\omega_t)$ and $x_t = X(x_{t-1},\omega_t)$. Given the appropriate complementarity in utility, the convergence of optimal processes follows in a straightforward manner. Additional references and details can be found in Mendelssohn and Sobel [1980] and Nyarko and Olson [1991, 1994].

B. Conjunctive groundwater use with stochastic surface supplies

Water allocation often involves joint management of groundwater stocks and stochastic surface supplies, and it may be possible to replenish groundwater through artificial recharge. Let x = groundwater stock, a = volume of groundwater withdrawal ($a \geq 0$) or artificial recharge ($a < 0$), and ω = volume of surface inflows, which may vary randomly from year to year. For simplicity assume there is no natural recharge to groundwater. The groundwater stock evolves according to the transition equation $x_t = x_{t-1} - a_t$. In each period, withdrawals from groundwater cannot exceed the available stock and the potential for artificial recharge is limited by available surface supplies. The period t state is $s_t = (x_{t-1},\omega_t)$. Net benefits from water allocation are given by U(s,a). The problem is to choose a sequence of water allocation decisions to maximize the expected discounted sum of net benefits over time. Under suitable assumptions on U optimal policies governing groundwater stocks and withdrawals can be characterized. Further analysis and references are given in Knapp and Olson [1995]. Similar techniques have also been applied to the problem of reservoir management (Sobel [1975]).

Irreversibility and the economics of environmental preservation

There is a significant literature in natural resource and environmental economics that focusses on problems of environmental preservation (see the survey by Fisher and Krutilla [1985]). A pervading theme in this literature is the "irreversibility effect" first noted by Henry [1974] and Arrow and Fisher [1974]. This refers to the fact that for decision problems satisfying certain conditions (Freixas and Laffont [1984]) a position of greater initial flexibility is desirable when choice constraints and the prospect of future learning are important. For decisions that involve irreversible development of the environment this means that preservation is made more desirable by the prospect of learning about future benefits of alternative uses for the environment. Care must be taken in interpreting this result, however, since the irreversibility effect is not necessarily one-sided in decision problems in $\mathbb{R}^n$. In a model of economic growth with both capital and environmental resources, environmental preservation may be either less or more desirable under irreversibility and learning, depending on society's preferences toward consumption and environmental benefits (Olson [1990]).

4. CONCLUSIONS

This paper surveys recent developments in the theory and applications of dynamic economic models characterized by uncertainty and irreversibility. Together, these 2 factors are capable of generating nonlinear decision rules on the part of optimizing agents. These have proved important in explaining a number of economic phenomena. Because irreversibility and uncertainty characterize many economic decisions there is much to be gained from further investigation of the nonlinear behavior caused by their interaction.

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